Degrees of Freedom for a two-cell relay network with soft handoffs

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***Abstract*—In this paper we investigate the degrees of freedom of a cellular relay network that consists of two base stations, two mobile stations and four decode-and-forward relays. The base stations and the mobile stations are equipped with M antennas each, whereas the relays are equipped with N antennas each. In addition, each base station has an independent message to each mobile station. The relays are used to froward the messages from the base stations to the mobile station as there is no direct link. We consider three different relaying architectures where the two relays associated with each base station simultaneously or alternately transmit their messages. We derive an upper bound on the degrees of freedom achievable by each relaying architecture as a function of the ratio between N and M. Furthermore, we propose an achievable scheme that uses interference alignment to achieve the upper bound on the DoF for all values of M and N except for $1 \leq \frac{N}{M} \leq \frac{5}{2}$.**

I. Introduction

Relaying is a promising technique for wireless cellular systems that can enhance the overall throughput, save the transmission power, and suppress the interference at cell edges [1]. Due to the half-duplex nature of network components, the degrees of freedom (DoF) or capacity pre-log factor of a relay channel are reduced by a factor of half. In order to compensate for this loss, alternate relaying has been proposed as in [2]–[6].

Throughout this paper we investigate the spatial DoF of the network, which can be defined as

$$\eta = \lim_{\rho \to \infty} \frac{C(\rho)}{\log(\rho)} \tag{1}$$

where ρ is signal-to-noise ratio (SNR) and $C(\rho)$ is the system capacity at SNR ρ [7]. The spatial DoF usually represent the total number of interference-free streams that can be transmitted from the sources to the destinations.

The DoF of wireless relaying networks have been studied in [3]–[6]. In [3], a relay-aided interference alignment and cancellation scheme was proposed for the single-antenna two-user X-channel, where the relay is equipped with 2 antennas. The proposed scheme achieves $\frac{4}{3}$ DoF. The MIMO case was considered but for only one user in [4]–[6]. In [4], [5], a network with one source, one destination and multiple amplify-and-forward (AF) relays equipped with M antennas was studied where alternate relaying was used to compensate for the loss of DoF due to half-duplex relaying. In [4], three AF relays were used to achieve $\frac{3M}{4}$ DoF compared with $\frac{M}{2}$ DoF in conventional relaying. In addition, it was shown in [5] that M DoF can be achieved using six AF relays. In [6], a cellular relaying network, that consists of two source-destination pairs, and four decode-and-forward relays where each node is equipped with N antennas, was considered. In this work, N DoF were achieved using simultaneous and alternate relaying transmission schemes. However, the case where the relays and source-destination pairs have different number of antennas was not considered.

In this paper, we consider a network consisting of two base stations and two mobile stations, where each node is equipped with M antennas. Each base station has a distinct message to each mobile station. Note that this scenario can represent a soft hand-off situation in a wireless cellular network where the mobile stations on the cell edge are attached to two base stations simultaneously. Each base station has two dedicated half-duplex decode-and-forward relays equipped with N antennas each. We assume that all relays can receive signals from all base stations, however, they can not decode the received signal from the other base station, e.g., they have access to the codebook of their designated base station only. The two relays associated with each base station can transmit either simultaneously or alternately. In the simultaneous transmission case the two relays receive in one time slot and transmit in the second. In contrast, in the alternate transmission case, one of the two relays receives while the other transmits. We consider all possible relaying architectures for the network in which the relays associated with each base station simultaneously or alternately transmit. For each relaying architecture, we derive an upper bound on the DoF of the network as a function of M and N. We also propose an achievable scheme that employs interference alignment/avoidance to deliver the messages from the base stations to mobile stations via the relays. The proposed scheme employs different relaying architectures based on the ratio between N and M. We show that the proposed scheme achieves the obtained upper bound on the DoF for all values of M and N except when $1 \leq \frac{N}{M} \leq \frac{5}{2}$.

The remainder of this paper is organized as follows: In Section II, we describe the system model. Next, we derive

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an outer bound on the DoF of different transmission schemes in Section III. An achievable schemes is proposed to seek the DoF upper bounds in Section IV. Finally, the paper is concluded in section V.

The following notation is used throughout the manuscript. We employ upper and lower case boldfaces to denote matrices and vectors, respectively. For any general matrix $\mathbf{X}$, $\mathbf{X^{-1}}$, $\mathbf{X^H}$, $\mathbf{X}^{\dagger}$, $\mathcal{S}\{\mathbf{X}\}$ and $\mathcal{N}\{\mathbf{X}\}$ denote the inverse, the hermitian transpose, the pseudo inverse, the span, and the null space of $\mathbf{X}$, respectively.

II. System Model

We consider a two-cell relaying network consisting of two base stations, two mobile stations, and four relays denoted respectively by B_i, M_i and R_i^j where i and $j \in 1, 2$ and R_i^j denotes the i^{th} relay dedicated to the j^{th} base station. We assume that no direct link exists between the mobile stations and the base stations as in [4], [8]. Each base station has two independent messages; one for each mobile station, that are transmitted through its two dedicated relays. The base stations and the mobile stations are equipped with M antennas, whereas the relays are half-duplex, decode-and-forward and equipped with N antennas each.

The channel between any two nodes is modelled as a block fading channel and is constant during one time slot. We denote the channel matrix from node X to node Y by $\boldsymbol{H}_{YX} \in \mathbb{C}^{N_Y \times N_X}$, where N_X denotes the number of antennas at node X. In addition, all channel matrices are assumed to be independent and identically distributed, thus they are full rank. Moreover, perfect channel state information is assumed to be available at all nodes.

