27. Miscellaneous Functions

27.1. Debye Functions

Series Representations

27.1.1

$$\int_0^x \frac{t^n dt}{e^t-1}=x^n\left[\frac{1}{n}-\frac{x}{2(n+1)}+\sum_{k=1}^{\infty}\frac{B_{2k}x^{2k}}{(2k+n)(2k)!}\right]$$

$$(|x|<2\pi, n\geq 1)$$

(For Bernoulli numbers B_{2k}, see chapter **23**.)

27.1.2

$$\int_x^\infty \frac{t^n dt}{e^t-1}=\sum_{k=1}^{\infty} e^{-kx}\left[\frac{x^n}{k}+\frac{nx^{n-1}}{k^2}+\frac{(n)(n-1)x^{n-2}}{k^3}+\ldots+\frac{n!}{k^{n+1}}\right](x>0, n\geq 1)$$

Relation to Riemann Zeta Function (see chapter 23)

27.1.3 $$\int_0^\infty \frac{t^n dt}{e^t-1}=n!\zeta(n+1).$$

[27.1] J. A. Beattie, Six-place tables of the Debye energy and specific heat functions, J. Math. Phys. **6**, 1–32 (1926).

$\frac{3}{x^3}\int_0^x \frac{y^3dy}{e^y-1}, \frac{12}{x^3}\left[\int_0^x \frac{y^3dy}{e^y-1}-\frac{3x}{e^x-1}\right], x=0(.01)24,$ 6S.

[27.2] E. Grüneisen, Die Abhängigkeit des elektrischen Widerstandes reiner Metalle von der Temperatur, Ann. Physik. (5) **16**, 530–540 (1933).

$\frac{20}{x^4}\int_0^x \frac{t^4dt}{e^t-1}-\frac{4x}{e^x-1},$

$x=0(.1)13(.2)18(1)20(2)52(4)80,$ 4S.

Table 27.1 **Debye Functions**

x	$\frac{1}{x}\int_0^x \frac{t\,dt}{e^t-1}$	$\frac{2}{x^2}\int_0^x \frac{t^2dt}{e^t-1}$	$\frac{3}{x^3}\int_0^x \frac{t^3dt}{e^t-1}$	$\frac{4}{x^4}\int_0^x \frac{t^4dt}{e^t-1}$
0.0	1.000000	1.000000	1.000000	1.000000
0.1	0.975278	0.967083	0.963000	0.960555
0.2	0.951111	0.934999	0.926999	0.922221
0.3	0.927498	0.903746	0.891995	0.884994
0.4	0.904437	0.873322	0.857985	0.848871
0.5	0.881927	0.843721	0.824963	0.813846
0.6	0.859964	0.814940	0.792924	0.779911
0.7	0.838545	0.786973	0.761859	0.747057
0.8	0.817665	0.759813	0.731759	0.715275
0.9	0.797320	0.733451	0.702615	0.684551
1.0	0.777505	0.707878	0.674416	0.654874
1.1	0.758213	0.683086	0.647148	0.626228
1.2	0.739438	0.659064	0.620798	0.598598
1.3	0.721173	0.635800	0.595351	0.571967
1.4	0.703412	0.613281	0.570793	0.546317
1.6	0.669366	0.570431	0.524275	0.497882
1.8	0.637235	0.530404	0.481103	0.453131
2.0	0.606947	0.493083	0.441129	0.411893
2.2	0.578427	0.458343	0.404194	0.373984
2.4	0.551596	0.426057	0.370137	0.339218
2.6	0.526375	0.396095	0.338793	0.307405
2.8	0.502682	0.368324	0.309995	0.278355
3.0	0.480435	0.342614	0.283580	0.251879
3.2	0.459555	0.318834	0.259385	0.227792
3.4	0.439962	0.296859	0.237252	0.205915
3.6	0.421580	0.276565	0.217030	0.186075
3.8	0.404332	0.257835	0.198571	0.168107
4.0	0.388148	0.240554	0.181737	0.151855
4.2	0.372958	0.224615	0.166396	0.137169
4.4	0.358696	0.209916	0.152424	0.123913
4.6	0.345301	0.196361	0.139704	0.111957
4.8	0.332713	0.183860	0.128129	0.101180
5.0	0.320876	0.172329	0.117597	0.091471
5.5	0.294240	0.147243	0.095241	0.071228
6.0	0.271260	0.126669	0.077581	0.055677
6.5	0.251331	0.109727	0.063604	0.043730
7.0	0.233948	0.095707	0.052506	0.034541
7.5	0.218698	0.084039	0.043655	0.027453
8.0	0.205239	0.074269	0.036560	0.021968
8.5	0.193294	0.066036	0.030840	0.017702
9.0	0.182633	0.059053	0.026200	0.014368
9.5	0.173068	0.053092	0.022411	0.011747
10.0	0.164443	0.047971	0.019296	0.009674
	$\left[\begin{smallmatrix}(-4)5\\5\end{smallmatrix}\right]$	$\left[\begin{smallmatrix}(-4)6\\5\end{smallmatrix}\right]$	$\left[\begin{smallmatrix}(-4)6\\5\end{smallmatrix}\right]$	$\left[\begin{smallmatrix}(-4)6\\5\end{smallmatrix}\right]$