On the Degrees of Freedom of the Two-Cell Two-Hop MIMO Network With Dedicated and Shared Relays

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***Abstract*—We investigate the degrees of freedom (DoF) of the downlink of a cellular relay network. In this network, two base stations transmit to two mobile stations via relays due to the absence of a direct communication link. Each base station and mobile station is equipped with M antennas. Each base station has two messages; one to each mobile station, and uses two relays to transmit to the mobile stations. The relays are half duplex, decode-and-forward and equipped with N antennas each. We consider two configurations of the relays; shared and dedicated relays. In the shared relays configuration, the system has two relays that are used by both base stations. Whereas, in the dedicated relays configuration, each base station has two dedicated relays, i.e., the system has four relays. We consider all possible relaying schemes where the base stations can use the relays either simultaneously or alternately. We derive an upper bound on the DoF achievable by each relaying scheme as a function of the ratio between N and M. Furthermore, we propose an achievable scheme that uses interference alignment to achieve the upper bound on the DoF for the shared relays configuration, and for all values of M and N except for $1 < \frac{N}{M} < \frac{5}{2}$ in the dedicated relays configuration.**

***Index Terms*—Channel capacity, MIMO, relays, decode-and-forward, cellular networks, interference channels, degrees of freedom, interference alignment.**

I. Introduction

DESPITE the fact that relays can enhance the performance of wireless systems, little progress has been made so far on characterizing the capacity regions of multihop networks. In wireless networks, the interference caused by the signals received at tended nodes limits the system performance especially in the high signal to noise ratio regime. In single hop wireless networks, interference management techniques have been developed with remarkable improvement in capacity gains [1]–[5]. However, developing interference management techniques for multihop wireless networks remains a challenge.

A variety of capacity approximations have been realized in the form of DoF [1]–[3] and generalized DoF [6]–[8]. The spatial DoF of a network can be defined as

$$\eta = \lim_{\rho\to\infty} \frac{C(\rho)}{\log(\rho)} \tag{1}$$

where ρ is the signal-to-noise ratio (SNR) and $C(\rho)$ is the system capacity at SNR ρ [2]. The spatial DoF usually represent the total number of interference-free streams that can be transmitted from the sources to the destinations.

In this work, we characterize the DoF region of a two-hop MIMO relay network. The relays are half-duplex decode-and-forward relays. Also, we provide achievable schemes that attain the upper bounds with arbitrary antenna configuration in most cases.

A. Related Work

The half-duplex nature of relay networks causes a loss in the capacity pre-log factor [9]–[13]. Alternate relaying schemes were proposed to overcome this loss [10]–[16], where the relay terminals transmit or receive a signal alternately. In [10], a network of one source and one destination and three amplify-and-forward relays was considered where each node was equipped with M antennas. An alternate transmission scheme was proposed to achieve $\frac{3M}{4}$ DoF instead of the $\frac{M}{2}$ DoF that can be achieved using conventional relaying where the relays receive simultaneously and transmit simultaneously too. Also, the same authors proposed an alternate scheme that can achieve M DoF when six relays with M antennas are used [11]. In this scheme the source transmits to three relays while the other three relays are transmitting to the destination.

In [12], a network consisting of two transmitters, two receivers and four decode-and-forward relays was studied, where each terminal was equipped with M antennas. Two different schemes were proposed based on alternate and conventional transmission to achieve M DoF. However, the case where different number of antennas at the relays was not discussed in the literature to the best of our knowledge.

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The $2 \times 2 \times 2$ relay network has attracted considerable attention, e.g., [17]–[19]. In this network, two transmitters make use of two relays to deliver their messages to two destination nodes. An interference neutralization technique was used in [17], where all the nodes were equipped with single antenna and the relays were full-duplex amplify-and-forward relays, to achieve 2 DoF asymptotically with infinite symbol extension. In addition, the case of different number of antennas in the $2 \times 2 \times 2$ network was studied in [18], where the relays were full-duplex and amplify-and-forward relays. The authors proposed a scheme that only uses linear transceivers to achieve the cut-set outer bound in most cases of arbitrary number of antennas at each node. {However, they did not achieve the outer bound in the case when all the nodes are equipped with the same number of antennas. In [19], the DoF region was established in the general case where the six terminals were equipped with arbitrary number of antennas each. The authors also showed that this outer bound is achievable using a concatenated communication scheme that consists of linear vector space joint beamforming. On the other hand, alternate transmission between the relays was not studied for the single antenna or for the multiple antenna case.

In [20], two shared multi-antenna relays were deployed in a network to assist the transmission from 2 groups of single antenna users towards two base stations equipped with M antennas each. Consequently, two scenarios were discussed throughout [20]. First, it was assumed that the relays are full-duplex and global channel state information (CSI) is available at the relays for the two hops. In this case, a scheme, that uses interference shaping at the receivers and interference neutralization at the transmitters, was introduced to achieve $2M$ DoF which is the outer bound of this network. Due to the difficulty of implementing full-duplex relays and to avoid the inter-relay interference caused by the full-duplex relays, half-duplex relaying was considered. However, only $\frac{2M}{3}$ DoFs were achieved if the relays are half-duplex and equipped with M antennas each.

B. Paper Contributions

In this paper, we consider a two-cell relaying network that consists of two base stations and two mobile stations, where each node is equipped with M antennas. Each base station aims to deliver a distinct message to each mobile station. We assume there is no direct link between the base stations and the mobile stations. Therefore, each base station transmission is assisted by two relays. We assume that the relays are half-duplex, decode-and-forward, and equipped with N antennas each. Note that, this scenario may arise in a wireless linear (highway) cellular network, where the geographical coverage area is divided into cells aligned in a line. The mobiles, which are located at the cell edges, are attached to two base stations during the soft handoff process. In addition, the excessive propagation path loss necessitates the use of relays. We consider two different relaying cases; namely, shared relaying and dedicated relaying. In the shared relays case, the base stations share two relays. On the other hand, in the dedicated relays case, each base station has two dedicated relays. Although the relays can receive the signals from both base stations, they are not capable of decoding the signal received from the other base station. As the transmission from each base station is aided by two relays in both cases, the relays can operate either simultaneously or alternately. In the simultaneous mode, the two relays receive in one time slot and transmit in the subsequent time slot simultaneously. In contrast, in the alternate transmission, while one relay is receiving, the other one transmits to the mobile stations.

Part of this work appeared in [21], in which we considered all possible relaying architectures for the dedicated relaying configuration, where the base station can use the relays either simultaneously or alternately. For those schemes we derived upper bounds on the DoF of the network as a function of the number of antennas at all nodes. Moreover, we proposed an achievable scheme based on interference alignment and zero forcing techniques to seek the upper bound. The proposed scheme achieves the upper bound except for $M < N < \frac{5M}{2}$.

Here, the shared relaying configuration is also considered, where the base stations are assisted by the same relays. We derive outer bounds on the DoF of this network. Also, we propose an achievable scheme that attains the derived upper bound for all values of M and N. In addition, we compare the achievable DoF of the shared and dedicated relaying networks. Hence, we prove that for the same total number of antennas at the relay stations, shared relaying can achieve higher or equal values of DoF than the dedicated relaying case.

C. Notation and Organization of the Paper

The remainder of this paper is organized as follows. We describe the system model in Section II. Then, we derive outer bounds on the DoF of different transmission modes in Section III. In Section IV, achievable schemes are proposed to seek the DoF upper bounds. Finally, we conclude the paper in Section V.

The following notation is used throughout the manuscript. We employ upper and lower case boldface symbols to denote matrices and vectors, respectively. For any general matrix $\mathbf{X}$, $\mathbf{X}^{-1}$, $\mathbf{X}^{\mathbf{H}}$, $\mathbf{X}^{\dagger}$, $\mathcal{S}\{\mathbf{X}\}$, $\mathcal{N}\{\mathbf{X}\}$ and $\Lambda^{(i)}(X)$ denote the inverse, the Hermitian transpose, the pseudo inverse, the span of the column vectors, the null space of $\mathbf{X}$, and eigenvectors that correspond to the maximum i eigenvalues of the matrix $\mathbf{X}$ respectively.

II. System Model

We consider a two-cell relaying network consisting of two base stations and two mobile stations denoted respectively by B_i, M_i where $i \in 1, 2$. We assume that there is no direct communication link between the base stations and the mobile stations as in [22], [23]. Each base station has two independent messages; one for each mobile station, that are transmitted through two relays in both networks. The base stations and the mobile stations are equipped with M antennas, whereas the relays are half-duplex, decode-and-forward and equipped with N antennas each.

We consider two configurations for the relays. In the first configuration, the communication is assisted by four relays denoted by R_i^j where i and $j \in \{1, 2\}$, and R_i^j denotes the i^{th} relay dedicated to the j^{th} base station. In the second configuration, two shared relays, denoted by R_i where $i \in \{1, 2\}$, are used to deliver the messages from the two base stations to the two mobiles.

The channel between any two nodes is modelled as a block fading channel and is constant during one time slot. We denote the channel matrix from node X to node Y by $\boldsymbol{H}_{YX} \in \mathbb{C}^{N_Y \times N_X}$, where N_X denotes the number of antennas at node X. In addition, all channel coefficients are assumed to be independent and identically distributed, thus they are full rank. In this work, global, perfect, and instantaneous CSI is assumed to be available at all nodes. In our future work, we will consider more practical assumptions on the CSI availability, e.g., when the CSI is available unavailable at some nodes or available with delay [24].

Since the relays are half-duplex, communication between the base stations and the mobile stations occurs in two hops where each relay decodes its received signal before retransmitting it in the subsequent time slot. Since each base station can use two relays to deliver its messages to the mobiles, there are only two possible ways for operating the relays: simultaneous and alternate relaying. In simultaneous relaying, the base station transmits to the two relays in one time slot while the relays transmit to the mobile stations in the next time slot. In contrast, in alternate relaying, each time slot the base station transmits to one of the relays while the second relay forwards the message it received from the base station in the previous time slot to the mobile stations. *Therefore, the transmitting relays cause interference at the receiving relays.*

For the first network configuration, there are four different combinations of choices to operate the relays at each cell. We denote these choices by the relaying mode of operating the relay by each base station. In the simultaneous-simultaneous relaying scheme, the two base stations use simultaneous relaying and transmit simultaneously to the relays in one time slot while all the relays transmit simultaneously to the base stations in the second time slot. In contrast, in the alternate-alternate relaying scheme, the two base stations use alternate relaying where in each time slot each base station transmits to one of its dedicated relays while the other relay transmits to the mobile users. Finally, there are two combinations that correspond to the simultaneous-alternate relaying scheme, where one base station employs alternate relaying while the second employs simultaneous relaying. From the symmetry of our network, it suffices to consider only one of these two combinations. Hence, a total of three relaying schemes will be considered.

