A Dantzig-Wolfe Decomposition-Based Approach for Distributed Storage Management

Ahmed S. Zamzam*, Emiliano Dall'Anese†, and Nicholas D. Sidiropoulos*‡
* Dept. of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN, USA
† National Renewable Energy Laboratory, Golden, CO, USA
‡ Dept. of Electrical and Computer Engineering, University of Virginia, Charlottesville, VA, USA

***Abstract*—This paper focuses on optimal management of energy storage systems in distribution grids. Performance and operation objectives are formalized in an optimal power flow formulation, where charging and discharging inefficiencies of the batteries are accounted for. Binary optimization variables are utilized to avoid meaningless solutions where a battery is required to simultaneously charge and discharge. Leveraging linearized AC power-flow models, the resultant problem is a mixed-integer linear program. A distributed solution method is then synthesized based on the Dantzig-Wolfe decomposition algorithm. The efficacy of the developed algorithm is demonstrated using an IEEE distribution feeder equipped with storage units and photovoltaic systems.**

***Index Terms*—Energy storage systems, distribution networks, distributed energy resources, Dantzig-Wolfe decomposition.**

I. Introduction

The technology for harnessing renewable energy has progressed rapidly in the past few decades. However, the contribution of renewable energy sources (RES) towards meeting Renewable Standard Portfolio targets is still limited [15]; this is mainly due to difficulties in coping with volatility of renewable generation and maintaining voltages within limits of current technological solutions. Energy storage (ES) units may play a pivotal role in increasing grid efficiency and coping with fluctuating wind and solar energy resources. ES units (e.g., batteries) can enhance grid performance in many additional ways, such as spatio-temporal energy arbitrage [16], peak shaving [17], frequency [18], and voltage support [19], as well as congestion management [16], [21].

Market-based approaches have been widely pursued in order to facilitate distributed control actions [22], [23]. End-customers aim at maximizing their profit by controlling their own energy assets like ES units and PV inverters. However, such individual behavior may lead to instable system operation, i.e., situations where network constraints are violated. Therefore, several approaches have been proposed to provide strategies to control energy assets in a way that maximizes a well-defined end-customer objective [1], [2], [17]. Although their efficacy in finding high quality solutions, the methods proposed in [1], [2] only focus on cases where continuous controllable assets are considered. On the other hand, the battery modeling considered in [3], [16], [17] neglects the fact that a battery can only charge or discharge at any point of time. Hence, the solution obtained using such methods does not guarantee feasibility of the solutions.

One challenge in formulating (and solving) optimization problems associated with energy storage systems is that the dynamical model of the state of charge leads to nonconvex constraints when charging and discharging efficiencies are not one (even for linearized or relaxed optimal power flow formulations). In fact, to indicate that charging and discharging are mutually exclusive, one has to resort to either binary variables or bilinear terms. Some prior works in the context [4]–[8] considered relaxing the binary variables (which are, in fact, dropped). Conditions where this is avoided in a relaxed problem (without binary variables) are provided in [9], but these conditions can be checked a posteriori. Relative to these works, the proposed method avoids meaningless results indicating simultaneous charging and discharging. Nonconvexity can be bypassed when the charging and discharging efficiencies are assumed to be 1 [10], [11]; however this is the case in practice.

The proposed technical approach builds on a linearized power flow model [12], [13] to formulate a mixed-integer linear programming problem for optimal ES management. The objective of the problem is to maximize the sum of the end-customer objectives while respecting the nodal voltage limits. The resulting optimization problem is hard to solve in a centralized fashion due to its huge size. Therefore, a distributed solver based on the Dantzig-Wolfe decomposition is put forth. The Dantzig-Wolfe decomposition [26], [27] is an algorithm that solves linear programs with block angular structure, i.e., the constraints are separable except for a small set of constraints that bind all blocks of the optimization variable. It builds upon the revised simplex method where a column is added to the basis only if it minimizes the cost function, and the columns are generated by solving smaller subproblems. We propose a relaxed Dantzig-Wolfe (RDW) decomposition where we relax the characterization constraints of the feasibility set of each subproblem. Hence, simple pivoting operations are done in order to include the vectors generated by the subproblems to the basis.

The work of A.S. Zamzam and N.D. Sidiropoulos was partially supported by NSF under grant CIF-1525194. The work of E. Dall'Anese was supported by the Advanced Research Projects Agency-Energy (ARPA-e) under the Network Optimized Distributed Energy Systems (NODES) program.

Emails: *ahmedz@umn.edu, emiliano.dallanese@nrel.gov, nikos@virginia.edu*.

The proposed approach facilitates parallel execution. In addition, the end-customers only share their power injection with the central solver. Therefore, the privacy of their own data is preserved. The developed approach utilizes the special structure of the problem to devise an elegant approach that keeps the local information regarding the energy assets secured at each bus. The efficacy of the RDW algorithm is demonstrated on an IEEE distribution network where the optimization is done over 24 hours. Superior performance results are shown compared to a relaxation approach as well as the centralized approach. While this paper focuses on the application of the Dantzig-Wolfe decomposition method applied to a linearized AC optimal power flow, future efforts will look at the formulation and solution of robust counterparts to account for linearization errors and forecasting errors.

