

# raport

November 28, 2023

## 1 Zadanie 1: metoda gradientu prostego

### 1.1 Import bibliotek

```
[2]: # %matplotlib ipynb
from wsilib.algorithms.gradient_descent.gradient_descent import (
    GradientDescentFunction as Function,
    GradientDescentSolver as Solver,
    GradientDescentStopConditions as StopConditions
)
from wsilib.utils.domain import Domain
from src.plotting import plot_function
from src.experiments import single_experiment, experiment_step_sizes
import autograd.numpy as np
```

### 1.2 Definicja funkcji celu i obliczanie gradientów

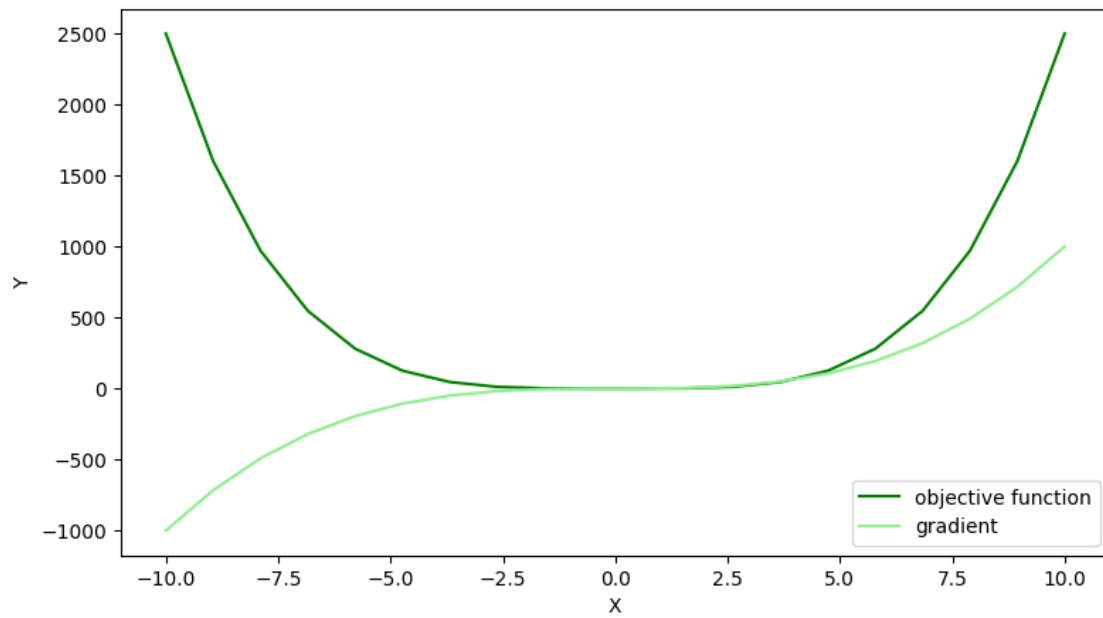
Gradyenty funkcji obliczam za pomocą biblioteki autograd. Jak widać są one bardzo zbliżone do tych obliczonych analitycznie.

```
[3]: domain_1 = Domain(x = np.linspace(-10, 10, 20))

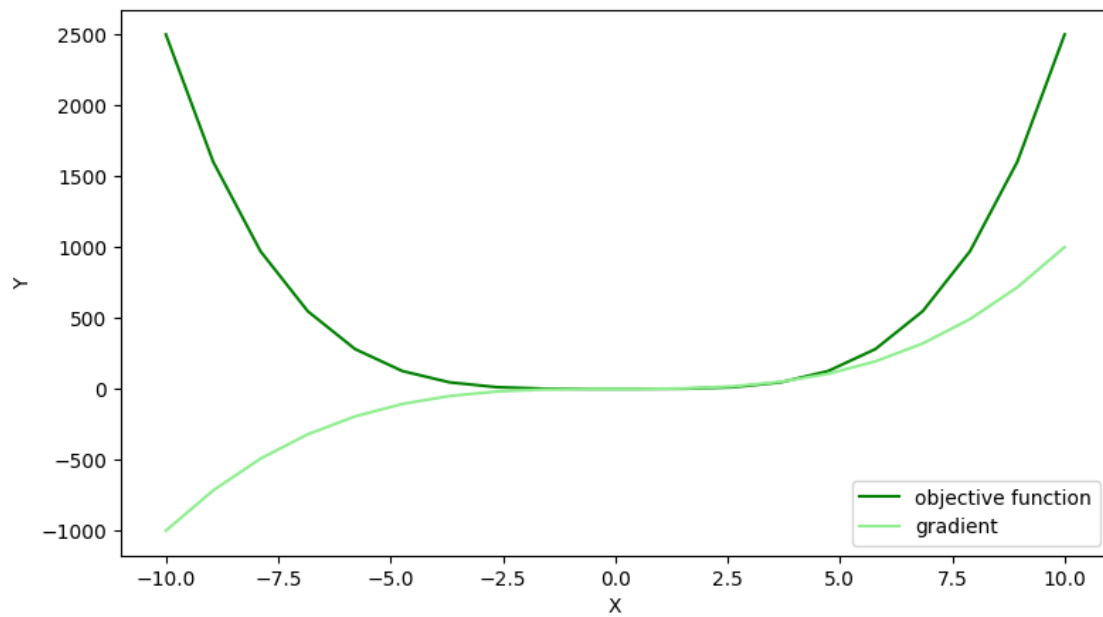
objective_function_1_given_grad = Function(
    f= lambda x: (1/4)*x**4,
    gradient= lambda x: np.array(x**3),
    dim=1,
    name="f(x) = 1/4 * x^4 [gradient analityczny]"
)
objective_function_1 = Function(
    f= lambda x: (1/4)*x**4,
    dim=1,
    name="f(x) = 1/4 * x^4"
)

plot_function(objective_function_1_given_grad, domain=domain_1)
plot_function(objective_function_1, domain=domain_1)
```

