Artificial Intelligence
Homework 5

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1. Probability Theorems

1. $P(\alpha|\beta \wedge \alpha) = 1$ whenever $P(\beta \wedge \alpha) \neq 0$

   Proof: Assume that $P(\beta \wedge \alpha) \neq 0$. Then $P(\alpha|\beta \wedge \alpha) = \frac{P(\alpha \wedge (\beta \wedge \alpha))}{P(\beta \wedge \alpha)}$ by the definition of conditional probability. By logical equivalence, this is $\frac{P(\beta \wedge \alpha)}{P(\beta \wedge \alpha)}$, which is equal to 1.

2. $P(\alpha) = P((\alpha \wedge \beta) \vee (\alpha \wedge \neg\beta))$

   Proof: By logical equivalence, we can rearrange terms from $P((\alpha \wedge \beta) \vee (\alpha \wedge \neg\beta))$ to $P(\alpha \wedge (\beta \vee \neg\beta))$. Since $\beta \vee \neg\beta$ is a tautology, we can further reduce this via logical equivalence to $P(\alpha)$, which is what we wanted to show.

3. $P(\alpha) = P(\alpha|\beta)P(\beta) + P(\alpha|\neg\beta)P(\neg beta)$

   Proof: By the definition of conditional probability, $P(\alpha|\beta)P(\beta) + P(\alpha|\neg\beta)P(\neg\beta) = \frac{P(\alpha \wedge \beta)}{P(\beta)}P(\beta) + \frac{P(\alpha \wedge \neg\beta)}{P(\neg\beta)}P(\neg\beta)$. This can be further reduced to $P(\alpha \wedge \beta) + P(\alpha \wedge \neg\beta)$; this is equivalent to $P((\alpha \wedge \beta) \vee (\alpha \wedge \neg\beta))$ since $P((\alpha \wedge \beta) \wedge (\alpha \wedge \neg\beta)) = 0$. By the previous problem, this is equivalent to $P(\alpha)$.

4. $P(\alpha|\beta) = 1 - P(\neg\alpha|\beta)$

   Proof: By the definition of conditional probability, $P(\alpha|\beta) = \frac{P(alpha \wedge beta)}{P(\beta)}$. Due to axiom 2, $P(\alpha) = 1 - P(\neg\alpha)$, and so $P(\alpha \wedge \beta) = P(\beta) - P(\neg\alpha \wedge \beta)$. Substituting this in the original equation gives $\frac{P(\beta) - P(\neg\alpha \wedge beta)}{P(\beta)}$, which reduces to $1 - \frac{P(\neg\alpha \wedge beta)}{P(\beta)}$. By the definition of conditional probability, this is equivalent to $1 - P(\neg\alpha|\beta)$, which is what we wanted to show.

2. Probabilistic Reasoning

Let's say that $P(\alpha)$ is the probability of the engine actually being cracked. The knowledge that only one in 10,000 cars of my specific model having this issue seems to place $P(\alpha)$ at $\frac{1}{10{,}000}$ - a fairly low chance!

However, I have extra information. The computer is reporting a cracked engine, with 99% accuracy. This extra information updates my model - now, rather than the naive belief that I have a 0.001% chance of my engine being cracked, I have a 99% chance of this being the case (since I know that the check engine light is on).

Given my knowledge of the situation (namely, the check engine light being on), and the consequences of such a fault (potential death for myself and others if I choose to ignore the alarm and am wrong, versus a monetary loss if I choose to replace the engine and don't need to) I cannot ignore the alarm and should replace the engine.

