Distributed Optimal Power Flow using Feasible Point Pursuit

Ahmed S. Zamzam*, Xiao Fu*, Emiliano Dall'Anese† and Nicholas D. Sidiropoulos*
*Dept. of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN
†National Renewable Energy Laboratory, Golden, CO

***Abstract*—The AC Optimal Power Flow (OPF) is a core optimization task in the domain of power system operations and control. It is known to be nonconvex (and, in fact, NP-hard). In general operational scenarios, identifying feasible (let alone optimal) power-flow solutions remains hard. This paper leverages the recently proposed Feasible Point Pursuit algorithm for solving the OPF problem to devise a fully distributed procedure that can identify AC OPF solutions. The paper considers a multi-area setting and develops an algorithm where all the computations are done locally withing each area, and then the local controllers have to communicate to only their neighbors a small amount of information pertaining to the boundary buses. The merits of the proposed approach are illustrated through an example of a challenging transmission network.**

I. Introduction

The Optimal Power Flow (OPF) problem is a main component of power systems operation. It aims at minimizing a specific operational cost while respecting demand constraints and engineering limits, such as bus voltage bounds and line thermal limits. Costs to minimize include, but are not limited to, the power generation cost from fossil-fuel cells, the power loss in the network, and the active power curtailed at the renewable energy resources. Since its introduction by Carpentier in 1962 [1], the OPF problem has been considered a cornerstone in power systems operation and planning. Yet, due to the nonlinear relations between voltage and power, the OPF problem is non-convex, and NP-hard in general [2], [3]. Many heuristics have been proposed to tackle the problem, however, they are not guaranteed to yield the optimal solution(s). On the other hand, a number of relaxations such as semidefinite relaxation [2], [4], and moment-based relaxation [5] have been proposed to to circumvent the non-convexity in the problem. However, tightness of these relaxations has only been shown for specific network classes under restrictive conditions [6].

Centralized OPF solutions have traditionally been considered in the power systems community, where all of the optimization and control are done by a central unit. Due to the rapid recent growth of distributed energy sources and the integration of storage units, attention partially shifted towards devising distributed OPF solvers [7]–[12]. The growing interest in distributed OPF solvers is also motivated by operational settings where different portions of the grid are managed by systems operators as well as security and privacy concerns. Developing an algorithm where neighboring local controllers only exchange limited amount of information relating to the nodes on the boundary is one of the main goals of decentralized OPF [13]–[15]. Relaxation-based decentralized solvers [7], [8], [11], [12] inherit the potential of finding the global optimal solution from their centralized counterparts, albeit only under restrictive conditions, including use of specific cost functions. One decentralization approach is to apply the alternating direction method of multipliers (ADMM) [16] directly to the OPF problem, but this requires locally solving non-convex and NP-hard optimization problems. In [10], the authors proposed using a sequential quadratic approximation procedure to solve the local problems. However, the feasibility of this approximation is not guaranteed, which hinders the algorithm's ability to find a globally feasible solution. We also note that many of the decentralized algorithms in the literature require a central coordination unit to perform global operations and facilitate the exchange of information between local solvers, which conflicts with privacy / security concerns.

Building on the so-called *Feasible Point Pursuit-Successive Convex Approximation* [17] (FPP-SCA) framework, a centralized OPF solver for multi-phase systems with renewables was proposed in [18]. The approach was later extended to handle OPF problems with wye- and delta-connected loads and distributed energy sources in [19]. The FPP algorithm replaces the non-convex parts of the constraints by convex restrictions. Then, slack variables are added to the constraints to ensure the feasibility of the iterates. Upon obtaining a feasible solution, the algorithm proceeds to minimize the predefined cost function. Building on the effectiveness of the centralized approach, this paper presents a distributed FPP-SCA approach to solve the OPF problem. Following the FPP-type approximation, an ADMM-based approach is used to facilitate separability of the cost function and the constraints. At each iteration, each local solver solves a relatively small *convex* (conic) program. Then, exchange of information relating the boundary buses is done only between neighboring areas. Finally, a dual step is taken locally by each local controller. The proposed algorithm is fully decentralized: it only requires exchanging small amounts of information between neighboring areas and it does not need any sort of central coordinator. The rest of this paper is organized as follows. Section II contains the OPF formulation considered, while section III introduces the proposed distributed FPP-SCA OPF algorithm. Section IV illustrates the effectiveness of the proposed algorithm via a challenging test case from the literature, and Section V summarizes conclusions and findings.

