MLPG Open Assessment

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1 Conditional independence in Bayesian networks

Independent pairs

$$I = \{(A,C),(A,E),(A,F),(B,C),(B,E),(B,F),(D,C),(D,E),(D,F)\}$$

Independent pairs conditioned on $Z = \{C,G\}$

$$I = \emptyset$$

Markov equivalent DAG Reverse edges but maintain immoralities, i.e. change B edges *fig 1*

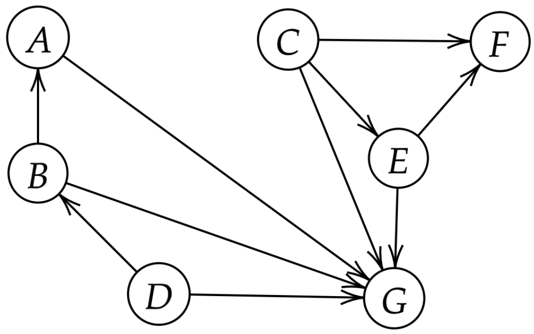


Figure 1: Question 1.3 Markov equivalent DAG

Non-Markov equivalent DAG Change immoralities. I.e. reverse all edges from G *fig 2*

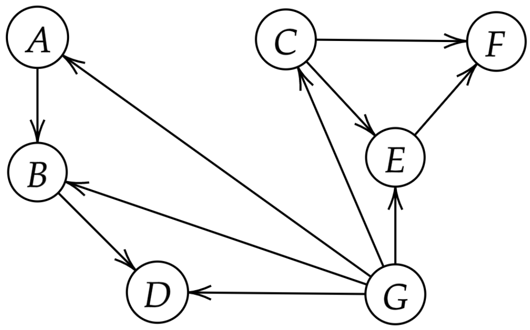


Figure 2: Question 1.4 Non-Markov equivalent DAG

2 House prices with STAN

2.1 A simple model (figs 4 and 3)

We can state that our estimated posteriors approximate the true distributions as all chains have converged $\hat{R} = 1$, which is also observed by the beta plot. The preference for MCMC sampling over variational inference was due to the fact that the size of our dataset and the length of this assessment permits the usage of the more computationally intensive method. In addition, the asymptotic correctness of the posterior justifies the larger computational expense [1]. Empirical tests show that the STAN default of 1000 iterations (2000 samples) and 4 chains is enough for convergence.

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
alpha	-23.43	0.11	3.51	-30.6	-25.63	-23.4	-21.21	-16.6	948	1.0
beta_L	58.26	0.07	2.27	53.82	56.75	58.28	59.75	62.65	1035	1.0
beta_A	0.1	8.2e-4	0.03	0.04	0.08	0.1	0.12	0.16	1253	1.0
beta_S	0.67	6.7e-4	0.02	0.63	0.66	0.67	0.68	0.71	1038	1.0
sigma	9.52	0.02	0.73	8.24	9.01	9.46	9.98	10.99	1618	1.0
lp__	-244.6	0.06	1.66	-248.7	-245.5	-244.3	-243.4	-242.5	717	1.0

Figure 3: Question 2.1 posterior table summary

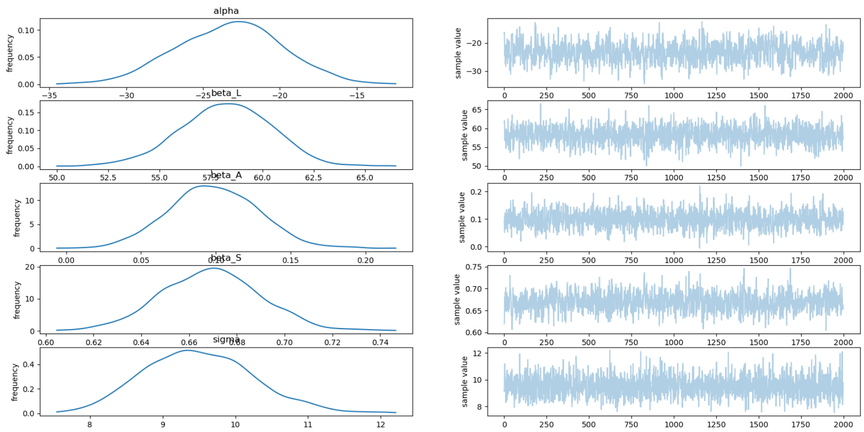


Figure 4: Question 2.1 plot summary

2.2 A less simple model (figs. 6 and 5)

Denoting that size has a positive effect on price does not affect performance. This is potentially due to the fact that the data already embodies this fact and explicitly stating it does not give us any new knowledge.

