1   Monte Carlo studies of simple integrals

Study various simulation methods for the simple test integral

$$\frac{\int_0^\infty x\,e^{-x^2}\,dx}{\int_0^\infty e^{-x^2}\,dx}.$$

Evaluate in four different ways (1) Using a Metropolis method, (2) Simple Monte Carlo, (3) Exactly using analytic integration. In (1), experiment with the maximum step size δ. Begin with $\delta = 1$, then compare with $\delta = 0.1$ and $\delta = 10$. Study the statistical error in the MC methods, by taking the average RMS error (error = result from (1) minus the result from (4)) as a function of the number of terms N (for example, $N = 10, 100, 1000, 10000, ...$) in the MC calculation. Is this a good measure of the statistical error in (1) and in (2)? Why or why not? Study the correlation time τ in (1). In (1), it is necessary to repeat the calculation many times, and calculate the RMS errors from the results of these independent runs. Plot the RMS error vs N in log-log plots. Compare the CPU times for (1)-(3) to reach a certain error level. Also study the more complicated oscillatory integral

$$\int_{-\infty}^\infty \sin^2 \frac{1}{x}\,dx.$$