*Emails: (ahmedz,xfu,nikos)@umn.edu. The work of A.S. Zamzam and N.D. Sidiropoulos was partially supported by NSF CIF-1525194. The work of X. Fu was partially supported by NSF ECCS-1608961.
†Email: emiliano.dallanese@nrel.gov. The work of E. Dall'Anese was supported by the U.S. Department of Energy under Contract No. DE-AC36-08GO28308 with the National Renewable Energy Laboratory; funding provided by the Advanced Research Projects Agency-Energy (ARPA-E) under the Network Optimized Distributed Energy Systems (NODES) program.

II. Problem Formulation

Consider a power network with n buses collected in the set $\mathcal{N} = \{1, 2, \ldots, n\}$, and lines represented by the set $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$. The voltage phasor of bus k is denoted by $v_k \in \mathbb{C}$. Real and reactive load at bus k are denoted by $p_k^{(L)}$ and $q_k^{(L)}$, respectively. Generators are assumed to be located at nodes $\mathcal{G} \subseteq \mathcal{N}$, and $p_k^{(G)}$ and $q_k^{(G)}$ represent the generated real and reactive powers at bus k, respectively. Additionally, the apparent power transferred from bus l to the rest of the network through the line $(l, m) \in \mathcal{L}$ is denoted by $s_{lm} = p_{lm} + jq_{lm}$. The $n \times n$ complex-valued network admittance matrix is denoted by $\mathbf{Y} = \mathbf{G} + j\mathbf{B}$. Therefore, stacking the voltages and the currents at all the nodes in the vectors $\mathbf{v} = [v_1, v_2, \ldots, v_n]$ and $\mathbf{i} = [i_1, i_2, \ldots, i_n]$, respectively, we can express Ohm's Law as $\mathbf{i} = \mathbf{Y}\mathbf{v}$, from which the apparent power s_{lm} can be written as [1]

$$s_{lm} = v_l(-[\mathbf{Y}]_{l,m}\ (v_l - v_m))^{\star}. \tag{1}$$

The power balance equations at bus $k \in \mathcal{G}$ can be formulated as

$$p_k^{(G)} - p_k^{(L)} = \sum_{l \in \mathcal{N}_k} p_{kl} = \sum_{l \in \mathcal{N}_k} \Re(s_{kl}), \tag{2}$$

$$q_k^{(G)} - q_k^{(L)} = \sum_{l \in \mathcal{N}_k} q_{kl} = \sum_{l \in \mathcal{N}_k} \Im(s_{kl}), \tag{3}$$

where $\mathcal{N}_k$ is a set that collects the neighboring buses to bus k. Note that, the power balance equations can be generalized for a bus that does not contain a generator by setting $p_k^{(G)}$ and $q_k^{(G)}$ to zero. Let $\mathbf{p}_G$, $\mathbf{q}_G$ be vectors that collect the active and reactive power generated by all of the generators in the system. The AC optimal power flow problem for a power network can be posed as follows

$$\min_{\mathbf{v}, \mathbf{p}_G, \mathbf{q}_G, \{p_{lm}, q_{lm}\}_{(l,m)\in\mathcal{L}}} \quad C_g(\mathbf{p}_G) \tag{4a}$$

subject to

$$p_k^{(G)} - p_k^{(L)} = \sum_{l \in \mathcal{N}_k} p_{kl} + p_{kk} \quad \forall k \in \mathcal{N} \tag{4b}$$

$$q_k^{(G)} - q_k^{(L)} = \sum_{l \in \mathcal{N}_k} q_{kl} + q_{kk} \quad \forall k \in \mathcal{N} \tag{4c}$$

$$\underline{p}_k^{(G)} \leq p_k^{(G)} \leq \overline{p}_k^{(G)} \quad \forall k \in \mathcal{N} \tag{4d}$$

$$\underline{q}_k^{(G)} \leq q_k^{(G)} \leq \overline{q}_k^{(G)} \quad \forall k \in \mathcal{N} \tag{4e}$$

$$p_{kl} + jq_{kl} = -v_k([\mathbf{Y}]_{k,l}\ (v_k - v_l))^{\star}\ \forall l \in \mathcal{N}_k, \forall k \in \mathcal{N} \tag{4f}$$

