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10. Petri Nets

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Roadmap

- Definition:
  - places, transitions, inputs, outputs
  - firing enabled transitions
- Modelling:
  - concurrency and synchronization
- Properties of nets:
  - liveness, boundedness
- Implementing Petri net models:
  - centralized and decentralized schemes

J. L. Peterson, *Petri Nets Theory and the Modelling of Systems*, Prentice Hall, 1983.

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Petri nets: a definition

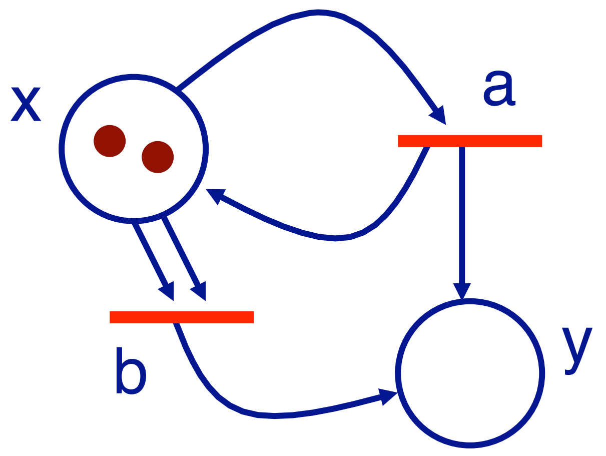
A <u>Petri net</u> C = ⟨P,T,I,O⟩ consists of:

1. A finite set P of *places*
2. A finite set T of *transitions*
3. An *input function* I: T → Nat^P (maps to bags of places)
4. An *output function* O: T → Nat^P

A <u>marking</u> of C is a mapping m: P → Nat

Example:

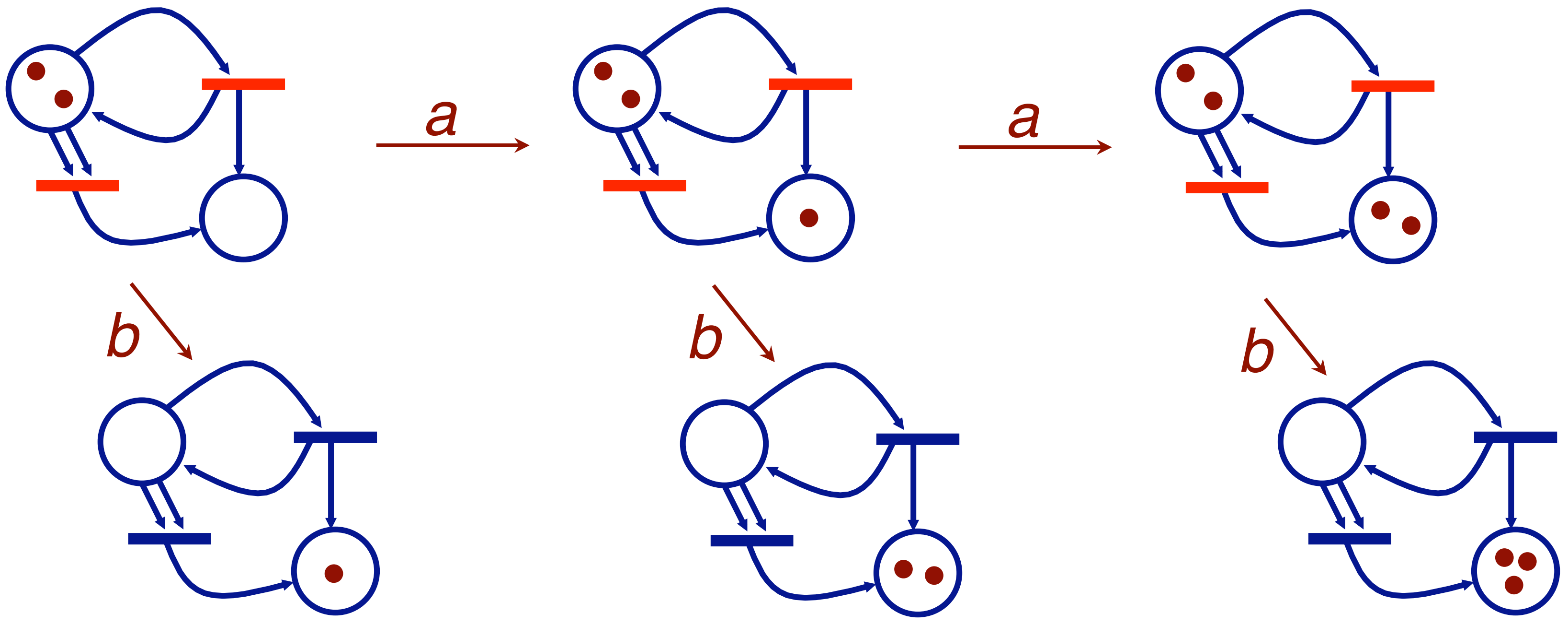
P = { x, y }
T = { a, b }
I(a) = { x }, I(b) = { x, x }
O(a) = { x, y }, O(b) = { y }
m = { x, x }



Firing transitions

To fire a transition t:

1. t must be enabled: $m \geq I(t)$
2. *consume* inputs and *generate* output: $m' = m - I(t) + O(t)$



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Modelling with Petri nets

Petri nets are good for modelling:

> concurrency

> synchronization

Tokens can represent:

> resource availability

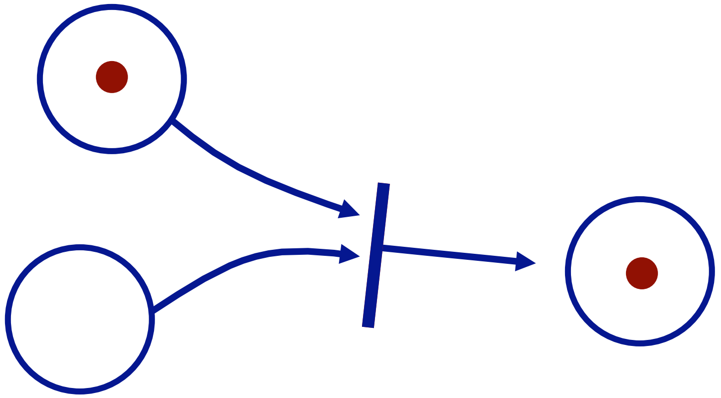
> jobs to perform

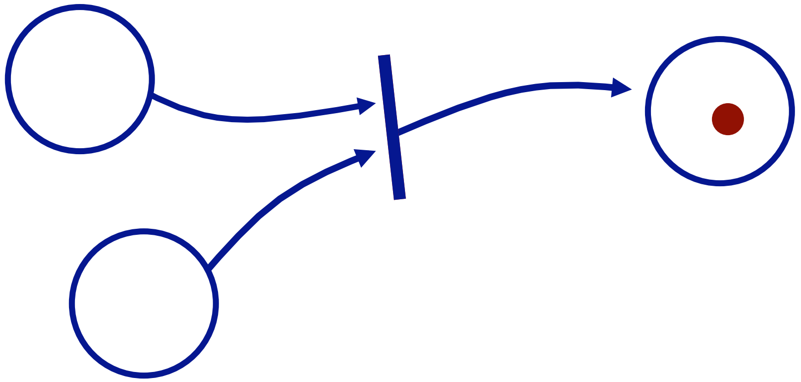
> flow of control

> synchronization conditions ...