Since the relays are half-duplex, communication between the base stations and the mobile stations occurs in two hops where each relay decodes its received signal before retransmitting it in the subsequent time slot. Since each base station has two dedicated relays, there are only two possible ways for operating the relays; simultaneous and alternate relaying. In simultaneous relaying, the base station transmits to the two relays in one time slot while the relays transmit to the mobile stations in the next time slot. In contrast, in alternate relaying, each time slot the base station transmits to one of its relays while the second relay forwards the message it received from the base station in the previous time slot to the mobile stations. Considering now the two-cell network, there are four different combinations of choices to operate the relay at each cell. We denote these choices by the number of receiving-transmitting relays in one time slot. In the 4-0 relaying scheme, the two base stations use simultaneous relaying and transmit simultaneously to the relays in one time slot while all the relays transmit simultaneously to the base stations in the second time slot. In contrast, in the 2-2 relaying scheme, the two base stations use alternate relaying where in each time slot each base station transmits to one of its dedicated relays while the other relay transmits to the mobile users. Finally, there are two combinations that correspond to the 3-1 relaying scheme, where one base station employs alternate relaying while the second employs simultaneous relaying. From the symmetry of our network, it suffices to consider only one of these two combinations. Hence, a total of three relaying schemes will be considered.

III. Upper Bounds on The Degrees of Freedom

A. Case 1: 2-2 Relaying

In this scheme, the two base stations employ alternate relaying. Let the k^{th} base station, $k \in \{1, 2\}$, transmits in the first time slot to the relay R_1^k while the other relays, namely R_2^k, transmit at simultaneously to the mobile stations. In the second time slot, the k^{th} base station transmits to relay R_2^k while the relays R_1^k forward the message received in the previous time slot to the mobile stations.

Theorem 1. *The total number of DoF of the 2-2 relaying scheme is upper bounded by*

$$\eta_{2-2} \leq \min\{N, 2M\} \tag{2}$$

Proof. Since the system is symmetric, we consider the first time slot only. Let us allow cooperation between the two base stations B_1 and B_2, the two receiving relays R_1^1 and R_1^2, the two transmitting relays R_2^1 and R_2^2 and the two mobile stations M_1 and M_2. This cooperation can not decrease the DoF. After cooperation, we obtain a Z-Channel with two sources (T_1; the cooperating base stations, and T_2; the cooperating transmitting relays), and 2 receivers (S_1; the cooperating receiving relays, and S_2; the cooperating mobile stations). Let $d_{i,j}$ denotes the number of non-interfering streams transmitted from T_i to S_j. From [9], the outer bound on the DoF of this Z-Channel is given by $\eta_z \leq \max N_{S_1}, N_{T_2}$ where $\eta_z = d_{1,1} + d_{2,1} + d_{2,2}$ and N_X denotes the number of antennas at node X. Hence,

$$d_{1,1} + d_{2,1} + d_{2,2} \leq 2N \tag{3}$$

As there are no intended messages from the transmitting relays to the receiving ones, we can set $d_{2,1}$ to be zero. Note that $d_{1,1}$ and $d_{2,2}$ represent the DoF of the first and second hops of transmission respectively. As the transmitted streams from the relays to the mobile stations in one time slot are those transmitted from the base stations to the relays in the previous time slot, the achievable DoF of the 2-2 relaying system is bounded by the minimum of $d_{1,1}$ and $d_{2,2}$. As a result, the DoF of the 2-2 alternate transmission in bounded by

$$\eta_{2-2} \leq N \tag{4}$$

Next, we consider the second hop where the relays R_2^1 and R_2^2 transmit at the same time to the mobile stations. Since each relay has a message to each mobile station, the second hop transmission can be considered as a two-user MIMO X-Channel, i.e., two transmitters equipped with M antennas each and two receivers equipped with N antennas each, where each transmitter has two independent messages for each receiver. Since the DoF of the 2-2 relaying scheme are less than or equal to the DoF achieved in the second hop, we have

$$\eta_{2-2} \leq \min\{2M, 2N, \frac{4}{3}\max\{M, N\}\} \tag{5}$$

where we have utilized the outer bound on the DoF of the two-user X-Channel in [9]. Using (5) and (4), we can get (2). □

B. Case 2: 3-1 Relaying

In this scheme, one base station employs alternate relaying while the second employs simultaneous relaying. For the sake of symmetry, we assume that each base station switches between alternate and simultaneous relaying every two time slots. In particular, one complete cycle consists of four time slots. In the first time slot, B_1 transmits to R_1^1 and R_2^1 while B_2 transmits to R_1^2. At the same time, R_2^2 transmits to the mobile stations. In the second time slot, R_1^1, R_2^1 and R_1^2 transmit their signals to the mobile stations while B_2 transmits to R_2^2. In the third time slot, B_1 transmits to R_1^1 while B_2 transmits to R_1^2 and R_2^2 as R_2^1 transmits to mobile stations. In the fourth time slot, B_1 transmits to R_2^1 while R_1^1, R_1^2 and R_2^2 transmit to the mobile stations.

Theorem 2. *The total number of DoF of the 3-1 relaying scheme is upper bounded by*

$$\eta_{3-1} \leq \min\left\{\frac{3N}{2}, \frac{N+M}{2}, \frac{3M}{2}, \max\left\{\frac{5N}{6}, \frac{N}{2}+\frac{M}{3}\right\}\right\} \tag{6}$$

Proof. Let $d_{i,j}^k$ denotes the number of interference-free streams transmitted from B_i in the k^{th} time slot and intended for M_j. Due to symmetry, we need to consider only the first and second time slots. Since the two base stations transmit simultaneously in the first time slot while only B_2 transmits in the second time slot, we have

$$\eta_{3-1} \leq \frac{1}{2}\left(d_{1,1}^1 + d_{1,2}^1 + d_{2,1}^1 + d_{2,2}^1 + d_{2,1}^2 + d_{2,2}^2\right) \tag{7}$$

where the inequality results from considering one hop only and the factor $\frac{1}{2}$ is due to considering two time slots.

