## CS 533: Natural Language Processing

(Due: 03/10/20)

# Assignment 3

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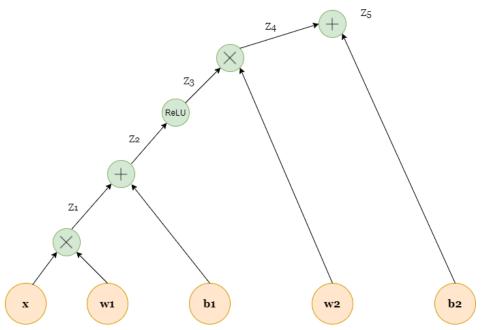
#### Problem 1: Backpropagation

$$(((1+1+3+1+1)+(1+1+4)=13 \text{ points}))$$

#### 1. (Scalar-Valued Variables):

(a) Computational graph:

 $z_5 = w_2$ . ReLU ( $w_1.x + b_1$ ) +  $b_2$ 



(b) Running the forward pass on the graph using the input values:

$$z_1 = w_1 x = \frac{1}{4}(1) = \frac{1}{4}$$

$$z_2 = z_1 + b_1 = \frac{1}{4} + 0 = \frac{1}{4}$$

$$z_3 = RELU(z_2) = \frac{1}{4}$$

$$z_4 = w_2 z_3 = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

$$z_5 = z_4 + b_2 = \frac{1}{12} + 0 = \frac{1}{12}$$

(1)

Output of forward pass:  $z_5 = 1/12$ 

(c) BackPropagation:

• 
$$\frac{dz_5}{db_2} = \frac{d(z_4 + b_2)}{db_2} = 1$$

• 
$$\frac{dz_5}{dw_2} = \frac{d(z_4 + b_2)}{d(z_4)} \cdot \frac{d(z_4)}{dw_2} = 1 \cdot \frac{d(w_2 \cdot z_3)}{dw_2} = z_3 = ReLU(w_1x + b_1) = 1/4$$

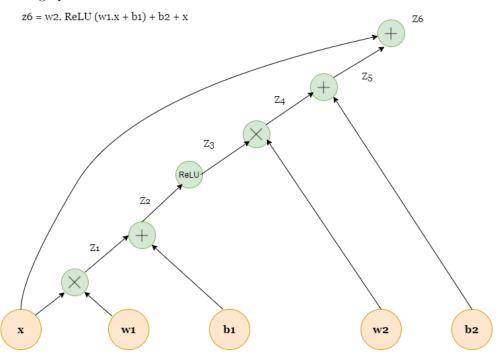
• 
$$\frac{dz_5}{dx} = \frac{d(z_4 + b_2)}{d(z_4)} \cdot \frac{d(z_4)}{dz_3} \cdot \frac{dz_3}{dz_2} \cdot \frac{dz_2}{dz_1} \cdot \frac{dz_1}{dx} = 1.w_2 \cdot 1. \frac{d(w_1 \cdot x)}{dx} = w_2 \cdot w_1 = 1/12$$

• 
$$\frac{dz_5}{db_2} = \frac{d(z_4 + b_2)}{db_2} = 1$$
  
•  $\frac{dz_5}{dw_2} = \frac{d(z_4 + b_2)}{d(z_4)} \cdot \frac{d(z_4)}{dw_2} = 1 \cdot \frac{d(w_2.z_3)}{dw_2} = z_3 = ReLU(w_1x + b_1) = 1/4$   
•  $\frac{dz_5}{dx} = \frac{d(z_4 + b_2)}{d(z_4)} \cdot \frac{d(z_4)}{dz_3} \cdot \frac{dz_3}{dz_2} \cdot \frac{dz_2}{dz_1} \cdot \frac{dz_1}{dx} = 1 \cdot w_2 \cdot 1 \cdot \frac{d(w_1.x)}{dx} = w_2 \cdot w_1 = 1/12$   
•  $\frac{dz_5}{dw_1} = \frac{d(z_4 + b_2)}{d(z_4)} \cdot \frac{d(z_4)}{dz_3} \cdot \frac{dz_3}{dz_2} \cdot \frac{dz_2}{dz_1} \cdot \frac{dz_1}{dw_1} = 1 \cdot w_2 \cdot 1 \cdot \frac{d(w_1.x)}{dw_1} = w_2 \cdot x = 1/3$   
•  $\frac{dz_5}{db_1} = \frac{d(z_4 + b_2)}{d(z_4)} \cdot \frac{d(z_4)}{dz_3} \cdot \frac{dz_2}{dz_2} \cdot \frac{dz_2}{db_1} = 1 \cdot w_2 \cdot 1 \cdot \frac{d(z_1 + b_1)}{db_1} = 1 \cdot w_2 \cdot 1 \cdot 1 = 1/3$ 