Concurrency

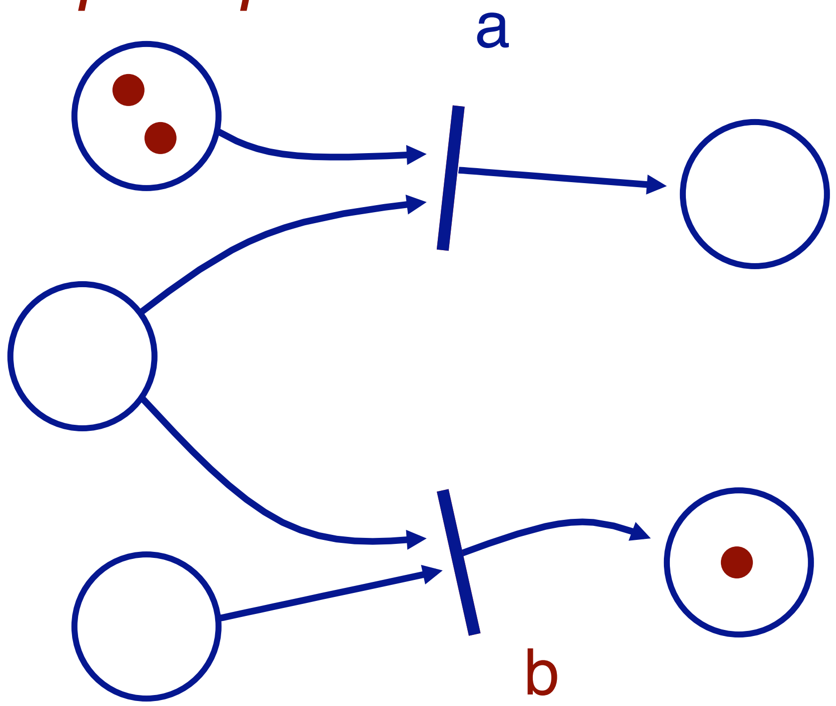
Independent inputs permit "concurrent" firing of transitions





Conflict

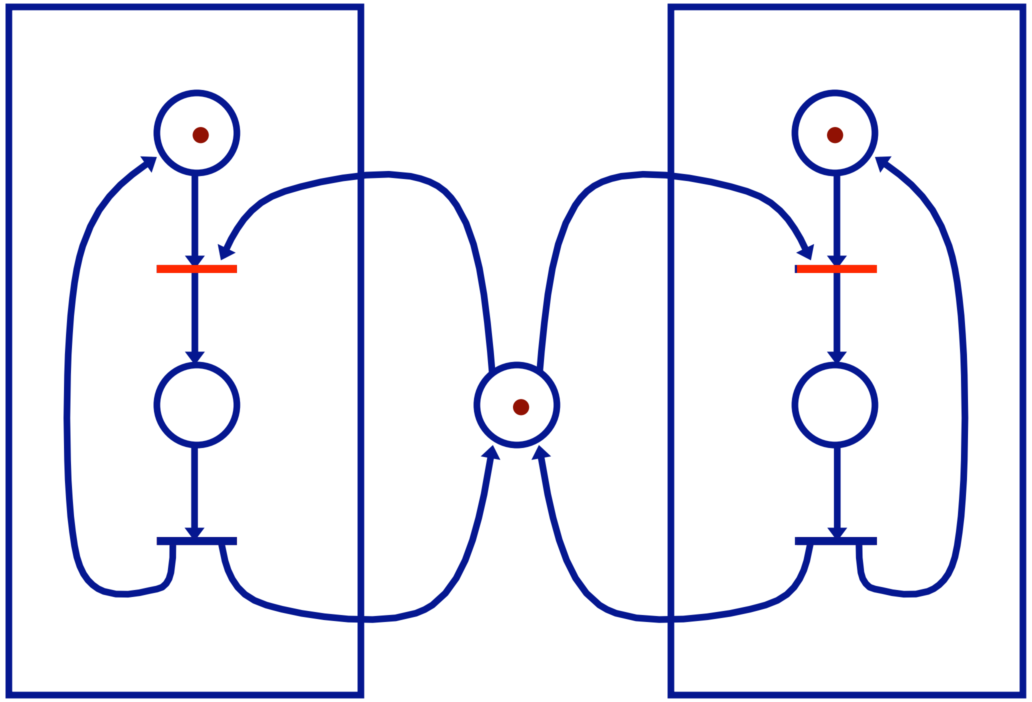
Overlapping inputs put transitions in conflict