In the first time slot, let us allow full cooperation between terminals R_1^1 and R_2^1. Furthermore, let us remove the interfering signal transmitted from R_2^2. As result, we have a 2-user MIMO interference channel (IC) with 2 sources ($T_1 = B_1$ and $T_2 = B_2$) and 2 destinations (S_1; the co-operating relays, and $S_2 = R_1^2$), In [7], the DoF of the MIMO-IC was shown to be $\eta_{IC} = \min\{N_{T_1} + N_{T_2}, N_{S_1} + N_{S_2}, \max\{N_{T_1}, N_{S_2}\}, \max\{N_{T_2}, N_{S_1}\}\}$. Applying this result on the resulting IC we get

$$d_{1,1}^1 + d_{1,2}^1 + d_{2,1}^1 + d_{2,2}^1 \leq \min\{2M, 3N, \max\{M, N\}\} \tag{8}$$

where the inequality is due to ignoring the interference from R_2^2. Next, let us consider the transmitted signal from the R_2^2 in the first time slot. This signal represents an interference at the receiving relays. The interference signal spans $d_{2,1}^2 + d_{2,2}^2$ dimensions at each receiving relay. As a result, the receiving relays, (R_1^1, R_2^1, and, R_1^2), can not decode more than $N - d_{2,1}^2 + d_{2,2}^2$ streams. Therefore, the number of the transmitted streams is bounded by

$$\tau(d_{1,1}^1 + d_{1,2}^1) + d_{2,1}^2 + d_{2,2}^2 \ \leq \ N \tag{9}$$

$$(1-\tau)(d_{1,1}^1 + d_{1,2}^1) + d_{2,1}^2 + d_{2,2}^2 \ \leq \ N \tag{10}$$

$$d_{2,1}^1 + d_{2,2}^1 + d_{2,1}^2 + d_{2,2}^2 \ \leq \ N \tag{11}$$

where τ is the fraction of $d_{2,1}^1 + d_{2,2}^1$ streams decoded and forwarded by R_1^1.

In the second time slot, let us consider the transmission between B_2 and R_2^2 is a point to point transmission by removing all interfering signals from other relays, then the outer bound for $d_{2,1}^2 + d_{2,2}^2$ is given by

$$d_{2,1}^2 + d_{2,2}^2 \leq \min(N, M) \tag{12}$$

Next, We define $D_1^1 = d_{1,1}^1 + d_{1,2}^1$, $D_2^1 = d_{2,1}^1 + d_{2,2}^1$ and $D_2^2 = d_{2,1}^2 + d_{2,2}^2$. By adding (9), (10), and (11) we obtain

$$D_1^1 + D_2^1 + 3D_2^2 \leq 3N \tag{13}$$

On the other hand, from (8), $D_1^1 + D_2^1$ is bounded by

$$D_1^1 + D_2^1 \leq \min\{2M, 3N, \max\{M, N\}\} \tag{14}$$

and from (12), D_2^2 is bounded by

$$D_2^2 \leq \min\{N, M\} \tag{15}$$

The inequalities in (13)–(15) describe a polyhedron in $\mathbb{R}^3$. Adding (14) to (15) we get

$$D_1^1 + D_2^1 + D_2^2 \leq \min\{2M, 3N, \max\{M, N\}\} + \min\{N, M\} \tag{16}$$

on the other hand, multiplying (13) by $\frac{1}{3}$ and adding it to (14) multiplied by $\frac{2}{3}$ we can get

$$D_1^1 + D_2^1 + D_2^2 \leq \frac{2}{3}(\min\{2M, 3N, \max\{M, N\}\}) + N \tag{17}$$

As a result, $D_1^1 + D_2^1 + D_2^2$ is upper bounded by the minimum of two boundaries in (16) and (17), which we can find in the following cases, depending on the values of N and M

$$D_1^1 + D_2^1 + D_2^2 \leq \begin{cases} 3N & \text{for } N \leq \frac{M}{3} \\ N + \frac{2M}{3} & \text{for } \frac{M}{3} \leq N \leq M \\ \frac{5N}{3} & \text{for } M \leq N \leq \frac{3M}{2} \\ M + N & \text{for } \frac{3M}{2} \leq N \leq 2M \\ 3M & \text{for } N \geq 2M \end{cases} \tag{18}$$

By substituting with the above upper bound on $D_1^1 + D_2^1 + D_2^2$ in (7), we can get (6).

□

C. Case 3: 4-0 Relaying

In this case, both base stations transmit to their relays simultaneously, i.e., in the first hop B_k transmits to R_k^j where $k, j \in \{1, 2\}$ and in the second hop the relays transmit to the mobile stations.

Theorem 3. *The total number of degrees of freedom of the 4-0 relaying scheme is upper bounded by*

$$\eta_{4-0} \leq \min\{2N, M, \frac{1}{2}\max\{M, 2N\}\} \tag{19}$$

Proof. The total DoF of the 4-0 relaying scheme is upper

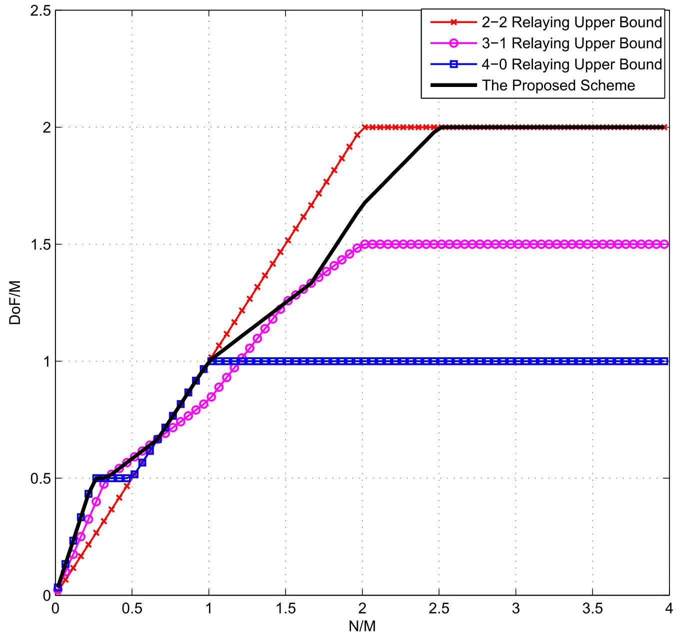

Fig. 1: Outer bounds for alternate relaying schemes and the achievable DoF using the proposed scheme.

bounded by the minimum of the achievable DoF of two hops. In the first hop, let us we allow full cooperation between R_1^1 and R_2^1 and between R_1^2 and R_2^2. As result, we obtain an IC with two transmitters and two receivers. Each transmitter is equipped with M antennas while the receivers have $2N$ antennas each. From [7], the outer bound of DoF of 4-0 relaying scheme is given by (19). □

IV. Proposed Achievable Scheme

Fig. 1 shows the upper bound on the DoF region of the network for different relay architectures versus the ratio $\frac{N}{M}$. We can see from this figure that the 4-0 relaying scheme can provide higher DoF at small values of $\frac{N}{M}$ while the 2-2 relaying scheme is superior at high values of $\frac{N}{M}$. As a result, the proposed achievable scheme will depend on the ratio between N and M.