3. Constructing Belief Networks I

Belief Network

Flames

P(Flames)
0.1

Smoke

Flames	P(Smoke)
T	0.8
F	0.1

Sprinkler

Smoke	P(Sprinkler)
T	0.9
F	0.01

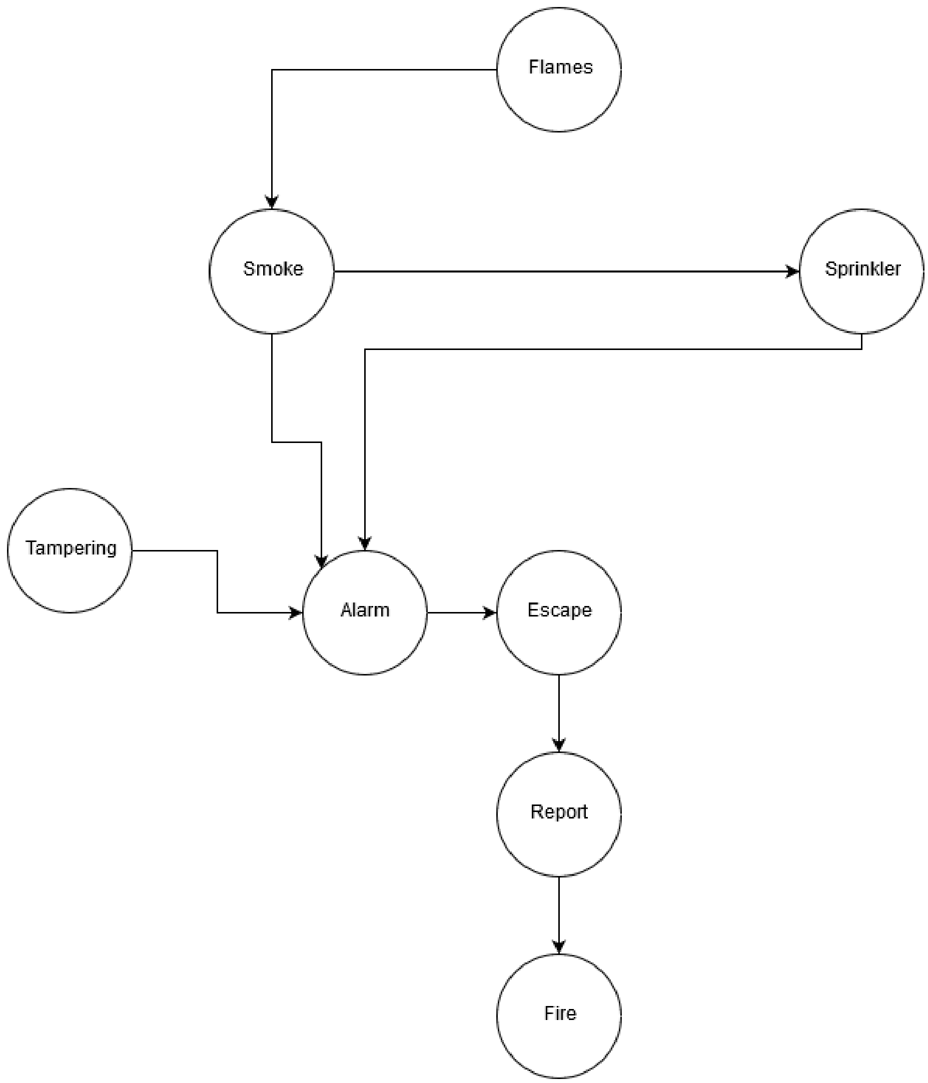

Figure 1: Belief Network

Tampering

P(Tampering)
0.1

Alarm

Smoke	Sprinkler	Tampering	P(Alarm)
T	T	T	0.9
T	T	F	0.99
T	F	T	0.9
T	F	F	0.92
F	T	T	0.8
F	T	F	0.89
F	F	T	0.8
F	F	F	0.01

Escape

Alarm	P(Escape)
T	0.9
F	0.2

Report

Escape	P(Report)
T	0.9
F	0.1

Fire

Report	P(Fire)
T	0.7
F	0.1

4. Constructing Belief Networks II

5. Querying Belief Networks

1. $P(W|S \wedge \neg R \wedge C)$

   From the CPT, we see that $P(W|S \wedge \neg R)$ is 0.90. Due to the conditional independence of belief networks, $P(W|S \wedge \neg R) = P(W|S \wedge \neg R \wedge C)$. Thus, the end result is 0.90.

2. $P(\neg R|C)$

   From the CPT, we see that $P(R|C) = 0.80$; since $P(\neg A) = 1 - P(A)$, we can say that $P(\neg R|C) = 1 - 0.8 = 0.2$.

3. $P(S|R \wedge C)$

   Since S and R are conditionally independent, $P(S|R \wedge C) = P(S|C) = 0.10$.

4. $P(S)$

   From 1.3 above, we see that $P(S) = P(S|C)P(C) + P(S|\neg C)P(\neg C) = 0.1 * 0.5 + 0.5 * 0.5 = 0.3$.

5. $P(R)$

   Again, $P(R) = P(R|C)P(C) + P(R|\neg C)P(\neg C) = 0.8 * 0.5 + 0.2 * 0.5 = 0.5$.