$$|s_{kl}| \leq |\overline{s}_{kl}| \quad \forall l \in \mathcal{N}_k, \forall k \in \mathcal{N} \tag{4g}$$

$$|\underline{v}_k| \leq |v_k| \leq |\overline{v}_k| \quad \forall k \in \mathcal{N} \tag{4h}$$

where $\underline{p}_k^{(G)}$ and $\overline{p}_k^{(G)}$ are the maximum and minimum real power generation limits at bus k, and $\underline{q}_k^{(G)}$ and $\overline{q}_k^{(G)}$ denote the reactive power generation limits at bus k; constraints (4h) confine the voltage magnitude levels within prescribed limits; and, $|\overline{s}_{kl}|$ is the apparent power limit for line $(k, l) \in \mathcal{L}$. Note that, for every line $(k, l) \in \mathcal{L}$ there are two different variables s_{kl} and s_{lk} that represent the injected apparent power at both ends of the line. For buses $\mathcal{N} \backslash \mathcal{G}$, one has that $\overline{p}_k^{(G)} = \underline{p}_k^{(G)} = \overline{q}_k^{(G)} = \underline{q}_k^{(G)} = 0$. If several generator are placed at the same bus, then additional variables will be added to the problem that represent the generated active and reactive power by each generator. Then, the constraints (4b) and (4e) need to be modified by summing up the generated power from all available generators, and associated generation limit constraints must also be added. A formal NP-hardness proof for (4) can be found in e.g., [2], [3].

[1] $(.)^{\star}$ and $(.)^T$ denote the complex conjugate and the transpose of (.), respectively.

In problem (4), the cost function represents the cost of power generation, and it is denoted by $C_g(\mathbf{p}_g)$. Other cost functions can be handled using the proposed framework, which is flexible in this regard. For example, a cost function that aims at minimizing the power loss in the network, or cost functions related to renewables as in [18] can be used here as well.

Consider partitioning the network into K non-overlapping areas $\{\mathcal{A}^{(k)} \subset \mathcal{N}\}_{k=1}^{K}$, such that $\bigcup_{k=1}^{K} \mathcal{A}^{(k)} = \mathcal{N}$. Suppose that each area has a local controller that controls the buses in this specific area. Let $\overline{\mathbf{v}}_k$ represent a complex vector that consists of voltages of the buses $i \in \overline{\mathcal{A}^{(k)}}$, where $\overline{\mathcal{A}^{(k)}}$ is a set that also collect the buses that belong to different areas but connected to a bus in $\mathcal{A}^{(k)}$ by a line, i.e., $\overline{\mathcal{A}^{(k)}} = \mathcal{A}^{(k)} \cup \{n | (n, m) \in \mathcal{L}, m \in \mathcal{A}^{(k)}, n \in \mathcal{A}^{(j)}, j \neq k\}$. Let $\overline{\mathbf{E}}_k \in \mathbb{R}^{n \times |\overline{\mathcal{A}^{(k)}}|}$ be a matrix that collects the i-th standard basis vector in $\mathbb{R}^n$ for all $i \in \overline{\mathcal{A}}_k$. Let $\mathbf{v}_k = [\Re(\overline{\mathbf{v}}_k)^T \quad \Im(\overline{\mathbf{v}}_k)^T]^T$, and define

$$\mathbf{E}_k = \begin{bmatrix} \overline{\mathbf{E}}_k^T & \mathbf{0} \\ \mathbf{0} & \overline{\mathbf{E}}_k^T \end{bmatrix} \tag{5}$$

Therefore, using the square matrices $\mathbf{Y}_{ij}$, $\overline{\mathbf{Y}}_{ij}$ and $\mathbf{M}_i$ defined in [2], the OPF problem in (4) can be re-written as

$$\min_{\substack{\mathbf{v}, \{\mathbf{v}_k\}_{1\leq k \leq K}, \mathbf{p}_G, \\ \mathbf{q}_G, \{p_{lm}, q_{lm}\}_{(l,m)\in\mathcal{L}}}} \quad C_g(\mathbf{p}_G) \tag{6a}$$

