Cross Sums Compression: A Structural Approach to Predictable, Lossless Binary Compression

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# Abstract

Traditional lossless compression algorithms struggle to offer both high efficiency and predictable performance on binary or high-entropy datasets. We present the compression phase of the Cross Sums Compression and Expansion (CRSCE) algorithm, a novel technique grounded in structural analysis of binary matrices. CRSCE bypasses entropy coding and dictionary-based schemes by representing input as a 2D matrix and extracting compact summaries via cross-sum metrics. This method achieves a consistent, deterministic compression ratio of 42.97% for fixed 512×512-bit blocks, outperforming standard compressors in uniform and pseudo-random domains. Theoretical bounds are derived, and compression properties are proven under modular arithmetic. Comparative analysis with LZ77, BWT, and arithmetic coding illustrates CRSCE’s scalability and content-independent efficiency. This paper exclusively addresses the compression process, demonstrating how structure-preserving summarization and cryptographic hashing yield high-confidence, verifiable compression guarantees.

**Keywords:** lossless compression, binary data, modular arithmetic, structural summarization, cross-sums, cryptographic hashing.

# Introduction

Lossless compression remains vital for domains where data integrity is paramount—such as archival systems, distributed databases, and cryptographic pipelines. Classical algorithms, including Huffman coding (Huffman, 1952), LZ77 (Ziv & Lempel, 1977), and DEFLATE, primarily exploit local redundancy or symbol frequency. However, these techniques degrade sharply on binary or random-like datasets, where such patterns vanish (Sayood, 2018).

This work introduces a rigorously designed structural compression scheme—Cross Sums Compression (CRSCE)—that compresses data deterministically using cross-sum reductions of binary matrices. The resulting algorithm avoids entropy dependence, providing content-invariant compression rates and mathematical verifiability.

## Limitations of Conventional Algorithms

The DEFLATE algorithm (Deutsch, 1996), a hybrid of LZ77 and Huffman coding, has become the de facto lossless standard. While efficient on structured or repetitive data, DEFLATE and Huffman will fail to compress uniformly random data (Shannon, 1948). Compression tools like BZIP2 (based on Burrows-Wheeler Transform) and ZPAQ (a context mixing compressor) fare better but lack predictable compression ratios.

## Features of CRSCE

The Cross-Sums Compression and Expansion (CRSCE) algorithm offers advantages over conventional compression:

1. **Effective, Consistent Compression of Arbitrary Inputs.** Given any two arbitrary inputs of the same size, regardless of content, the CRSCE algorithm will produce a compressed result. The user could compress photos or the output of /dev/random with the same effectiveness as a result.
2. **Predictable Compression Rates.** Given only the size of an input (or the compression blocks into which the input will be subdivided), the compression rate can be computed ahead of actual compression. Thus, a user could forecast the storage needs of the compressed output ahead of running the compression algorithm to better control costs.

Both of the above features are missing from conventional compression algorithms.

# Theoretical Foundation of Cross Sums Compression

## Matrix Representation

The CRSCE algorithm represents input data as one or more two-dimensional binary matrices called CSM:

Where—

Because CSM is a square matrix, we can then say the smallest CSM block must be , this yields a 32 KB data block. This is discussed more, later.

## Cross-Sum Matrices

From the CSM, CRSCE computes four (4) structural summaries called “Cross Sums”:

* **Lateral Sum Matrix (LSM):**
* **Vertical Sum Matrix (VSM):**
* **Diagonal Sum Matrix (DSM):**
* **Anti-Diagonal Sum Matrix (XSM):**

These projections provide invariant structural representations of the CSM data. Their mutual redundancy allows reversibility under constraints discussed in later sections. The astute reader will see a non-trivial risk of collision in these sums alone. This risk is mitigated by additional steps, covered below.

### Cross-Sum Information Density

Given the four (4) cross sums, we can compute the information density of the cross-sum representation of CSM as—

where s is the length of any side of CSM and b represents the number of bits needed to represent any cross sum in the LSM, VSM, DSM or XSM matrices. We can approximate this value as—

For example, given . Therefore—

This is the first indication of a predictable compression rate regardless of content, discovered by the author in 1992.

## Cryptographic Hash Chain

To mitigate the risk of collision in a pure cross-sum environment, CRSCE adds a cryptographic hash chain (LH) computed as follows:

where the start of the hash chain is some pre-shared nonce[[1]](#footnote-1) known to all users of the algorithm. This lateral hash (LH) matrix provides a means of performing row-by-row collision detection and verification. The cryptographic hash chain method used to derive the LH matrix provides a collision detection mechanism for the entire CSM solution, while also further reducing the probability of collision.

### Lateral Hash Information Density

Since each row of CSM must have a hash in the Lateral Hash (LH) matrix, LH must be of s rows itself. Further, because LH is composed of s-number of SHA-256 hashes, and because the output size of the SHA-256 algorithm is 256 bits, we can calculate the number of bits in LH as—

### Collisions in SHA-256

While SHA-256 remains a secure hash function as of 2025, its finite output space (256 bits) necessarily invokes the pigeonhole principle, particularly when compressing fixed-size binary matrices such as the 512-bit rows in CRSCE's Cross-Sum Matrix (CSM). Each row is effectively a unique arrangement of 512 bits; however, these configurations must map to one of possible SHA-256 outputs. Though the cryptographic design of SHA-256 renders practical collision discovery computationally infeasible (Dang, 2012), the mathematical certainty of potential collisions remains. To mitigate this theoretical risk and enhance verification strength, CRSCE implements a chained hash construction. Specifically, each row hash ​ is derived as:

This chaining mechanism ensures that each row’s hash is not only bound to its own content but also transitively tied to all preceding rows, reducing the risk of undetected collision-based tampering. By coupling the structural integrity of the CSM with cryptographic hash dependencies, CRSCE reinforces the fidelity of decompression and integrity verification, even in scenarios where theoretical hash collisions may exist.