• 
$$\frac{dz_5}{db_1} = \frac{d(z_4+b_2)}{d(z_4)} \cdot \frac{d(z_4)}{dz_3} \cdot \frac{dz_3}{dz_2} \cdot \frac{dz_2}{db_1} = 1.w_2.1 \cdot \frac{d(z_1+b_1)}{db_1} = 1.w_2.1.1 = 1/3$$

# (d) After adding $z_6 = z_5 + x$ :

i. Computation graph:



- ii. Forward Propagation: (Additional computation)  $z_6 = z_5 + x = (1/12 + 1) = 13/12$
- iii. BackPropagation:

$$\bullet \ \frac{dz_6}{db_2} = \frac{dz_6}{dz_5} \cdot \frac{d(z_4 + b_2)}{db_2} = 1.1 = 1$$

• 
$$\frac{dz_6}{db_2} = \frac{dz_6}{dz_5} \cdot \frac{d(z_4 + b_2)}{db_2} = 1.1 = 1$$
  
•  $\frac{dz_6}{dw_2} = \frac{dz_6}{dz_5} \cdot \frac{d(z_4 + b_2)}{d(z_4)} \cdot \frac{dz_4}{dw_2} = 1 \cdot \frac{d(w_2 \cdot z_3)}{dw_2} = z_3 = ReLU(w_1x + b_1) = 1/4$   
•  $\frac{dz_6}{dw_1} = \frac{dz_6}{dz_5} \cdot \frac{d(z_4 + b_2)}{d(z_4)} \cdot \frac{z_4}{dz_3} \cdot \frac{dz_3}{dz_2} \cdot \frac{dz_2}{dz_1} \cdot \frac{dz_1}{dw_1} = 1 \cdot w_2 \cdot 1 \cdot \frac{d(w_1 \cdot x)}{dw_1} = w_2 \cdot x = 1/3$   
•  $\frac{dz_6}{db_1} = \frac{dz_6}{dz_5} \cdot \frac{d(z_4 + b_2)}{dz_4} \cdot \frac{dz_4}{dz_3} \cdot \frac{dz_3}{dz_2} \cdot \frac{dz_2}{db_1} = 1 \cdot w_2 \cdot 1 \cdot \frac{d(z_1 + b_1)}{db_1} = 1 \cdot w_2 \cdot 1 \cdot 1 = 1/3$ 

• 
$$\frac{dz_6}{dw_1} = \frac{dz_6}{dz_1} \cdot \frac{d(z_4 + b_2)}{d(z_1)} \cdot \frac{z_4}{dz_2} \cdot \frac{dz_3}{dz_2} \cdot \frac{dz_2}{dw_1} \cdot \frac{dz_1}{dw_2} = 1.w_2 \cdot 1. \frac{d(w_1 \cdot x)}{dw_2} = w_2 \cdot x = 1/3$$

• 
$$\frac{dz_6}{db_1} = \frac{dz_6}{dz_5} \cdot \frac{d(z_4 + b_2)}{dz_4} \cdot \frac{dz_4}{dz_3} \cdot \frac{dz_3}{dz_2} \cdot \frac{dz_2}{db_1} = 1.w_2 \cdot 1 \cdot \frac{d(z_1 + b_1)}{db_1} = 1.w_2 \cdot 1 \cdot 1 = 1/3$$

$$\frac{dz_6}{dx} = \frac{d(z_5 + x)}{dx} = \frac{dz_5}{dx} + 1$$

Now,

$$\frac{dz_5}{dx} = \frac{d(z_4 + b_2)}{d(z_4)} \cdot \frac{dz_4}{dz_3} \cdot \frac{dz_3}{dz_2} \cdot \frac{dz_2}{dz_1} \cdot \frac{dz_1}{dx} = w_2 \cdot w_1 = \frac{1}{12}$$

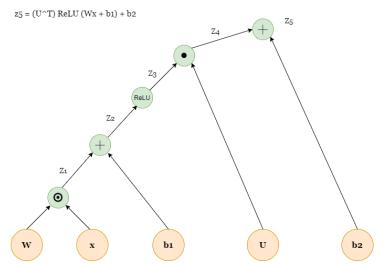
Therefore,

$$\frac{dz_6}{dx} = \frac{1}{12} + 1 = \mathbf{13/12} \tag{2}$$

(e) After adding  $z_6$ , the sensitivity of the variable x increases. This is because, as compared to  $z_5$ , x is connected to  $z_6$  directly via linear combination. Any change in the variable x will have a greater change in  $z_6$  which is function of linear combination of x and function of x.  $z_5$  is a function of x and does not have direct influence of x. We can verify this from the gradient of  $z_6$  and  $z_5$  w.r.t x. The gradient of  $z_6$  w.r.t x is 13 times greater than gradient of  $z_5$  w.r.t x.