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
alpha	-23.09	0.11	3.43	-29.71	-25.33	-23.07	-20.8	-16.37	1020	1.0
beta_L	58.27	0.06	2.29	53.61	56.77	58.26	59.83	62.66	1245	1.0
beta_A	0.1	8.0e-4	0.03	0.04	0.08	0.1	0.12	0.16	1288	1.0
beta_S	0.67	6.4e-4	0.02	0.63	0.65	0.67	0.68	0.71	1065	1.0
sigma	9.48	0.02	0.7	8.17	8.99	9.43	9.92	10.95	1632	1.0
lp__	-245.0	0.06	1.6	-248.8	-245.8	-244.7	-243.8	-242.9	775	1.0

Figure 5: Question 2.2 posterior table summary

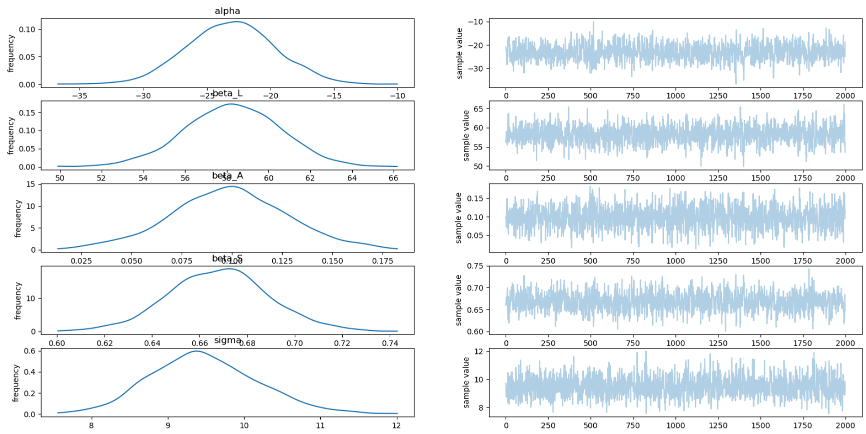


Figure 6: Question 2.2 plot summary

2.3 Two models (figs. 7,8)

Houses in 0 get cheaper with age, which was obfuscated in 2.2. Splitting also reduces noise. The higher certainty in our split models is also reflected by the superior `lp__`.

2.4 A compromise model (figs. 9, 10 11)

We create a hierarchical model [2] that has variant slopes, but maintains the intercept across locales $y_i = \alpha + \beta_{j[i]} x_i + \varepsilon_i$ with all $\varepsilon \sim N(0, \sigma^2)$ [3]. With it we are able to capture the Age difference and still share knowledge