subject to

- $\forall k \in \{1, 2, \ldots, K\}$

$$\mathbf{v}_k = \mathbf{E}_k \mathbf{v} \tag{6b}$$

  - $\forall i \in \mathcal{A}^{(k)}$

$$p_i^{(G)} - p_i^{(L)} = p_{ii} + \sum_{j \in \mathcal{N}_k} p_{ij} \tag{6c}$$

$$q_i^{(G)} - q_i^{(L)} = q_{ii} + \sum_{j \in \mathcal{N}_k} q_{ij} \tag{6d}$$

$$\underline{p}_i^{(G)} \leq p_i^{(G)} \leq \overline{p}_i^{(G)} \tag{6e}$$

$$\underline{q}_i^{(G)} \leq q_i^{(G)} \leq \overline{q}_i^{(G)} \tag{6f}$$

$$p_{ij} = \mathbf{v}_k^T \mathbf{Y}_{ij} \mathbf{v}_k \quad \forall j \in \mathcal{N}_i \cup i \tag{6g}$$

$$q_{ij} = \mathbf{v}_k^T \overline{\mathbf{Y}}_{ij} \mathbf{v}_k \quad \forall j \in \mathcal{N}_i \cup i \tag{6h}$$

$$p_{ij}^2 + q_{ij}^2 \leq |\overline{s}_{ij}|^2 \quad \forall j \in \mathcal{N}_i \tag{6i}$$

  - $\forall i \in \overline{\mathcal{A}^{(k)}}$

$$|\underline{v}_i|^2 \leq \mathbf{v}_k^T \mathbf{M}_i \mathbf{v}_k \leq |\overline{v}_i|^2 \tag{6j}$$

Notice that, the cost function is also decomposable in terms of the generated active power at each area. Therefore, the previous formulation suggests solving the OPF problem in a distributed fashion where every local controller has to solve only for the local variables. To this end, an ADMM-like solver that utilizes the FPP algorithm will be presented in the next section.

III. FPP-based Distributed OPF solver

In this section, the FPP-based solver proposed in [18] is utilized to devise a distributed ADMM-like algorithm for solving the OPF problem. The FPP solver consists of two phases. In the first phase, the algorithm tries to find a feasible point of the problem by minimizing slack variables that relate to the violation of the constraints. Utilizing the feasible solution obtained in the first phase as initialization, a sequence of restricted problems are solved to attain a solution of the OPF.

The OPF problem formulation (6) is non-convex due to the constraints (6g), (6h) and the lower bound on the voltage magnitude in (6j). Therefore, applying the ADMM algorithm directly requires solving a non-convex optimization problem at every iteration optimally, which is NP-hard in general. Instead, bringing into play the FPP-SCA procedure allows leveraging its insightful handling of the non-convex constraints. In what follows, the distributed FPP-SCA solver will be discussed.

A. Feasibility Pursuit Phase

The FPP algorithm solves a sequence of inner-approximations of the non-convex feasibility set. Since the approximation point is not feasible, the inner-approximations might be empty sets. Therefore, slack variables are added to each approximated constraint in order to ensure feasibility of the iterates. Then, the summation of the slack sizes is minimized to approach feasibility. A feasible point is revealed when the values of all the slacks are zeros, and then, the feasible point can be refined to obtain an operating point that optimizes the cost function. In the first phase, the inner-approximations are constructed as follows. Constraint (6g) can be written as

$$\mathbf{v}_k^T \mathbf{Y}_{ij} \mathbf{v}_k \le p_{ij}, \tag{7a}$$

$$\mathbf{v}_k^T (-\mathbf{Y}_{ij}) \mathbf{v}_k \le -p_{ij}. \tag{7b}$$

Both constraints are non-convex as the matrix $\mathbf{Y}_{kl}$ is indefinite. Consider (7a) where the inequality can be rewritten as

$$\mathbf{v}_k^T \mathbf{Y}_{ij}^{(+)} \mathbf{v}_k + \mathbf{v}_k^T \mathbf{Y}_{ij}^{(-)} \mathbf{v}_k \le p_{ij} \tag{8}$$

where $\mathbf{Y}_{ij}^{(+)}$ and $\mathbf{Y}_{ij}^{(-)}$ are the positive semidefinite and the negative semidefinite parts of the matrix $\mathbf{Y}_{ij}$, respectively. For $\mathbf{Y}_{ij}^{(-)}$, the following inequality holds. $(\mathbf{v}_k - \mathbf{z})^T \mathbf{Y}_{ij}^{(-)} (\mathbf{v}_k - \mathbf{z}) \le 0$. Then, expanding the left hand side, it follows that $\mathbf{v}_k^T \mathbf{Y}_{ij}^{(-)} \mathbf{v}_k \le 2\mathbf{z}^T \mathbf{Y}_{ij}^{(-)} \mathbf{v}_k - \mathbf{z}^T \mathbf{Y}_{ij}^{(-)} \mathbf{z}$. Therefore, the surrogate function for the non-convex quadratic constraint (8) can be defined as

