Coursework

This coursework relates to the properties of spike train. These questions are adapted from the exercises given in Part 1.1 of

`http://www.gatsby.ucl.ac.uk/~dayan/book/exercises.html`

in their original form they are exercises for the book Dayan and Abbott. These questions explore the properties of spike trains and give a more data-led perspective on spike trains than the one we have looked at before.

Poisson processes

The first question relates to Poisson processes. A **Poisson process** is a memoryless random process producing a time series of events; In the application to spike trains an event corresponds to a spike. **Memorylessness** means that the chance of an event depends only on a rate[1] function $\lambda(t)$ and not on the history of past events. The probability of an event in a small interval Δt wide is $\lambda(t) \cdot \Delta t$.

Imagine a person fishing in the sea. The chance of catching a fish might depend on the time of day but it doesn't depend on how many fish the person has already caught. However, if they are fishing in a small pond the chance of catching a fish would diminish as they catch fish. Fishing in the sea is a Poisson process; fishing in a pond is not.

We will begin by exploring a **homogeneous** Poisson process, where here *homogeneous* means that its rate λ is constant. Of course, if there is a refractory period (a time during which the neuron cannot spike), the rate is not constant in time, and the spike train is not a homogeneous Poisson process. In neuroscience, it is common to treat spike trains as Poisson processes with time-varying *"inhomogeneous"* rates. This is called an ***in*homogeneous Poisson process.**

For a homogeneous Poisson process, one can prove—with a nice argument you are urged to look up—that the distribution of inter-event intervals t,

[1]some sources may use the term "intensity" rather than "rate"

here inter-spike intervals, is an exponential distribution given by

$$p(t) = \lambda e^{-\lambda t} \tag{1}$$

Spike-train statistics

For a homogeneous Poisson process, the Fano factor "F" of binned spike counts, and the coefficient of variation C_v of the inter-spike-interval distribution, are both 1.

These quantities measure how regular a neuron's spiking is. Neurons that have a Fano factor larger than one may have additional processes that cause their firing rate to vary over time. Neurons that have a Fano factor less than one are more regular than expected for a homogeneous Poisson process. We will calculate both Fano factor and coefficient of variation for simulated and real spike trains.

Fano factor: To calculate the Fano factor, we count the number of spikes k occurring in bins over a longer interval of time Δt_{bin}. The Fano factor is then defined as

$$F = \frac{\sigma_k^2}{\mu_k} \tag{2}$$

where σ_k^2 is the variance of binned spike counts K, and μ_k the average.

Coefficient of variation: For spike trains, we usually apply the coefficient of variation to the *inter-spike interval* (or ISI for short) distribution. This is the distribution of times between consecutive spikes. The coefficient of variation is defined as:

$$C_v = \frac{\sigma_{\text{ISI}}}{\mu_{\text{ISI}}} \tag{3}$$

where σ_{ISI} is a standard deviation of the time between consecutive spikes, and μ_{ISI} is an average time between spikes.

Question 1 [10 marks]

(i) Write code to generate spikes using a Poisson process and sample a homogeneous Poisson spike train with firing rate of 35 Hz.

(ii) Extend your code to sample from an inhomogeneous Poisson neuron that has a 5 ms absolute refractory period, keeping the overall firing rate at 35 Hz (35 spikes per second).

(iii) Calculate the spike-count Fano factor and ISI coefficient of variation for the homogeneous and refractory-inhomogeneous neurons sampled in parts **i** and **ii**. In the case of the Fano factor the count should be performed over windows of width 10 ms, 50 ms and 100 ms.

(iv) Plot how the ISI coefficient of variation and spike-count Fano factor in 100 ms bins change over refractory periods ranging from `0` to `28` milliseconds. Include a descriptive figure title and a caption with enough detail to understand the context of what is being plotted and the effect that the figure is demonstrating.

Question 2 [10 marks]

In the `spike_trains` folder you will find the data file `rho.dat`. This is contains data collected and provided by Rob de Ruyter van Steveninck from a fly H1 neuron responding to an approximate white-noise visual motion stimulus. Data were collected for 20 minutes at a sampling rate of 500 Hz. In the file, rho is a vector that gives the sequence of spiking events or non-events at the sampled time, that is, every 2 ms.