## 2. (Vector-Valued Variables):

(a) Computation graph:



(b) Forward pass on the graph using the given input values:

$$z_{1} = Wx = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$z_{2} = z_{1} + b_{1} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$z_{3} = ReLU(z_{2}) = ReLU(\begin{bmatrix} -2 \\ 2 \end{bmatrix}) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$z_{4} = U^{T}z_{3} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = 2$$

$$z_{5} = z_{4} + b_{2} = 2 + 0 = 2$$

Output of forward pass:  $z_5 = 2$ 

(c) Backpropagation

$$z_{5} = z_{4} + b_{2}$$

$$\Rightarrow \frac{\partial z_{5}}{\partial z_{4}} = \frac{\partial z_{4}}{\partial z_{4}} + \frac{\partial b_{2}}{\partial z_{4}} = 1 + 0 = 1$$

$$\Rightarrow \frac{\partial z_{5}}{\partial b_{2}} = \frac{\partial z_{4}}{\partial b_{2}} + \frac{\partial b_{2}}{\partial b_{2}} = 0 + 1 = 1$$
(3)

$$z_{4} = u^{T} z_{3}$$

$$\Rightarrow \frac{\partial z_{5}}{\partial z_{3}} = u \frac{\partial z_{5}}{\partial z_{4}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} (UsingLemma1)$$

$$z_{4} = u^{T} z_{3} = z_{3}^{T} u$$

$$\Rightarrow \frac{\partial z_{5}}{\partial u} = \frac{\partial z_{5}}{\partial z_{4}} z_{3} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$z_{3} = ReLU(z_{2})$$

$$\Rightarrow \frac{\partial z_{5}}{\partial z_{2}} = \frac{\partial z_{5}}{\partial z_{3}} \frac{\partial z_{3}}{\partial z_{2}}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \odot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$z_{2} = z_{1} + b_{1}$$

$$\Rightarrow \frac{\partial z_{2}}{\partial z_{1}} = \frac{\partial z_{1}}{\partial z_{1}} + \frac{\partial b_{1}}{\partial z_{1}}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \frac{\partial z_{5}}{\partial z_{1}} = \frac{\partial z_{5}}{\partial z_{2}} \frac{\partial z_{2}}{\partial z_{1}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \odot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \frac{\partial z_{5}}{\partial b_{1}} = \frac{\partial z_{5}}{\partial z_{2}} \frac{\partial z_{2}}{\partial b_{1}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \odot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

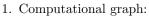
$$z_{1} = Wx$$

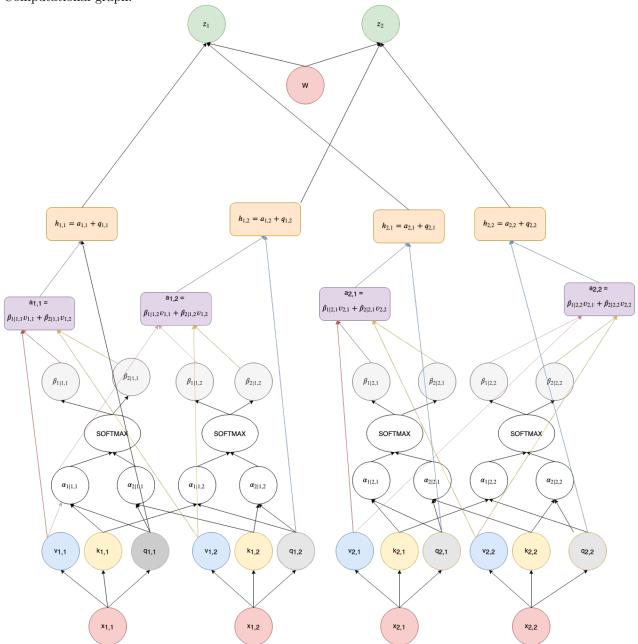
$$\Rightarrow \frac{\partial z_{5}}{\partial x} = W^{T} \frac{\partial z_{5}}{\partial z_{1}} = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\frac{\partial z_{5}}{\partial x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \frac{\partial z_{5}}{\partial W} = \frac{\partial z_{5}}{\partial z_{1}} x^{T} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\frac{\partial z_{5}}{\partial W} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$