A. Achievable Scheme for $N \leq \frac{M}{3}$

From the previous section, the outer bound for the DoF of the 4-0 relaying scheme surpasses the other outer bounds for $N \leq \frac{M}{3}$. Hence, the 4-0 relaying scheme will be used for this region. The outer bound for the DoF in this region is given by $\eta_{4-0} \leq \min\{2N, \frac{M}{2}\}$. The proposed scheme in this region is based on zero-forcing. Each base station sends $2d$ symbols to its dedicated relays in the first time slot. Each relay decodes d symbols and then transmits them to the two mobile stations in the second time slot. The transmitted signal from the k^{th} base station is given by

$$\boldsymbol{x}_{B_k} = \boldsymbol{W}_{R_1^k B_k} \boldsymbol{x}_{R_1^k B_k} + \boldsymbol{W}_{R_2^k B_k} \boldsymbol{x}_{R_2^k B_k} \qquad k \in \{1,2\} \quad (20)$$

where $\boldsymbol{W}_{YX} \in \mathbb{C}^{N_x \times d}$ denotes the linear precoding matrix employed by terminal X with N_x antennas to transmit to terminal Y and $\boldsymbol{x}_{YX} \in \mathbb{C}^d$ is the the transmitted symbol vector. The received signal at the relay R_i^j is given by

$$\boldsymbol{y}_{R_i^j} = \boldsymbol{H}_{R_i^j B_1} \boldsymbol{x}_{B_1} + \boldsymbol{H}_{R_i^j B_2} \boldsymbol{x}_{B_2} + \boldsymbol{z}_{R_i^j} \qquad i,j \in \{1,2\} \quad (21)$$

where $\boldsymbol{z}_X \in \mathbb{C}^{N_x}$ is the received noise vector at terminal X.

In the second time slot, the transmitted signals from relay R_i^j is given by

$$\boldsymbol{x}_{R_i^j} = \boldsymbol{W}_{M_1 R_i^j} \boldsymbol{x}_{M_1 R_i^j} + \boldsymbol{W}_{M_2 R_i^j} \boldsymbol{x}_{M_2 R_i^j} \qquad i,j \in \{1,2\} \quad (22)$$

where $\boldsymbol{W}_{M_n R_i^j} \in \mathbb{C}^{N \times d_n}$ for $n \in \{1,2\}$ and d_n denotes the number of streams transmitted from the relay to M_n. Note that $d_1 + d_2 = d$. Thus, each relay distributes its received/transmitted between the two mobile users. Therefore, the received signal at mobile station M_n in the second time slot is given by

$$\boldsymbol{y}_{M_n} = \sum_{j=1}^{2} \sum_{i=1}^{2} \boldsymbol{H}_{M_n R_i^j} \boldsymbol{x}_{R_i^j} + \boldsymbol{z}_{M_n} \qquad n = \{1,2\} \quad (23)$$

For $N \leq \frac{M}{4}$, the outer bound for DoF region is $\eta_{4-0} < 2N$. Therefore, in the proposed scheme, the precoder matrices $\boldsymbol{W}_{R_i^j B_k}$ where $i,j \in \{1,2\}$, are chosen to be of dimension N to achieve the outer bound, i.e., $d = N$. The precoder matrices at B_1 are chosen such that

$$\mathcal{S}\{\boldsymbol{W}_{R_1^1 B_1}\} \subseteq \left\{\mathcal{N}\{\boldsymbol{H}_{R_2^1 B_1}\} \cap \mathcal{N}\{\boldsymbol{H}_{R_1^2 B_1}\} \cap \mathcal{N}\{\boldsymbol{H}_{R_2^2 B_1}\}\right\} \quad (24)$$

$$\mathcal{S}\{\boldsymbol{W}_{R_1^2 B_1}\} \subseteq \left\{\mathcal{N}\{\boldsymbol{H}_{R_1^1 B_1}\} \cap \mathcal{N}\{\boldsymbol{H}_{R_1^2 B_1}\} \cap \mathcal{N}\{\boldsymbol{H}_{R_2^2 B_1}\}\right\} \quad (25)$$

Since all the channel matrices are full rank, $\boldsymbol{W}_{R_i^j B_k} \in \mathbb{C}^{M \times N}$ can be found since the dimension of the intersection between the three nullspaces in each of the above two equations is $M - 3N$ which is always greater than or equal to N.

Similarly, we can choose $\boldsymbol{W}_{R_1^2 B_2}$ and $\boldsymbol{W}_{R_2^2 B_2}$ to lie in the intersection of the nullspaces of the channel matrices to the nontarget relays. As a result, the received signal at R_1^1 in (21) will be

$$\boldsymbol{y}_{R_1^1} = \boldsymbol{H}_{R_1^1 B_1} \boldsymbol{W}_{R_1^1 B_1} \boldsymbol{x}_{R_1^1 B_1} + \boldsymbol{z}_{R_1^1} \quad (26)$$

Since $\boldsymbol{H}_{R_1^1 B_1} \boldsymbol{W}_{R_1^1 B_1} \in \mathbb{C}^{N \times N}$ is full rank as can be derived easily from Lemma 2 in [10], the relay can decode N streams from the received signal. The same can be applied to other relays.

For the second hop, the received signal at each mobile station has a dimension of $4N$ which is less than or equal to M. Hence, the mobile stations can decode $4N$ streams perfectly. The decoding matrices at the n^{th} mobile station, $\boldsymbol{U}_{M_n} \in \mathbb{C}^{M \times 4d_n}$, where $d_1 + d_2 = N$, will be designed such that

$$\boldsymbol{U}_{M_n} = \mathcal{N}\left\{\left[\boldsymbol{J}_{n,m}^{(1,1)}, \ \boldsymbol{J}_{n,m}^{(2,1)}, \ \boldsymbol{J}_{n,m}^{(1,2)}, \ \boldsymbol{J}_{n,m}^{(2,2)}\right]\right\} \quad (27)$$

where $\boldsymbol{J}_{n,m}^{(i,j)} = \boldsymbol{H}_{M_n R_i^j} \boldsymbol{W}_{M_m R_i^j}$ is the subspace of containing the interference received at the n^{th} mobile user due to the transmission of the relay R_i^j and intended for mobile user m and $n, m \in \{1, 2\}, n \neq m$.