In the second network configuration where two shared relays are employed by the two base stations, the choices for operating the relays are either to use them simultaneously or alternately. In the first scenario, the base stations transmit their signals to the relays in one time slot and the relays simultaneously forward the received messages to the mobiles in the next time slot. We refer to this scheme as the *simultaneous relaying scheme*. On the other hand, in the *alternate relaying scheme*, the relays forward their messages to one relay while the other relay transmits the signal received during the previous time slot.

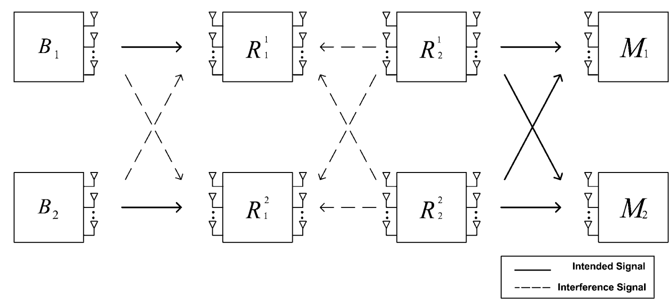

Fig. 1. First time slot transmission in alternate-alternate relaying.

For each relaying scheme in the dedicated and shared relays configurations, we derive an information-theoretic upper bound on the total DoF of the network. Due to the symmetry between the two hops of all relaying schemes except simultaneous-alternate relaying scheme, we assume that the network operates in both hops of these relaying schemes for equal periods(time slots). In contrast, for simultaneous-alternate relaying scheme, we assume that the network operates for different periods in each hop. Afterwards, for each value of M/N, we derive the proposed achievable scheme seeking the highest upperbound among those derived for all possible relaying schemes. The proposed schemes achieve the upper bound except for $M \leq N \leq \frac{5M}{2}$ in dedicated relays configuration.

III. Upper Bounds on the DoF

A. Dedicated Relays Configuration

In this case, each base station uses two dedicated relays. The relays can decode only the messages received from their designated base station. This case was considered in [25] and termed "one way relaying." It might arise due to the unavailability of the codebook of the neighboring base station or due to proximity or power control considerations. As discussed before, each base station can use its dedicated relays either simultaneously or alternately leading to three operation cases for the network. In this subsection, we derive an upper bound on the DoF in each case.

1) Case 1: Alternate-Alternate Relaying: In this scheme, the two base stations employ alternate relaying. Let the two base stations transmit in the first time slot to the relays R_1^k, where the k^{th} base station is targeting the relay R_1^k, while the other relays, namely R_2^k, transmit simultaneously to the mobile stations as shown in Fig. 1. In the second time slot, the k^{th} base station transmits to relay R_2^k while the relays $\{R_1^k\}_{k=1}^2$ forward the message received in the previous time slot to the mobile stations.

Theorem 1: The total number of DoF of the alternate-alternate relaying scheme is upper bounded by

$$\eta_{a-a} \leq \min\{N, 2M\}. \tag{2}$$

Proof: Since the system is symmetric, we consider the first time slot only. Let us allow cooperation between the two base stations B_1 and B_2, the two receiving relays R_1^1 and R_1^2, the two transmitting relays R_2^1 and R_2^2 and the two mobile stations

M_1 and M_2. This cooperation can not decrease the DoF. After cooperation, we obtain a Z-Channel with two sources (T_1; the cooperating base stations, and T_2; the cooperating transmitting relays), and 2 receivers (S_1; the cooperating receiving relays, and S_2; the cooperating mobile stations). Let $d_{i,j}$ denotes the number of non-interfering streams transmitted from T_i to S_j. From [1], the outer bound on the DoF of this Z-Channel is given by $\eta_z \leq \max\{N_{S_1}, N_{T_2}\}$ where $\eta_z = d_{1,1} + d_{2,1} + d_{2,2}$ and N_X denotes the number of antennas at node X. Hence,

$$d_{1,1} + d_{2,1} + d_{2,2} \leq 2N. \tag{3}$$

As there are no intended messages from the transmitting relays to the receiving ones, we can set $d_{2,1}$ to be zero. Note that $d_{1,1}$ and $d_{2,2}$ represent the DoF of the first and second hops of transmission respectively. As the transmitted streams from the relays to the mobile stations in one time slot are those transmitted from the base stations to the relays in the previous time slot, the achievable DoF of the alternate-alternate relaying system is bounded by the minimum of $d_{1,1}$ and $d_{2,2}$. As a result, the DoF of the alternate-alternate transmission in bounded by

$$\eta_{a-a} \leq N. \tag{4}$$

Next, we consider the second hop where the relays R_2^1 and R_2^2 transmit in the same time to the mobile stations. Since each relay has a message to each mobile station, the second hop transmission can be considered as a two-user MIMO X-channel, i.e., two transmitters equipped with N antennas each and two receivers equipped with M antennas each, where each transmitter has two independent messages for each receiver. Since the DoF of the alternate-alternate relaying scheme are less than or equal to the DoF achieved in the second hop, we have

$$\eta_{a-a} \leq \min\left\{2M, 2N, \frac{4}{3}\max\{M, N\}\right\} \tag{5}$$

where we have utilized the outer bound on the DoF of the two-user X-channel in [1]. Using (5) and (4), we can get (2). □

2) Case 2: Simultaneous-Alternate Relaying: For the sake of symmetry, we assume that each base station switches between alternate and simultaneous relaying every two transmission periods. That is, one complete cycle consists of four transmission periods. Each transmission period goes on for α_i where $i \in \{1, 2, 3, 4\}$, i.e., we divide the total transmission time into four periods.

In the first period, B_1 transmits to R_1^1 and R_2^1 while B_2 transmits to R_1^2. At the same time, R_2^2 transmits to the mobile stations as depicted in Fig. 2(a). In the second transmission period, R_1^1, R_2^1 and R_1^2 transmit their signals to the mobile stations while B_2 transmits to R_2^2 as shown in Fig. 2(b). In the third period, B_1 transmits to R_1^1 while B_2 transmits to R_1^2 and R_2^2 as R_2^1 transmits to mobile stations. In the fourth transmission period, B_1 transmits to R_2^1 while R_1^1, R_1^2 and R_2^2 transmit to the mobile stations.

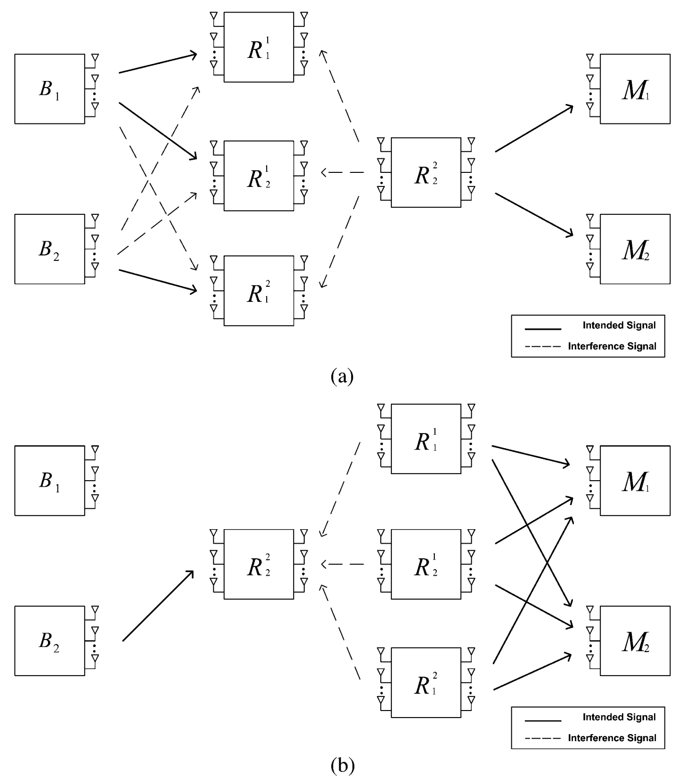

Fig. 2. Transmission in simultaneous-alternate relaying: (a) First time slot, and (b) second time slot.

Theorem 2: The total number of DoF of the simultaneous-alternate relaying scheme is upper bounded by

$$\eta_{s-a} \leq \min\left\{\frac{3N}{2}, \frac{N+M}{2}, \frac{3M}{2}, \max\left\{\frac{5N}{6}, \frac{N}{2} + \frac{M}{3}\right\}\right\}. \tag{6}$$

Proof: Let $d_{i,j}^k$ denote the number of interference-free streams transmitted from B_i in the k^{th} time slot and intended for M_j. Due to symmetry, we need to consider only the first and second transmission periods. Since the two base stations transmit simultaneously in the first transmission period while only B_2 transmits in the second transmission period, we have

$$\eta_{s-a} \leq \left(d_{1,1}^1 + d_{1,2}^1 + d_{2,1}^1 + d_{2,2}^1 + d_{2,1}^2 + d_{2,2}^2\right). \tag{7}$$

In the first transmission period, let us allow full cooperation between relays R_1^1 and R_2^1. Furthermore, let us remove the interfering signal transmitted from R_2^2. As result, we have a 2-user MIMO interference channel (IC) with 2 sources ($T_1 = B_1$ and $T_2 = B_2$) and 2 destinations (S_1; the cooperating relays, and $S_2 = R_1^2$), In [2], the DoF of the MIMO-IC was shown to be $\eta_{IC} = \min\{N_{T_1} + N_{T_2}, N_{S_1} + N_{S_2}, \max\{N_{T_1}, N_{S_2}\}, \max\{N_{T_2}, N_{S_1}\}\}$. Applying this result on the resulting IC we get

$$d_{1,1}^1 + d_{1,2}^1 + d_{2,1}^1 + d_{2,2}^1 \leq \alpha_1 \min\{2M, 3N, \max\{M, N\}\} \tag{8}$$

where the inequality is due to ignoring the interference from R_2^2. Thereafter, we consider the transmitted signal from R_2^2 in the first transmission period. This transmission can be considered as a broadcast channel (BC). Hence, the outer bound for $d^2_{2,1} + d^2_{2,2}$ can be written as

$$d^2_{2,1} + d^2_{2,2} \leq \alpha_1 \min\{N, 2M\}. \tag{9}$$

The transmitted signal from the R^2_2 in the first transmission period represents an interference at the receiving relays. This interference signal spans $d^2_{2,1} + d^2_{2,2}$ dimensions at each receiving relay. As a result, the receiving relays R^1_1, R^1_2, and, R^2_1 can not decode more than $N - d^2_{2,1} - d^2_{2,2}$ streams. Therefore, the number of the transmitted streams is bounded by

$$\tau\left(d^1_{1,1} + d^1_{1,2}\right) + d^2_{2,1} + d^2_{2,2} \leq \alpha_1 N, \tag{10}$$

$$(1-\tau)\left(d^1_{1,1} + d^1_{1,2}\right) + d^2_{2,1} + d^2_{2,2} \leq \alpha_1 N, \tag{11}$$

$$d^1_{2,1} + d^1_{2,2} + d^2_{2,1} + d^2_{2,2} \leq \alpha_1 N \tag{12}$$

where τ is the fraction of $d^1_{2,1} + d^1_{2,2}$ streams decoded and forwarded by R^1_1.