Function:  $f(x) = 1/4 * x^4$  [gradient analityczny]



Function:  $f(x) = 1/4 * x^4$



```
[4]: domain_2 = Domain(x = np.linspace(-2, 3, 20), y=np.linspace(-4, 2, 20))
```

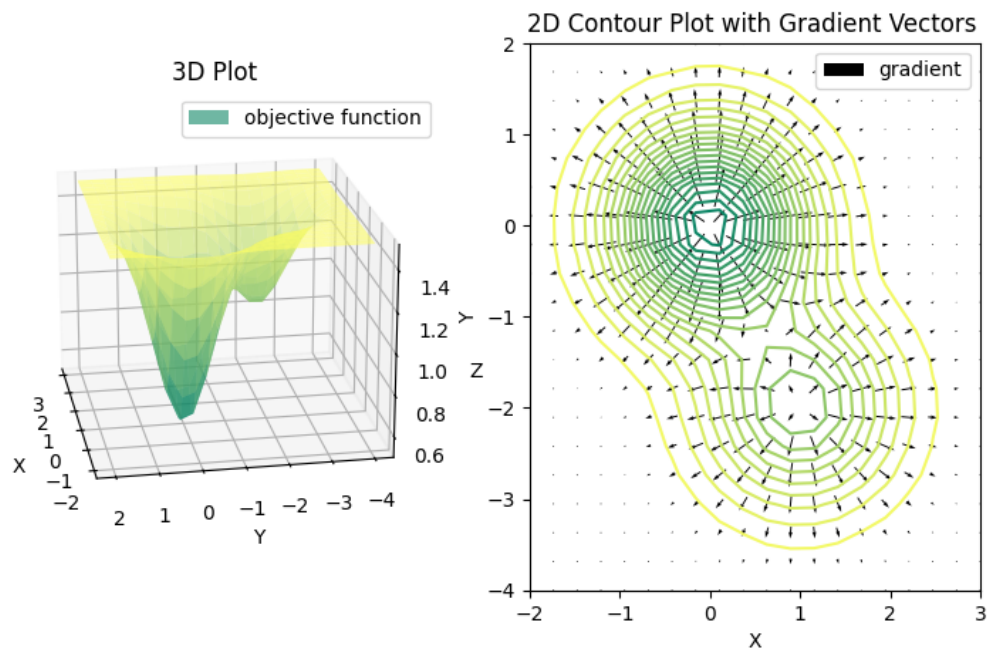
```

objective_function_2_given_grad = Function(
    f=lambda x: 1.5 - np.exp(-x[0]**2 - x[1]**2) - 0.5*np.exp(-(x[0]-1)**2 -
    ↪(x[1]+2)**2),
    gradient=lambda x: np.array([2*x[0]*np.exp(-x[0]**2 - x[1]**2) + (x[0]-1)*np.
    ↪exp(-(x[0]-1)**2 - (x[1]+2)**2), 2*x[1]*np.exp(-x[0]**2 - x[1]**2) +