The remainder of this paper is organized as follows. In Section II, the system model used in the paper is described. Then, the problem formulation is introduced in Section III. The relaxed Dantzig-Wolfe decomposition algorithm is derived in Section IV, with its performance being assessed using an IEEE distribution feeder in Section V. Finally, Section VI concludes the paper.

II. System Model

Consider a distribution feeder with $N+1$ nodes collected in the set $\overline{\mathcal{N}} := \{0, 1, \ldots, N\}$. Let the node 0 be the point of coupling with the rest of the grid (e.g., the substation), and define the set $\mathcal{N} := \overline{\mathcal{N}} \backslash \{0\}$. Further, let the nodes with distributed energy sources installed be collected in the set $\mathcal{D}$ and let the set $\mathcal{E}$ collect distribution lines. The temporal axis is discretized into time slots of duration of δ_t. For each node $n \in \mathcal{N}$, $p_n(t)$ and $q_n(t)$ denote the net active and reactive power injection at time t. The non-controllable demand at node n is denoted by $s_{n,l}(t) = p_{n,l}(t) + i\ q_{n,l}(t)$. The maximum available active power at the node $n \in \mathcal{D}$ from the renewable energy sources is denoted by $p_{av,n}(t)$. Also, let $p_{n,r}(t)$ and $q_{n,r}(t)$ denote the active and reactive power injection from the energy source installed at node $n \in \mathcal{D}$ at time slot t, respectively.

The nodes with storage units installed are collected in the set $\mathcal{B}$. For each bus $n \in \mathcal{B}$, the charging and discharging rate of the battery at the t-th time slot are denoted by $p_{n,c}(t)$ and $p_{n,d}(t)$, respectively. The state of charge of the battery located at node $n \in \mathcal{B}$ at the end of the t-th time slot is denoted by $e_n(t)$, with $e_n(0)$ representing the initial state of charge. Let $\overline{e}_n$ denote the maximum capacity of the storage unit installed at bus $n \in \mathcal{B}$. With these definitions in place, the dynamics of the state of charge can be described by the following equation:

$$e_n(t) = e_n(t-1) + \delta_t(\eta_c\ p_{n,c}(t) - \frac{1}{\eta_d}\ p_{n,d}(t)) \tag{1}$$

where η_c and η_d denote the charging and discharging efficiency coefficients. The maximum charging and discharging rates of the storage unit installed at bus $n \in \mathcal{B}$ are denoted by $\overline{p}_{n,c}$ and $\overline{p}_{n,d}$, respectively. One critical challenge in solving optimization problems associated with energy storage systems is that $p_{n,c}(t)$ and $p_{n,d}(t)$ *must not be simultaneously different than zero* at a given time t; that is, a solution where a battery is supposed to charge and discharge within the same slot *is not acceptable* from physical and operational standpoints. Relative to prior works in context [4]–[8], the proposed method will resolve this issue; conditions for avoiding results indicating simultaneous charging and discharging are provided in [9], but they can be checked a posteriori. Relative to e.g., [10], [11], our method allows one to consider charging and discharging efficiencies that are different than 1.

Let the complex voltage at node $n \in \mathcal{N}$ during the t-th time slot be denoted by $v_n(t) \in \mathbb{C}$. Then, define $\mathbf{v}(t) \in \mathbb{C}^N$ to be the vector collecting the voltages at all buses during the t-th time slot. To facilitate the derivation of computationally more affordable optimization problems, we leverage the approximate model proposed in [12], [13] to linearly relate voltage magnitudes with net injected powers; that is,

$$|\mathbf{v}(t)| \approx \mathbf{R}\ \mathbf{p}(t) + \mathbf{X}\ \mathbf{q}(t) + \mathbf{a} \tag{2}$$

where the vector $\mathbf{a}$ and the matrices $\mathbf{R}$ and $\mathbf{X}$ are built as specified in [12], [13], and the vectors $\mathbf{p}(t)$ and $\mathbf{q}(t)$ collect the active and reactive nodal injections at all buses $n \in \mathcal{N}$ at time t, respectively. Notice that the matrices $\mathbf{R}$ and $\mathbf{X}$ and the vector $\mathbf{a}$ are not be time varying if a fixed-point linearization method is utilized [13]. On the other hand, these linear models are time varying if a Taylor-type approximation is utilized.

III. Problem Formulation

Consider the following cost associated with the n-th end-customer:

$$C_n(\{p_n(t)\}_{t=1}^T) = \sum_{t=1}^{T} -\alpha_t \delta_t p_n(t) \tag{3}$$

where T represents the total number of time slots considered in the multi-period optimization task, and α_t is the hourly price of the active power at time slot t. Aligned with a social welfare formulation, the following optimization problem aims at minimizing the net payment of the end-customers, subject to network constraints:

$$\min_{\left\{\substack{\mathbf{p}(t),\mathbf{q}(t)\\ \{p_{n,r}(t),q_{n,r}(t)\}_{n\in\mathcal{D}}\\ \{p_{n,c}(t),p_{n,d}(t),e_n(t),z_n(t)\}_{n\in\mathcal{B}}}\right\}_{t=1}^{T}} \quad \sum_{t=1}^{T} -\alpha_t \delta_t \mathbf{1}^{\mathcal{T}} \mathbf{p}(t) \tag{4a}$$

subject to

- $\forall t \in \{1, \ldots, T\}$

$$\underline{\mathbf{v}} \leq \mathbf{R}\mathbf{p}(t) + \mathbf{X}\mathbf{q}(t) + \mathbf{a} \leq \overline{\mathbf{v}} \tag{4b}$$