In the second transmission period, let us consider the transmission between B_2 and R^2_2 as a point to point transmission by removing all interfering signals from other relays, then the outer bound for $d^2_{2,1} + d^2_{2,2}$ is given by

$$d^2_{2,1} + d^2_{2,2} \leq \alpha_2 \min\{N, M\}. \tag{13}$$

Thereafter, we consider the transmission from the relays R^1_1, R^1_2 and R^2_1 in the second transmission period as a 3×2 X-Network. Let us allow full cooperation between R^1_1 and R^1_2, thus we get a MIMO X-Channel with 2 sources (T_1; the cooperating relays, $T_2 = R^2_1$) and 2 destinations ($R_1 = M_1$ and $R_2 = M_2$), In [1], the DoF of the MIMO X-Channel have been characterized, and hence we can apply the derived bounds and get

$$\begin{aligned} &d^1_{1,1} + d^1_{1,2} + d^1_{2,1} + d^1_{2,2} \\ &\quad\leq \alpha_2 \min\left\{2M, 3N, \max\left\{\frac{N}{2} + M, \frac{5N}{2}\right\}, \frac{2}{3}(2N+M)\right\}. \end{aligned} \tag{14}$$

In addition, the number of DoF achieved by all messages associated with transmitter n and receiver m in the X-Network is upper bounded by $\max\{N_m, N_n\}$, where N_X is the number of antennas at terminal X. Applying this bound on all possible pair we get

$$d^1_{1,1} + d^1_{1,2} + d^1_{2,1} + d^1_{2,2} \leq \alpha_2 \frac{3}{2} \max\{M, N\}. \tag{15}$$

The receiving relay R^2_2 suffers from interference signals that are caused by the transmission of the relays R^1_1, R^1_2 and R^2_1. Hence, the relay R^2_2 can not decode streams more than $N - \max\{\tau(d^1_{1,1} + d^1_{1,2}), (1-\tau)(d^1_{1,1} + d^1_{1,2}), d^1_{2,1} + d^1_{2,2}\}$ streams. As a result, the received streams are upper bounded by

$$d^2_{2,1} + d^2_{2,2} + \tau\left(d^1_{1,1} + d^1_{1,2}\right) \leq \alpha_2 N, \tag{16}$$

$$d^2_{2,1} + d^2_{2,2} + (1-\tau)\left(d^1_{1,1} + d^1_{1,2}\right) \leq \alpha_2 N, \tag{17}$$

$$d^2_{2,1} + d^2_{2,2} + d^1_{2,1} + d^1_{2,2} \leq \alpha_2 N. \tag{18}$$

Next, we define $D^1_1 = d^1_{1,1} + d^1_{1,2}$, $D^1_2 = d^1_{2,1} + d^1_{2,2}$ and $D^2_2 = d^2_{2,1} + d^2_{2,2}$. By adding (10), (11), and (12), we obtain

$$D^1_1 + D^1_2 + 3D^2_2 \leq 3\alpha_1 N. \tag{19}$$

Similarly, by adding (16), (17), and (18), we get

$$D^1_1 + D^1_2 + 3D^2_2 \leq 3\alpha_2 N. \tag{20}$$

On the other hand, from (8), (15) and (15), $D^1_1 + D^1_2$ is bounded by

$$D^1_1 + D^1_2 \leq \alpha_1 \min\{2M, 3N, \max\{M, N\}\}, \tag{21}$$

$$D^1_1 + D^1_2 \leq \alpha_2 \min\left\{2M, 3N, \max\left\{\frac{N+2M}{2}, \frac{5N}{2}\right\}, \frac{2}{3}(2N+M)\right\}, \tag{22}$$

$$D^1_1 + D^1_2 \leq \alpha_2 \frac{3}{2} \max\{M, N\}. \tag{23}$$

and from (13) and (9), D^2_2 is bounded by

$$D^2_2 \leq \alpha_1 \min\{N, M\}, \tag{24}$$

$$D^2_2 \leq \alpha_2 \min\{N, 2M\}. \tag{25}$$

The theorem is proved by solving the following linear program

$$\begin{aligned} \max_{D^1_1, D^1_2, D^2_2, \alpha_1, \alpha_2} \quad & D^1_1 + D^1_2 + D^2_2 \\ \text{subject to} \quad & (19), (20), \cdots, (25) \\ & D^j_i \geq 0 \quad (i,j) \in \{(1,1), (2,1), (2,2)\} \\ & \alpha_1 + \alpha_2 = 1. \end{aligned} \tag{26}$$

We replace α_2 in (26) by $1 - \alpha_1$. Hence, this linear program in $\mathbb{R}^4$ can be reformulated as

$$\max_{\mathbf{x}} \ \mathbf{c}^{\mathbf{T}} \mathbf{x} \quad \text{subject to } \mathbf{A}\,\mathbf{x} \leq \mathbf{b},$$

where $\mathbf{x} = [D^1_1 \quad D^1_2 \quad D^2_2 \quad \alpha_1]$. The problem is solved by same procedure as (79) in the Appendix which yields the following upper bound on $D^1_1 + D^1_2 + D^2_2$

$$D^1_1 + D^1_2 + D^2_2 \leq \begin{cases} 3N & \text{for } N \leq \frac{M}{3} \\ N + \frac{2M}{3} & \text{for } \frac{M}{3} \leq N \leq M \\ \frac{5N}{3} & \text{for } M \leq N \leq \frac{3M}{2} \\ M + N & \text{for } \frac{3M}{2} \leq N \leq 2M \\ 3M & \text{for } N \geq 2M \end{cases}. \tag{27}$$

□

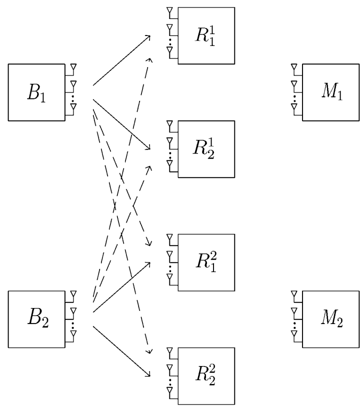

Fig. 3. First time slot transmission in Simultaneous-Simultaneous relaying.

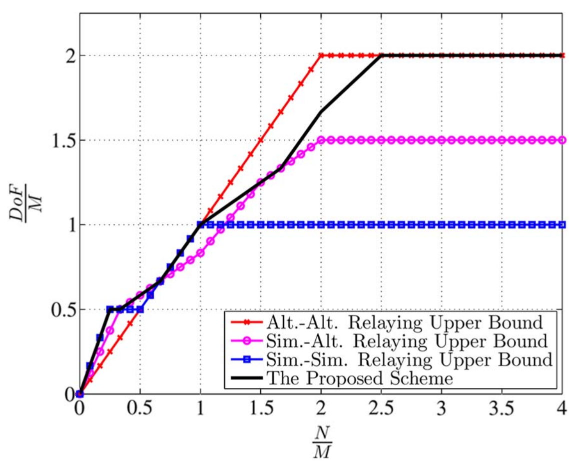

Fig. 4. Outer bounds and the achievable DoF for the dedicated relays configuration.

3) *Case 3: Simultaneous-Simultaneous Relaying:* In this case, both base stations transmit to their relays simultaneously, i.e., in the first hop B_k transmits to R_k^j where $k, j \in \{1, 2\}$, as shown in Fig. 3, and in the second hop the relays transmit to the mobile stations.

Theorem 3: The total number of DoF of the simultaneous-simultaneous relaying scheme is upper bounded by

$$\eta_{s-s} \leq \min\left\{2N, M, \frac{1}{2}\max\{M, 2N\}\right\}. \tag{28}$$

Proof: The total DoF of the simultaneous-simultaneous relaying scheme is upper bounded by the minimum of the achievable DoF of two hops. In the first hop, let us we allow full cooperation between R_1^1 and R_2^1 and between R_1^2 and R_2^2. As result, we obtain an IC with two transmitters and two receivers. Each transmitter is equipped with M antennas while the receivers have $2N$ antennas each. From [2], the outer bound of DoF of simultaneous-simultaneous relaying scheme is given by (28). □

The upper bounds in (2), (6) and (28) are shown in Fig. 4 for different values of $\frac{N}{M}$.