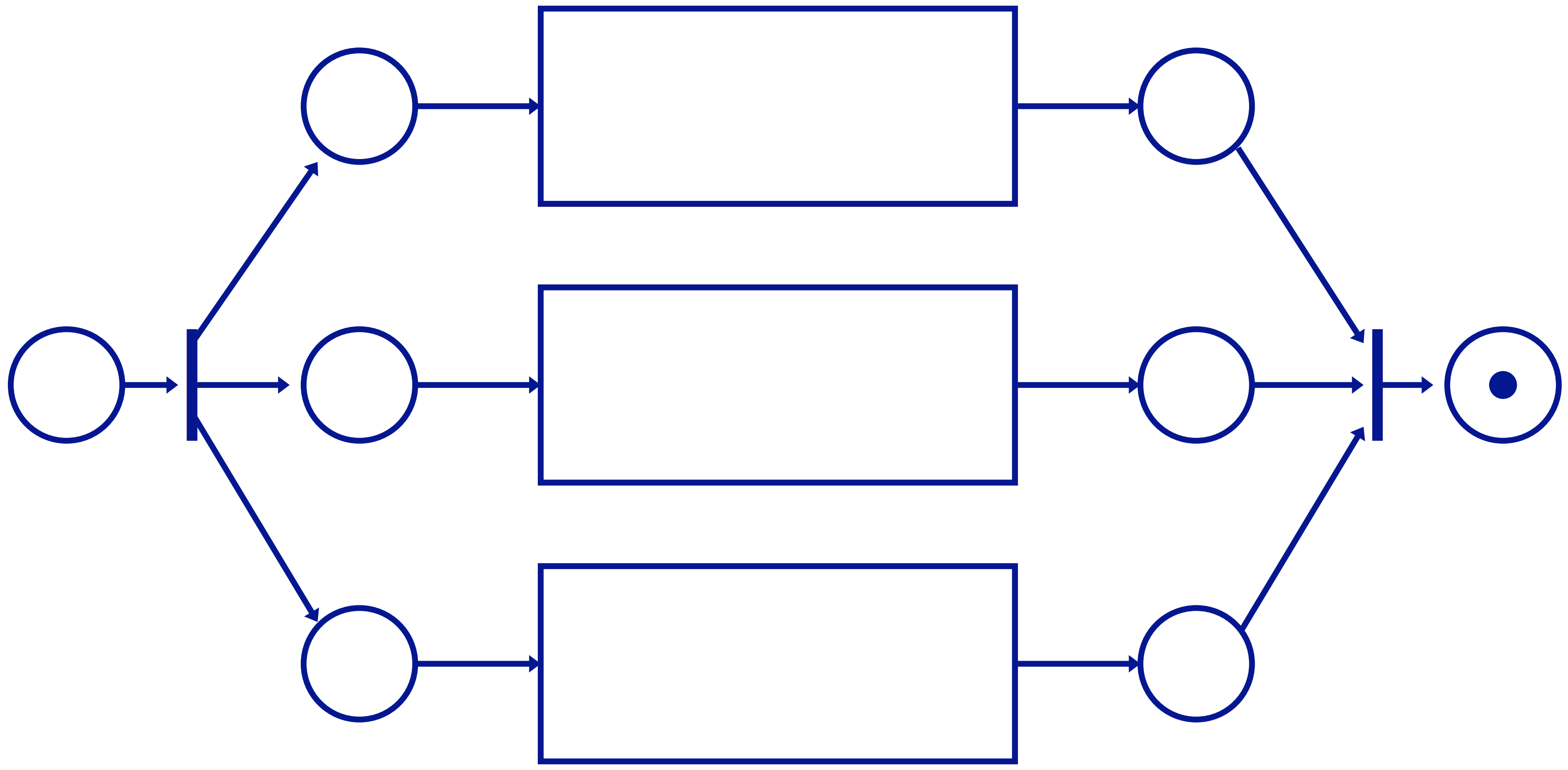
Only one of a or b may fire

Mutual Exclusion

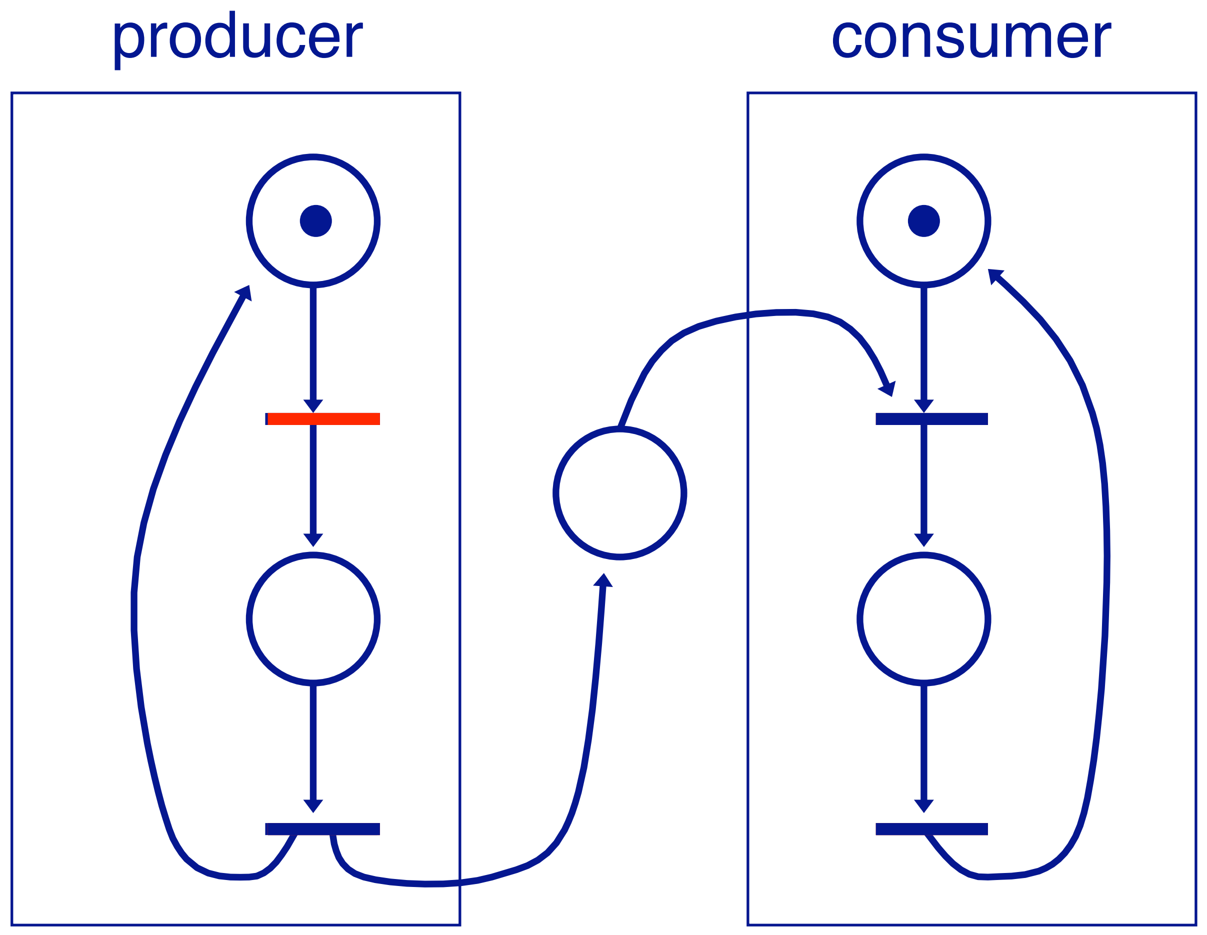
The two subnets are forced to synchronize



