CS:4420 Artificial Intelligence
Spring 2019

Homework 5

Due: Friday, April 12 by 11:59pm

This is a written assignment to be done individually. Write your solutions in a text editor or word processor and submit on ICON a printout of the file in PDF format. Make sure you write your name at the beginning of the file.

1 Probability Theorems

For each of the equalities (2–4) below, provide a mathematically rigorous proof that the equalities hold for any pair of events α and β. Base your proofs on the axioms of probability given in the class notes and on the definition of conditional probability. The proof of Equality (1) is provided as an example of the sort of proofs you are expected to write.

You can use any of the equalities below to prove a later equality. Also, it will help you to keep in mind that two logically equivalent expressions have the same probability simply because such expressions denote the same event.

1. $P(\alpha \mid \beta \wedge \alpha) = 1$ whenever $P(\beta \wedge \alpha) \neq 0$

   Proof: Assume that $P(\beta \wedge \alpha) \neq 0$. Then

$$\begin{aligned} P(\alpha \mid \beta \wedge \alpha) &= \tfrac{P(\alpha \wedge (\beta \wedge \alpha))}{P(\beta \wedge \alpha)} && \text{[by def. of cond. prob.]} \\ &= \tfrac{P(\beta \wedge \alpha)}{P(\beta \wedge \alpha)} && \text{[by logical equivalence]} \\ &= 1 \end{aligned}$$

2. $P(\alpha) = P((\alpha \wedge \beta) \vee (\alpha \wedge \neg\beta))$

3. $P(\alpha) = P(\alpha \mid \beta)P(\beta) + P(\alpha \mid \neg\beta)P(\neg\beta)$

4. $P(\alpha \mid \beta) = 1 - P(\neg\alpha \mid \beta)$

You may want to refer back to these equations in some of the following problems.

2 Probabilistic Reasoning

Modern cars have sophisticated on-board computers that continuously monitor the state of the engine. When the computer detects a problem, it turns on the "check engine" warning light on the car's dashboard and stores a problem code in its memory, which a technician can then read with a proper scanner. For a number of reasons, the detectors used by the computer are not 100% accurate. This causes the "check engine" light at times to be on when there is no problem at all with the engine, or to be off when there is a problem with the engine.

Suppose your car's "check engine" light is on, you take it to the mechanic and he tells you that the computer is reporting a crack in the engine. He also tells you that the accuracy of such a report on that car model is 99%.[1] A cracked engine cannot be fixed and must be replaced. On the other hand, only one over 10,000 cars of that model ever experience an engine crack.

What would you do based on this information? Would you replace the engine, and spend a lot of money on it, or would you decide that the warning is a fluke and do nothing, risking an accident later on? Base your decision on the probability that the engine is actually cracked, given what you know. Justify your answer.

3 Constructing Belief Networks I

Suppose you are to build an application for diagnosing whether there is a fire in a building. Since the diagnosis will be based on noisy sensory data and on possibly conflicting explanations, you decide to implement the application using a belief network.

The application will use *only sensors that monitor the entrances* of the building and can tell if a large crowd is escaping the building. You know that if a building is on fire, smoke and sometimes also flames may come out of the building. The building has a fire sprinkler system automatically activated by smoke and a fire alarm system independently activated by smoke or by the sprinkler system itself.[2] The fire alarm can be activated manually as well; doing that, however, does not activate the sprinkler system. If the fire alarm goes off, you expect that everybody will leave. Fire alarms occasional malfunction or are activated manually in the absence of fire, maliciously or by mistake. Hence the fire alarm may go off when there is no fire or it may not go off if there is a fire. Sprinklers systems may malfunction in a similar way. The entrance sensor is not 100% accurate. It may report a mass escape even if there is none, or it may fail to report one. Finally, people may leave the building in large numbers for reasons other than fire, or may not not leave in case of fire because they are trapped inside.

1. Draw a belief network with a *minimal* number of links that correctly formalizes the knowledge above. Use the Boolean random variables *Alarm*, *Escape*, *Fire*, *Flames*, *Smoke*, *Sprinkler*, *Report*, and *Tampering*, where *Report* is true iff the sensors report a mass escape from the building, *Tampering* is true iff the alarm has been tampered with in any way (causing the alarm to go off when there is no fire or preventing it to go off when there is fire), and each of the other variables has the obvious meaning.