    ↪(x[1]+2)*np.exp(-(x[0]-1)**2 - (x[1]+2)**2)]),
    dim=2,
    name="f(x, y) = 1.5 - exp(-x^2 - y^2) - 0.5*exp(-(x-1)^2 - (y+2)^2) [gradient_
    ↪analityczny]"
)
objective_function_2 = Function(
    f=lambda x: 1.5 - np.exp(-x[0]**2 - x[1]**2) - 0.5*np.exp(-(x[0]-1)**2 -
    ↪(x[1]+2)**2),
    dim=2,
    name="f(x, y) = 1.5 - exp(-x^2 - y^2) - 0.5*exp(-(x-1)^2 - (y+2)^2)"
)

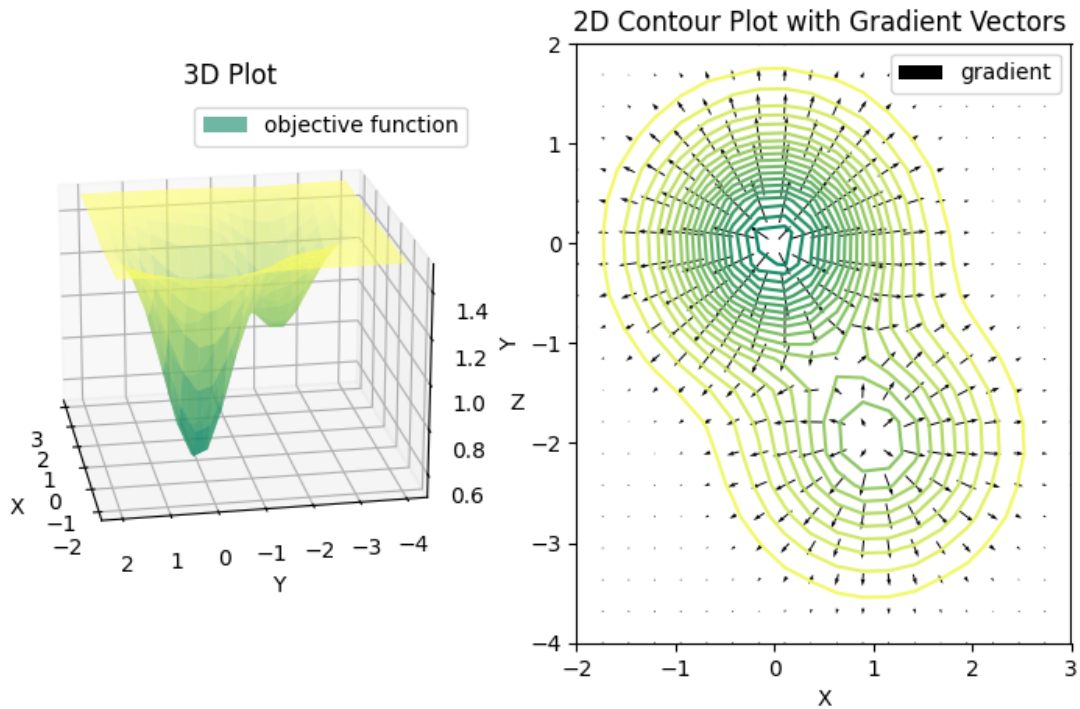
plot_function(objective_function_2_given_grad, domain=domain_2)
plot_function(objective_function_2, domain=domain_2)