For $\frac{M}{4} \leq N \leq \frac{M}{3}$, we can achieve $\frac{M}{2}$ DoF by simply powering-off $(N - \frac{M}{4})$ antennas at the relays and then applying the same scheme with $N = \frac{M}{4}$.

B. Achievable Scheme for $\frac{M}{3} \leq N \leq \frac{2M}{3}$

When $\frac{M}{3} \leq N \leq \frac{2M}{3}$, the DoF for the 3-1 relaying scheme is bounded by $\eta_{3-1} \leq \frac{M}{3} + \frac{N}{2}$ which surpasses the DoF outer bound of the 4-0 and 2-2 relaying schemes. Consequently, the proposed scheme will use 3-1 relaying in this region. Due to symmetry, we will focus only on the first and second time slots. In the first time slot, R_1^1, R_2^1 and R_1^2 will receive from their associated base stations, while R_2^2 transmits to the mobile stations.

In the first time slot, let R_2^2 pick randomly its precoding matrix as $\boldsymbol{W}_{R_2^2} = [\boldsymbol{W}_{M_1 R_2^2} \quad \boldsymbol{W}_{M_2 R_2^2}] \in \mathbb{C}^{N \times (N - \frac{M}{3})}$. Therefore, the transmitted signal from R_2^2 is given by

$$\boldsymbol{x}_{R_2^2} = \boldsymbol{W}_{M_1 R_2^2}\, \boldsymbol{x}_{M_1 R_2^2} + \boldsymbol{W}_{M_2 R_2^2}\, \boldsymbol{x}_{M_2 R_2^2} \tag{28}$$

The received signal at each mobile station has a dimension of $N - \frac{M}{3}$ which is less than or equal to M. Therefore, the mobile stations can decode $N - \frac{M}{3}$ streams perfectly. The decoding matrices at the n^{th} mobile station, $\boldsymbol{U}_{M_n} \in \mathbb{C}^{M \times d_n}$, where $d_1 + d_2 = N - \frac{M}{3}$, are designed such that

$$\boldsymbol{U}_{M_n} \subseteq \mathcal{N}\{\boldsymbol{H}_{M_n R_2^2}\, \boldsymbol{W}_{M_m R_2^2}\} \quad m, n \in \{1, 2\}, n \neq m \tag{29}$$

and hence, the two mobile stations can decode a total of $N - \frac{M}{3}$ streams.

Note that the signal transmitted by R_2^2 is considered as interference at the other receiving relays. Let

$$\boldsymbol{I}_{R_n^m} = \boldsymbol{H}_{R_n^m R_2^2} \boldsymbol{W}_{R_2^2} \quad (n, m) \in \{(1,1), (1,2), (2,1)\} \tag{30}$$

denote the interference subspace at R_n^m due to the transmission of R_2^2. We define the matrix $\boldsymbol{N}_{B_j R_n^m}$ which combines the null space of the channel from B_j to R_n^m and the subspace $\boldsymbol{H}^{\dagger}_{B_j R_n^m} \boldsymbol{I}_{R_n^m}$ that represents the image of the interference subspace of R_n^m at B_j, i.e.,

$$\boldsymbol{N}_{B_j R_n^m} = (\boldsymbol{H}^{\dagger}_{R_n^m B_j} \boldsymbol{I}_{R_n^m}) \cup \mathcal{N}\{\boldsymbol{H}_{R_n^m B_j}\} \tag{31}$$

Hence, the transmission from B_j using a precoding matrix that belongs to the subspace $\boldsymbol{N}_{B_j R_n^m}$ lies entirely in the interference subspace of R_n^m. Note that the dimension of $\boldsymbol{N}_{B_j R_n^m}$ is given by $N - \frac{M}{3} + M - N = \frac{2M}{3}$.

At the base stations, we direct the transmitted signal such that it lies in the interference subspace of the unintended relays, i.e., we design $\boldsymbol{W}_{R_i^j B_j} \in \mathbb{C}^{M \times \frac{N}{3}}$ such that

$$\boldsymbol{W}_{R_i^j B_j} = \bigcap_{\forall (n,m) \neq (i,j)} \boldsymbol{N}_{B_j R_n^m} \tag{32}$$

where (i, j) and $(n, m) \in \{(1,1), (1,2), (2,1)\}$ in (31) and (32). Since the dimension of $\boldsymbol{N}_{B_j R_n^m}$ is given by $\frac{2M}{3}$ and the dimension of the union of any two of them is M, then the dimension of $\boldsymbol{W}_{R_i^j B_j}$ is given by $\frac{M}{3}$.

Since the interference signals at the receiving relays are already aligned to lie in null spaces of the channel matrices or at its interference subspaces, the decoding matrices at the receiving relays, $\boldsymbol{U}_{R_i^j} \in \mathbb{C}^{N \times \frac{M}{3}}$, are designed such that

$$\boldsymbol{U}_{R_i^j} \subseteq \mathcal{N}\{\boldsymbol{I}_{R_i^j}\} \tag{33}$$

Hence, each of R_1^1, R_2^1, and R_1^2 can decode $\frac{M}{3}$ interference-free streams.