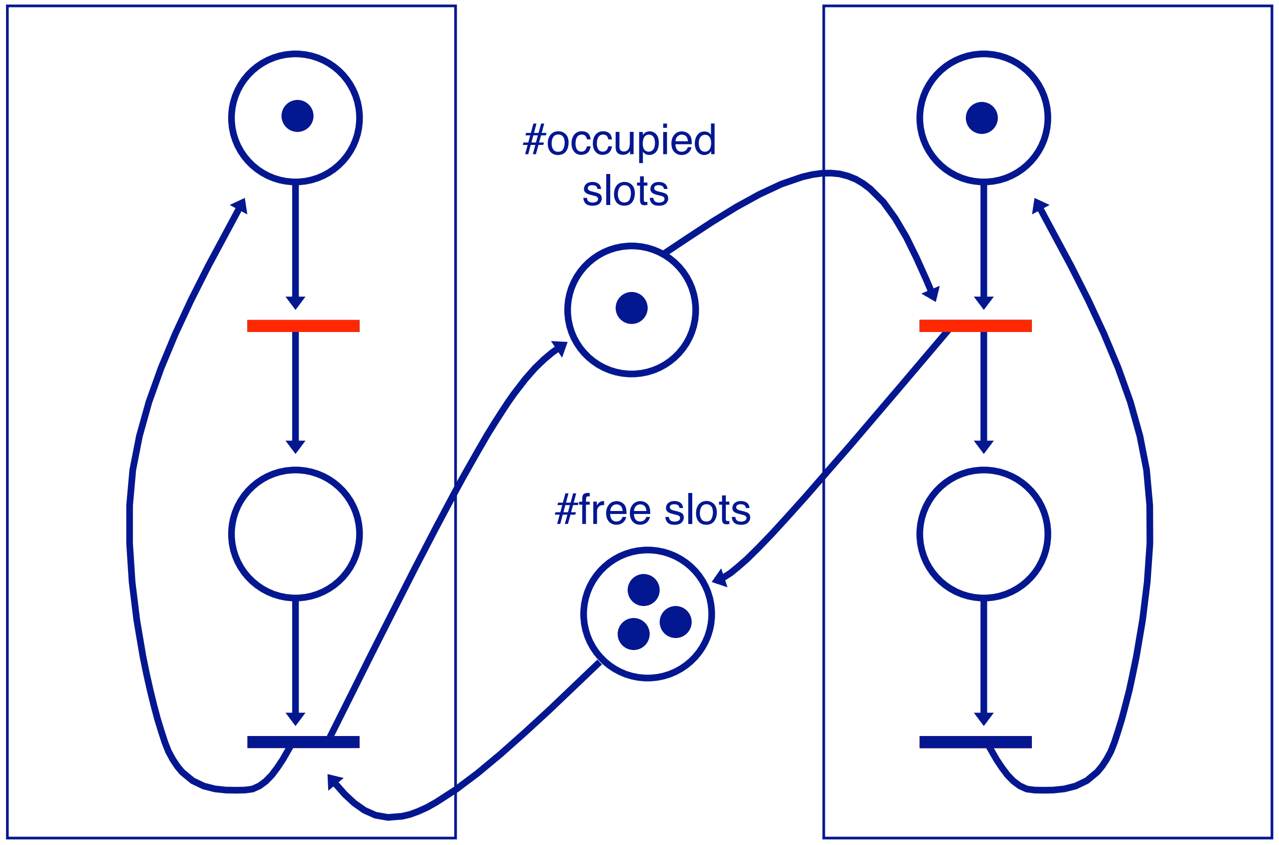
Fork and Join



Producers and Consumers



Bounded Buffers



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Reachability and Boundedness

Reachability:

> The reachability set $R(C,\mu)$ of a net C is the set of all markings μ' *reachable from initial marking m*.

Boundedness:

> A net C with initial marking μ is safe if places always hold *at most 1 token*.

> A marked net is (k-)bounded if places *never hold more than k tokens*.

> A marked net is conservative if the number of tokens is *constant*.

Liveness and Deadlock

Liveness:

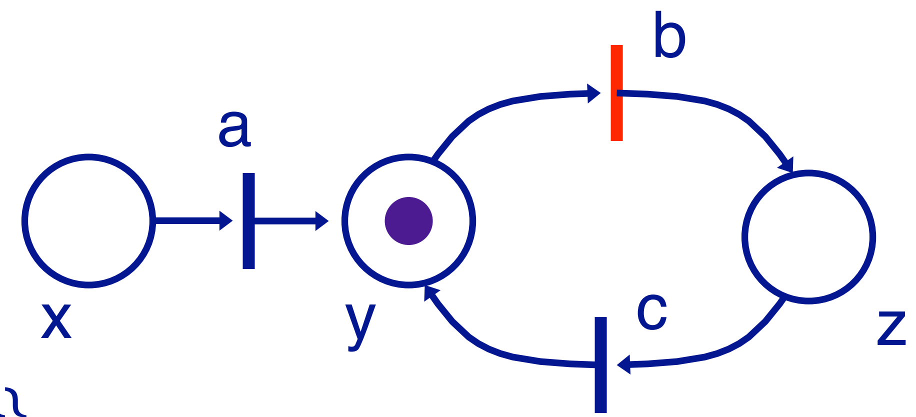
> A transition is deadlocked if it can *never fire.*
> A transition is live if it can *never deadlock.*

This net is both *safe and conservative*.

Transition a is *deadlocked*.

Transitions b and c are *live*.

The reachability set is {{y}, {z}}.



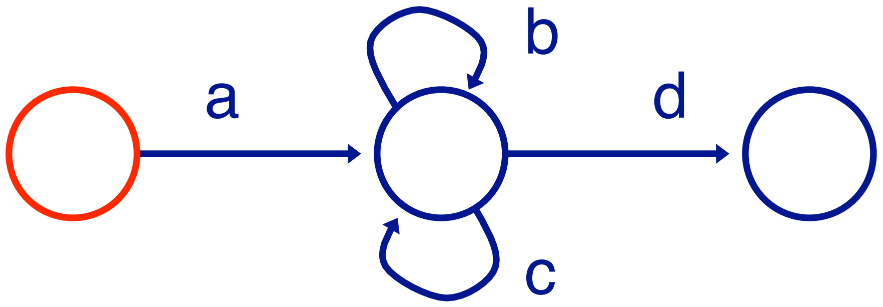
Are the examples we have seen bounded? Are they live?

Related Models

Finite State Processes

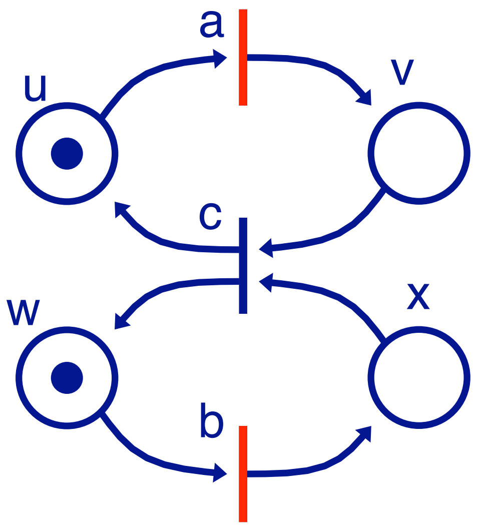
- Equivalent to *regular expressions*
- Can be modelled by *one-token conservative nets*

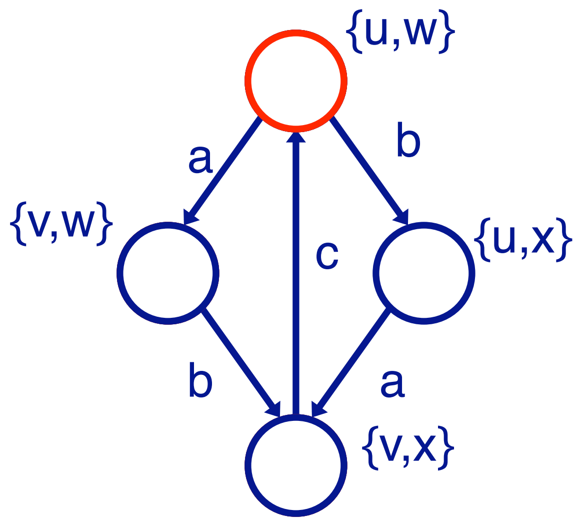
The FSA for: a(b|c)*d



Finite State Nets

Some Petri nets can be modelled by FSPs





Precisely which nets can (cannot) be modelled by FSPs?