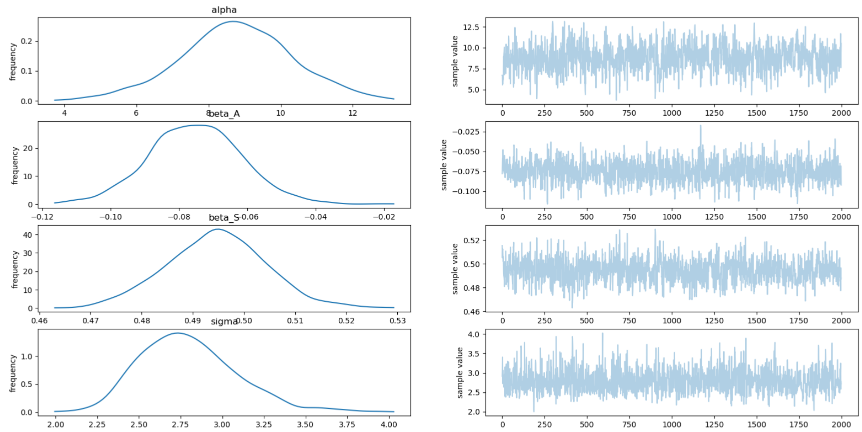
3 VB vs MCMC

In fewer than 200 words overall: (i) describe Hamiltonian MCMC, (ii) describe variational inference as done in Stan and (iii) discuss the pros and cons of both approaches. (Any equations or figures do not count towards the word count. HMCMC approximates posteriors. VS approximates posteriors as a product of Gaussians **[[citation needed]]**. VS is faster and has been optimised for large datasets **[[citation needed]]**, but if the true underlying distribution is not well approximated by a product of gaussians, VS will fail. MCMC is asymptotically precise yet is computationally intensive and multimodal distributions can confuse it. **[[citation needed]]**.

4 Hidden Markov models

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
alpha	8.75	0.05	1.48	5.78	7.8	8.78	9.71	11.61	874	1.0
beta_A	-0.07	3.7e-4	0.01	-0.1	-0.08	-0.07	-0.07	-0.05	1238	1.0
beta_S	0.49	3.0e-4	9.6e-3	0.47	0.49	0.49	0.5	0.51	1002	1.0
sigma	2.82	9.2e-3	0.31	2.29	2.6	2.79	3.0	3.5	1146	1.0
lp__	-75.57	0.06	1.44	-79.08	-76.28	-75.24	-74.5	-73.79	667	1.0

(a) posterior table summary

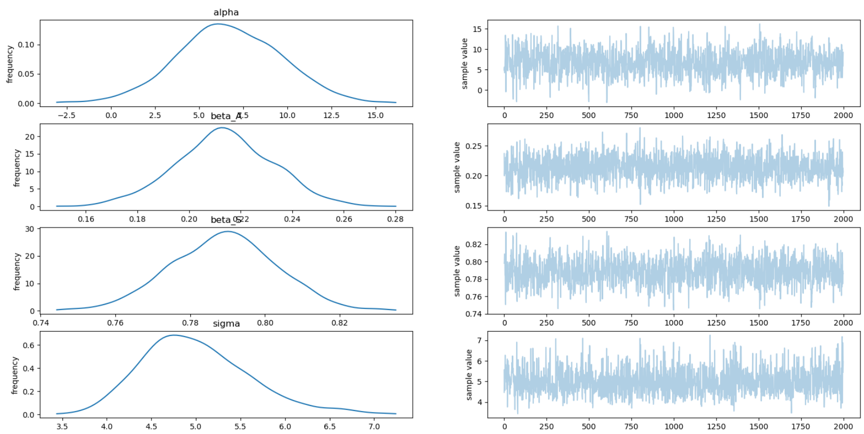


(b) plot summary

Figure 7: Question 2.3 Locale 0

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
alpha	6.62	0.11	2.97	0.84	4.67	6.5	8.52	12.78	787	1.0
beta_A	0.21	5.9e-4	0.02	0.18	0.2	0.21	0.23	0.25	1133	1.0
beta_S	0.79	4.8e-4	0.01	0.76	0.78	0.79	0.8	0.82	896	1.0
sigma	5.0	0.02	0.6	4.0	4.58	4.93	5.38	6.31	1447	1.0
lp__	-82.27	0.06	1.49	-86.0	-83.02	-81.92	-81.14	-80.4	731	1.0