(i) Calculate the Fano factor and coefficient of variation for this spike train as for the simulated spike trains above (also exploring 10 ms, 50 ms, 100 ms bin size).

(ii) Does the spike train exhibit more variability or less variability than you'd expect for a homogeneous Poisson process? How is this reflected in the count Fano factor and ISI coefficient of variation? Conjecture on why the data may deviate from a homogeneous Poisson process in this experiment. (25-100 words)

Question 3 (open ended)* [15 marks]

Consider a (leaky) integrate and fire neuron, such as the one described in the formative coursework. With a constant input this neuron spikes periodically: The variation in spike intervals is zero (any variation in spike counts it a trivial consequence of a mismatch in the spike period and the interval over which counting is being performed).

However, it is possible to add noise to the input. One way to do this would be to include a synaptic input where pre-synaptic neurons are modelled by Poisson processes.

Examine what happens in this case. Try to keep the firing rate of the post-synaptic cell (the LIF neuron) constant but experiment with lots of weak Poisson input or a small amount of strong Poisson input. Consider having both inhibitory and excitatory input.

Report on the Fano factor and coefficient of variation; for the highest marks consider plotting these against a parameter like the synapse strength or excitatory to inhibitory ratio.

The spike triggered average (STA)

The Spike Triggered Average (STA) is used to identify what is causing the neuron to spike. Imagine the neuron spikes at times $\{t_1, t_2, \ldots, t_n\}$ and the stimulus is $x(t)$ then the spike triggered average is

$$s(\tau) = \frac{1}{n}\sum_{i=1}^{n} x(t_i - \tau) \tag{4}$$

In other words it is the average value of stimulus a time offset τ relative to when the neuron spikes.

Question 4 [10 marks]

In the `spike_trains` folder you will find the data file `stim.dat`. This gives the motion stimulus that evoked the spike train in `rho.dat`. Calculate and plot the spike triggered average over a 100 ms window.

Question 5 [10 marks]

Calculate the stimulus triggered by pairs of spikes: For intervals of 2 ms, 4 ms, 10 ms, 20 ms and 50 ms calculate the average stimulus before a pair of spikes separated by that interval; Do this for both the case where the spikes are not necessarily adjacent and the case where they are. Speculate on the role of the refractory period in any differences in the STA between the two scenarios (adjacent vs. non-adjacent spikes).

Question 6* [15 marks]

Again, compare your results to an integrate and fire neuron: the idea is that the integrate and fire would receive an input based on the stimulus file; to ensure the neuron fires you will need to also include the direct current, but this can be modified with the addition of the stimulus. What is the spike triggered average? You can experiment with changing the strength of the stimulus contribution and parameters like the membrane constant.

Report instructions

Submit your coursework as a professionally formatted document in the `.pdf` file format.

There are 70 marks in total, this will be used to calculate a percentage. The two questions marked with an asterisk are harder and more open ended; do not spend too much time on these, the marking will give some credit for any attempt at the modelling they involve, but to get most or all of the 15 marks will require some experimentation.

Scientific communication relies strongly on graphs. You should take care in producing nice graphs; marks will be deducted for graphs where the text is tiny, or relevant units, titles, or axis labels are missed. Do not include graphs that have been produced by screenshots. Neuroscience as a subject favours long figure captions that provide detailed information about what the graphs show, without giving interpretation (that is left to the main text). Try to follow this convention.

Please do not exceed six pages, however, you should not regard six pages as a target length, a much shorter submission could still earn all or most of the marks. Exceptionally, seven-page submissions may be accepted provided you have checked with course instructors to ensure you are not doing more work than expected. The reason the length is not shorter is to make it easier to write without having to worry about being very precise—it is not to indicate that a long response is required.

Do not use a font size less than 11 pt and have normal looking margins. Submission is in pdf. You should include small code extracts where appropriate, any coding language can be used. You will also be required to upload your code. Submission is through blackboard upload.