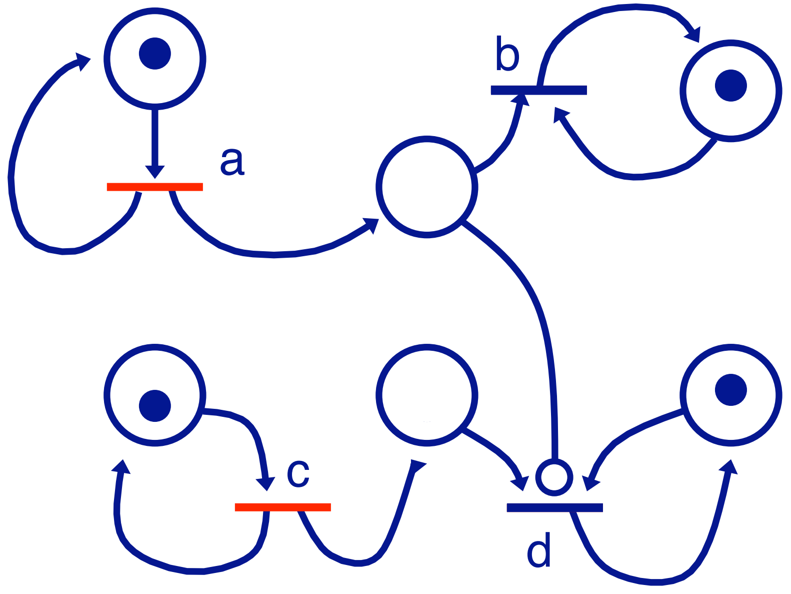
Zero-testing Nets

Petri nets are not computationally complete

> Cannot model “zero testing”

> Cannot model priorities

A zero-testing net: An equal number of a and b transitions may fire as a sequence during any sequence of matching c and d transitions.
(#a ≥ #b, #c ≥ #d)



Other Variants

There exist countless variants of Petri nets

Coloured Petri nets:

> Tokens are “coloured” to represent different kinds of resources

Augmented Petri nets:

> Transitions additionally depend on external conditions

Timed Petri nets:

> A duration is associated with each transition

Applications of Petri nets

Modelling information systems:

> Workflow
> Hypertext (possible transitions)
> Dynamic aspects of OODB design

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  - **centralized and decentralized schemes**

Implementing Petri nets

We can implement Petri net structures in either *centralized* or *decentralized* fashion:

Centralized:

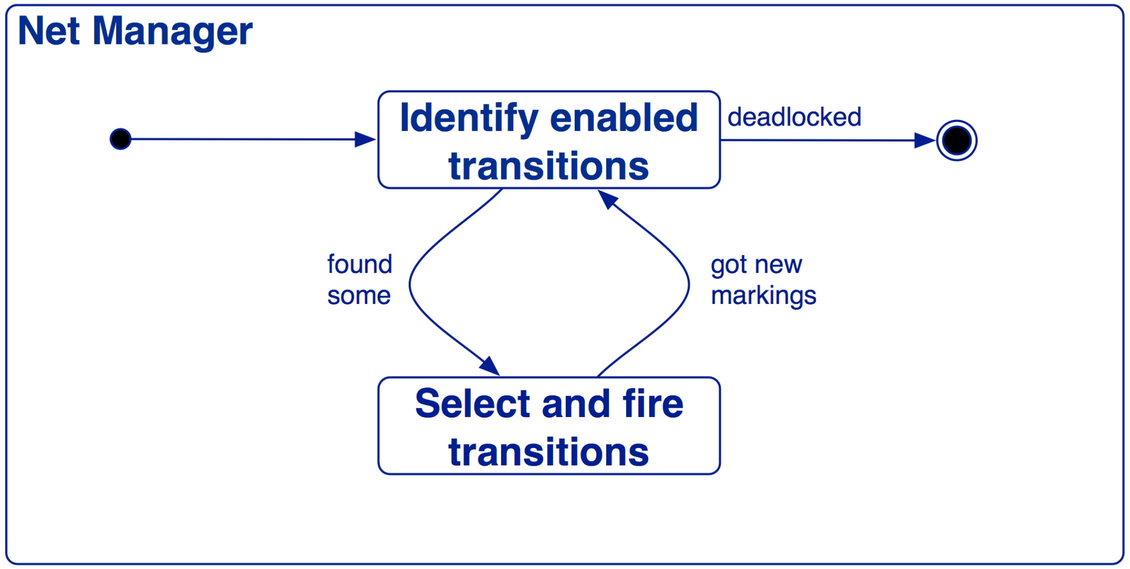
> A single "net manager" monitors the current state of the net, and fires enabled transitions.

Decentralized:

> Transitions are processes, places are shared resources, and transitions compete to obtain tokens.

Centralized schemes

In one possible centralized scheme, the Manager selects and fires enabled transitions.

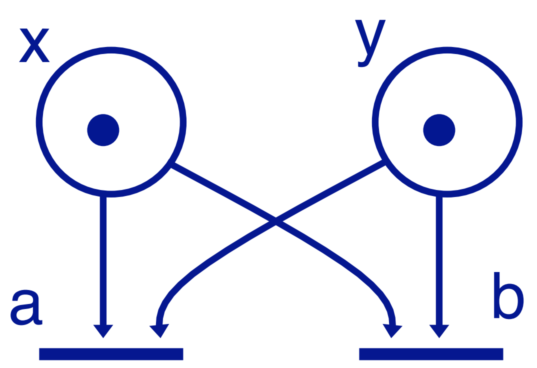


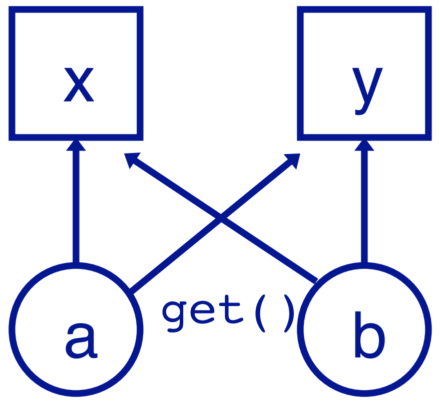
Concurrently enabled transitions can be fired in parallel.

What liveness problems can this scheme lead to?

Decentralized schemes

In decentralized schemes transitions are processes and tokens are resources held by places:

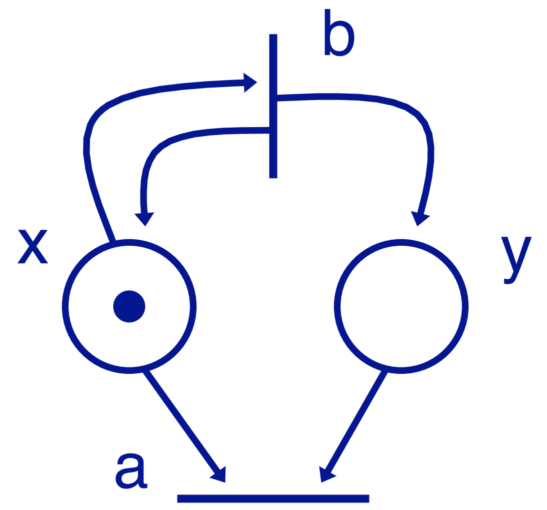




Transitions can be implemented as *thread-per-message gateways* so the same transition can be fired more than once if enough tokens are available.