In the second time slot, only R_2^2 receives from B_2 while the other relays transmit to the mobile stations. In this case, B_2 picks randomly the precoding matrix $\boldsymbol{W}_{R_2^2 B_2} \in \mathbb{C}^{M \times N - \frac{M}{3}}$. Therefore, the signal space at R_2^2 is $\boldsymbol{H}_{R_2^2 B_2} \boldsymbol{W}_{R_2^2 B_2}$ whose dimension is $N - \frac{M}{3}$ and the dimension of the available interference subspace at R_2^2 is $\frac{M}{3}$. The transmitting relays aim to align their signal at the interference subspace of R_2^2. Accordingly, the precoding matrices at transmitting relays, $\boldsymbol{W}_{R_i^j} \in \mathbb{C}^{N \times \frac{M}{3}}$ where $(i, j) \in \{(1,1), (2,1), (1,2)\}$, must satisfy the following condition

$$\boldsymbol{H}_{R_2^2 R_1^1} \boldsymbol{W}_{R_1^1} = \boldsymbol{H}_{R_2^2 R_1^2} \boldsymbol{W}_{R_1^2} = \boldsymbol{H}_{R_2^2 R_2^1} \boldsymbol{W}_{R_2^1} \tag{34}$$

In addition, the decoding matrix at R_2^2 is designed basically to cancel the interference from the other relays, which is already aligned to a subspace of dimension $\frac{M}{3}$ as follows

$$\boldsymbol{U}_{R_2^2} \subseteq \mathcal{N}\{\boldsymbol{H}_{R_2^2 R_1^1} \boldsymbol{W}_{R_1^1}\} \tag{35}$$

At each mobile station, the dimension of the received signal is M as each transmitting relay uses $\frac{M}{3}$ dimensions to transmit its messages. Therefore, these M streams can be decoded perfectly at each mobile station. The decoder of each mobile station can extract its intended message by zero-forcing the unintended ones.

In summary, using the proposed scheme, the mobile users can decode $N - \frac{M}{3}$ streams in the first time slot and M streams in the second one via the relays without any interference. Thus, the proposed scheme achieves $\frac{M}{3} + \frac{N}{2}$ DoF, and hence, it is DoF-optimal.

C. Achievable Scheme for $N \geq \frac{2M}{3}$

From the results of the previous section, we can see that for $\frac{2M}{3} \leq N$, the upper bound for the 2-2 relaying scheme surpasses those for the other schemes. Therefore, we will use the 2-2 relaying scheme in this region.

We will focus on the odd time slots only due to the symmetry between odd and even time slots. In the odd time slots, R_1^1 and R_1^2 are receiving from B_1 and B_2 respectively while R_2^1 and R_2^2 are transmitting to the mobile stations. In the proposed scheme, the precoding matrices of the transmitting relays, each of size $N \times d$, are divided into eight submatrices, i.e., the relays use the first four submatrices to precode the intended signal to M_1 and the last four to precode the messages intended to M_2. The precoding matrix of the relay $\boldsymbol{W}_{R_2^j}$ can be written as

$$\boldsymbol{W}_{R_2^j} = [\boldsymbol{W}^{(1)}_{R_2^j},\ \boldsymbol{W}^{(2)}_{R_2^j}, \cdots \boldsymbol{W}^{(8)}_{R_2^j}] \qquad j \in \{1, 2\} \tag{36}$$

where each submatrix is of size $N \times d_i$ and $i \in \{1, 2, \cdots, 8\}$.

As a result, the total number of the transmitted streams from R_2^j is given by $d = \sum_{i=1}^{8} d_i$ and the total number of streams transmitted to the mobile users is $2d$.

The proposed scheme aligns parts of the interference caused by the transmission of R_2^1 and R_2^2 at both R_1^1 and R_1^2. The interference alignment conditions at the receiving relays are given by

$$\mathcal{S}\{\boldsymbol{H}_{R_1^1R_2^1}\boldsymbol{W}^{(i)}_{R_2^1}\} \subseteq \mathcal{S}\{\boldsymbol{H}_{R_1^1R_2^2}\boldsymbol{W}^{(i)}_{R_2^2}\} \quad i \in \{1,2,5,6\} \tag{37}$$

$$\mathcal{S}\{\boldsymbol{H}_{R_1^2R_2^1}\boldsymbol{W}^{(i)}_{R_2^1}\} \subseteq \mathcal{S}\{\boldsymbol{H}_{R_1^2R_2^2}\boldsymbol{W}^{(i)}_{R_2^2}\} \quad i \in \{1,3,5,7\} \tag{38}$$

where (37) aligns the interference caused by the transmission of $\boldsymbol{W}^{(i)}_{R_2^1}$ with that caused by the transmission of $\boldsymbol{W}^{(i)}_{R_2^2}$ at R_1^1. From equation (37), we can get

$$\boldsymbol{W}^{(1)}_{R_2^1} \subseteq \mathcal{S}\{\boldsymbol{H}^{-1}_{R_1^1R_2^1}\boldsymbol{H}_{R_1^1R_2^2}\boldsymbol{W}^{(1)}_{R_2^2}\} \tag{39}$$

likewise, from equation (38), we can find

$$\boldsymbol{W}^{(1)}_{R_2^2} \subseteq \mathcal{S}\{\boldsymbol{H}^{-1}_{R_1^2R_2^2}\boldsymbol{H}_{R_1^2R_2^1}\boldsymbol{W}^{(1)}_{R_2^1}\} \tag{40}$$

As a result, the precoding matrix $\boldsymbol{W}^{(1)}_{R_2^1}$ must satisfy the following condition

$$\boldsymbol{W}^{(1)}_{R_2^1} \subseteq \mathcal{S}\{\boldsymbol{H}^{-1}_{R_1^1R_2^1}\boldsymbol{H}_{R_1^1R_2^2}\boldsymbol{H}^{-1}_{R_1^2R_2^2}\boldsymbol{H}_{R_1^2R_2^1}\boldsymbol{W}^{(1)}_{R_2^1}\} \tag{41}$$

Therefore, the columns of the matrix $\boldsymbol{W}^{(1)}_{R_2^1}$ are given by any d_1 eigenvectors of the matrix $\boldsymbol{H}^{-1}_{R_1^1R_2^1}\boldsymbol{H}_{R_1^1R_2^2}\boldsymbol{H}^{-1}_{R_1^2R_2^2}\boldsymbol{H}_{R_1^2R_2^1}$ while the precoding matrix $\boldsymbol{W}^{(1)}_{R_2^2}$ can be determined from (40). Similarly, we can get $\boldsymbol{W}^{(5)}_{R_2^1}$ and $\boldsymbol{W}^{(5)}_{R_2^2}$.