$$\mathbf{v}_k^T \mathbf{Y}_{ij}^{(+)} \mathbf{v}_k + 2\mathbf{z}^T \mathbf{Y}_{ij}^{(-)} \mathbf{v}_k \le p_{ij} + \mathbf{z}^T \mathbf{Y}_{ij}^{(-)} \mathbf{z} + s_{ij}^{(P)} \tag{9}$$

where the nonnegative slack variable $s_{ij}^{(P)}$ is added to ensure feasibility. Similarly, (7b) is replaced by

$$\mathbf{v}_k^T (-\mathbf{Y}_{ij}^{(-)}) \mathbf{v}_k - 2\mathbf{z}^T \mathbf{Y}_{ij}^{(+)} \mathbf{v}_k \le -p_{ij} - \mathbf{z}^T \mathbf{Y}_{ij}^{(+)} \mathbf{z} + s_{ij}^{(P)}. \tag{10}$$

Similar to (6g), the constraint (6h) is also replaced by a surrogate where a slack variable $s_{ij}^{(Q)}$ is added to the right hand side of the surrogates. Finally, the lower bound in the constraint (6j) is replaced by the convex restriction

$$2\mathbf{z}^T(-\mathbf{M}_i)\mathbf{v}_k \le -(|\underline{v}_l|)^2 + \mathbf{z}^T(-\mathbf{M}_i)\mathbf{z} + s_i^{(V)} \tag{11}$$

where $s_i^{(V)}$ is a non-negative slack variable that is added to the constraint to ensure feasibility.

Define the vector $\mathbf{u}_k$ for $k \in \{1, 2, \dots, K\}$ that collects all the variables at buses $i \in \mathcal{A}^{(k)}$ except $\mathbf{v}_k$. Let the non-convex set described by the constraints (6c)–(6i) be denoted by $\mathcal{X}_k$. Therefore, (6c)–(6j) can written as $(\mathbf{v}_k, \mathbf{u}_k) \in \mathcal{X}_k$. By replacing the non-convex constraints by their respective convex restrictions and adding the nonnegative slack variables, the resulting convex set is denoted by $\hat{\mathcal{X}}_k(\mathbf{z})$ where $\mathbf{z}$ is the approximation point. Let $\mathbf{s}_k$ be defined as a vector that collects all the slack variables at the k-th area. Then, the problem that needs to be solved at each iteration in the first phase of the algorithm (FPP) can written as

$$\min_{\mathbf{v}, \{\mathbf{v}_k, \mathbf{u}_k, \mathbf{s}_k\}_{k=1}^K} \sum_{k=1}^K \|\mathbf{s}_k\|_1 \tag{12a}$$

$$\text{subject to} \quad (\mathbf{v}_k, \mathbf{u}_k, \mathbf{s}_k) \in \hat{\mathcal{X}}_k(\mathbf{z}) \quad \forall k \in \{1, 2, \dots, K\} \tag{12b}$$

$$\mathbf{v}_k = \mathbf{E}_k \mathbf{v} \qquad \forall k \in \{1, 2, \dots, K\} \tag{12c}$$

The cost function (12a) aims at decreasing the overall violation of the constraints by minimizing the sum of the slack variables. The partial augmented Langrangian function with respect to the consistency constraint (12c) can be defined as follows. $L_\rho(\mathbf{v}, \{\mathbf{v}_k, \mathbf{u}_k, \mathbf{s}_k, \lambda_k\}_{k=1}^K) = \sum_{k=1}^K \|\mathbf{s}_k\|_1 + \lambda_k^T(\mathbf{v}_k - \mathbf{E}_k\mathbf{v}) + \frac{\rho}{2}\|\mathbf{v}_k - \mathbf{E}_k\mathbf{v}\|_2^2$, where $\lambda_\mathbf{k}$ is the dual variable that corresponds to the constraint (12c). The proposed ADMM FFP-based algorithm is summarized in Algorithm 1 where the value of the variable $(.)$ at the i-th iteration is denoted by $(.)^{(i)}$.