Transactions

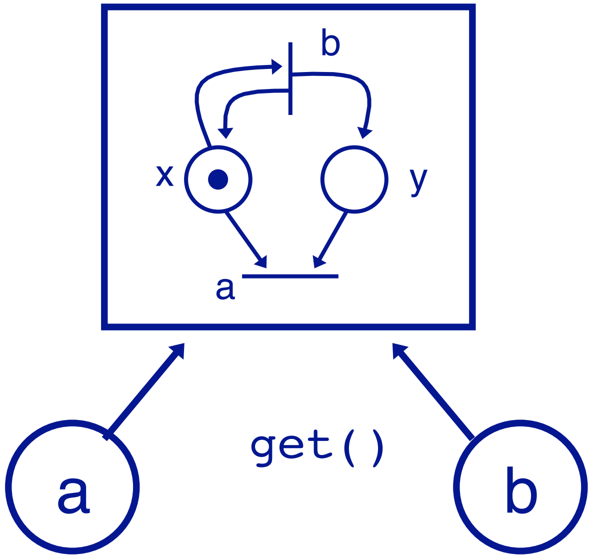
Transitions attempting to fire must grab their input tokens as an atomic transaction, or the net may deadlock even though there are enabled transitions!



If a and b are implemented by independent processes, and x and y by shared resources, this net can deadlock even though b is enabled if a (incorrectly) grabs x and waits for y.

Coordinated interaction

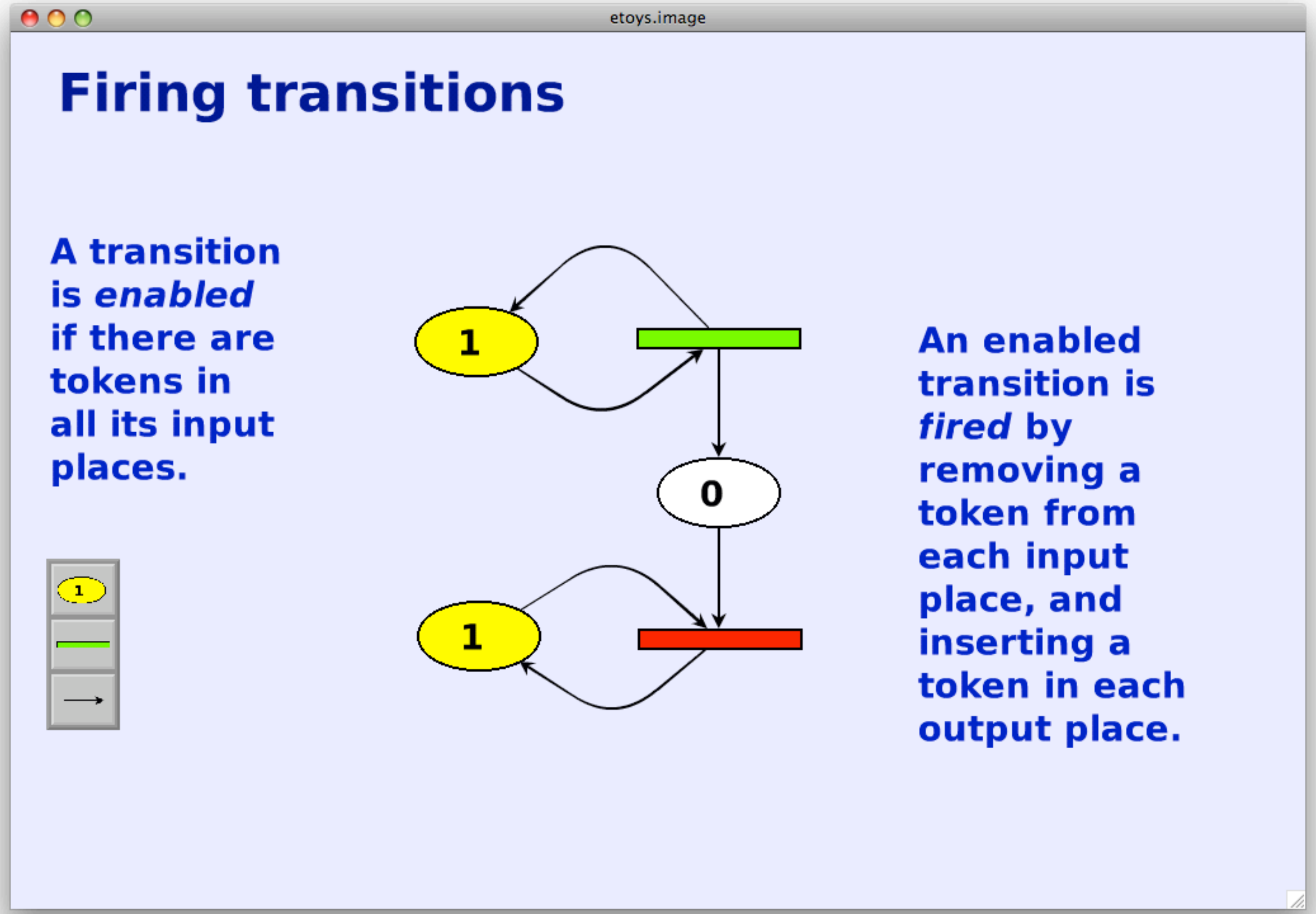
A simple solution is to treat the state of the entire net as a single, shared resource:



After a transition fires, it notifies waiting transitions.

How could you refine this scheme for a distributed setting?