(a) posterior table summary



(b) plot summary

Figure 8: Question 2.3 Locale 1

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
alpha	7.79	0.05	1.58	4.73	6.69	7.84	8.83	10.94	905	1.0
beta_A[1]	-0.07	4.3e-4	0.02	-0.1	-0.08	-0.07	-0.06	-0.04	1653	1.0
beta_A[2]	0.21	3.5e-4	0.01	0.18	0.2	0.21	0.22	0.24	1531	1.0
beta_S[1]	0.5	3.5e-4	0.01	0.48	0.49	0.5	0.51	0.52	1112	1.0
beta_S[2]	0.78	2.8e-4	9.0e-3	0.77	0.78	0.78	0.79	0.8	1033	1.0
sigma_beta_A	2.29	1.28	33.77	0.1	0.19	0.33	0.68	7.13	701	1.0
sigma_beta_S	15.39	7.2	189.6	0.43	0.83	1.45	3.29	52.89	694	1.01
sigma_y	3.86	7.2e-3	0.3	3.33	3.65	3.84	4.05	4.52	1703	1.0
y_hat[1]	95.94	0.02	1.04	93.83	95.28	95.95	96.63	97.94	1742	1.0
y_hat[2]	105.98	0.03	1.42	103.16	105.05	106.04	106.93	108.68	1747	1.0
y_hat[3]	52.9	0.02	0.63	51.63	52.48	52.9	53.31	54.14	1393	1.0
y_hat[4]	60.82	0.02	0.84	59.18	60.25	60.8	61.36	62.51	1818	1.0
y_hat[5]	37.65	0.02	0.84	35.93	37.11	37.67	38.19	39.32	1435	1.0
y_hat[6]	55.53	0.02	0.83	53.86	54.98	55.52	56.08	57.18	1829	1.0
y_hat[7]	48.36	0.02	0.71	46.92	47.9	48.37	48.82	49.78	1687	1.0
y_hat[8]	43.9	0.03	1.06	41.78	43.21	43.91	44.59	46.01	1774	1.0
y_hat[9]	41.97	0.03	1.07	39.82	41.27	41.98	42.68	44.08	1760	1.0
y_hat[10]	36.21	0.02	0.83	34.52	35.69	36.24	36.78	37.84	1304	1.0
y_hat[11]	101.88	0.03	1.09	99.66	101.15	101.88	102.62	104.01	1652	1.0
y_hat[12]	54.87	0.02	0.81	53.26	54.33	54.88	55.41	56.47	1364	1.0
y_hat[13]	40.03	0.03	0.82	38.38	39.48	40.07	40.57	41.68	1041	1.0
y_hat[14]	36.17	0.02	0.97	34.25	35.55	36.18	36.8	38.08	1619	1.0
y_hat[15]	71.44	0.02	0.73	70.05	70.95	71.44	71.91	72.91	1674	1.0
y_hat[16]	69.32	0.02	0.92	67.54	68.7	69.32	69.92	71.17	1637	1.0
y_hat[17]	41.59	0.03	0.87	39.88	41.01	41.62	42.17	43.34	1064	1.0
y_hat[18]	67.52	0.02	0.71	66.15	67.03	67.52	67.96	68.93	1772	1.0
y_hat[19]	73.95	0.02	0.82	72.38	73.39	73.94	74.47	75.62	1572	1.0
y_hat[20]	29.81	0.03	1.06	27.72	29.14	29.82	30.52	31.89	1518	1.0
y_hat[21]	60.79	0.02	0.88	59.04	60.22	60.8	61.38	62.5	1577	1.0
y_hat[22]	84.23	0.02	0.86	82.58	83.66	84.22	84.8	85.98	1358	1.0
y_hat[23]	56.32	0.02	0.86	54.6	55.75	56.31	56.87	58.05	1824	1.0
y_hat[24]	44.16	0.03	0.85	42.5	43.58	44.19	44.73	45.86	1103	1.0
y_hat[25]	83.23	0.02	0.8	81.66	82.72	83.24	83.76	84.78	1846	1.0
y_hat[26]	48.58	0.02	0.74	47.08	48.1	48.59	49.07	50.07	1729	1.0
y_hat[27]	68.36	0.01	0.6	67.17	67.96	68.36	68.75	69.56	1887	1.0
y_hat[28]	79.43	0.02	0.93	77.58	78.8	79.44	80.07	81.26	1821	1.0
y_hat[29]	98.08	0.03	1.06	96.05	97.37	98.07	98.79	100.24	1122	1.0
y_hat[30]	71.82	0.02	0.93	70.05	71.19	71.81	72.42	73.68	1585	1.0
y_hat[31]	63.87	0.02	0.66	62.57	63.43	63.86	64.29	65.2	1877	1.0
y_hat[32]	59.94	0.02	0.81	58.33	59.41	59.95	60.49	61.52	1571	1.0
y_hat[33]	52.17	0.02	0.72	50.75	51.69	52.18	52.64	53.57	1293	1.0
y_hat[34]	75.67	0.02	0.88	74.0	75.07	75.65	76.23	77.44	1519	1.0
y_hat[35]	50.21	0.02	0.63	48.93	49.81	50.22	50.62	51.44	1551	1.0