Initialization: *set* $i = 0$, $\lambda_k^{(0)} = \mathbf{0}$, $\mathbf{v}^{(0)}$ to be the flat voltage profile, and $\mathbf{v}_k^{(0)} = \mathbf{E}_k \mathbf{v}^{(0)}$.
repeat
- **for** $k \in \{1, 2, \dots, K\}$ **do**
  - $(\mathbf{v}_k^{(i+1)}, \mathbf{u}_k^{(i+1)}, \mathbf{s}_k^{(i+1)}) \leftarrow \arg\min_{(\mathbf{v}_k, \mathbf{u}_k, \mathbf{s}_k) \in \hat{\mathcal{X}}_k(\mathbf{v}_k^{(i)})} L_\rho(\mathbf{v}^{(i)}, \mathbf{v}_k, \mathbf{u}_k, \mathbf{s}_k, \lambda_k^{(i)})$
- **end**
- $\mathbf{v}^{(i+1)} \leftarrow \arg\min_{\mathbf{v}} L_\rho(\mathbf{v}, \{\mathbf{v}_k^{(i+1)}, \mathbf{u}_k^{(i+1)}, \mathbf{s}_k^{(i+1)}, \lambda_k^{(i)}\}_{k=1}^K)$.
- **for** $k \in \{1, 2, \dots, K\}$ **do**
  - $\lambda_k^{(i+1)} = \lambda_k^{(i)} + \rho(\mathbf{v}_k - \mathbf{E}_k \mathbf{v})$
- **end**
- $i \leftarrow i + 1$.

until *convergence criteria is met*
Output: $\mathbf{v}_f \leftarrow \mathbf{v}^{(i)}$

Algorithm 1: Distributed algorithm for feasibility phase.

In this algorithm, three different types of updates are performed at each iteration. In the first step, the local variables $\mathbf{v}_k, \mathbf{u}_k, \mathbf{s}_k$ are updated for every area $\mathcal{A}^{(k)}$. The problem to be solved at this step is a convex QCQP that can be formulated as a convex conic program, and hence, be solved efficiently using off-the-shelf solvers. In the second step, the consistency variable $\mathbf{v}$ is updated by minimizing the Lagrangian function while fixing all the other variables. The optimal solution is to simply average all the values computed at different areas for the variables.[2]. For example, if node $i \in \mathcal{A}^{(k)}$ has connections

[2]Given that the dual variables λ_k are initialized to be all zeros [8].

to nodes $j_1 \in \mathcal{A}^{(k')}$ and $j_2 \in \mathcal{A}^{(k'')}$, then three values will be computed for the value of v_i at $\mathcal{A}^{(k)}$, $\mathcal{A}^{(k')}$, and $\mathcal{A}^{(k'')}$. The optimal solution of the i-th element $\mathbf{v}$ is the average of these three computed values in the first step. Updating the dual variables λ_k requires knowledge of only local variables and the averages calculated at the neighboring areas.

B. Cost Refinement Phase

Once a feasible voltage profile is obtained, the procedure continues to the second phase where the solution is refined. In this step, the feasible set is again restricted, but this time at feasible points – hence, the slack variables are not needed. The convex inner-approximation feasibility set of the non-convex feasible set $\mathcal{X}_k$ at the feasible voltage profile $\mathbf{v}_k^{(i)}$ is denoted by $\check{\mathcal{X}}(\mathbf{v}_k^{(i)})$. In addition, the generation cost function can be decomposed to be sum of the generation cost at each bus, i.e., $C_g(\mathbf{p}_g) = \sum_{k=1}^{K} C_{g_k}(\mathbf{u}_k)$. Accordingly, the problem that needs to be solved at each iteration in the second phase can be written as follows.

$$\min_{\mathbf{v}, \{\mathbf{v}_k, \mathbf{u}_k, \mathbf{s}_k\}_{k=1}^{K}} \sum_{k=1}^{K} C_{g_k}(\mathbf{u}_k) \tag{13a}$$

$$\text{subject to} \quad (12\text{b}) - (12\text{c}). \tag{13b}$$

The distributed algorithm for solving the above is very similar to that in Algorithm 1 except that 1) the augmented Lagrangian replaced by $\Lambda_\rho(\mathbf{v}, \{\mathbf{v}_k, \mathbf{u}_k, \lambda_k\}_{k=1}^{K}) = \sum_{k=1}^{K} C_{g_k}(\mathbf{u}_k) + \lambda_k^T(\mathbf{v}_k - \mathbf{E}_k\mathbf{v}) + \frac{\rho}{2}\|\mathbf{v}_k - \mathbf{E}_k\mathbf{v}\|_2^2$ and 2) there is no slack variable $\mathbf{s}_k$ involved.