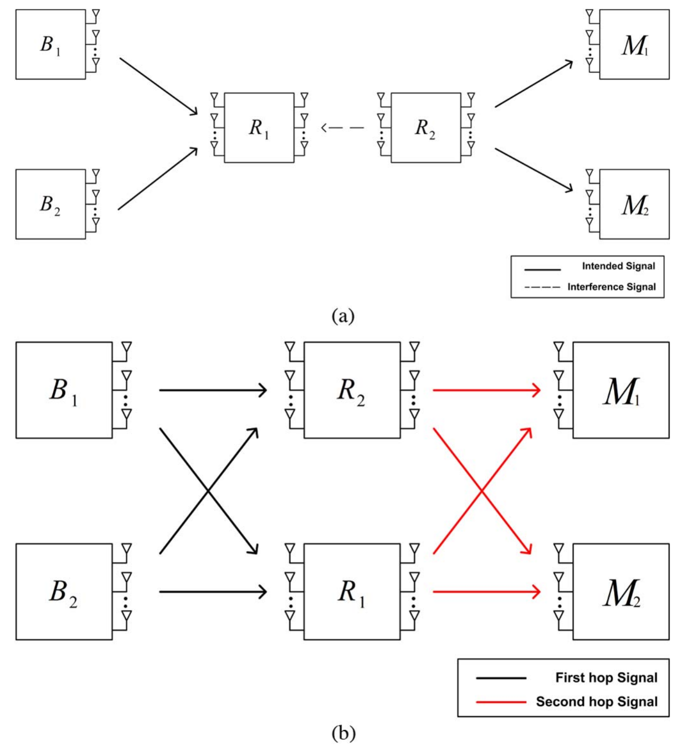

Fig. 5. Transmission in shared relays configuration: (a) Alternate relaying, and (b) simultaneous relaying.

B. Shared Relays Configuration

In this configuration, both base stations are aided by both relays. Hence, the relays can be used either simultaneously or alternately. The upper bounds for these cases are obtained as follows:

1) *Case 1: Alternate Relaying:* In this scheme, the base stations alternately use the two relays to transmit to the mobile stations. In the first time slot, the base stations transmit to the relay R_1 while the other relay R_2 forwards its received messages in the preceding time slot to the mobile stations. During the second time slot, the relay R_2 receives from the base stations while the relay R_1 transmits to the mobiles. Fig. 5(a) shows the transmission in the first time slot.

Theorem 4: The total number of DoF of the alternate relaying scheme is upper bounded by

$$\eta_a \leq \min\left\{\frac{N}{2}, 2M\right\}. \tag{29}$$

Proof: We can consider the second hop as a Broadcast Channel (BC) with one transmitter having N antennas (the relay), and two receivers having M antennas each (the mobile stations). The DoF of this network is upper bounded by $\min\{N, 2M\}$. In addition, the receiving relay receives inevitable interference caused by the transmitting relay. Note that the signal subspace and the interference subspace at the receiving relay can not exceed N dimensions. Since these subspaces represent the total streams transmitted from the base stations in two consecutive time slots, the total DoF of this scheme is bounded by $\frac{N}{2}$. Combining the two bounds above we can get (29). □

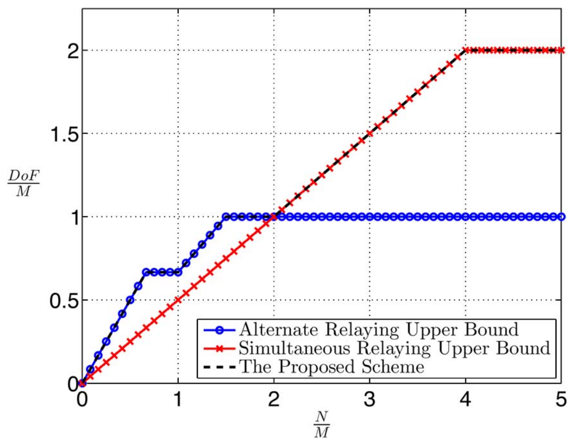

Fig. 6. Outer bounds and the achievable DoF for the shared relays configuration.

2) Case 1: Simultaneous Relaying: In this case, both base stations transmit to the relays simultaneously and in the second hop the relays transmit to the mobile stations, as shown in Fig. 5(b).

Theorem 5: The total number of DoF of the simultaneous relaying scheme is upper bounded by

$$\eta_s \leq \min\left\{N, M, \frac{2}{3}\max\{M, N\}\right\}. \tag{30}$$

Proof: In the first hop, the transmission from the base station to the relays can be considered as a two-user X-channel. Accordingly, the DoF of this channel, where two transmitters with M antennas each transmit to two receivers with N antennas each, are bounded by $\eta \leq \left\{2M, 2N, \frac{4}{3}\max\{M, N\}\right\}$ [1]. As the total DoF of the network is bounded by the minimum DoF of the two hops and taking into consideration that the transmission takes place in two hops, the DoF of this scheme is bounded as in (30). □

Fig. 6 shows the upper bounds (29) and (30) for different values of $\frac{N}{M}$.

IV. Achievable Schemes

A. Dedicated Relays Configuration

Fig. 4 shows the upper bound on the DoF of the network for different relay architectures versus the ratio $\frac{N}{M}$. We can see from this figure that the simultaneous-simultaneous relaying scheme has an upper bound with higher DoF at small values of $\frac{N}{M}$ while the alternate-alternate relaying scheme is superior at high values of $\frac{N}{M}$. As a result, the proposed achievable scheme will depend on the ratio between N and M.

1) Achievable Scheme for $N \leq \frac{M}{3}$: From the previous section, the outer bound for the DoF of the simultaneous-simultaneous relaying scheme, given by $\eta_{s-s} \leq \min\left\{2N, \frac{M}{2}\right\}$, surpasses the other outer bounds for $N \leq \frac{M}{3}$. Hence, the simultaneous-simultaneous relaying scheme will be used for this region. The proposed scheme in this region is based on zero-forcing. Each base station sends $2d$ symbols to its dedicated relays in the first time slot. Each relay decodes d symbols and then transmits them to the mobile stations in the second time slot. The transmitted signal from the k^{th} base station is given by

$$\boldsymbol{x}_{B_k} = \boldsymbol{W}_{R_1^k B_k}\boldsymbol{x}_{R_1^k B_k} + \boldsymbol{W}_{R_2^k B_k}\boldsymbol{x}_{R_2^k B_k} \tag{31}$$

where $\boldsymbol{W}_{YX} \in \mathbb{C}^{N_x \times d}$ denotes the linear precoding matrix employed by terminal X with N_x antennas to transmit to terminal Y and $\boldsymbol{x}_{YX} \in \mathbb{C}^d$ is the transmitted symbol vector. The received signal at the relay R_i^j is given by

$$\boldsymbol{y}_{R_i^j} = \boldsymbol{H}_{R_i^j B_1}\boldsymbol{x}_{B_1} + \boldsymbol{H}_{R_i^j B_2}\boldsymbol{x}_{B_2} + \boldsymbol{z}_{R_i^j} \tag{32}$$

where $\boldsymbol{z}_X \in \mathbb{C}^{N_x}$ is the received noise vector at terminal X.

In the second time slot, the transmitted signals from relay R_i^j is given by

$$\boldsymbol{x}_{R_i^j} = \boldsymbol{W}_{M_1 R_i^j}\boldsymbol{x}_{M_1 R_i^j} + \boldsymbol{W}_{M_2 R_i^j}\boldsymbol{x}_{M_2 R_i^j} \tag{33}$$

where $\boldsymbol{W}_{M_n R_i^j} \in \mathbb{C}^{N \times d_n}$ for $n \in \{1, 2\}$ and d_n denotes the number of streams transmitted from the relay to M_n. Note that $d_1 + d_2 = d$. Thus, each relay distributes its received/transmitted streams between the two mobile users. Since the relays are identical, we use the same distribution for all the relays. The received signal at mobile station M_n in the second time slot is given by

$$\boldsymbol{y}_{M_n} = \sum_{j=1}^{2}\sum_{i=1}^{2}\boldsymbol{H}_{M_n R_i^j}\boldsymbol{x}_{R_i^j} + \boldsymbol{z}_{M_n}. \tag{34}$$

For $N \leq \frac{M}{4}$, the outer bound for the DoF of the system is $\eta_{s-s} < 2N$. Therefore, in the proposed scheme, the precoder matrices $\boldsymbol{W}_{R_i^j B_k}$ where $i, j \in \{1, 2\}$, are chosen to be of dimension N to achieve the outer bound. The precoder matrices at B_1 are chosen such that

$$\mathcal{S}\left\{\boldsymbol{W}_{R_1^1 B_1}\right\} \subseteq \left\{\mathcal{N}\left\{\boldsymbol{H}_{R_2^1 B_1}\right\} \cap \mathcal{N}\left\{\boldsymbol{H}_{R_1^2 B_1}\right\} \cap \mathcal{N}\left\{\boldsymbol{H}_{R_2^2 B_1}\right\}\right\}, \tag{35}$$

$$\mathcal{S}\left\{\boldsymbol{W}_{R_1^2 B_1}\right\} \subseteq \left\{\mathcal{N}\left\{\boldsymbol{H}_{R_1^1 B_1}\right\} \cap \mathcal{N}\left\{\boldsymbol{H}_{R_1^2 B_1}\right\} \cap \mathcal{N}\left\{\boldsymbol{H}_{R_2^2 B_1}\right\}\right\}. \tag{36}$$

Since all the channel matrices are full rank, $\boldsymbol{W}_{R_i^j B_k} \in \mathbb{C}^{M \times N}$ can be found since the dimension of the intersection between the three nullspaces in each of the above two equations is $M - 3N$ which is always greater than or equal to N.

Similarly, we choose $\boldsymbol{W}_{R_1^2 B_2}$ and $\boldsymbol{W}_{R_2^2 B_2}$ to lie in the intersection of the nullspaces of the channel matrices to the nontarget relays. As a result, the received signal at R_1^1 in (32) will be

$$\boldsymbol{y}_{R_1^1} = \boldsymbol{H}_{R_1^1 B_1}\boldsymbol{W}_{R_1^1 B_1}\boldsymbol{x}_{R_1^1 B_1} + \boldsymbol{z}_{R_1^1}. \tag{37}$$

Since $\boldsymbol{H}_{R_1^1 B_1}\boldsymbol{W}_{R_1^1 B_1} \in \mathbb{C}^{N \times N}$ is full rank as can be derived easily from Lemma 2 in [26], the relay can decode N streams from the received signal. The same can be applied to other relays.

For the second hop, the received signal at each mobile station has a dimension of $4N$ which is less than or equal to M. Hence, the mobile stations can decode $4N$ streams perfectly. The decoding matrices at the n^{th} mobile station, $\boldsymbol{U}_{M_n} \in \mathbb{C}^{M\times 4d_n}$, where $d_1 + d_2 = N$, will be designed such that

$$\boldsymbol{U}_{M_n} \subseteq \mathcal{N}\left\{\left[\boldsymbol{J}_{n,m}^{(1,1)},\ \boldsymbol{J}_{n,m}^{(2,1)},\ \boldsymbol{J}_{n,m}^{(1,2)},\ \boldsymbol{J}_{n,m}^{(2,2)}\right]\right\} \quad (38)$$

where $\boldsymbol{J}_{n,m}^{(i,j)} = \boldsymbol{H}_{M_nR_i^j}\boldsymbol{W}_{M_mR_i^j}$ is the subspace containing the interference received at the n^{th} mobile user due to the transmission of the relay R_i^j that is intended for mobile user m and $n, m \in \{1, 2\}, n \neq m$.