  - $\forall n \in \mathcal{N}$

$$p_n(t) = p_{n,r}(t) - p_{n,l}(t) + p_{n,d}(t) - p_{n,c}(t) \tag{4c}$$

$$q_n(t) = q_{n,r}(t) - q_{n,l}(t) \tag{4d}$$

- $\forall n \in \mathcal{B}$

$$e_n(t) = e_n(t-1) + \delta_t(\eta_c p_{n,c}(t) - ({}^1/_{\eta_d}) p_{n,d}(t)) \quad (4e)$$
$$\underline{e}_n \le e_n(t) \le \overline{e}_n \quad (4f)$$
$$0 \le p_{n,c}(t) \le z_n(t)\overline{p_c} \quad (4g)$$
$$0 \le p_{n,d}(t) \le (1 - z_n(t))\overline{p_d} \quad (4h)$$
$$z_n(t) \in \{0, 1\} \quad (4i)$$

- $\forall n \in \mathcal{D}$

$$0 \le p_{n,r}(t) \le p_{av,n}(t) \quad (4j)$$
$$-\kappa_n p_{n,r}(t) \le q_{n,r}(t) \le \kappa_n p_{n,r}(t) \quad (4k)$$

where $\overline{\mathbf{v}}$ and $\underline{\mathbf{v}}$ are vectors containing the maximum and minimum voltage magnitude allowed at all the nodes in the network. The positive constant κ_n defines the power factor constraint for the power injected from the renewable energy source installed at bus n. For the buses with no renewable energy sources or storage units, the values of $p_{n,r}(t)$ and $q_{n,r}(t)$, or $p_{n,c}(t)$ and $p_{n,d}(t)$ are set to zero in (4c) and (4d). The binary variables $z_n(t)$ are used to ensure that the problem yields a feasible solution, i.e., each battery is either charging or discharging (not both) at any time point. Lastly, notice that alternative cost functions can be utilized; however, for simplicity of exposition, we consider (3) in this paper. The optimization problem (4) is a mixed-integer linear program due to the binary variables $\{z_n(t)\}$.

Define the vectors $\mathbf{p}_n, \mathbf{q}_n, \mathbf{p}_{n,c}, \mathbf{p}_{n,d}, \mathbf{e}_n, \mathbf{z}_n, \mathbf{p}_{n,r}, \mathbf{q}_{n,r}$ of length T to be the nodal variables over all time slots. We stack the vectors $\mathbf{p}_n, \mathbf{q}_n$ in a vector $\mathbf{y}_n \in \mathbb{R}^{2T}$, and similarly we stack all the vectors $\mathbf{p}_{n,c}, \mathbf{p}_{n,d}, \mathbf{e}_n, \mathbf{p}_{n,r}, \mathbf{q}_{n,r}$ in a vector $\mathbf{x}_n \in \mathbb{R}^{5T}$. Then, the optimization problem (4) can be reformulated as

$$\text{P1:} \quad \min_{\{\mathbf{x}_n,\mathbf{y}_n,\mathbf{z}_n\}\forall n} \quad \sum_{n=1}^{N} \mathbf{c}_n^{\mathcal{T}} \mathbf{y}_n \quad (5a)$$

subject to

$$\sum_{n=1}^{N} \mathbf{A}_n \mathbf{y}_n \le \mathbf{b} \quad (5b)$$
$$(\mathbf{x}_n, \mathbf{y}_n, \mathbf{z}_n) \in \mathbf{\Omega}_n \qquad \forall n \in \mathcal{N} \quad (5c)$$

where the matrices $\mathbf{A}_n$ and the vector $\mathbf{b}$ are chosen to encode the constraints in (4b). Similarly, the vectors $\mathbf{c}_n$ are obtained based on cost function in (4a). The sets Ω_n are defined as follows

$$\mathbf{\Omega}_n = \left\{ (\mathbf{x}_n, \mathbf{y}_n, \mathbf{z}_n) \middle| \begin{array}{c} \mathbf{D}_n\mathbf{x}_n + \mathbf{G}_n\mathbf{z}_n \le \mathbf{u}_n \\ \mathbf{F}_n\mathbf{x}_n + \mathbf{H}_n\mathbf{y}_n = \mathbf{v}_n \\ \mathbf{z}_n \in \{0,1\}^T \end{array} \right\}. \quad (6)$$