C. Distributed Implementation

Due to the separability of the proposed problem formulation and the simple updating rules, distributed execution comes naturally. In such implementation, the first step can be solved locally at each area by a local controller. Then, every local controller broadcasts the values calculated for the voltages of nodes that have connections with other areas to the corresponding neighboring areas. Accordingly, every area will receive the values calculated for the voltages of its buses at the neighbors. Then, all areas perform *local averaging*, where the voltage of each node is calculated by averaging out all the values that pertain to this specific node. Then, the averages are broadcasted to the neighbors in order to use them in further calculations. Therefore, two phases of communication are used. In each phase, a complex variable is transferred along every edge $\{(i,j) \in \mathcal{L} | i \in \mathcal{A}^{(k_1)}, j \in \mathcal{A}^{(k_2)}, k_1 \neq k_2\}$ twice. Hence, the communication cost relates to the number of edges that connect two buses in two different areas. Hence, for a given number of areas, the partitioning that minimizes the communication cost is given by solving the graph partitioning (Min K-cut) problem over the network graph. Since this problem is NP-hard, an approximation algorithm can be used to find a good partitioning of the network, as detailed in the next section.

IV. Test Case

In this section, the effectiveness of the devised algorithm is demonstrated using a test network of moderate size. We consider a conventional OPF problem for a 118-bus transmission network (*case 118Q*). The network was divided into 20 areas using a spectral clustering algorithm. The number of buses in each area has a mean of 5.9, with maximum and minimum of 11 and 2, respectively. We initialized our algorithm with the flat voltage profile, and chose ρ to be $2\sqrt{n}$ and 2×10^8 for the first and second phase, respectively. To stop the algorithm, we chose the convergence criteria to be the difference between the cost function of two consecutive steps be less than 10^{-10} and 10^{-4} for the feasibility phase and the refinement phase, respectively. The local conic problem in each area can be solved in few milliseconds, and both phase need a few hundreds of iterations to converge. Fig. 1 shows the convergence of the algorithm in the feasibility phase. An operating point that satisfies all the network constraints was identified after 100 iterations. Then, the second phase of the algorithm was used to refine the value of the cost function at the operating point. Convergence of the second phase of the algorithm is depicted in Fig. 2. The algorithm was able to find an operating point that achieves cost function very close the SDR lower bound.

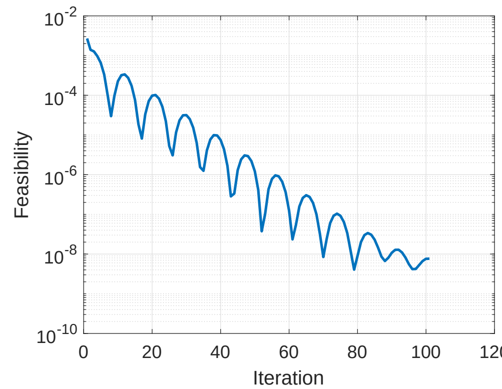

Fig. 1. Convergence of the feasibility phase

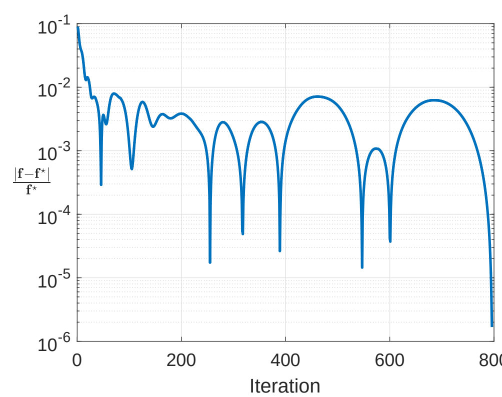

Fig. 2. Convergence of the refinement phase

V. Concluding Remarks

This paper put forth a decentralized algorithm for solving the OPF problem benefiting from the FPP-SCA procedure, which was previously demonstrated to be successful in solving the OPF problem for different setups. The new algorithm uses an ADMM-like approach to enable separability of the problem over several local controllers. Efficacy of the fully distributed algorithm was shown on an IEEE 118-bus network. In the future, we will further explore the applicability of the algorithm to generic network setups, and study the convergence behavior of the algorithm especially under changing algorithm parameters.