y_hat[36]	71.44	0.02	0.73	70.05	70.95	71.44	71.91	72.91	1674	1.0
y_hat[37]	79.0	0.02	0.94	77.12	78.36	79.01	79.65	80.84	1816	1.0
y_hat[38]	95.88	0.03	1.12	93.73	95.14	95.85	96.64	98.12	1124	1.0
y_hat[39]	105.04	0.04	1.5	102.12	104.05	105.11	106.04	107.9	1766	1.0
y_hat[40]	77.62	0.01	0.63	76.38	77.21	77.62	78.05	78.88	1847	1.0
y_hat[41]	61.23	0.02	0.68	59.87	60.78	61.21	61.67	62.64	1872	1.0
y_hat[42]	97.14	0.03	1.15	94.84	96.38	97.18	97.91	99.32	1766	1.0
y_hat[43]	61.84	0.01	0.54	60.79	61.48	61.83	62.2	62.89	1871	1.0
y_hat[44]	69.87	0.02	0.74	68.45	69.38	69.87	70.34	71.34	1703	1.0
y_hat[45]	46.78	0.03	1.01	44.82	46.09	46.81	47.47	48.8	1185	1.0
y_hat[46]	52.41	0.02	0.6	51.22	52.01	52.41	52.8	53.6	1479	1.0
y_hat[47]	102.4	0.03	1.04	100.3	101.7	102.4	103.06	104.47	1382	1.0
y_hat[48]	78.88	0.02	0.76	77.4	78.39	78.89	79.38	80.38	1868	1.0
y_hat[49]	90.42	0.02	0.84	88.79	89.85	90.42	90.97	92.09	1430	1.0
y_hat[50]	49.99	0.02	0.62	48.71	49.58	49.99	50.4	51.23	1484	1.0
y_hat[51]	76.53	0.03	0.91	74.76	75.91	76.52	77.15	78.34	1086	1.0
y_hat[52]	163.73	0.02	0.72	162.31	163.26	163.72	164.2	165.21	1691	1.0
y_hat[53]	143.44	0.02	0.94	141.55	142.8	143.43	144.08	145.26	1745	1.0
y_hat[54]	187.79	0.02	0.8	186.22	187.27	187.79	188.3	189.4	1479	1.0
y_hat[55]	117.06	0.02	0.99	115.07	116.38	117.07	117.74	118.99	1679	1.0
y_hat[56]	84.25	0.03	0.92	82.34	83.63	84.28	84.87	85.98	1082	1.0
y_hat[57]	98.65	0.03	1.13	96.4	97.9	98.63	99.43	100.86	1508	1.0
y_hat[58]	74.16	0.03	0.94	72.23	73.52	74.19	74.79	75.96	1015	1.0
y_hat[59]	189.88	0.02	0.8	188.32	189.36	189.89	190.39	191.5	1477	1.0
y_hat[60]	195.37	0.02	0.86	193.68	194.78	195.39	195.91	197.02	1473	1.0
y_hat[61]	190.28	0.03	1.21	187.88	189.48	190.28	191.1	192.64	1465	1.0
y_hat[62]	223.8	0.03	1.12	221.56	223.08	223.79	224.57	226.06	1223	1.0
y_hat[63]	179.24	0.02	0.81	177.65	178.68	179.25	179.77	180.86	1676	1.0
y_hat[64]	179.91	0.02	1.01	177.98	179.22	179.93	180.58	181.88	1682	1.0
y_hat[65]	144.06	0.02	0.81	142.44	143.51	144.05	144.61	145.68	1765	1.0
y_hat[66]	190.57	0.03	1.18	188.3	189.75	190.6	191.36	192.87	1635	1.0
y_hat[67]	87.89	0.02	0.81	86.29	87.33	87.87	88.44	89.47	1134	1.0
y_hat[68]	140.02	0.02	0.64	138.77	139.58	140.01	140.43	141.31	1794	1.0
y_hat[69]	140.26	0.03	1.27	137.76	139.41	140.27	141.15	142.64	1723	1.0
y_hat[70]	188.71	0.03	1.05	186.64	188.02	188.7	189.42	190.81	1457	1.0
y_hat[71]	162.1	0.02	0.69	160.73	161.65	162.1	162.55	163.51	1717	1.0
y_hat[72]	161.42	0.03	1.07	159.34	160.69	161.44	162.16	163.51	1716	1.0
y_hat[73]	169.29	0.03	1.04	167.2	168.6	169.29	170.0	171.33	1602	1.0
y_hat[74]	190.52	0.02	0.87	188.79	189.96	190.52	191.09	192.29	1434	1.0
y_hat[75]	212.78	0.03	1.23	210.36	211.98	212.79	213.59	215.16	1317	1.0
y_hat[76]	134.55	0.02	0.68	133.22	134.1	134.56	135.01	135.85	1639	1.0
y_hat[77]	192.89	0.02	0.96	191.05	192.26	192.9	193.52	194.81	1568	1.0
y_hat[78]	97.04	0.02	0.74	95.57	96.54	97.04	97.52	98.48	1185	1.0