2. Forward pass with the input values, H=2, t=2:

$$q_{h,t} = (W_h)^Q x_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$
$$k_{h,t} = (W_h)^K x_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$
$$v_{h,t} = (W_h)^V x_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Now,  $\alpha_{t'|h,t} = k_{h,t'}q_{h,t}$ . Thus for t = 1...T and t' = 1...T,

$$\alpha_{1|h,t} = \begin{bmatrix} 1 & -1 \\ 4 & 6 \end{bmatrix}$$
$$\alpha_{2|h,t} = \begin{bmatrix} -1 & 1 \\ 6 & 9 \end{bmatrix}$$

Now for t = 1....T,

$$(\beta_{1|h,t}...\beta_{T|h,t}) = softmax(\alpha_{1|h,t}...\alpha_{T|h,t})$$
  

$$\implies (\beta_{1|h,t},\beta_{2|h,t}) = softmax(\alpha_{1|h,t},\alpha_{2|h,t})$$

Therefore, for t = 1:

$$\beta_{1|h,t} = \begin{bmatrix} 0.8808 & 0.1192 \\ 0.1192 & 0.8808 \end{bmatrix}$$

Therefore, for t = 2:

$$\beta_{2|h,t} = \begin{bmatrix} 0.1192 & 0.8808 \\ 0.4742 & 0.9525 \end{bmatrix}$$

Now, for t = 1....T,

$$a_{h,t} = \sum_{t'=1}^{T} \beta_{t'|h,t} v_{h,t'}$$

For t = 1:

$$h1: a_{11} = \beta_{1|1,1}v_{1,1} + \beta_{2|1,1}v_{1,2} = (0.8808)(1) + (0.1192)(-1) = 0.7616$$
$$h2: a_{21} = \beta_{1|2,1}v_{2,1} + \beta_{2|2,1}v_{2,2} = (0.1192)(2) + (0.8808)(3) = 2.8808$$

For t = 2:

$$h1: a_{12} = \beta_{1|1,2}v_{1,1} + \beta_{2|1,2}v_{1,2} = (0.1192)(1) + (0.8808)(-1) = -0.7616$$
$$h2: a_{22} = \beta_{1|2,2}v_{2,1} + \beta_{2|2,2}v_{2,2} = (0.4742)(2) + (0.9525)(3) = 2.9525$$

Therefore,

$$a_{h,t} = \begin{bmatrix} 0.7616 & -0.7616 \\ 2.8808 & 2.9525 \end{bmatrix}$$

For t = 1...T,

$$h_{h,t} = a_{h,t} + q_{h,t}$$

$$h_{h,t} = \begin{bmatrix} 0.7616 & -0.7616 \\ 2.8808 & 2.9525 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$h_{h,t} = \begin{bmatrix} 1.7616 & -1.7616 \\ 4.8808 & 5.9525 \end{bmatrix}$$

Now,

$$z_{t} = W.(h_{1,t} + \dots + h_{T,t})$$

$$z_{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1.7616 \\ 4.8808 \end{bmatrix} = \begin{bmatrix} 1.7616 \\ 4.8808 \end{bmatrix}$$

$$z_{2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1.7616 \\ 5.9525 \end{bmatrix} = \begin{bmatrix} -1.7616 \\ 5.9525 \end{bmatrix}$$

```
Problem 3: Programming
```

```
((1+2+(1+1+1)+(1+1+1)+5+4+1+1+3=23 \text{ points}))
```

1. Implementation of get\_ngram\_counts:

Fig 1:

2. Implementation of compute\_bleu:

After running the tests in *test\_blue.py*:

BLEU score for small test: 0.5920778868801042 BLEU score for large test: 0.010382904408270435

3. (a) In case of continuous batch method, the data is processed into a single block by dividing the data according to the number of batches parameter. The function epoch\_continuous\_data in the class control

uses the method  $build\_itr$  to make the examples for training. The model will run forward pass on chunks of sub-blocks of size  $max(bptt, block\_size)$ . The gradients are updated based after processing one batch the training data divided into batches. The function  $count\_batches$  divides the training data into batches such that each batch example has bptt size. Thus, bptt determines the length of sequence that is being processing in the model at a time.