6. $P(S \wedge R|\neg C)$

   Since S and R are conditionally independent given C, $P(S \wedge R|C) = P(S|C)P(R|C) = 0.1 * 0.8 = 0.08$.

7. $P(S \wedge R)$

   $P(S \wedge R) = P(S|R)P(R)$; since S and R are conditionally independent, $P(S|R) = P(S)$, so $P(S \wedge R) = P(S)P(R) = 0.3 * 0.5 = 0.15$.

8. $P(S \wedge \neg R)$

   Since $P(\neg R) = 1 - P(R)$, the expression in 5.7 above is $P(S \wedge \neg R) = P(S)P(\neg R) = P(S)(1 - P(R)) = P(S) - P(S)P(R) = 0.3 - 0.15 = 0.15$.

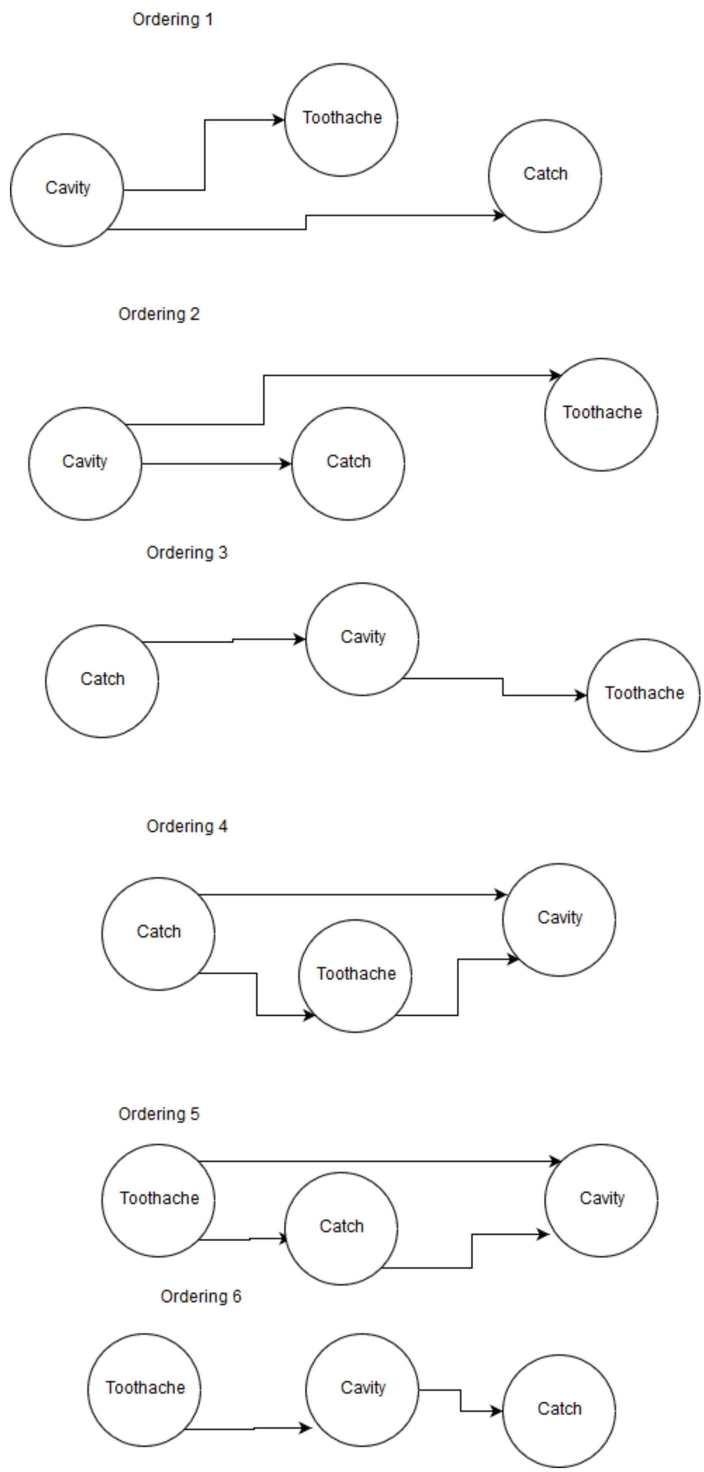

Figure 2: Dentist Belief Networks