For $\frac{M}{4} \le N \le \frac{M}{3}$, we can achieve $\frac{M}{2}$ DoF by simply powering-off $\left(N - \frac{M}{4}\right)$ antennas at the relays and then applying the same scheme with $N = \frac{M}{4}$.

2) Achievable Scheme for $\frac{M}{3} \le N \le \frac{2M}{3}$: In the case when $\frac{M}{3} \le N \le \frac{2M}{3}$, the DoF for the simultaneous-alternate relaying scheme is bounded by $\eta_{s-a} \le \frac{M}{3} + \frac{N}{2}$ which surpasses the DoF outer bound of the simultaneous-simultaneous and alternate-alternate relaying schemes. Consequently, the proposed scheme will use simultaneous-alternate relaying in this region. Due to symmetry, we will focus only on the first and second time slots. In the first time slot, R_1^1, R_2^1 and R_1^2 will receive from their associated base stations, while R_2^2 transmits to the mobile stations.

In the first time slot, let R_2^2 pick randomly its precoding matrix as $\boldsymbol{W}_{R_2^2} = [\boldsymbol{W}_{M_1R_2^2}\ \boldsymbol{W}_{M_2R_2^2}] \in \mathbb{C}^{N\times\left(N-\frac{M}{3}\right)}$. Therefore, the transmitted signal from R_2^2 is given by

$$\boldsymbol{x}_{R_2^2} = \boldsymbol{W}_{M_1R_2^2}\,\boldsymbol{x}_{M_1R_2^2} + \boldsymbol{W}_{M_2R_2^2}\,\boldsymbol{x}_{M_2R_2^2}. \quad (39)$$

The received signal at each mobile station has a dimension of $N - \frac{M}{3}$ which is less than or equal to M. Hence, the mobile stations can decode $N - \frac{M}{3}$ streams perfectly. The decoding matrices at the n^{th} mobile station, $\boldsymbol{U}_{M_n} \in \mathbb{C}^{M\times d_n}$, where $d_1 + d_2 = N - \frac{M}{3}$, are designed such that

$$\boldsymbol{U}_{M_n} \subseteq \mathcal{N}\left\{\boldsymbol{H}_{M_nR_2^2}\ \boldsymbol{W}_{M_mR_2^2}\right\} \quad m, n \in \{1, 2\}, n \neq m, \quad (40)$$

and hence, the two mobile stations can decode a total of $N - \frac{M}{3}$ streams. Note that the signal transmitted by R_2^2 is considered as interference at the other receiving relays. Let

$$\boldsymbol{I}_{R_n^m} = \boldsymbol{H}_{R_n^mR_2^2}\boldsymbol{W}_{R_2^2} \quad (n, m) \in \{(1, 1), (1, 2), (2, 1)\} \quad (41)$$

denote the interference subspace at R_n^m due to the transmission of R_2^2. We define the matrix $\boldsymbol{N}_{B_jR_n^m}$ which combines the null space of the channel from B_j to R_n^m and the subspace $\boldsymbol{H}^{\dagger}_{B_jR_n^m}\boldsymbol{I}_{R_n^m}$ that represents the image of the interference subspace of R_n^m at B_j, i.e.,

$$\mathcal{S}\left\{\boldsymbol{N}_{B_jR_n^m}\right\} = \mathcal{S}\left\{\left(\boldsymbol{H}^{\dagger}_{R_n^mB_j}\boldsymbol{I}_{R_n^m}\right) \cup \mathcal{N}\left\{\boldsymbol{H}_{R_n^mB_j}\right\}\right\}. \quad (42)$$

Hence, the transmission from B_j using a precoding matrix that belongs to the subspace $\boldsymbol{N}_{B_jR_n^m}$ lies entirely in the interference subspace of R_n^m. Note that the dimension of $\boldsymbol{N}_{B_jR_n^m}$ is given by $N - \frac{M}{3} + M - N = \frac{2M}{3}$.

At the base stations, we align the transmitted signal such that it lies in the interference subspace of the non-intended relays, i.e., we design $\boldsymbol{W}_{R_i^jB_j} \in \mathbb{C}^{M\times\frac{M}{3}}$ such that

$$\mathcal{S}\left\{\boldsymbol{W}_{R_i^jB_j}\right\} = \bigcap_{\forall\,(n,m)\neq(i,j)} \boldsymbol{N}_{B_jR_n^m} \quad (43)$$

where (i, j) and $(n, m) \in \{(1, 1), (1, 2), (2, 1)\}$ in (42) and (43). Since the dimension of $\boldsymbol{N}_{B_jR_n^m}$ is given by $\frac{2M}{3}$ and the dimension of the union of any two of them is M, then the dimension of $\boldsymbol{W}_{R_i^jB_j}$ is given by $\frac{M}{3}$.

Since the interference signals at the receiving relays are already aligned to lie in null spaces of the channel matrices or in its interference subspaces, the decoding matrices at the receiving relays, $\boldsymbol{U}_{R_i^j} \in \mathbb{C}^{N\times\frac{M}{3}}$, are designed such that

$$\boldsymbol{U}_{R_i^j} \subseteq \mathcal{N}\left\{\boldsymbol{I}_{R_i^j}\right\}. \quad (44)$$

Hence, each of R_1^1, R_2^1, and R_1^2 can decode $\frac{M}{3}$ interference-free streams.

In the second time slot, only R_2^2 receives from B_2 while the other relays transmit to the mobile stations. In this case, B_2 picks randomly the precoding matrix $\boldsymbol{W}_{R_2^2B_2} \in \mathbb{C}^{M\times N-\frac{M}{3}}$. Therefore, the signal space at R_2^2 is $\mathcal{S}\{\boldsymbol{H}_{R_2^2B_2}\boldsymbol{W}_{R_2^2B_2}\}$ whose dimension is $N - \frac{M}{3}$ and the dimension of the available interference subspace at R_2^2 is $\frac{M}{3}$. The transmitting relays aim to align their signal at the interference subspace of R_2^2. Accordingly, the precoding matrices at transmitting relays, $\boldsymbol{W}_{R_i^j} \in \mathbb{C}^{N\times\frac{M}{3}}$ where $(i, j) \in \{(1, 1), (2, 1), (1, 2)\}$, must satisfy the following condition

$$\mathcal{S}\left\{\boldsymbol{H}_{R_2^2R_1^1}\boldsymbol{W}_{R_1^1}\right\} = \mathcal{S}\left\{\boldsymbol{H}_{R_2^2R_1^2}\boldsymbol{W}_{R_1^2}\right\} = \mathcal{S}\left\{\boldsymbol{H}_{R_2^2R_2^1}\boldsymbol{W}_{R_2^1}\right\}. \quad (45)$$

In addition, the decoding matrix at R_2^2 is designed basically to cancel the interference from the other relays, which is already aligned to a subspace of dimension $\frac{M}{3}$ as follows

$$\boldsymbol{U}_{R_2^2} \subseteq \mathcal{N}\left\{\boldsymbol{H}_{R_2^2R_1^1}\boldsymbol{W}_{R_1^1}\right\}. \quad (46)$$

At each mobile station, the dimension of the received signal is M as each transmitting relay uses $\frac{M}{3}$ dimensions to transmit its messages. Therefore, these M streams can be decoded perfectly at each mobile station. The decoder of each mobile station can extract its intended message by zero-forcing the non-intended ones.

In summary, using the proposed scheme, the mobile users can decode $N - \frac{M}{3}$ streams in the first time slot and M streams in the second one via the relays without any interference. Thus, the proposed scheme achieves $\frac{M}{3} + \frac{N}{2}$ DoF, and hence, it is DoF-optimal.

3) Achievable Scheme for $N \ge \frac{2M}{3}$: Guided by the results of the upper bound in the previous section, the alternate-alternate relaying scheme will be used in this region.

We will focus on the odd time slots only due to the symmetry between odd and even time slots. In the odd time slots, R_1^1 and R_1^2 are receiving from B_1 and B_2 respectively while R_2^1 and R_2^2

are transmitting to the mobile stations. In the proposed scheme, the precoding matrices of the transmitting relays, each of size $N \times d$, are divided into eight submatrices, i.e., the relays use the first four submatrices to precode the intended signal to M_1 and the last four to precode the messages intended to M_2. The precoding matrix of the relay $\boldsymbol{W}_{R_2^j}$ can be written as

$$\boldsymbol{W}_{R_2^j} = \left[\boldsymbol{W}_{R_2^j}^{(1)},\ \boldsymbol{W}_{R_2^j}^{(2)}, \cdots \boldsymbol{W}_{R_2^j}^{(8)}\right] \quad j \in \{1, 2\} \tag{47}$$

where each submatrix is of size $N \times d_i$ and $i \in \{1, 2, \cdots, 8\}$. As a result, the total number of the transmitted streams from R_2^j is given by $d = \sum_{i=1}^{8} d_i$ and the total number of streams transmitted to the mobile users is $2d$.

