﻿Dear Editor,

Thank you for taking the time to arrange the reviews of this paper. Please find below our responses to all of the comments made by the reviewers. We would like to send our appreciation to them for taking the time to review this paper and for making their helpful suggestions for our revised submission. We believe their feedback has strengthened the paper.

Where possible, we have indicated the page number and paragraph of the revised manuscript where changes have been made. We have also highlighted the corresponding text in our revised manuscript in red.

Please do not hesitate to contact us if you require anything further.

Yours sincerely,

A. L. Hawa, R, Lewis, and J. M. Thompson (28th October 2020)

**Reviewer #2**

The authors tackle a variant of the one dimensional Bin Packing Problem (BPP): the Score-Constrained Packing Problem (SCPP). They propose a number of exact and approximate algorithms to solve the SCPP, which are evaluated across some sets of randomly generated instances. The code of the algorithms and the random instances generator are publicly available.

The paper is generally very well written and all the proposals are very well explained and justified. Besides, the confronted problem is really interesting. The experimental study establishes a comparison of the proposed methods, but there is not a comparison to other results from the literature. Overall, the paper makes a good contribution to the state of the art, but some points could be improved. More detailed comments and suggestions are in the following:

Thank you for your positive comments regarding this work.

1.- The recombination operator works on solutions instead of on encodings (which is the most usual in EA). This fact deserves some comment and maybe some cite to similar approaches.

Thank you for this suggestion. We have now introduced an additional subsection, “Representation”, in Section 4, which explains our reasoning behind using the description of a solution $\mathcal{S}$ for an instance of the SCPP in our EA framework as opposed to a standard encoding. *Page 16, Section 4.1.*

2.- The LS seems to use hill-climbing as acceptation criteria; this should be clarified. Besides, some theoretical or experimental analysis of the neighbourhood size could be included.

Yes, you are correct. We have amended the local search procedure explanation by clarifying the acception criteria. *Page 19, paragraph 1.*

3.- The preliminary tests described in lines 40-48 (page 6) requires more detailed explanation. I think that some illustrating example would be good here.

We have re-written the description of the preliminary tests to avoid confusion and have provided an example. We feel that the addition of an example dismisses the requirement for an illustration, however one can be included if the reviewer prefers. *Page 6, paragraph 6.*

4.- In definition 3, the statement “.. each pair is a tuple” sounds rather informal. It could be rewritten in more formal way.

The statement in the COP definition “... in which each pair is a tuple” has been re-written as “... in which each element is an ordered pair”. We hope that this is acceptable. *Page 6, Definition 3.*

5.- In the description of MCM algorithm, the statement “At this point, … terminates.” (lines 36-39 in page 8) requires some formal proof. In the same paragraph, It should be clarified if AHC is always capable of forming a single alternating Hamiltonian cycle.

The proof for the MCM algorithm is lengthy (over a page) and so we have added a statement to the manuscript regarding the proof for the procedure, namely that the matching $R'$ returned by MCM is of maximum cardinality, along with a reference to a publication by the main author of this manuscript which contains the full proof. Nevertheless, if the reviewer would prefer that the full proof is included in the manuscript then we would be more than happy to accommodate the request. *Page 10, paragraph 2.*

6.- the statement “$\mathcal{R}’’$ is said to cover … an edge in $C\_j$” (lines 23-24 page 9) may be misleading if the notion of component is that induced from Fig. 3bc.

We have rephrased this sentence to clarify the explanation. It now reads “... if an edge from a component $C\_j$ of $G'$ is in a subset in the collection $\mathcal{R}''$, then $\mathcal{R}''$ is said to *cover* the component $C\_j$.” We have also updated the description of BCR to help the reader understand the aim of the procedure. *Page 11, paragraph 1.*

7.- Maybe algorithm BCR requires a pseudocode in the same way as MCM. Also, its completeness property could be justified.

The pseudocode for the BCR is very long, and so we have included it in this submission as Supplementary Material. This pseudocode may be omitted if the reviewer feels it is not required. The proof for the BCR procedure is also long; thus we have added a reference to a publication by the main author of this manuscript which details the full proof. As with the MCM proof, this too can be added to the manuscript if preferred. *Page 11, paragraph 3.*

8.- All the instances used in the experiments should be made available for the purpose of fair comparison to other methods. Even though the instance generator is of public use, different sets of random instances could give rise to different results.

Thank you for this observation. The problem instances used in the experiments are also available for public use along with the problem instance generator, however this was not made clear in the manuscript. We have amended the text to clarify that both the generator *and* the problem instances used are available online. *Page 20, paragraph 1.*

9.- The convergence of the algorithms could be analysed in some way. Giving all of them the same time is good for the purpose of comparison, but to get insight on the capability of a given algorithm it is good to leave then running until it converges.

To obtain this information we would have to re-run 36,000 experiments for the EA. To run the experiments for the original manuscript we had access to a dozen computers, which allowed us to run a large number of trials in months as opposed to over 4 years on a single machine. Unfortunately, due to the current pandemic we no longer have access to the additional machines, and so we are unable to re-run our experiments to assess the convergence of the algorithms. Instead, we have commented on the convergence of the algorithms using information from our original results. *Page 21, paragraph 5.*

10.- The authors have considered a complete set of references, many of them quite recent. In spite of that, and given that the confronted problem is actually a variant of the well-known cutting stock problem, the could consider some references about this problem.

Thank you for this remark. We have added more information regarding the CSP in Section 1 along with references. *Page 5, paragraph 2.*

11.- If possible, the authors should compare their results with those from other methods in the state of the art. Otherwise, they should justify why it is not possible.