References

[1] J. Carpentier, "Optimal power flows," *International Journal of Electrical Power and Energy Systems*, vol. 1, no. 1, pp. 3 – 15, 1979.

[2] J. Lavaei and S. Low, "Zero Duality Gap in Optimal Power Flow Problem," *IEEE Trans. on Power Systems*, vol. 27, no. 1, pp. 92–107, Feb 2012.

[3] K. Lehmann, A. Grastien, and P. V. Hentenryck, "AC-Feasibility on Tree Networks is NP-Hard," *IEEE Trans. on Power Systems*, vol. 31, no. 1, pp. 798–801, Jan 2016.

[4] R. Jabr, "Radial distribution load flow using conic programming," *IEEE Trans. on Power Systems*, vol. 21, no. 3, pp. 1458–1459, Aug 2006.

[5] D. Molzahn and I. Hiskens, "Moment-based relaxation of the optimal power flow problem," in *Power Systems Computation Conference*, Wrocaw, Poland, Aug 2014.

[6] J. Lavaei, D. Tse, and B. Zhang, "Geometry of power flows in tree networks," in *IEEE Power and Energy Society General Meeting*, San Diego, CA, July 2012.

[7] A. Y. Lam, B. Zhang, and D. N. C. Tse, "Distributed algorithms for optimal power flow problem," in *IEEE 51st Annual Conference on Decision and Control (CDC)*, 2012, pp. 430–437.

[8] E. Dall'Anese, H. Zhu, and G. B. Giannakis, "Distributed optimal power flow for smart microgrids," *IEEE Trans. on Smart Grid*, vol. 4, no. 3, pp. 1464–1475, Sep 2013.

[9] T. Erseghe, "Distributed optimal power flow using ADMM," *IEEE Trans. on Power Systems*, vol. 29, no. 5, pp. 2370–2380, 2014.

[10] S. Magnússon, P. C. Weeraddana, and C. Fischione, "A Distributed Approach for the Optimal Power-Flow Problem Based on ADMM and Sequential Convex Approximations," *IEEE Trans. on Control of Network Systems*, vol. 2, no. 3, pp. 238–253, Sept 2015.

[11] M. Bazrafshan and N. Gatsis, "Decentralized Stochastic Optimal Power Flow in Radial Networks With Distributed Generation," *IEEE Trans. on Smart Grid*, vol. 8, no. 2, pp. 787–801, March 2017.

[12] Q. Peng and S. H. Low, "Distributed Optimal Power Flow Algorithm for Radial Networks, I: Balanced Single Phase Case," *IEEE Trans. on Smart Grid*, vol. PP, no. 99, pp. 1–1, 2017.

[13] B. H. Kim and R. Baldick, "Coarse-grained distributed optimal power flow," *IEEE Trans. on Power Systems*, vol. 12, no. 2, pp. 932–939, May 1997.

[14] R. Baldick, B. H. Kim, C. Chase, and Y. Luo, "A fast distributed implementation of optimal power flow," *IEEE Tran. on Power Systems*, vol. 14, no. 3, pp. 858–864, Aug 1999.

[15] G. Hug-Glanzmann and G. Andersson, "Decentralized optimal power flow control for overlapping areas in power systems," *IEEE Trans. on Power Systems*, vol. 24, no. 1, pp. 327–336, Feb 2009.

[16] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2011.

[17] O. Mehanna, K. Huang, B. Gopalakrishnan, A. Konar, and N. Sidiropoulos, "Feasible point pursuit and successive approximation of non-convex QCQPs," *IEEE Signal Processing Letters*, vol. 22, no. 7, pp. 804–808, July 2015.

[18] A. S. Zamzam, N. D. Sidiropoulos, and E. Dall'Anese, "Beyond relaxation and newton-raphson: Solving ac opf for multi-phase systems with renewables," *IEEE Trans. on Smart Grid*, vol. PP, no. 99, pp. 1–1, 2016.

[19] A. S. Zamzam, C. Zhao, E. Dall'Anese, and N. D. Sidiropoulos, "A QCQP Approach for OPF in Multiphase Radial Networks with Wye and Delta Connections," in *IREP Symposium Bulk Power System Dynamics and Control*, Espinho, Portugal, Aug 2017.