Since the submatrices $\boldsymbol{W}^{(i)}_{R_2^1}$ and $\boldsymbol{W}^{(i)}_{R_2^2}$, where $i \in \{1,5\}$, were designed to achieve interference alignment at the receiving relays only, the transmission using these submatrices causes an interference at the unintended mobiles. We can use this interference subspace to align the unintended signal from the remaining submatrices of the transmitting relays. Hence, we constrain the transmission of the submatrices $\boldsymbol{W}^{(i)}_{R_2^1}$ $\left(\text{and } \boldsymbol{W}^{(i)}_{R_2^2}\right)$, where $i \in \{2,3,4\}$, to either lie in the null space of the channel $\boldsymbol{H}_{M_2R_2^1}$ $\left(\text{and } \boldsymbol{H}_{M_2R_2^2}\right)$ or be aligned with the interference signal at M_2 that results from the transmission of $\boldsymbol{W}^{(1)}_{R_2^1}$ $\left(\text{and } \boldsymbol{W}^{(1)}_{R_2^2}\right)$. The following equations describe this constraint

$$\boldsymbol{W}^{(i)}_{R_2^j} \subseteq \mathcal{S}\{\boldsymbol{N}_{M_2R_2^j}\} \qquad i \in \{2,3,4\}, j \in \{1,2\} \tag{42}$$

where $\boldsymbol{N}_{M_2R_2^j} = \mathcal{N}\{\boldsymbol{H}_{M_2R_2^j}\} \cup \mathcal{S}\{\boldsymbol{H}^{\dagger}_{M_2R_2^j}\boldsymbol{H}_{M_2R_2^{3-j}}\boldsymbol{W}^{(1)}_{R_2^{3-j}}\}$ and $j \in \{1,2\}$. Note that the dimension of the subspace $\boldsymbol{N}_{M_2R_2^j}$ is given by $(N-M+d_1)^+$ where $(x)^+ = \max\{0,\ x\}$.

Similarly, we choose $\boldsymbol{W}^{(i)}_{R_2^1}$ and $\boldsymbol{W}^{(i)}_{R_2^2}$ such that

$$\boldsymbol{W}^{(i)}_{R_2^j} \subseteq \mathcal{S}\{\boldsymbol{N}_{M_1R_2^j}\} \qquad i \in \{6,7,8\}, j \in \{1,2\} \tag{43}$$

where $\boldsymbol{N}_{M_1R_2^j} = \mathcal{N}\{\boldsymbol{H}_{M_1R_2^j}\} \cup \mathcal{S}\{\boldsymbol{H}^{-1}_{M_1R_2^j}\boldsymbol{H}_{M_1R_2^{3-j}}\boldsymbol{W}^{(5)}_{R_2^{3-j}}\}$ and its dimension is $(N-M+d_5)^+$. Note that the union of null space and the image of the interference subspace at the transmitter could potentially increase the achievable DoF as we merge two subspaces that can be used for transmission. As a result, in order to suppress the interference at the mobiles, the decoders $\boldsymbol{U}_{M_1}$ and $\boldsymbol{U}_{M_2}$ are designed such that

$$\boldsymbol{U}^H_{M_1}\boldsymbol{H}_{M_1R_2^i}\boldsymbol{W}^{(5)}_{R_2^i} = 0 \quad i \in \{1,2\} \tag{44}$$

$$\boldsymbol{U}^H_{M_2}\boldsymbol{H}_{M_2R_2^i}\boldsymbol{W}^{(1)}_{R_2^i} = 0 \quad i \in \{1,2\} \tag{45}$$

Next, we consider the design of $\boldsymbol{W}^{(i)}_{R_2^1}$ and $\boldsymbol{W}^{(i)}_{R_2^2}$ for $i \in \{2,3,6,7\}$. In order to satisfy (37) for $i = 2$, the required condition is

$$\boldsymbol{W}^{(2)}_{R_2^2} \subseteq \mathcal{S}\{\boldsymbol{H}^{-1}_{R_1^1R_2^2}\boldsymbol{H}_{R_1^1R_2^1}\boldsymbol{W}^{(2)}_{R_2^1}\} \tag{46}$$

By substituting from (42) for $i=2$ and $j=1$ into (46), we get

$$\boldsymbol{W}^{(2)}_{R_2^2} \subseteq \mathcal{S}\{\boldsymbol{H}^{-1}_{R_1^1R_2^2}\boldsymbol{H}_{R_1^1R_2^1}\boldsymbol{N}_{M_2R_2^1}\} \tag{47}$$

In addition to the above condition, the submatrix $\boldsymbol{W}^{(2)}_{R_2^2}$ has to satisfy (42) for $i=2$ and $j=2$. Therefore, we design $\boldsymbol{W}^{(2)}_{R_2^2}$ such that

$$\boldsymbol{W}^{(2)}_{R_2^2} \subseteq \mathcal{S}\{\boldsymbol{H}^{-1}_{R_1^1R_2^2}\boldsymbol{H}_{R_1^1R_2^1}\boldsymbol{N}_{M_2R_2^1}\} \cap \mathcal{S}\{\boldsymbol{N}_{M_2R_2^2}\} \tag{48}$$

The intersection between the two subspaces in the above equation exists if $N < 2N - 2M + 2d_1$, and hence, its dimension is given by $(N - 2M + 2d_1)^+$. The submatrix $\boldsymbol{W}^{(2)}_{R_2^1}$ can be determined from (46) as

$$\boldsymbol{W}^{(2)}_{R_2^1} \subseteq \mathcal{S}\{\boldsymbol{H}^{-1}_{R_1^1R_2^1}\boldsymbol{H}_{R_1^1R_2^2}\boldsymbol{W}^{(2)}_{R_2^2}\} \tag{49}$$

Similarly, we can find $\boldsymbol{W}^{(i)}_{R_2^1}$ and $\boldsymbol{W}^{(i)}_{R_2^2}$ for $i \in \{3,6,7\}$.