The proposed scheme aligns parts of the interference caused by the transmission of R_2^1 and R_2^2 at both R_1^1 and R_1^2. The interference alignment conditions at R_1^1 and R_1^2 are given by

$$\mathcal{S}\left\{\boldsymbol{H}_{R_1^1R_2^1}\boldsymbol{W}_{R_2^1}^{(i)}\right\} \subseteq \mathcal{S}\left\{\boldsymbol{H}_{R_1^1R_2^2}\boldsymbol{W}_{R_2^2}^{(i)}\right\} \quad i \in \{1, 2, 5, 6\}, \tag{48}$$

$$\mathcal{S}\left\{\boldsymbol{H}_{R_1^2R_2^1}\boldsymbol{W}_{R_2^1}^{(i)}\right\} \subseteq \mathcal{S}\left\{\boldsymbol{H}_{R_1^2R_2^2}\boldsymbol{W}_{R_2^2}^{(i)}\right\} \quad i \in \{1, 3, 5, 7\} \tag{49}$$

where (48) aligns the interference caused by the transmission of $\boldsymbol{W}_{R_2^1}^{(i)}$ with that caused by the transmission of $\boldsymbol{W}_{R_2^2}^{(i)}$ at R_1^1. From equations (48) and (49), we can get

$$\boldsymbol{W}_{R_2^1}^{(1)} \subseteq \mathcal{S}\left\{\boldsymbol{H}_{R_1^1R_2^1}^{-1}\boldsymbol{H}_{R_1^1R_2^2}\boldsymbol{W}_{R_2^2}^{(1)}\right\}, \tag{50}$$

$$\boldsymbol{W}_{R_2^2}^{(1)} \subseteq \mathcal{S}\left\{\boldsymbol{H}_{R_1^2R_2^2}^{-1}\boldsymbol{H}_{R_1^2R_2^1}\boldsymbol{W}_{R_2^1}^{(1)}\right\}. \tag{51}$$

As a result, the precoding matrix $\boldsymbol{W}_{R_2^1}^{(1)}$ must satisfy the following condition

$$\boldsymbol{W}_{R_2^1}^{(1)} \subseteq \mathcal{S}\left\{\boldsymbol{H}_{R_1^1R_2^1}^{-1}\boldsymbol{H}_{R_1^1R_2^2}\boldsymbol{H}_{R_1^2R_2^2}^{-1}\boldsymbol{H}_{R_1^2R_2^1}\boldsymbol{W}_{R_2^1}^{(1)}\right\}. \tag{52}$$

Therefore, the columns of the matrix $\boldsymbol{W}_{R_2^1}^{(1)}$ are given by any d_1 eigenvectors of the matrix $\boldsymbol{H}_{R_1^1R_2^1}^{-1}\boldsymbol{H}_{R_1^1R_2^2}\boldsymbol{H}_{R_1^2R_2^2}^{-1}\boldsymbol{H}_{R_1^2R_2^1}$ while the precoding matrix $\boldsymbol{W}_{R_2^2}^{(1)}$ can be determined from (51). Similarly, we can get $\boldsymbol{W}_{R_2^1}^{(5)}$ and $\boldsymbol{W}_{R_2^2}^{(5)}$.

Since the submatrices $\boldsymbol{W}_{R_2^1}^{(i)}$ and $\boldsymbol{W}_{R_2^2}^{(i)}$, where $i \in \{1, 5\}$, were designed to achieve interference alignment at the receiving relays only, transmission using these submatrices causes an interference at the non-intended mobiles. We can use this interference subspace to align the non-intended signal from the remaining submatrices. Hence, we constrain the transmission of the submatrices $\boldsymbol{W}_{R_2^1}^{(i)}$ $\left(\text{and } \boldsymbol{W}_{R_2^2}^{(i)}\right)$, where $i \in \{2, 3, 4\}$, to either lie in the null space of the channel $\boldsymbol{H}_{M_2R_2^1}$ $\left(\text{and } \boldsymbol{H}_{M_2R_2^2}\right)$ or be aligned with the interference signal at M_2 that results from the transmission of $\boldsymbol{W}_{R_2^2}^{(1)}$ $\left(\text{and } \boldsymbol{W}_{R_2^1}^{(1)}\right)$. The following equations describe this constraint

$$\boldsymbol{W}_{R_2^j}^{(i)} \subseteq \mathcal{S}\left\{\boldsymbol{N}_{M_2R_2^j}\right\} \qquad i \in \{2, 3, 4\}, j \in \{1, 2\} \tag{53}$$

where $\boldsymbol{N}_{M_2R_2^j} = \mathcal{N}\{\boldsymbol{H}_{M_2R_2^j}\} \cup \mathcal{S}\{\boldsymbol{H}_{M_2R_2^j}^{\dagger}\boldsymbol{H}_{M_2R_2^{3-j}}\boldsymbol{W}_{R_2^{3-j}}^{(1)}\}$ and $j \in \{1, 2\}$. Note that the dimension of the subspace $\boldsymbol{N}_{M_2R_2^j}$ is given by $(N - M + d_1)^+$ where $(x)^+ = \max\{0,\ x\}$.

Similarly, we choose $\boldsymbol{W}_{R_2^1}^{(i)}$ and $\boldsymbol{W}_{R_2^2}^{(i)}$ such that

$$\boldsymbol{W}_{R_2^j}^{(i)} \subseteq \mathcal{S}\left\{\boldsymbol{N}_{M_1R_2^j}\right\} \qquad i \in \{6, 7, 8\}, j \in \{1, 2\} \tag{54}$$

where $\boldsymbol{N}_{M_1R_2^j} = \mathcal{N}\{\boldsymbol{H}_{M_1R_2^j}\} \cup \mathcal{S}\{\boldsymbol{H}_{M_1R_2^j}^{-1}\boldsymbol{H}_{M_1R_2^{3-j}}\boldsymbol{W}_{R_2^{3-j}}^{(5)}\}$ and its dimension is $(N - M + d_5)^+$. Note that the union of null space and the image of the interference subspace at the transmitter could potentially increase the achievable DoF as we merge two subspaces that can be used for transmission. As a result, in order to suppress the interference at the mobiles, the decoders $\boldsymbol{U}_{M_1}$ and $\boldsymbol{U}_{M_2}$ are designed such that

$$\boldsymbol{U}_{M_1}^H\boldsymbol{H}_{M_1R_2^i}\boldsymbol{W}_{R_2^i}^{(5)} = 0 \quad i \in \{1, 2\}, \tag{55}$$

$$\boldsymbol{U}_{M_2}^H\boldsymbol{H}_{M_2R_2^i}\boldsymbol{W}_{R_2^i}^{(1)} = 0 \quad i \in \{1, 2\}. \tag{56}$$

Next, we consider the design of $\boldsymbol{W}_{R_2^1}^{(i)}$ and $\boldsymbol{W}_{R_2^2}^{(i)}$ for $i \in \{2, 3, 6, 7\}$. In order to satisfy (48) for $i = 2$, the required condition is

$$\boldsymbol{W}_{R_2^2}^{(2)} \subseteq \mathcal{S}\left\{\boldsymbol{H}_{R_1^1R_2^2}^{-1}\boldsymbol{H}_{R_1^1R_2^1}\boldsymbol{W}_{R_2^1}^{(2)}\right\} \tag{57}$$

By substituting from (53) for $i = 2$ and $j = 1$ into (57), we get

$$\boldsymbol{W}_{R_2^2}^{(2)} \subseteq \mathcal{S}\left\{\boldsymbol{H}_{R_1^1R_2^2}^{-1}\boldsymbol{H}_{R_1^1R_2^1}\boldsymbol{N}_{M_2R_2^1}\right\}. \tag{58}$$

In addition to the above condition, the submatrix $\boldsymbol{W}_{R_2^2}^{(2)}$ has to satisfy (53) for $i = 2$ and $j = 2$. Therefore, we design $\boldsymbol{W}_{R_2^2}^{(2)}$ such that

$$\boldsymbol{W}_{R_2^2}^{(2)} \subseteq \mathcal{S}\left\{\boldsymbol{H}_{R_1^1R_2^2}^{-1}\boldsymbol{H}_{R_1^1R_2^1}\boldsymbol{N}_{M_2R_2^1}\right\} \cap \mathcal{S}\left\{\boldsymbol{N}_{M_2R_2^2}\right\}. \tag{59}$$

The intersection between the two subspaces in the above equation exists if $N < 2N - 2M + 2d_1$, and hence, its dimension is given by $(N - 2M + 2d_1)^+$. The submatrix $\boldsymbol{W}_{R_2^1}^{(2)}$ can be determined from (57) as

$$\boldsymbol{W}_{R_2^1}^{(2)} \subseteq \mathcal{S}\left\{\boldsymbol{H}_{R_1^1R_2^1}^{-1}\boldsymbol{H}_{R_1^1R_2^2}\boldsymbol{W}_{R_2^2}^{(2)}\right\}. \tag{60}$$

Similarly, we can find $\boldsymbol{W}_{R_2^1}^{(i)}$ and $\boldsymbol{W}_{R_2^2}^{(i)}$ for $i \in \{3, 6, 7\}$.

In addition, to suppress the interference at the receiving relay R_1^1, we design the decoding matrix $\boldsymbol{U}_{R_1^1}$ such that

$$\boldsymbol{U}_{R_1^1}^H\left[\boldsymbol{H}_{R_1^1R_2^1}\boldsymbol{W}_{R_2^1}, \boldsymbol{H}_{R_1^1R_2^2}\left[\boldsymbol{W}_{R_2^2}^{(3)}, \boldsymbol{W}_{R_2^2}^{(4)}, \boldsymbol{W}_{R_2^2}^{(7)}, \boldsymbol{W}_{R_2^2}^{(8)}\right]\right] = \boldsymbol{0} \tag{61}$$

where the interference from the remaining submatrices of $W_{R_2^2}$ is aligned into the interference space of R_1^1 according to (48). Similarly, the decoding matrix $\boldsymbol{U}_{R_1^2}$ is designed such that

$$\boldsymbol{U}_{R_1^2}^H \left[\boldsymbol{H}_{R_1^2 R_2^1}\boldsymbol{W}_{R_2^1}, \boldsymbol{H}_{R_1^2 R_2^2}\left[\boldsymbol{W}_{R_2^2}^{(2)}, \boldsymbol{W}_{R_2^2}^{(4)}, \boldsymbol{W}_{R_2^2}^{(6)}, \boldsymbol{W}_{R_2^2}^{(8)}\right]\right] = \boldsymbol{0}. \tag{62}$$

Therefore, the receiving relays do not suffer from interference.