```

Function:  $f(x, y) = 1.5 - \exp(-x^2 - y^2) - 0.5 \cdot \exp(-(x-1)^2 - (y+2)^2)$  [gradient analityczny]



Function:  $f(x, y) = 1.5 - \exp(-x^2 - y^2) - 0.5 \cdot \exp(-(x-1)^2 - (y+2)^2)$

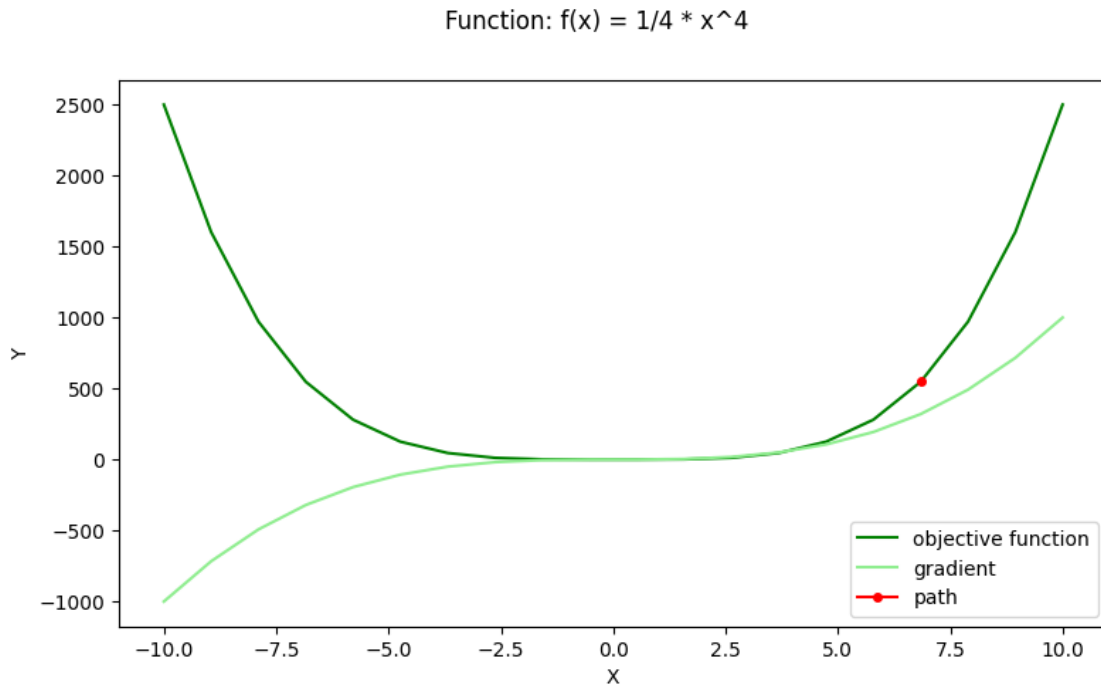


### 1.3 Przykładowy eksperyment dla losowych punktów startowych

```
[5]: s1 = Solver(
    step_size=0.07,
    stop_conditions=[
        StopConditions.max_time(2),
    ]
)
single_experiment(objective_function_1, domain_1, s1, log=True)

# Można też zdefiniować stały punkt startowy
# single_experiment(objective_function_1, domain_1, s1, log=True,
# ↪ starting_point=np.array([5.]))
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([6.84210526]),
f_value=547.8971155838274, gradient_value=array([320.30908296]))
```