(b) The model computes the loss and calculates the gradients in the function:  $step-on\_batch$ . The flow is: main class initiates and calls control.train. The training is done by calling  $do\_epoch$  which in turn calls  $epoch\_continuous\_data$  if the batch method is continuous and calls  $epoch\_translation\_data$  if the batch method is translation. The  $step\_on\_batch$  method is called for each batch. The method resets the gradients to zero using torch function  $zero\_grad()$ . The method then runs a forward pass and calculates the loss using cross entropy loss. The gradients are updated using torch.backward(). This calculates the gradients for all the parameters in the model. The next two steps are regularization using gradient clipping and gradient update step.

- (c) The model maintains the state from the previous batch in *self.state* in the decoder. The methods *detach\_state*, *update\_state*, *init\_state* in the *decoder* class help maintain and initialize the state of the hidden states of the model. The forward method of the model initializes the state is start of the batch and detaches states using repackaging of the hidden states using detach\_state.
- 4. (a) In case of translation batch method, the data is processed in terms of bundles of source-target sequences taking the longest sentence and padding the rest of the sentences. During training, the method epoch\_translation\_data calls the count\_batches which builds training examples from the blocks of the data. Thus, the encoding and decoding is done on the sub-block (containing source and target sequences) of the block in the step\_on\_batch method. Thus, bbpt is not longer used in translation data.
  - (b) The model encodes the source sentence in the forward method of the encoder class. During training using translation as batch method, the epoch\_translation\_data in the control class calls the step\_on\_batch for forward and backproprogation. This method uses the encoder of the s2s model to encode the source sequence. The encoder uses torch.nn.Embeddings to encode the sequence.
  - (c) In the class *model*, the method forward encodes the source sentence and returns its final state if the batch method is translation. This final state of the encoder stored in the variable *encoder\_final*, is passed on to the decoder while initializing the state for decoder. Thus, the *dec.init\_state* conditions on the final encoding of the source sentence.
- 5. Implementation of score in attention:

6. Implementation of forward in attention:

```
h_t = torch.cat((c,queries),dim =2)
attn_h = torch.tanh(self.linear_out(h_t))
```

- 7. Passed the tests.
- 8. Training session:

```
use attention: 1
```

9. After training the model for 1000 steps, the model converged at around 227th epoch with validation perplexity as 33.23

```
| end of epoch 225 | time: 7.26s | valid loss 3.50 | valid ppl 33.24 | valid sqxent 74.52 | |
| epoch 226 | 20/ 29 batches | lr 0.00 | ms/batch 206.43 | loss 2.13 | ppl 8.45 |
| end of epoch 226 | time: 7.15s | valid loss 3.50 | valid ppl 33.23 | valid sqxent 74.52 |
| epoch 227 | 20/ 29 batches | lr 0.00 | ms/batch 209.06 | loss 2.15 | ppl 8.58 |
| end of epoch 227 | time: 7.26s | valid loss 3.50 | valid ppl 33.23 | valid sqxent 74.52 |
| epoch 228 | 20/ 29 batches | lr 0.00 | ms/batch 208.10 | loss 2.14 | ppl 8.53 |
| end of epoch 228 | time: 7.16s | valid loss 3.50 | valid ppl 33.23 | valid sqxent 74.52 |
| epoch 229 | 20/ 29 batches | lr 0.00 | ms/batch 205.33 | loss 2.15 | ppl 8.59 |
| end of epoch 229 | time: 7.10s | valid loss 3.50 | valid ppl 33.23 | valid sqxent 74.52 |
| epoch 230 | 20/ 29 batches | lr 0.00 | ms/batch 205.33 | loss 2.14 | ppl 8.49 |
| end of epoch 230 | time: 7.21s | valid loss 3.50 | valid ppl 33.23 | valid sqxent 74.52 |
| epoch 231 | 20/ 29 batches | lr 0.00 | ms/batch 205.80 | loss 2.14 | ppl 8.49 |
| end of epoch 231 | 20/ 29 batches | lr 0.00 | ms/batch 205.80 | loss 2.14 | ppl 8.49 |
```

Yes, the attention weights of the decoder are as expected. After exploring input sequences with test\_decoder, text\_encoder and test\_model and observing the weights, the attention weights highlight the words which are relevant to the current word or give strong negative weights to the word which are less correlated. These attention weights are a probability distribution over the vocab or input sequence(in case of self attention) given the current context word.