Petit Petri — a Petri Net Editor built with Etoys

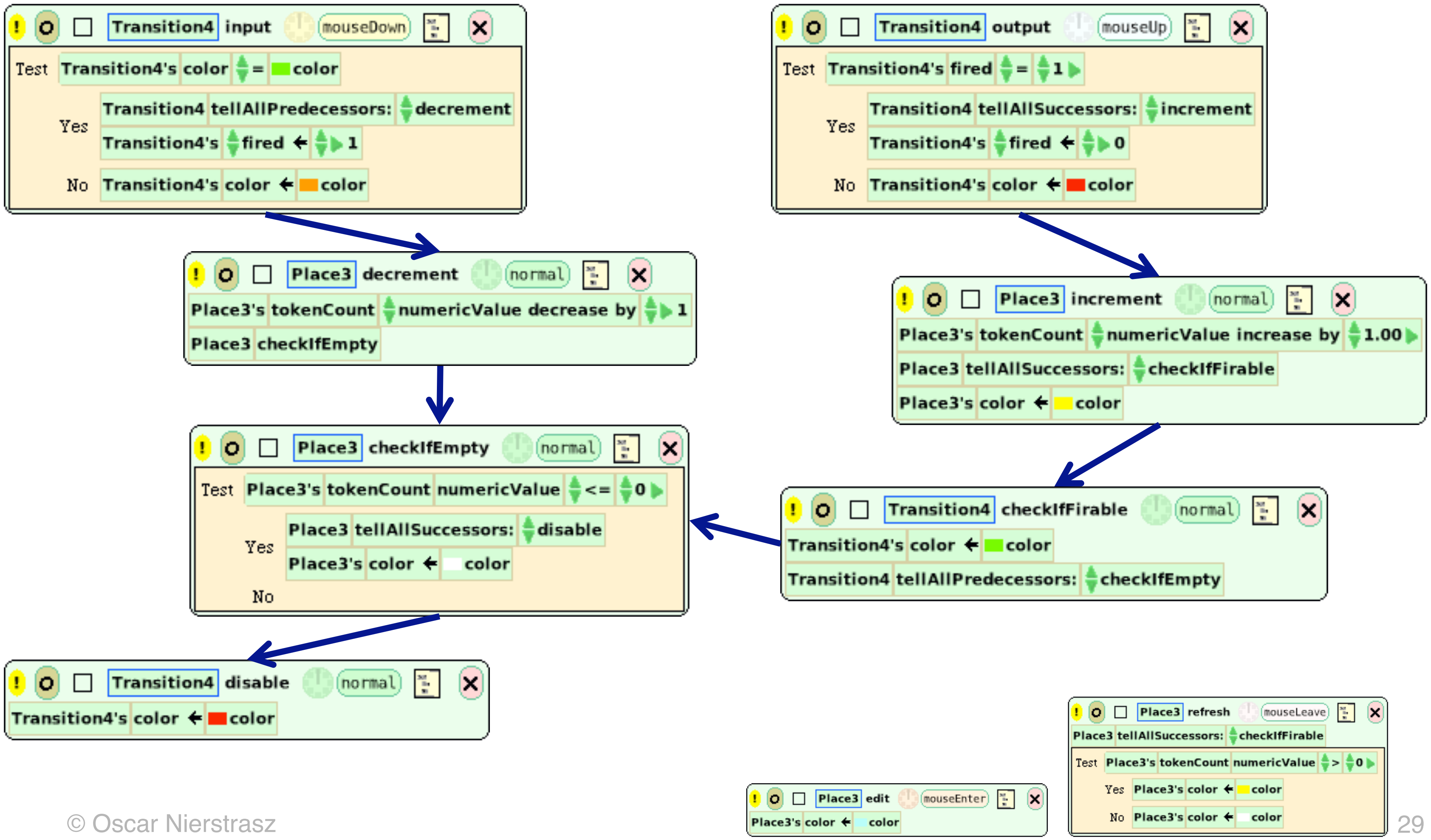


Firing transitions

A transition is *enabled* if there are tokens in all its input places.

An enabled transition is *fired* by removing a token from each input place, and inserting a token in each output place.

Etoys implementation



```
Transition4 input  mouseDown
Test  Transition4's color = 🟩 color
  Yes  Transition4 tellAllPredecessors: decrement
       Transition4's fired ← 1
  No   Transition4's color ← 🟧 color
```

```
Transition4 output  mouseUp
Test  Transition4's fired = 1
  Yes  Transition4 tellAllSuccessors: increment
       Transition4's fired ← 0
  No   Transition4's color ← 🟥 color
```

```
Place3 decrement  normal
Place3's tokenCount numericValue decrease by 1
Place3 checkIfEmpty
```

```
Place3 increment  normal
Place3's tokenCount numericValue increase by 1.00
Place3 tellAllSuccessors: checkIfFirable
Place3's color ← 🟨 color
```

```
Place3 checkIfEmpty  normal
Test  Place3's tokenCount numericValue <= 0
  Yes  Place3 tellAllSuccessors: disable
       Place3's color ← ⬜ color
  No
```

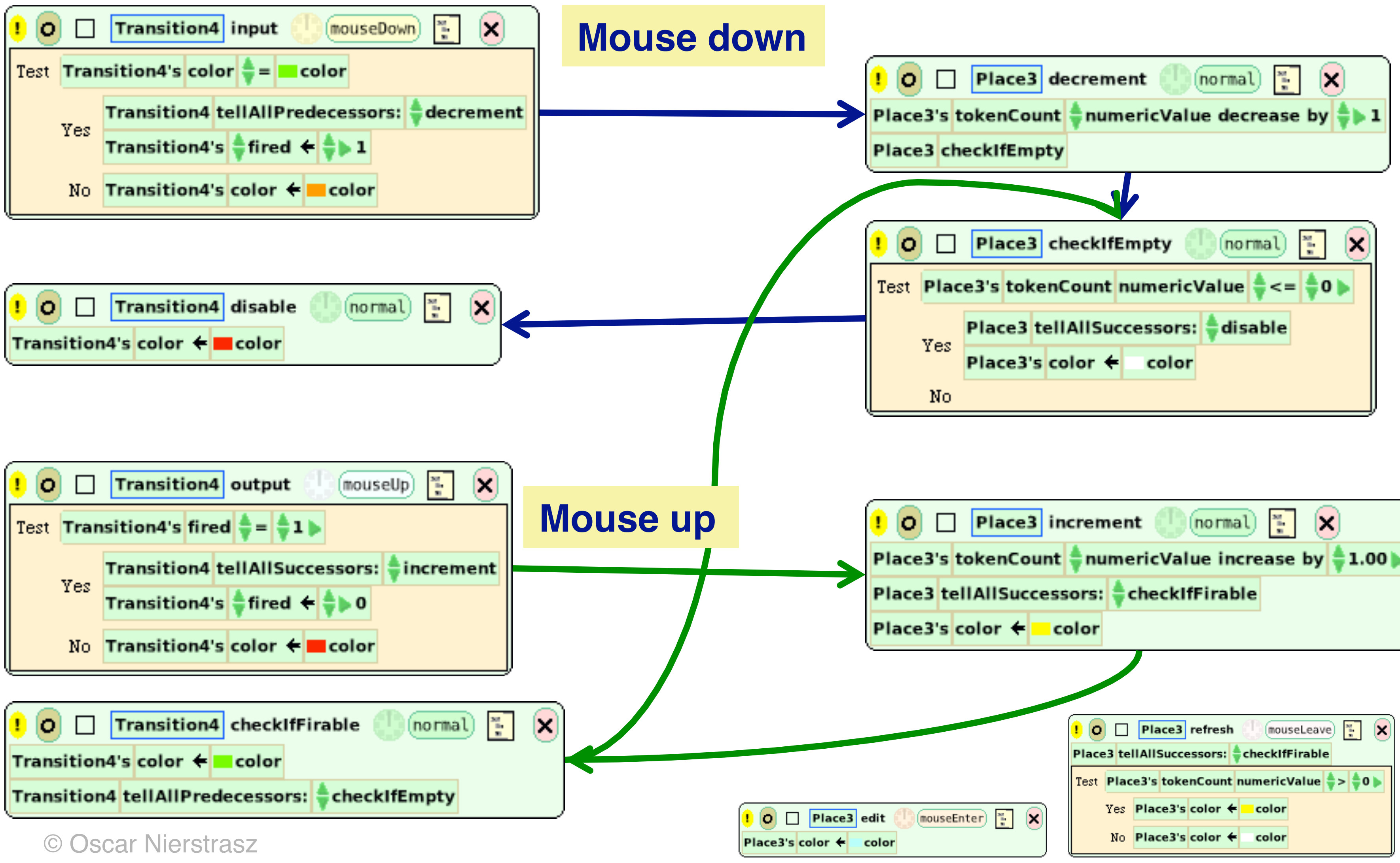
```
Transition4 checkIfFirable  normal
Transition4's color ← 🟩 color
Transition4 tellAllPredecessors: checkIfEmpty
```

```
Transition4 disable  normal
Transition4's color ← 🟥 color
```

```
Place3 edit  mouseEnter
Place3's color ← 🟦 color
```

```
Place3 refresh  mouseLeave
Place3 tellAllSuccessors: checkIfFirable
Test  Place3's tokenCount numericValue > 0
  Yes  Place3's color ← 🟨 color
  No   Place3's color ← ⬜ color
```