Next, we derive the necessary conditions on $\{d_i\}_{i=1}^8$ to ensure the achievability of the proposed design. From equations (53) and (54), we can obtain the following bounds, which are required to be able to choose $\boldsymbol{W}_{R_2^1}^{(i)}$ and $\boldsymbol{W}_{R_2^2}^{(i)}$ for $i \in \{2, 3, 4, 6, 7, 8\}$

$$d_2 + d_3 + d_4 \leq (N - M + d_1)^+, \tag{63}$$

$$d_6 + d_7 + d_8 \leq (N - M + d_5)^+. \tag{64}$$

Also, From (59) and its similar equations for $\boldsymbol{W}_{R_2^j}^{(i)}$, where $i \in \{2, 3, 6, 7\}$, the required conditions to be able to choose these precoding submatrices are given by

$$d_i \leq (N - 2M + 2d_1)^+ \qquad i \in \{2, 3\}, \tag{65}$$

$$d_i \leq (N - 2M + 2d_5)^+ \qquad i \in \{6, 7\}. \tag{66}$$

Moreover, in order to be able to choose $\boldsymbol{U}_{M_1}$ and $\boldsymbol{U}_{M_2}$ from equations (55) and (56) respectively, we obtain the following conditions on the received streams at the mobile stations

$$M - 2d_5 \geq 2(d_1 + d_2 + d_3 + d_4), \tag{67}$$

$$M - 2d_1 \geq 2(d_5 + d_6 + d_7 + d_8) \tag{68}$$

where the R.H.S. represents the desired signal subspace at the mobile station while the L.H.S. is the difference between the number of antennas at the mobile station and the dimension of the interference subspace. Similarly, in order to be able to choose $\boldsymbol{U}_{R_1^1}$ and $\boldsymbol{U}_{R_1^2}$ from equations (61) and (62) respectively, we obtain the following conditions on the received streams at the receiving relays

$$N - d_3 - d_4 - d_7 - d_8 - \sum_{i=1}^{8} d_i \geq \sum_{i=1}^{8} d_i, \tag{69}$$

$$N - d_2 - d_4 - d_6 - d_8 - \sum_{i=1}^{8} d_i \geq \sum_{i=1}^{8} d_i. \tag{70}$$

The maximum achievable DoF can be obtained by maximizing the summation of the transmitted streams by each of R_2^1 and R_2^2 subject to the aforementioned constrains. The solution of the following optimization problem yields the achievable DoF using the proposed scheme

$$\begin{aligned} &\text{maximize} && \sum_{i=1}^{8} d_i \\ &\text{subject to} && (63), (64), \cdots, (69) \\ &&& d_i \geq 0 \qquad i \in \{1, 2, \cdots, 8\} \end{aligned} \tag{71}$$

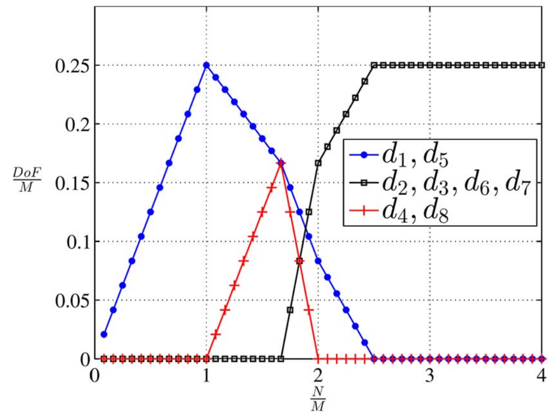

Fig. 7. Values of $d_1, d_2, \cdots, d_8$ that achieve the DoF for all regions.

Note that the above problem can be solved analytically using linear programming, where we obtain the following closed form expression for the achievable DoF

$$\eta = \begin{cases} N & \text{for } N \leq M \\ \frac{1}{2}(N+M) & \text{for } M \leq N \leq \frac{5M}{3} \\ N - \frac{M}{3} & \text{for } \frac{5M}{3} \leq N \leq 2M \\ \frac{2N}{3} + \frac{M}{3} & \text{for } 2M \leq N \leq \frac{5M}{2} \\ 2M & \text{for } N \geq \frac{5M}{2} \end{cases}. \tag{72}$$

where $\eta = 2\sum_{i=1}^{8} d_i$. Fig. 4 shows the number of achievable DoF by the proposed scheme versus the ratio $\frac{N}{M}$ and their relationship to the upper bounds. The derivation of (72) can be found in the Appendix. In addition, Fig. 7 shows the values of $d_1, d_2, \cdots, d_8$ that achieve the DoF versus the ratio $\frac{N}{M}$. At $\frac{N}{M} \leq 1$, the channel matrix between the relay and the mobiles is a thin matrix. Hence, the second hop relays can not exploit the transmission to the null space of the channel matrices of the non-intended mobiles and d_1 and d_5 are used in this region. For d_2, d_3, d_6 and d_7, they have to be less than $(N - 2M + d_1)^+$ or $(N - 2M + 2d_5)^+$. Thus, these d's can not be exploited for $\frac{N}{M} \leq \frac{5}{3}$ where $d_1 = d_5 = \frac{1}{6}$ at $\frac{N}{M} = \frac{5}{3}$. On the other hand, for higher values of $\frac{N}{M}$, we aim to achieve $2M$ DoF so the relays have to avoid causing interference at the mobiles. Hence, d_1 and d_5 can not be used as they cause interference subspaces at the mobiles.

B. Shared Relays Configuration

Fig. 6 shows the outer bounds on the DoF of the shared relays configuration. We can see from this figure that the outer bound for the DoF of the simultaneous transmission scheme surpasses that for alternate transmission for $N \leq 2M$. However, the outer bound for alternate transmission is higher when $N > 2M$.

1) Case 1: $N \leq 2M$: We will use the simultaneous transmission is this region. The outer bound of the region is given by (30). Note that both hops can be considered as two consecutive MIMO X-channels. The bound can be achieved using the interference alignment and cancellation scheme used in [1] for the MIMO X-channel. The transmission schemes depend on the ratio between the number of antennas at the transmitters and the receivers of the X-channel, which denoted by L_T, L_R respectively. If $L_R \leq L_T$, then each transmitter transmits $\min\left\{\frac{L_R}{2}, \frac{2L_T}{6}\right\}$ streams to each receiver. The transmitted streams to a specific receiver are directed to lie totally in the null space of the channel matrix between the transmitter and the non-intended receiver, and each receiver powers on only $\min\left\{L_R, \frac{2L_T}{3}\right\}$ antennas. However, if $L_R > L_T$, each transmitter transmits $\min\left\{\frac{L_T}{2}, \frac{2L_R}{6}\right\}$ streams to each receiver. The precoding matrices at the transmitters are chosen such that the interference signals from both transmitters are aligned to lie in the same subspace, which increases the signal subspaces at both receivers. These scheme can be used for both hops, where $L_T = M$ and $L_R = N$ for the first hop and vice versa for the next hop.

2) Case 2: $N > 2M$: In this region, the outer bound for the alternate transmission is higher than that for simultaneous transmission. Thus, alternate transmission will be used. The outer bound can be described as $\eta \leq \{N/2, 2M\}$. For $2M < N \leq 4M$, the total number of DoF is $\frac{N}{2}$. Therefore, each source aims to transmit an $N/8$-dimensional message to each destination. Due to the symmetry between the time slots, we will describe only the transmission scheme during the time slot where R_1 receives from the base stations and R_2 transmits to the mobile stations. The same scheme can be applied in the other time slots.

The transmitting relay R_2 chooses the predocing matrices $\boldsymbol{W}_{M_iR_2} \in \mathbb{C}^{N\times N/4}$ as follows

$$\boldsymbol{W}_{M_1R_2} \subseteq \mathcal{N}\left\{\boldsymbol{H}_{M_2R_2}\right\}, \tag{73}$$

$$\boldsymbol{W}_{M_2R_2} \subseteq \mathcal{N}\left\{\boldsymbol{H}_{M_1R_2}\right\}. \tag{74}$$

Note that, the null space of the channel matrix between the relay and the mobile has a dimension $N - M$ which is larger that $N/4$ for $N \geq 2M$. Hence, the mobiles do not suffer from interference and only receive their intended messages. Further, the base stations choose their precoding matrices $\boldsymbol{W}_{R_1B_i} \in \mathbb{C}^{M\times\frac{N}{4}}$ randomly. At the receiving relay R_1, zero-forcing is used to eliminate the inter-relay interference, i.e., the decoding matrix $\boldsymbol{U}_{R_1} \in \mathbb{C}^{N\times\frac{N}{2}}$ is designed such that

$$\boldsymbol{U}_{R_1}^H \boldsymbol{H}_{R_1R_2}\left[\boldsymbol{W}_{M_1R_2} \quad \boldsymbol{W}_{M_2R_2}\right] = 0. \tag{75}$$

Hence, R_1 can decode $\frac{N}{2}$ streams received from both base stations perfectly. For $N > 4M$, we can directly achieve $2M$ DoF by powering-off $N - 4M$ antennas at the relays and applying the same scheme as that at $N = 4M$.

Fig. 6 shows the number of achievable DoF versus $\frac{N}{M}$. We can see from this figure that the proposed scheme can achieve the upper bound for all values of $\frac{N}{M}$.

C. Discussion

The maximum number of achievable DoF at both configurations is $2M$. Nevertheless, we can achieve this bound at different values of N in the two configurations, i.e., $N = \frac{5M}{2}$ for dedicated relays case and $N = 4M$ for shared relays configuration. Recall that the total number of relays is 4 in the dedicated relays case while only 2 relays are used in the shared relays case. Fig. 8 shows that the shared relays configuration achieves the same DoF or higher than that can be achieved in the other configuration for the same number of total antennas at the relays. We can refer this to two reasons. First, in the shared relays configuration, the relays can decode the signal received from both base stations. On the other hand, the dedicated relays can decode the signal received from their designated base station only. Second, the shared relays take advantage of the co-located antennas that can jointly process the received signal. Instead of distributing the antennas over four relays in dedicated relays configuration, they are distributed over two relays only in the shared relay configuration.

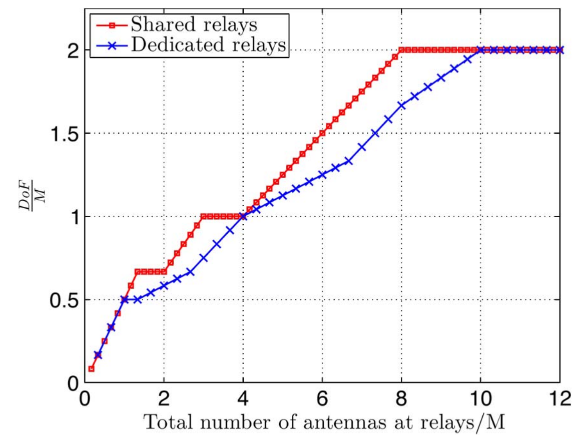

Fig. 8. The total achieved DoF versus the total number of antennas at the relays.