In addition, to suppress the interference at the receiving relay R_1^1, we design the decoding matrix $\boldsymbol{U}_{R_1^1}$ such that

$$\boldsymbol{U}^H_{R_1^1}\left[\boldsymbol{H}_{R_1^1R_2^1}\boldsymbol{W}_{R_2^1}, \boldsymbol{H}_{R_1^1R_2^2}[\boldsymbol{W}^{(3)}_{R_2^2}, \boldsymbol{W}^{(4)}_{R_2^2}, \boldsymbol{W}^{(7)}_{R_2^2}, \boldsymbol{W}^{(8)}_{R_2^2}]\right] = \boldsymbol{0} \tag{50}$$

where the interference from the remaining submatrices of $W_{R_2^2}$ is aligned into the interference space of R_1^1 according to (37). Similarly, the decoding matrix $\boldsymbol{U}_{R_1^2}$ is designed such that

$$\boldsymbol{U}^H_{R_1^2}\left[\boldsymbol{H}_{R_1^2R_2^1}\boldsymbol{W}_{R_2^1}, \boldsymbol{H}_{R_1^2R_2^2}[\boldsymbol{W}^{(2)}_{R_2^2}, \boldsymbol{W}^{(4)}_{R_2^2}, \boldsymbol{W}^{(6)}_{R_2^2}, \boldsymbol{W}^{(8)}_{R_2^2}]\right] = \boldsymbol{0} \tag{51}$$

therefore, the receiving relays do not suffer from interference.

Next, we derive the necessary conditions on $\{d_i\}_{i=1}^{8}$ to ensure the achievability of the proposed design. From equations (42) and (43), we can obtain the following bounds, which is required to be able to choose $\boldsymbol{W}^{(i)}_{R_2^1}$ and $\boldsymbol{W}^{(i)}_{R_2^2}$ for $i \in \{2,3,4,6,7,8\}$

$$d_2 + d_3 + d_4 \leq (N - M + d_1)^+ \tag{52}$$

$$d_6 + d_7 + d_8 \leq (N - M + d_5)^+ \tag{53}$$

Also, From (48) and its similar equations for $\boldsymbol{W}^{(i)}_{R_2^j}$, where $i \in \{2,3,6,7\}$, the required conditions to be able to choose these precoding submatrices can be given by

$$d_i \leq (N - 2M + 2d_1)^+ \qquad i \in \{2,3\} \tag{54}$$
$$d_i \leq (N - 2M + 2d_5)^+ \qquad i \in \{6,7\} \tag{55}$$

Moreover, in order to be able to choose $\boldsymbol{U}_{M_1}$ and $\boldsymbol{U}_{M_2}$ from equations (44) and (45) respectively, we obtain the following condition on the received streams at the mobile stations

$$M - 2d_5 \geq 2(d_1 + d_2 + d_3 + d_4) \tag{56}$$
$$M - 2d_1 \geq 2(d_5 + d_6 + d_7 + d_8) \tag{57}$$

where the R.H.S. represents the desired signal subspace at the mobile station while the L.H.S. is the difference between the number of antennas at the mobile station and the dimension of the interference subspace. Similarly, in order to be able to choose $\boldsymbol{U}_{R_1^1}$ and $\boldsymbol{U}_{R_1^2}$ from equations (50) and (51) respectively, we obtain the following conditions on the received streams at the receiving relays

$$N - d_3 - d_4 - d_7 - d_8 - \sum_{i=1}^{8} d_i \geq \sum_{i=1}^{8} d_i \tag{58}$$
$$N - d_2 - d_4 - d_6 - d_8 - \sum_{i=1}^{8} d_i \geq \sum_{i=1}^{8} d_i \tag{59}$$

The maximum achievable DoF can be obtained by maximizing the summation of the transmitted streams by each of R_2^1 and R_2^2 subject to the constrains we stated above. The solution of the following optimization problem yields the DoF that can be achieved using the proposed scheme

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{8} d_i \\ \text{subject to} \quad & (52), (53), \cdots, (58) \\ & d_i \geq 0 \qquad i \in \{1, 2, \cdots, 8\} \end{aligned} \tag{60}$$

Note that the above problem can be solved using linear programming. Solving the above problem yields

$$\eta = \begin{cases} N & \text{for } N \leq M \\ \frac{1}{2}(N+M) & \text{for } M \leq N \leq \frac{5M}{3} \\ N - \frac{M}{3} & \text{for } \frac{5M}{3} \leq N \leq 2M \\ \frac{2N}{3} + \frac{M}{3} & \text{for } 2M \leq N \leq \frac{5M}{2} \\ 2M & \text{for } N \geq \frac{5M}{2} \end{cases} \tag{61}$$

where $\eta = 2\sum_{i=1}^{8} d_i$. Fig. 1 shows the number of achievable DoF by the proposed scheme versus the ratio $\frac{N}{M}$ and their relationship to the upper bounds. Due to space limitations, we omit the derivation of (61).

V. Conclusions

In this paper, we investigated the DoF for a network consisting of two base station and two mobiles without a direct communication link. In this network each base station has two dedicated decode-and-forward relays that are used to transmit to the two mobiles. The base stations and the mobile station are equipped with M antenna, whereas the relays are equipped with N antenna. We have derived upper bounds on the DoF by considering all possible transmission modes; namely, both base stations transmit simultaneously to both relays, both transmit alternately to both relays in two time slots, and a third mode where one base station transmits alternately to its relays and the other base station transmits simultaneously to its respective relays. We have also presented an achievable scheme based on the ratio between N and M. For $\frac{N}{M} \leq \frac{1}{3}$, the upper bound on the DoF for the first mode is higher than those for the other modes. Hence, a simultaneous transmission scheme has been proposed in this region that achieves the upper bound. On the other hand, the upper bound on the DoF for the third mode surpasses the upper bounds on the DoF for the other modes for $\frac{1}{3} \leq \frac{N}{M} \leq \frac{2}{3}$. As a result, the proposed scheme uses this transmission mode and also achieves the upper bound of DoF in this region. Finally, for $\frac{N}{M} \geq \frac{2}{3}$, the upper bound for the second mode is higher than the other two upper bounds. Therefore, the proposed scheme uses this mode and achieves the upper bound except for $1 \leq \frac{N}{M} \leq \frac{5}{2}$. Nevertheless, the achieved DoF using the second mode for $1 \leq \frac{N}{M} \leq \frac{5}{2}$ are greater than or equal to the upper bounds on the achievable DoF using the first and the third modes which proves the optimality of the second mode in this region.

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