The constraints (4f), (4g), (4h), (4j), and (4k) are represented by a single inequality constraint in the matrix form using the matrices $\mathbf{D}_n$, and $\mathbf{G}_n$ and the vector $\mathbf{u}_n$. Similarly, the matrices $\mathbf{F}_n$ and $\mathbf{H}_n$ and the vector $\mathbf{v}_n$ represent the constraints (4c), (4d), and (4e) in a matrix form. Clearly, the optimization problem **P1** is a mixed-integer linear program which is NP-hard in general. A straightforward relaxation approach can be adopted by relaxing the integrality constraint in (6). Therefore the relaxed problem can be formulated as

$$\text{P2:} \quad \min_{\{\mathbf{x}_n,\mathbf{y}_n,\mathbf{z}_n\}\forall n} \quad \sum_{n=1}^{N} \mathbf{c}_n^{\mathcal{T}} \mathbf{y}_n \quad (7a)$$

subject to

$$\sum_{n=1}^{N} \mathbf{A}_n \mathbf{y}_n \le \mathbf{b} \quad (7b)$$
$$(\mathbf{x}_n, \mathbf{y}_n, \mathbf{z}_n) \in \tilde{\mathbf{\Omega}}_n \qquad \forall n \in \mathcal{N} \quad (7c)$$

where the set $\tilde{\Omega}$ can be defined as

$$\tilde{\mathbf{\Omega}}_n = \left\{ (\mathbf{x}_n, \mathbf{y}_n, \mathbf{z}_n) \middle| \begin{array}{c} \mathbf{D}_n\mathbf{x}_n + \mathbf{G}_n\mathbf{z}_n \le \mathbf{u}_n \\ \mathbf{F}_n\mathbf{x}_n + \mathbf{H}_n\mathbf{y}_n = \mathbf{v}_n \\ \mathbf{z}_n \in [0,1]^T \end{array} \right\}. \quad (8)$$

The feasibility of the solution given by solving **P2** can be examined a posteriori. If the solution is feasible, i.e., the optimal values of $\mathbf{z}_n$ are all integers, then the obtained solution is guaranteed to be optimal. However, the integrality constraint does not often hold, and hence the solution of **P2** provides only a lower bound on the optimal cost function that can be attained. In the next section, a distributed approach to tackle **P1** will be put forward.

IV. Relaxed Dantzig-Wolfe Decomposition

The Dantzig-Wolfe decomposition is an algorithm for solving linear programming problems with block-angular structure. The procedure utilizes ideas from delayed column generation in order to facilitate the solution of large-scale optimization problems. When using the simplex method to find the optimal solution of the master problem, most columns (variables) are not present in the basis at each step. In such algorithm, a master problem containing the currently active basis uses subproblems solutions to generate columns for entry into the basis such that their addition reduces the cost function.

The block-angular structure of **P1** suggests breaking the problem into N independent subproblems and then adjust the solution to take into account the coupling constraint (5b). Note that, the cost function is also decomposable in terms of the nodal variables $\mathbf{y}_n$. Focusing on the feasibility set $\mathbf{\Omega}_n$, the set is clearly nonconvex due to the binary variables $\mathbf{z}_n$. For a fixed $\mathbf{z}_n = \mathbf{z}^i$, the projection $\mathbf{\Omega}_{n,\mathbf{z}^i}$ into $\mathbf{y}_n$ space is a bounded affine set. Let the $L_{n,i}$ extreme points (vertices) of the set $\mathbf{\Omega}_{n,\mathbf{z}^\star}$ that correspond to the vector $\mathbf{y}_n$ be denoted by $\{\mathbf{y}_{n,i}^{\ell}\}_{\ell=1}^{L_{n,i}}$. Therefore, any point inside the set $\mathbf{\Omega}_{n,\mathbf{z}^i}$ can be expressed as a convex combination of the extreme points of the set. There are 2^T different realizations of $\mathbf{z}_n$ which result from considering charging and discharging scenarios at each time slot. Therefore, any feasible point $\mathbf{y}_n$ in $\mathbf{\Omega}_n$ can be expressed as

$$\mathbf{y}_n = \sum_{i=1}^{2^T} \sum_{\ell=1}^{L_{n,i}} \mu_{n,i}^{\ell} \mathbf{y}_{n,i}^{\ell} \quad (9)$$

where the following constraint has to be satisfied

$$\sum_{\ell}^{L_{n,i}} \mu_{n,i}^{\ell} = \lambda_{n,i} \tag{10}$$

for all $i \in \{1, \ldots, L_{n,i}\}$, where $\lambda_{n,i}$ are binary variables that achieve

$$\sum_{i=1}^{2^T} \lambda_{n,i} = 1. \tag{11}$$

The representation of the set $\mathbf{\Omega}_n$ in (9)–(11) is again nonconvex and hard to enforce in the master problem. Thus, we relax the binary constraint on $\lambda_{n,i}$, and hence the constraints (10) and (11) can be replaced by

$$\sum_{i=1}^{2^T} \sum_{\ell}^{L_{n,i}} \mu_{n,i}^{\ell} = 1. \tag{12}$$

Therefore, the subscript i that relates to every realization of $\mathbf{z}_n$ can be removed for simplicity. Let L_n be the sum of $L_{n,i}$ over all $i \in \{1, \ldots, 2^T\}$, and hence any feasible $\mathbf{y}_n$ in this relaxed model can be written as

$$\mathbf{y}_n = \sum_{\ell=1}^{L_n} \mu_n^{\ell} \mathbf{y}_n^{\ell} \tag{13}$$

such that $\sum_{\ell=1}^{L_n} \mu_n^{\ell} = 1$.