Etoys implementation



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Place3 decrement   normal
Place3's tokenCount numericValue decrease by 1
Place3 checkIfEmpty
```

```
Place3 checkIfEmpty   normal
Test  Place3's tokenCount numericValue <= 0
  Yes  Place3 tellAllSuccessors: disable
       Place3's color ← ⬜ color
  No
```

```
Transition4 disable   normal
Transition4's color ← 🟥 color
```

```
Transition4 output   mouseUp
Test  Transition4's fired = 1
  Yes  Transition4 tellAllSuccessors: increment
       Transition4's fired ← 0
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```

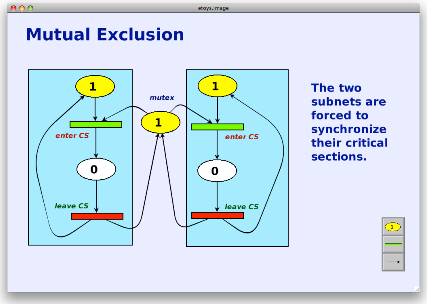
```
Place3 increment   normal
Place3's tokenCount numericValue increase by 1.00
Place3 tellAllSuccessors: checkIfFirable
Place3's color ← 🟨 color
```

```
Transition4 checkIfFirable   normal
Transition4's color ← 🟩 color
Transition4 tellAllPredecessors: checkIfEmpty
```

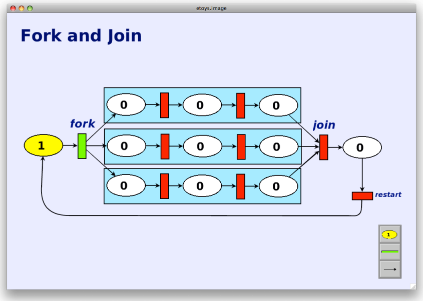
```
Place3 edit   mouseEnter
Place3's color ← 🟦 color
```

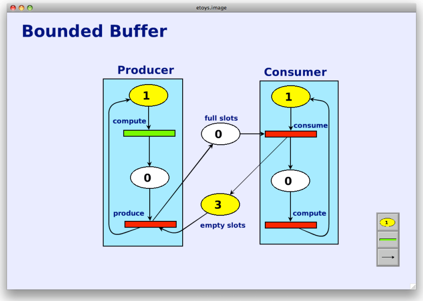
```
Place3 refresh   mouseLeave
Place3 tellAllSuccessors: checkIfFirable
Test  Place3's tokenCount numericValue > 0
  Yes  Place3's color ← 🟨 color
  No   Place3's color ← ⬜ color
```

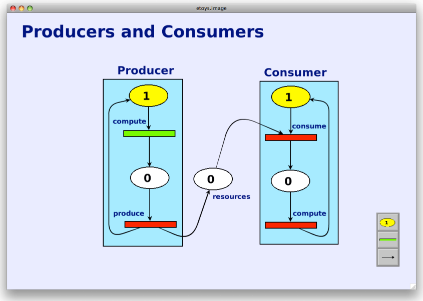
Examples

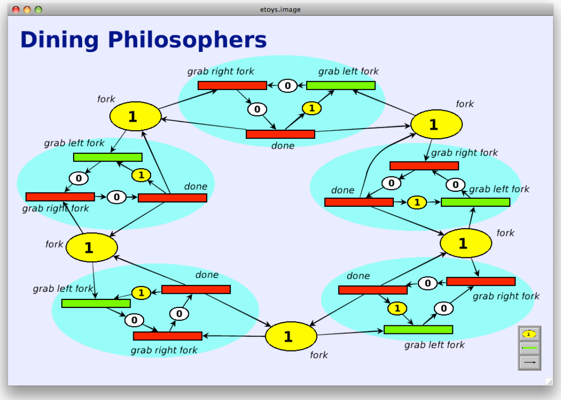


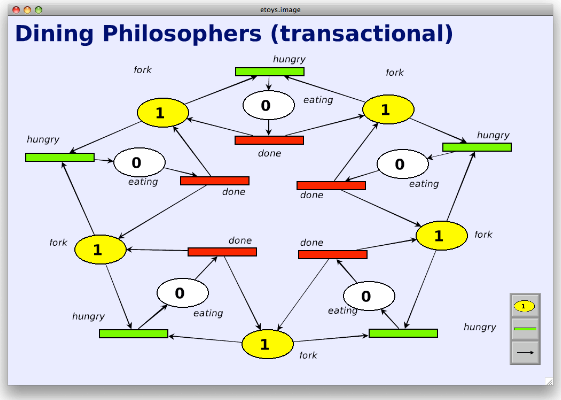
The two subnets are forced to synchronize their critical sections.

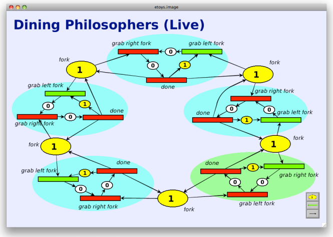


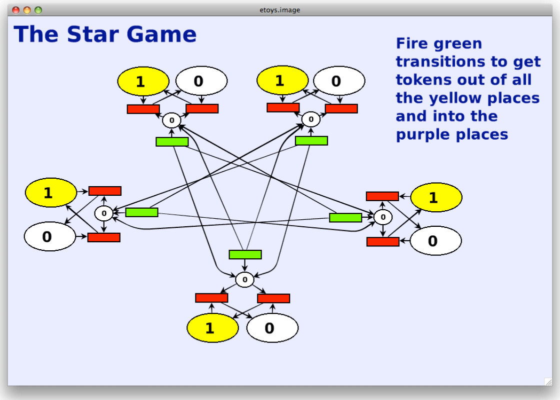












Fire green transitions to get tokens out of all the yellow places and into the purple places

What you should know!

- *How are Petri nets formally specified?*
- *How can nets model concurrency and synchronization?*
- *What is the "reachability set" of a net? How can you compute this set?*
- *What kinds of Petri nets can be modelled by finite state processes?*
- *How can a (bad) implementation of a Petri net deadlock even though there are enabled transitions?*
- *If you implement a Petri net model, why is it a good idea to realize transitions as "thread-per-message gateways"?*

Can you answer these questions?

- *What are some simple conditions for guaranteeing that a net is bounded?*
- *How would you model the Dining Philosophers problem as a Petri net? Is such a net bounded? Is it conservative? Live?*
- *What could you add to Petri nets to make them Turing-complete?*
- *What constraints could you put on a Petri net to make it fair?*

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