For low values of the total number of antennas at the relays, the simultaneous transmission scheme outperforms alternate transmission. In this scheme, the base stations use the null space of the non-targeted relays to avoid causing interference at the these relays. In the dedicated relays case, the dimension of the intersection between the null spaces at the non-intended relays is given by $M - 3N$. In the shared relays case, the base stations force the transmitted signals to lie completely at the null space of the non-intended relays. Hence, the dimension of the null space in this case is $M - N$. Comparing the dimensions that can be exploited by the base stations in the two configurations, we justify that the shared relays can still align the transmitted interference in the null space of non-intended relays for higher values of total number of antennas.

Unsurprisingly, both configurations can achieve the same DoF when the total number of antennas at relays equals $4M$. In the dedicated relays configuration, the transmitting relays can align their transmission at both receiving relays. Hence, this decreases the dimension of the interference subspace to $\frac{M}{2}$ at the receiving relays and increase the signal subspace which can be used to receive from the base stations. However, in the shared relays configuration, the dimension of the interference subspace at the receiving relay is M. Therefore, dedicated and shared relays configurations can achieve M DoF using alternate transmission.

It is worth mentioning that some of the proposed schemes need time extension and can not be implemented only by linear precoding over the spatial domain. For instance, for $N = 3$ and $M = 5$ in the shared relays configuration, the DoF outer bound for simultaneous relaying case surpasses the other outer bounds. Hence, we use simultaneous relaying to achieve 3 DoF in two hops, that is, the total transmitted streams in each hop is 6 streams. In the first hop, each base station aims at transmitting $\frac{3}{2}$ streams to each relay. Accordingly, each base station uses two time slots to form a new channel matrix of dimension 6×10. Then, the new channel matrices can be used to transmit 3 streams to each relay from each base station. As result, the upper bound can be achieved by using the time extension approach.

V. Conclusion

In this paper, we have studied the DoF region for a cellular network consisting of two base stations and two mobile stations in the absence of a direct communication link, where the base stations are obliged to use relays to deliver their messages to the mobiles. The base stations and the mobile stations are equipped with M antennas, whereas the relays are equipped with N antenna each. We have considered shared and dedicated relays configurations. We have derived upper bounds on the DoF by considering all possible relying architectures in both configurations. In the dedicated relays case, we have studied three transmission modes. In the shared relays case, we have studied two transmission modes. We have also presented an achievable scheme based on the ratio between N and M for both configurations. In each of the two configurations, the choice of the transmission mode used in the achievable scheme for a certain ration of $\frac{N}{M}$ was inspired by the prevailing upper bound in the same region. We conclude that if the same total number of antennas is used at the relays in the two configurations, the achieved DoF of the shared relays are greater than or equal to that for the dedicated relays configuration. This can be attributed to the benefits achieved due to cooperation and the reduction of inter-relay interference which yield DoF gain when the total number of antennas at the relays is between M and $10M$.

Appendix

In (71), we formulated an optimization problem that describes the DoF region of our proposed scheme. We can see the symmetry in the problem, i.e., between the dimensions of the submatrices used for transmission to the first mobile and those which are used for transmission to the second mobile. Hence, we can set $d_{i+4} = d_i$ where $i = 1, 2, 3, 4$. In that case, the problem can re-formulated to be

$$\begin{aligned}
&\max_{\{d_i\}_{i=1}^4} \quad d_1 + d_2 + d_3 + d_4 \\
&\text{subject to} \\
&d_2 + d_3 + d_4 \le (N - M + d_1)^+ \\
&d_i \le (N - 2M + 2d_1)^+ \quad i = 2, 3 \\
&4d_1 + 2d_2 + 2d_3 + 2d_4 \le M \\
&4d_1 + 6d_2 + 4d_3 + 6d_4 \le N \\
&4d_1 + 4d_2 + 6d_3 + 6d_4 \le N \\
&d_i \ge 0 \qquad i = 1, 2, 3, 4
\end{aligned} \tag{76}$$

Furthermore, We can see that the feasible set of the above problem is symmetric in the directions of d_2 and d_3. In addition, the objective function of the above problem is symmetric in d_2 and d_3. As a result, we can set $d_3 = d_2$ to reduce the dimensions of the problem without any loss of optimality. The reduced problem is given by

$$\begin{aligned}
&\max_{d_1,d_2,d_4} \quad d_1 + 2d_2 + d_4 \\
&\text{subject to} \\
&2d_2 + d_4 \le (N - M + d_1)^+ \\
&d_2 \le (N - 2M + 2d_1)^+ \\
&4d_1 + 4d_2 + 2d_4 \le M \\
&4d_1 + 10d_2 + 6d_4 \le N \\
&d_i \ge 0 \qquad i = 1, 2, 4
\end{aligned} \tag{77}$$

Since $(x)^+$ has two values depending on the value of x, we can solve the problem by considering the four combinations of the first two constraints in (77). For example, the second constraint has two forms. In the first form, it will be $d_2 \le N - 2M + 2d_1$ when $N - 2M + 2d_1$ is greater than or equal to 0. Otherwise, the constraint will be $d_2 \le 0$ when $N - 2M + 2d_1$ is negative.

For the first case, the problem reduces to

$$\begin{aligned}
&\max_{d_1,d_2,d_4} \quad d_1 + 2d_2 + d_4 \\
&\text{subject to} \\
&-d_1 + 2d_2 + d_4 \le N - M \\
&-2d_1 + d_2 \le N - 2M \\
&4d_1 + 4d_2 + 2d_4 \le M \\
&4d_1 + 10d_2 + 6d_4 \le N \\
&d_1 \ge M - N \\
&2d_1 \ge 2M - N \\
&d_i \ge 0 \qquad i = 1, 2, 4
\end{aligned} \tag{78}$$

The above problem is a linear program in $\mathbb{R}^3$ and its solution depends on the value of M and N. Note that for some values of $\frac{N}{M}$ the problem can be infeasible. For example, adding the third constraint and the sixth constraint multiplied by 4 yields $4d_2 + 2d_4 \le 4N - 7M$. Hence, the problem is infeasible for $N < \frac{7M}{4}$ as d_2 and d_4 can not be negative. As in [27], if the duality gap of a primal dual feasible pair is zero, then the pair is the optimal solution. To solve this problem, we establish the dual problem, which is a linear program, and find a point that makes the duality gap zero. For (78), we can write the problem and its dual, respectively, as

$$\max_{\mathbf{d}} \ \mathbf{c}^{\mathbf{T}} \mathbf{d} \qquad \text{subject to } \mathbf{A}\,\mathbf{d} \le \mathbf{b}, \tag{79}$$

$$\min_{\nu} \ \mathbf{b}^{\mathbf{T}} \nu \qquad \text{subject to } \mathbf{A}^{\mathbf{T}} \nu \ge \mathbf{c} \tag{80}$$

where $\mathbf{d}$ and ν are the vectors of original and dual variables respectively, and $\mathbf{A}$, $\mathbf{b}$ and $\mathbf{c}$ can be written as

$$\mathbf{A}^{\mathbf{T}} = \begin{bmatrix} -1 & -2 & 4 & 4 & -1 & -1 & -1 & 0 & 0 \\ 2 & 1 & 4 & 10 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 2 & 6 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\mathbf{b}^{\mathbf{T}} = [u \quad v \quad M \quad N \quad u \quad v \quad 0 \quad 0 \quad 0],$$

$$\mathbf{c}^{\mathbf{T}} = [1 \quad 2 \quad 1]$$

where $[u \quad v] = [N - M \quad N - 2M]$. One of the vertices of the simplex is an optimal solution of the problem. At this point, three constraints have to be satisfied with equality. If we identify these constraints, we can find a closed-form solution of the problem and prove its optimality. For instance,[1] let us consider the case where $2M \leq N \leq \frac{5M}{2}$. We assume that the third, fourth and ninth constrains are active. As a result, we get a system of equations $\mathbf{d} = \left[\frac{1}{4}(\frac{5M}{2} - N) \quad \frac{1}{6}(N - M) \quad 0\right]^T$. On the other hand, using the complementary slackness condition in [27], only the third, fourth and ninth elements of the vector ν can be non zero. Therefore, the dual problem can be reduced to a three dimensional problem. Again, we can solve system of equations and get the closed form solution as $\nu = \left[0\ 0\ \frac{1}{12}\ \frac{1}{6}\ 0\ 0\ 0\ 0\ \frac{1}{6}\right]^T$. For the values of $\mathbf{d}$ and ν, the duality gap is given by

$$\mathbf{c}^{\mathbf{T}}\mathbf{d} - \mathbf{b}^{\mathbf{T}}\nu = 0 \tag{81}$$

which proves the optimality of this solution. By the same way, we can find the solution for all values of M and N. The problem optimal solution is given by

$$\mathbf{d}^{\mathbf{T}} = \begin{cases} \left[\frac{2M-N}{2}, 0, \frac{2N-3M}{2}\right] & \text{for } \frac{3M}{2} \leq N \leq \frac{5M}{3} \\ \left[\frac{7M-3N}{12}, \frac{3N-5M}{6}, \frac{2M-N}{2}\right] & \text{for } \frac{5M}{3} \leq N \leq 2M \\ \left[\frac{5M-2N}{8}, \frac{N-M}{6}, 0\right] & \text{for } 2M \leq N \leq \frac{5M}{2} \\ \left[0, \frac{M}{4}, 0\right] & \text{for } N \geq \frac{5M}{2} \end{cases}. \tag{82}$$

Similarly, we can solve the four problems which result from (77). By taking the maximum of all feasible solutions of the four problems, $\mathbf{d}^*$ that achieves the maximum objective value is found to be

$$\mathbf{d}^{*T} = \begin{cases} \left[\frac{N}{4}, 0, 0\right] & \text{for } N \leq M \\ \left[\frac{3M-N}{8}, 0, \frac{N-M}{4}\right] & \text{for } M \leq N \leq \frac{5M}{3} \\ \left[\frac{7M-3N}{12}, \frac{3N-5M}{6}, \frac{2M-N}{2}\right] & \text{for } \frac{5M}{3} \leq N \leq 2M \\ \left[\frac{5M-2N}{8}, \frac{N-M}{6}, 0\right] & \text{for } 2M \leq N \leq \frac{5M}{2} \\ \left[0, \frac{M}{4}, 0\right] & \text{for } N \geq \frac{5M}{2} \end{cases}. \tag{83}$$

By substituting by this values in (71), we can get (72).

[1] Similarly, we can find the closed form solution. To avoid repetition, we only describe one region as an example.

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