Based on the new relaxed characterization of $\mathbf{y}_n$, the master problem can be written in the following form

$$\min_{\left\{ \{\mu_n^{\ell} \in \mathbb{R}_+\}_{\ell=1}^{L_n} \right\}_{\forall n}} \quad \sum_{n=1}^{N} \mathbf{c}_n^{\mathcal{T}} \sum_{\ell=1}^{L_n} \mu_n^{\ell} \mathbf{y}_n^{\ell} \tag{14a}$$

subject to

$$\sum_{n=1}^{N} \mathbf{A}_n \sum_{\ell=1}^{L_n} \mu_n^{\ell} \mathbf{y}_n^{\ell} \leq \mathbf{b} \tag{14b}$$

$$\sum_{\ell=1}^{L_n} \mu_n^{\ell} = 1 \qquad \forall n \in \mathcal{N} \tag{14c}$$

In addition, introduce a positive slack variable that allows us to write the constraint in the general form of the simplex algorithm. Therefore, the master problem in the proposed relaxed Dantzig-Wolfe (RDW) decomposition can be formulated as

$$\textbf{M:} \quad \min_{\left\{ \substack{\{\mu_n^{\ell} \in \mathbb{R}_+\}_{\ell=1}^{L_n}, \\ \mathbf{s} \in \mathbb{R}_+^{2TN}} \right\}_{\forall n}} \quad \sum_{n=1}^{N} \sum_{\ell=1}^{L_n} (\mathbf{c}_n^{\mathcal{T}} \mathbf{y}_n^{\ell}) \mu_n^{\ell} \tag{15a}$$

subject to

$$\boldsymbol{\pi}: \qquad \sum_{n=1}^{N} \sum_{\ell=1}^{L_n} (\mathbf{A}_n \mathbf{y}_n^{\ell}) \mu_n^{\ell} + \mathbf{s} = \mathbf{b} \tag{15b}$$

$$r_n: \qquad \sum_{\ell=1}^{L_n} \mu_n^{\ell} = 1 \qquad \forall n \in \mathcal{N} \tag{15c}$$

where π and r_n are the dual variables associated with the constraints (15b) and (15c), respectively. The master problem has only $N(2T+1)$ constraints, which is much less than the number of the constraints in the original problem (4). However, the number of variables can grow astronomically in the master problem $\mathbf{M}$. Therefore, the revised simplex method is used where a variable is introduced into the basis only when it is guaranteed to reduce the cost of the master problem. At any given iteration, the revised simplex method involves only $N(2T+1)$ variable, and hence the size of the basis matrix is $N(2T+1) \times N(2T+1)$. At any step of the revised simplex algorithm the reduced cost of the variable μ_n^{ℓ}, i.e., the reduction in the cost that results from adding μ_n^{ℓ} to the basis, can be calculated by

$$\Delta_n^{\ell} = \mathbf{c}_n^{\mathcal{T}} \mathbf{y}_n^{\ell} - \boldsymbol{\pi}^{\mathcal{T}} \mathbf{A}_n \mathbf{y}_n^{\ell} - r_n. \tag{16}$$

Still, calculating the reduction in the cost for all of the variables not included in the basis is prohibitive. Instead, the following subproblem is solved at each node $n \in \mathcal{N}$.

$$\textbf{SP:} \qquad \min_{(\mathbf{x}_n, \mathbf{y}_n, \mathbf{z}_n) \in \mathbf{\Omega}_n} \quad (\mathbf{c}_n^{\mathcal{T}} - \boldsymbol{\pi}^{\mathcal{T}} \mathbf{A}_n) \mathbf{y}_n - r_n \tag{17}$$

Notice that the optimal solution of $\mathbf{y}_n$ according to **SP** will always be one of the points $\{\mathbf{y}_n^{\ell}\}$. Upon solving **SP**, if the optimal value is negative, then we obtain an operating point $\mathbf{y}_n^{\ell}$ that can enter the basis. However, if the optimal solution is positive, then there is no additional points $\mathbf{y}_n^{\ell}$ which can enter the basis in order to decrease the cost function. Note that, the problem **SP** is nonconvex, however it is much smaller than the original problem and can be solved very efficiently. Algorithm 1 summarizes the overall RDW algorithm, where $\boldsymbol{\theta}_n$ denotes the n-th row vector in the identity matrix of size $N \times N$.

Algorithm 1: RDW Algorithm

Initialization: A total of $N(2T+1)$ variables are in the basis, which lead to a feasible solution of $\mathbf{M}$. set $i = 0$, and let $\mathbf{B}$, and $\mathbf{c}$ be the initial basis matrix and the corresponding cost vector.

repeat
- $[\boldsymbol{\pi}^{\mathcal{T}}, r_1, r_2, \ldots, r_N]^{\mathcal{T}} \leftarrow \mathbf{c}^{\mathcal{T}} \mathbf{B}^{-1}$
- Solve the subproblems **SP** for all $n \in \mathcal{N}$
- **for** *subproblems with negative optimal solution* **do**
  - add $[(\mathbf{A}_n \mathbf{y}_n^{\ell})^{\mathcal{T}},\ \boldsymbol{\theta}_n]^{\mathcal{T}}$ to the basis matrix,
  - add the cost term associated with the entering variable
- **end**

until *the optimal solutions of all the subproblems are nonnegative.*

Remark (Solving the subproblems)**.** *Each subproblem is a mixed-integer linear program of smaller dimension. In our experiments, off-the-shelf solvers were able to find an optimal solution efficiently.*

V. Experimental Results

In this section, the efficacy of the RDW approach is demonstrated. A single-phase model of the IEEE-37 distribution feeder, depicted in Fig. 1, is used to assess the performance of the developed algorithm. A total of 18 buses were assumed to be equipped with photovoltaic units and storage cells. The prices of active power are obtained from the Midcontinent

Independent System Operator[1] for June, 20, 2017. The simulation are done on an Intel CORE i5 processor computer equipped with 8 GB of RAM.