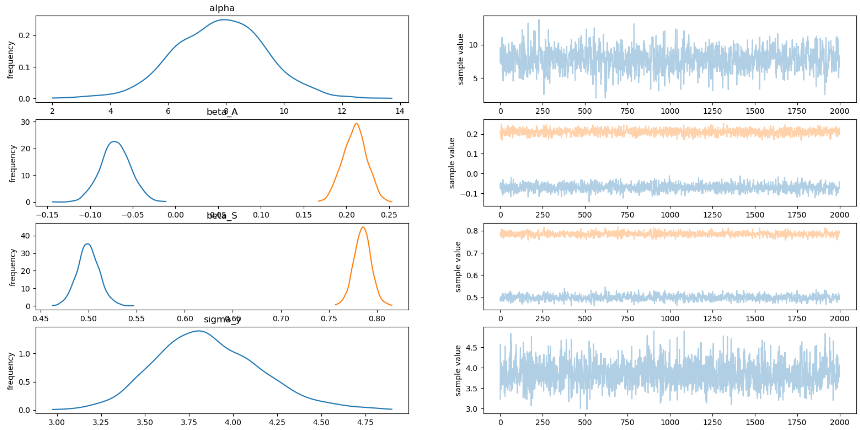


Figure 10: Question 2.4 plot summary

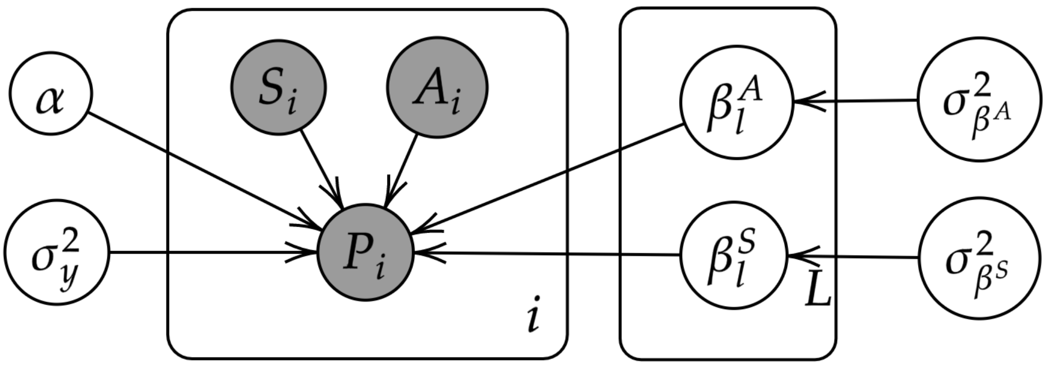


Figure 11: Question 2.4 plot summary