On page 4 we have mentioned that there exists very little literature on the SCPP. We have also emphasised this on page 20, where we have stated that no benchmark instances exist for the SCPP and therefore we analyse our results with respect to the theoretical minimum $t$. *Page 4, paragraph 2, and page 20, paragraph 2.*

12.- Even though that the English style is excellent in my opinion, the authors should revise the use of “consist of” along the paper, if I am not wrong sometimes should be “consists in” instead.

Yes, you are correct. Thank you for highlighting this error. We have rectified this appropriately according to the definitions from Collins Dictionary (<https://www.collinsdictionary.com/dictionary/english/consist>). *Page 4, Definition 2, and page 6, Definition 3.*

**Reviewer #3**

This paper studies a variant of the classical bin packing problem (BPP) where items have a width and a border of different widths on both sides. The objective is to minimize the number of used bins such that the sum of items' width in each bin respect the bin size. Also, items have to be orderly placed inside the bin so that borders' width of two items beside each other is greater than a threshold. Verifying this constraint isn't trivial and a polynomial algorithm is proposed. Several heuristics are presented to solve the problem.

In my opinion, the authors introduce an interesting variant of the BPP. Given all what is developed for its resolution, I think this paper is worthy of being published considering the following comments are taken care of.

Thank you for reviewing our work; we have addressed the comments below.

First, section 2 should be greatly improved. It contains the most important contribution of the paper. Many parts need to have more details or be explained better.

We have edited Section 2 and tightened the text. Some of the changes made are as follows:

* Updated the explanation of how the COP and the sub-SCPP are related. *Page 6, paragraph 4.*
* Clarified the preliminary test and added an example. *Page 6, paragraph 6.*
* Moved paragraphs describing the modelling of an instance $\mathcal{M}$ of the COP graphically into a separate subsection. *Page 7, Section 2.1.*
* Added figures to the above subsection illustrating the process of modelling $\mathcal{M}$ into a graph $G$. *Page 7, paragraph 3 and Fig. 3.*
* Confirmed that the edge sets $B$ and $R$ are disjoint. *Page 7, paragraph 4.*
* Improved the description regarding how an alternating Hamiltonian cycle corresponds to a feasible solution $\mathcal{T}$, and added relevant figures. *Page 8, paragraphs 2 and 3 and Fig. 4.*
* Further explained the structure of the subgraph $G'=(V, B \cup R')$. *Page 10, paragraph 2.*
* Expanded the BCR section by detailing what needs to happen to the cyclic components of $G'$ in order to form a single alternating Hamiltonian cycle. *Page 10, paragraph 3, and page 11, paragraphs 1 and 4.*
* Added pseudocode for the BCR algorithm (Supplementary Material).
* Added pseudocode for the overall AHC algorithm. *Page 14, Algorithm 2.*
* At the end of the section: restated the purpose of AHC, introduced Theorem 1, and explained the benefit of AHC for the sub-SCPP. *Page 12, paragraphs 3 and 4, and page 13, paragraph 4.*

Second, I recommend modeling as a MIP the covering problem of section 5. MIP solvers are probably much faster than MinDLX for solving these problems.

We agree that in some cases, an ILP solver could give better results than MinDLX. However, the MinDLX procedure is based on an algorithm by Donald Knuth, using the highly regarded “dancing links” method. Another advantage of using MinDLX is that we were able to design the procedure directly within our C++ code using our pre-existing data structure and running the program using the same compiler options as the rest of the CMSA algorithm. We could indeed replace MinDLX with an ILP solver, however this would require re-running the 600 experiments on the same computers used to obtain the EA results, which we no longer have access to due to the current pandemic. We have, however, made a note of this in the manuscript. *Page 23, paragraph 1.*

Third, I recommend using another bound than the continuous bin packing bound for estimating the optimal solution value. The literature contains many bounds. This would give a better idea of the optimal value. The linear relaxation of the set partition formulation typically gives the best bound.

Yes, we agree, and have updated the manuscript to state the existence of other lower bounds in the literature and explained our reasoning behind using the continuous lower bound $t$. We have also noted that using a different lower bound does not affect the analysis of the algorithms' performance. *Page 15, paragraph 3.*

Finally, I would move all algorithms into one section and perform all tests in another section. This means that CSMA should be explained before any test is performed.

We have considered this suggestion carefully and, with respect, still feel that the structure of the original manuscript is best for describing the progression of the procedures. Although we understand the reasoning behind this suggestion, we feel that having all algorithm descriptions in one section would be a large amount of information for the reader to consume before reaching the experimental analysis of the first algorithm (EA). The analysis of each algorithm may also appear to be disjoint and the reader may have a difficult time linking the results back to each algorithm. Furthermore, without the analysis of the EA results, it may not make sense as to why we opt for the CMSA algorithm. We have added further information to Section 5 of the manuscript clarifying that the task is to find a minimum cardinality exact cover $\mathcal{S}^\*$ of $\mathcal{B}$, and also explained that CMSA has potential as it focuses on collecting groups of high quality bins, which was seen to be a successful approach in our EA with the GGA recombination operator. We hope that this is satisfactory, and we will comply if the reviewer still feels strongly regarding this suggestion. *Page 22, Definition 6 and paragraphs 3, 5, and 6.*

Remarks :

On page 17 “descibed”

This has been rectified. *Page 20, paragraph 2.*

Again, we would like to thank the reviewers for their useful suggestions for improving this paper. Thank you very much for taking the time to go over both the initial submission and this revised submission, and we look forward to hearing your positive responses soon.