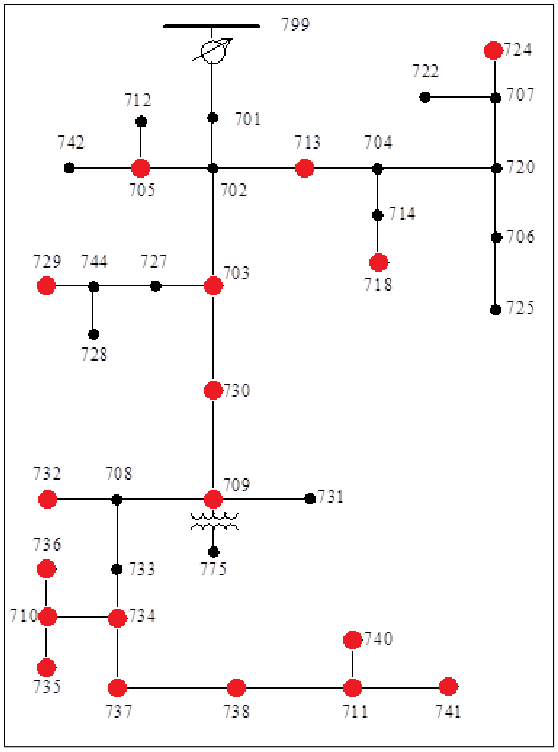

Figure 1. IEEE-37 distribution test feeder (the buses equipped with RES are colored in red).

The implementation of the RDW approach is done by modifying the *dwsolver* optimization package [28][2], which utilizes the GLPK solver [29]. The other approaches are similarly solved using the GLPK solver.

The generation profiles of the PV units are based on the real irradiance data available in [30] which has a granularity of 1 second. The installed batteries are assumed to have a common capacity of 250 kWh, and charging and discharging efficiency of 0.85. The initial state of the batteries is assumed to be 100 kWh for all units. The system is likely to experience overvoltage challenges because of the high PV penetrations. In this case, curtailment of the active power at the PV units is necessary to maintain voltage magnitudes within prescribed limits. A total of 48 optimization intervals of 30 minutes duration are used. In addition, the state of charge of all units at the end of the optimization time (24 hours) is constrained to be equal to the initial state of charge. A maximum of 50 kW charging and discharging rates are allowed for all storage units. For the power available at the PV units, the power is averaged over the each time slot and assumed to be constant during each time slot.

The ability of the proposed approach to yield injection profiles that do not cause over-voltage condition is tested. The voltage profile resulting from injecting all available power at all nodes during some time intervals is compared against the voltage profiles resulting from the RDW algorithm after installing batteries. It is clear that adding batteries to the system increases the system flexibility that can be utilized to minimize the wasted renewable energy from curtailment while respecting the network constraints. In Fig. 2, it can be seen that the system will experience over-voltage conditions if all available renewable energy is injected. However, exploiting the ability to curtail active power prevent such situations. The amount of active power curtailed when storage units are installed in the network is compared with case where no batteries are installed in Fig. 3. The great impact of installing storage units is demonstrated in terms of curtailed (wasted) power, and also in terms of the customer revenues where the cost function (4a) using the RDW is -814.811 while the minimum cost without storage units is -720.95.

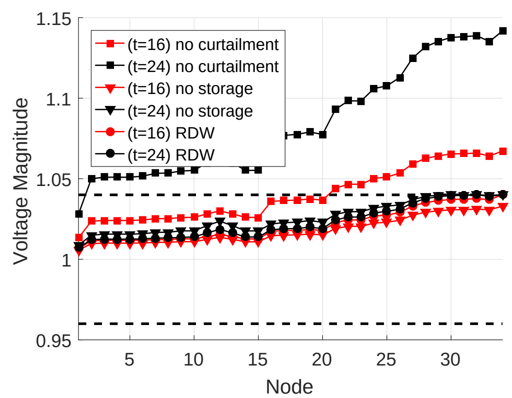

Figure 2. The voltage profile at two time instances using different approaches.

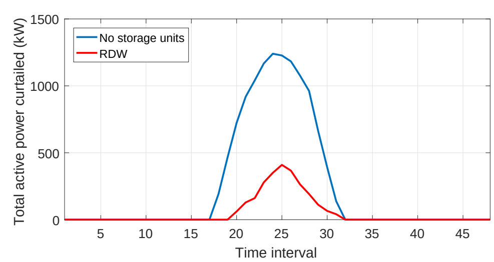

Figure 3. The total active power curtailed with and without storage units installed.