[1] Accuracy here is meant in the technical sense: the probability of getting a false positive is 0.01 and so is the probability of getting a false negative.

[2] That is, when the sprinkler system gets activated it also activates the fire alarm regardless of whether the fire alarm has already been activated by smoke or not. This redundancy is added in for greater safety.

2. Complete the network with a conditional probability table for each node in the network. You don't have to fill in the table with actual probabilities, but make sure you write the rest of the table correctly.

4 Constructing Belief Networks II

Consider the dentist example described in Chap 13 of our textbook, involving the Boolean random variables *Cavity*, *Toothache* and *Catch*. Using the algorithm seen in class, construct a belief network for this example for each of the possible initial orderings of the variables:

1. $(Cavity, Toothache, Catch)$
2. $(Cavity, Catch, Toothache)$
3. $(Catch, Cavity, Toothache)$
4. $(Catch, Toothache, Cavity)$
5. $(Toothache, Catch, Cavity)$
6. $(Toothache, Cavity, Catch)$

You do not need to build the nodes' CTPs as well. Draw just the networks.

5 Querying Belief Networks

Using the belief network with CPTs shown in Figure 1, compute the following probabilities:

1. $P(W \mid S \wedge \neg R \wedge C)$
2. $P(\neg R \mid C)$
3. $P(S \mid R \wedge C)$
4. $P(S)$
5. $P(R)$
6. $P(S \wedge R \mid \neg C)$
7. $P(S \wedge R)$
8. $P(S \wedge \neg R)$

Show all your work and justify each intermediate step by specifying which axioms or theorems of probability you have used to derive the expression in that step. You can refer directly to the equalities below, where the last one holds under the assumptions on $X_1, \ldots, X_n$ listed on pag. 14 of the Probabilistic Reasoning class notes.

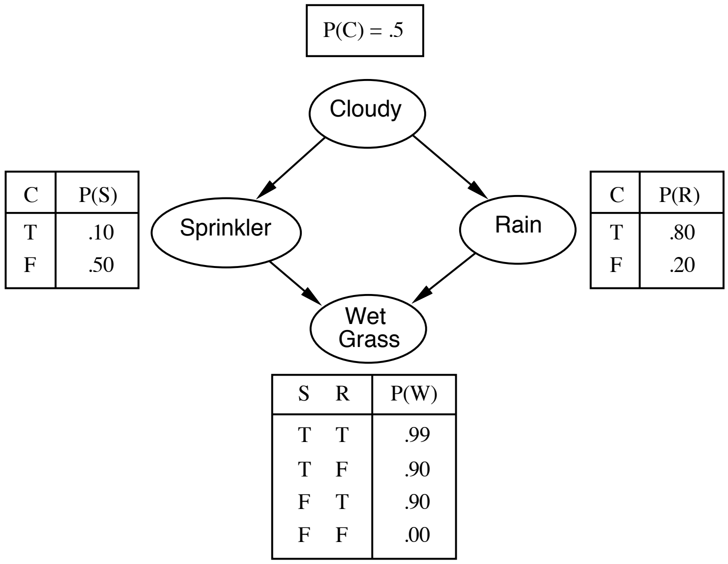

Figure 1: Recall that the first value in the $P(W)$ column of the CPT of Wet Grass, for instance, is $P(W \mid S \wedge R)$, the second element is $P(W \mid S \wedge \neg R)$, and so on.

$$
\begin{aligned}
P(\mathbf{False}) &= 0 && (1)\\
P(\neg A) &= 1 - P(A) && (2)\\
P(A \wedge B) &= P(A \mid B)P(B) && (3)\\
P(A \vee B) &= P(A) + P(B) - P(A \wedge B) && (4)\\
P(A \mid B) &= P(B \mid A)P(A)/P(B) \text{ if } P(B) \neq 0 && (5)\\
P(A \wedge B \mid C) &= P(A \mid C)P(B \mid C) && (6)\\
&\quad \text{if } A \text{ and } B \text{ are cond. independent given } C && (7)\\
P(X_1 = x_1 \wedge \cdots \wedge X_n = x_n) &= \prod_{i=1}^{n} P(X_i = x_i \mid Parents(X_i)) && (8)\\
P(X_1 = x_1 \mid X_2 = x_2 \wedge \cdots \wedge X_m = x_m) &= P(X_1 = x_1 \mid Parents(X_1)) && (9)
\end{aligned}
$$