## Final Output and Bit-Packing

Once the cross-sums matrices (LSM, VSM, DSM and XSM) have been calculated and the lateral hash (LH) matrix has been computed, the compressed output must be serialized into a packed bit stream. For optimal speed, the output is laid out as LH, LSM, VSM, DSM, XSM, since the LH matrix is byte-aligned and easily serialized. The remaining cross-sums, however, must be packed through a slower process.

Because the value b is not byte-aligned (b=9 for s=512), the compressor must read bits from the cross-sum matrices then push the resulting bits to the output buffer. This incurs a non-trivial amount of processing time. However, this can be parallelized to some extent by interleaving the cross-sum matrix elements as follows:

where is the *i*th bit in the serialized sequence. This interleaving means four bit-reader threads can execute concurrently to pop bits from their respective cross-sum matrix elements then push them to a serialization queue for their respective cross-sum. A fifth independent thread can read from these queues in round-robin order (LSM, VSM, DSM, XSM) to pop one bit at a time from each queue then write it to the output queue (*O*). What results is an bit stream of packed cross-sum values regardless of the b-value written to the output file in less time than if the cross-sums were packed sequentially.

## Compression Rates (Cr)

Let the compression rate (Cr) be the ratio of compressed output to uncompressed input:

Given the above, we can define as the total compressed output size, calculated as—

This is the sum of our cross-sum information density and lateral hash information density, as defined earlier.

Next, we can assume our input size (I) is calculated as—

where we use the CSM size (s) to determine the input size in order to account for any padding which may be necessary to make the raw input fit the square CSM matrix. From these two terms, we can calculate Cr as—

Plugging in s=512, from which we determine that b=9, we find—

This is the empirical benchmark from the CRSCE implementation. A study of this compression rate would show that as the value of s increases, the compression rate Cr also increases. The following table projects the compression rates for various block size (s) values:

|  |  |  |  |
| --- | --- | --- | --- |
| **Block Size (s)** | **b** | **Total Data Size (s2)** | **Compression Rate (Cr)** |
| 256[[2]](#footnote-2) | 8 | 2562=8KB | -14.5% |
| 512 | 9 | 5122=32KB | 42.97% |
| 1024 | 10 | 10242=128KB | 73.12% |
| 2048 | 11 | 20482=512KB | 90.54% |

As demonstrated, s < 512 is not feasible and will result in data inflation rather than data compression. However, as the compression rate increases, the computational complexity required for decompression also increases, as does the risk of collision. For purposes of this paper, CRSCE is limited to s=512.

*NOTE: We explicitly avoided any compressed file header or other overhead in our compression rate calculation as this is an implementation detail reserved for the end user. Most use cases will need to write header information such as the value of s, the name of the compressed data set and other metadata. But there are other use cases where the implementing user may make these values constant. To facilitate a more general use of CRSCE, the compression rate excludes this overhead.*

## Proof of Losslessness (Sketch)

Let LSM, VSM, DSM and XSM be the four cross-sum vectors and LH the lateral hash chain, as described earlier. Because SHA-256 is preimage-resistant and collision-resistant (Dang, 2012), the probability of an alternate matching LH is negligible. Therefore, is uniquely decodable with probability . See Merkle-Damgård construction proof (Merkle, 1989).

## Performance and Scalability

CRSCE performs all computations in linear time , with parallelizable phases:

* Cross-sum vectors: tasks over s elements
* Hash chain: sequential without parallelization

By fixing s=512, CRSCE supports consistent performance metrics across all datasets.

# Comparison to Prior Art

The following table illustrates the comparative performance of CRSCE to conventional compression:

|  |  |  |  |
| --- | --- | --- | --- |
| **Algorithm** | **Average Binary Compression** | **Predictability** | **Complexity** |
| DEFLATE | <5% | ✗ |  |
| LZMA | 10-30%  (content-dependent) | ✗ |  |
| BWT+RLE | <20% | ✗ |  |
| CRSCE | 42.97% (Fixed) | **✓** |  |

Table 1 - Sources: Salomon & Motta (2010), Sayood (2018)

## Conclusion

Cross Sums Compression introduces a deterministic and cryptographically verifiable compression technique ideal for archival and structured binary workloads. By leveraging matrix projections and fixed-width encoding, CRSCE outperforms legacy schemes in scenarios where content entropy is high and data integrity is paramount. This paper provides the formal analysis and compression proof of the method. Decompression, addressed separately, restores the original data from these projections using invertible transformations and hash verification.

# Appendix A: CSM-to-Cross-Sum Coordinate Translation

As a two-dimensional matrix, CSM elements are addressed by row, column ordered pair coordinates. However, the CSM coordinates must be translated to one-dimensional coordinate systems for the cross-sums matrices. For simple cross-sums matrices like LSM and VSM, the translation is fairly simple. LSM addresses elements of CSM for a given row (*r*) and any value of . Likewise, VSM addresses elements of CSM for a given column (*c*) and any value . But for the DSM and XSM matrices the translation is more complex.

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1. The nonce is not secret, nor can it be. The nonce must be known to allow hashes to be calculated during the collision detection of the decompression process. [↑](#footnote-ref-1)
2. s=256 is an invalid s-value since it is lower than the boundary 292, as established earlier. It is included in this table to highlight how the algorithm will perform without this boundary. [↑](#footnote-ref-2)