In addition, the solution obtained using the RDW method is compared against the solutions of **P1** and **P2**. In order to solve the mixed-integer linear program in **P1**, the GLPK package is used where a branch and bound algorithm is utilized to find an approximate integer solution of the problem. In order to compare the feasibility of the obtained solutions, we define γ to be a measure of the solution feasibility and is given by

$$\gamma = \|\mathbf{P}_c \circledast \mathbf{P}_d\|_F \tag{18}$$

where $\circledast$ represents an element by element multiplication of the matrices $\mathbf{P}_c, \mathbf{P}_d \in \mathbb{R}^{N \times T}$ which collect the charging

[1] Available at: https://www.misoenergy.org

[2] Available at: https://github.com/alotau/dwsolver

TABLE I
COMPARISON BETWEEN DIFFERENT APPROACHES TO SOLVE THE PROBLEM.

Algorithm	Time (secs)	γ	Cost
P1	252.3	0	−814.811
P2	22.8	1077.6	−814.854
RDW	38.3	0	−814.811

and discharging rates at all buses during all time intervals, respectively. In Table I, the cost function value obtained using different approaches are presented. While solving the exact problem formulation in **P1** gives a feasible solution, it takes a very long time to find the optimal solution. On the other hand, the solution of the relaxed problem **P2** is easy to obtain but is not feasible, i.e., the value of γ is very large. The proposed RDW algorithm can find a solution with a cost that is *very close* to the lower bound obtained from the relaxed problem **P2**. In addition, the value of γ is zero for the solution obtained using the RDW which guarantees its feasibility. In Fig. 4, the state of charge of all batteries in the system is depicted where it is clear that customers exploit different pricing to make a profit from their installed assets.

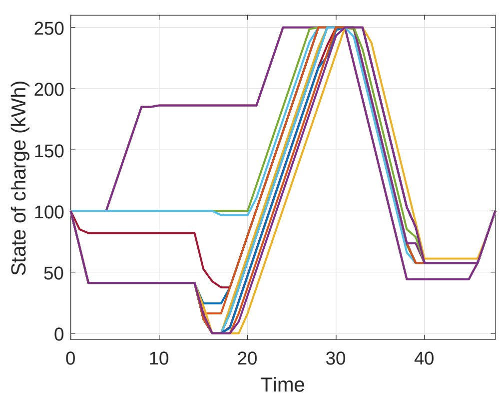

Figure 4. State of charge of the batteries installed in the system.

Finally, the RDW algorithm was used to solve the considered problem with different durations of the time slots δ_t. In Table II, the resulting number of iterations of the RDW decomposition algorithms are listed where it can be noticed that the number of iterations does not grow rapidly when smaller time durations are considered. The cost function is very close to the lower bound (LB) obtained by solving **P2** which yields infeasible solutions for all the considered cases. This shows that the results obtained using the RDW decomposition approach are almost optimal.

VI. CONCLUSIONS

A distributed energy storage management scheme was designed in order to maximize customers objectives and respect the network-wide constraints. Based on the Dantzig-Wolfe decomposition algorithm, a novel idea was put forward that utilizes a linearized flow model to formulate the problem as a mixed-integer linear program. Therefore, the special structure

TABLE II
COMPARISON BETWEEN DIFFERENT TIME SLOT DURATIONS.

δ (mins)	# Slots	# Iter.	Cost	LB
10	144	268	−814.4908	−814.5160
15	96	163	−814.5250	−814.5659
20	72	112	−814.5580	−814.6000
30	48	71	−814.8110	−814.8542
45	32	45	−814.2786	−814.3504
60	24	32	−815.2133	−815.2846

of the problem was used to devise the so-called *relaxed Dantzig-Wolfe* algorithm. The procedure respects the privacy of the customers, and provides solutions that are very close to optimality. The merits of the proposed algorithm were demonstrated using a single-phase model of the IEEE-37 bus feeder where PV units and batteries are installed. The use of the proposed approach can be extended to other optimization problems where only discrete control actions are allowed.

REFERENCES

[1] E. Dall'Anese and A. Simonetto, "Optimal power flow pursuit," *IEEE Transactions on Smart Grid*, vol. PP, no. 99, pp. 1–1, 2016.

[2] X. Zhou, E. Dall'Anese, L. Chen, and A. Simonetto, "An incentive-based online optimization framework for distribution grids," *arXiv preprint arXiv:1705.01482*, 2017.

[3] K. Baker, J. Guo, G. Hug, and X. Li, "Distributed mpc for efficient coordination of storage and renewable energy sources across control areas," *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 992–1001, March 2016.

[4] Y. M. Atwa and E. El-Saadany, "Optimal allocation of ess in distribution systems with a high penetration of wind energy," *IEEE Transactions on Power Systems*, vol. 25, no. 4, pp. 1815–1822, 2010.

[5] A. Gabash and P. Li, "Active-reactive optimal power flow in distribution networks with embedded generation and battery storage," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 2026–2035, 2012.

[6] M. Ghofrani, A. Arabali, M. Etezadi-Amoli, and M. S. Fadali, "A framework for optimal placement of energy storage units within a power system with high wind penetration," *IEEE Transactions on Sustainable Energy*, vol. 4, no. 2, pp. 434–442, 2013.

[7] S. Wogrin and D. F. Gayme, "Optimizing storage siting, sizing, and technology portfolios in transmission-constrained networks," *IEEE Transactions on Power Systems*, vol. 30, no. 6, pp. 3304–3313, 2015.

[8] K. Baker, J. Guo, G. Hug, and X. Li, "Distributed mpc for efficient coordination of storage and renewable energy sources across control areas," *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 992–1001, 2016.

[9] A. Castillo and D. Gayme, "Evaluating the effects of real power losses in optimal power flow based storage integration," *IEEE Transactions on Control of Network Systems*, 2017.

[10] D. Gayme and U. Topcu, "Optimal power flow with large-scale storage integration," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 709–717, 2013.

[11] K. M. Chandy, S. H. Low, U. Topcu, and H. Xu, "A simple optimal power flow model with energy storage," in *Decision and Control (CDC), 2010 49th IEEE Conference on*. IEEE, 2010, pp. 1051–1057.

[12] K. Christakou, J.-Y. Le Boudec, M. Paolone, and D.-C. Tomozei, "Efficient Computation of Sensitivity Coefficients of Node Voltages and Line Currents in Unbalanced Radial Electrical Distribution Networks," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 741–750, 2013.

[13] A. Bernstein and E. Dall'Anese, "Linear power-flow models in multiphase distribution networks," in *The 7th IEEE Intl. Conf. on Innovative Smart Grid Technologies*, Sep. 2017.

[14] J. Bank and J. Hambrick, "Development of a high resolution, real time, distribution-level metering system and associated visualization, modeling, and data analysis functions," National Renewable Energy Laboratory (NREL), Golden, CO., Tech. Rep., 2013.

[15] W. Braff, J. M. Mueller, and J. E. Trancik, "Value of storage technologies for wind and solar energy," *Nature Climate Change*, vol. 6, no. 10, pp. 964–969, Oct. 2016.

[16] H. Pandzic, Y. Wang, T. Qiu, Y. Dvorkin, and D. S. Kirschen, "Near-optimal method for siting and sizing of distributed storage in a transmission network," *IEEE Transactions Power Syst.*, vol. 30, no. 5, pp. 2288-2300, Sep. 2015.
[17] D. Gayme and U. Topcu, "Optimal power flow with large-scale storage integration," *IEEE Transactions Power Systems*, vol. 28, no. 2, pp. 709-717, May 2013.
[18] Y. V. Makarov, P. Du, M. C. W. Kintner-Meyer, C. Jin, and H. F. Illian, "Sizing energy storage to accommodate high penetration of variable energy resources," *IEEE Transactions Sust. Energy*, vol. 3, no. 1, pp. 34-40, Jan. 2012.
[19] B. M. Grainger, G. F. Reed, A. R. Sparacino and P. T. Lewis, "Power Electronics for Grid-Scale Energy Storage," *Proceedings of IEEE*, vol. 102, no. 6, pp. 1000–1013, June 2014.
[20] J. P. Barton and D. G. Infield, "Energy storage and its use with intermittent renewable energy," *IEEE Transactions on Energy Conversion*, vol. 19, no. 2, pp. 441-448, June 2004.
[21] L. S. Vargas, G. Bustos-Turu, and F. Larrain, "Wind power curtailment and energy storage in transmission congestion management considering power plants ramp rates," *IEEE Transactions Power Systems*, vol. 30, no. 5, pp. 2498-2506, Sep. 2015.
[22] S. Maharjan, Q. Zhu, Y. Zhang, S. Gjessing, and T. Basar, "Dependable demand response management in the smart grid: A Stackelberg game approach," *IEEE Transactions on Smart Grid*, vol. 4, no. 1, pp. 120-132, Feb. 2013.
[23] A. Mohsenian-Rad, V. W. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE Transactions on Smart Grid*, vol. 1, no. 3, pp. 320-331, Nov. 2010.
[24] D. Bienstock, C. Matke, G. Munoz, and S. Yang, "Robust linear control of nonconvex battery operation in transmission systems." *arXiv preprint* arXiv:1610.09432, 2016.
[25] S. S. Guggilam, E. Dall'Anese, Y. C. Chen, S. V. Dhople and G. B. Giannakis, "Scalable Optimization Methods for Distribution Networks With High PV Integration," *IEEE Transactions on Smart Grid*, vol. 7, no. 4, pp. 2061-2070, July 2016.
[26] G. B. Dantzig , and P. Wolfe, "Decomposition principle for linear programs," *Operations research*, vol. 8, no. 1, pp. 101-111, Feb. 1960.
[27] F. Vanderbeck, and M. Savelsbergh, "A generic view of DantzigWolfe decomposition in mixed integer programming," *Operations Research Letters*, vol. 34, no. 3, pp. 296-306, May 2006.
[28] J. Rios, and K. Ross, "Massively Parallel Dantzig-Wolfe Decomposition Applied to Traffic Flow Scheduling," *Journal of Aerospace Computing, Information, and Communication*, vol. 7, no. 1, pp. 32-45, Jan 2010.
[29] Makhorin, Andrew, "GLPK (GNU linear programming kit)," 2008.
[30] J. Bank and J. Hambrick, "Development of a High Resolution, Real Time, Distribution-level Metering System and Associated Visualization, Modeling, and Data Analysis Functions," *National Renewable Energy Laboratory* (NREL), Golden, CO, 2013.