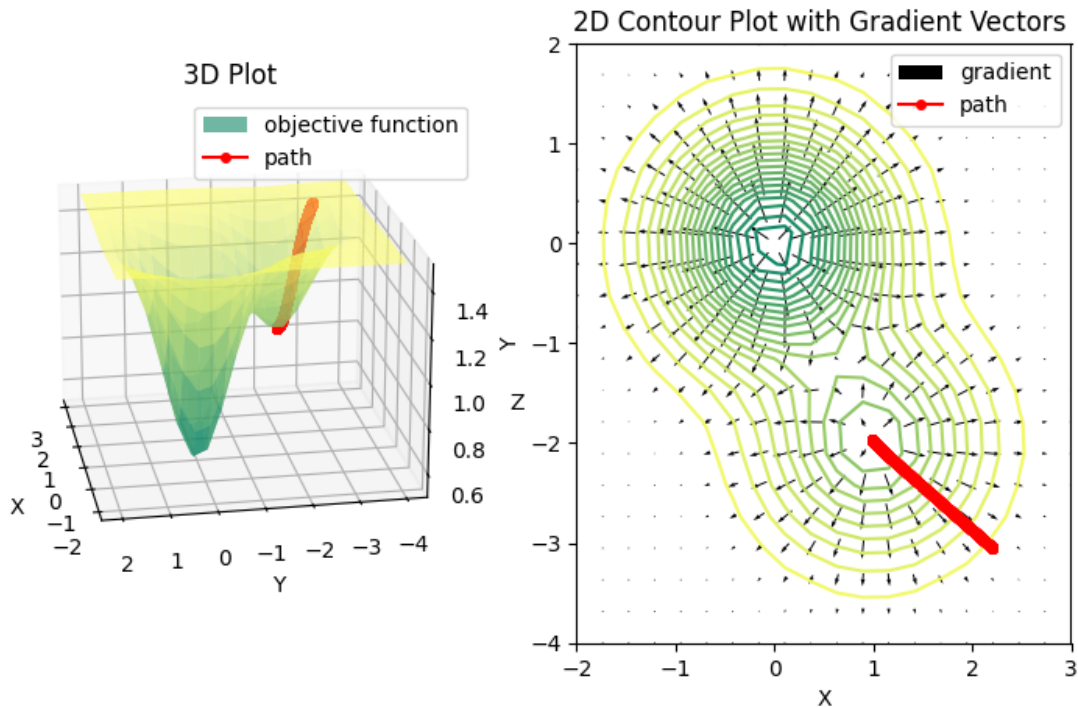
```
GradientDescentResult(n_iter=1, time_running=0.0012, x=array([-15.57953054]),
stop_condition='X_OUT_OF_DOMAIN', f_value=14728.464646523058,
gradient_value=array([-3781.49125999]))
```

```
[6]: s2 = Solver(
    stop_conditions=[
        StopConditions.max_time(5),
    ]
)
single_experiment(objective_function_2, domain_2, s2, log=True)
```

```
GradientDescentIteration(n_iter=0, time_running=0.0013, x=array([ 2.21052632,
-3.05263158]), f_value=1.4618617140422694, gradient_value=array([ 0.09233615,
-0.08029384]))
GradientDescentIteration(n_iter=2459, time_running=1.0002, x=array([ 0.98451007,
-1.96902013]), f_value=0.992742092022676, gradient_value=array([ 3.72285841e-09,
-7.06457008e-09]))
GradientDescentIteration(n_iter=7226, time_running=2.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.54764568e-15,
-1.10952914e-14]))
GradientDescentIteration(n_iter=12032, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
```

```
GradientDescentIteration(n_iter=16933, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
```

Function:  $f(x, y) = 1.5 - \exp(-x^2 - y^2) - 0.5 \cdot \exp(-(x-1)^2 - (y+2)^2)$



```
GradientDescentResult(n_iter=21712, time_running=5.0, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 5.54764568e-15, -1.10952914e-14]))
```

## 1.4 Eksperyment dla różnych rozmiarów kroku i losowych punktów startowych

```
[7]: step_sizes_1 = [0.005, 0.01, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1]
conditions_1 = [
    [StopConditions.max_time(5), StopConditions.min_gradient(1e-6)],
    [StopConditions.min_gradient(1e-4)],
    [StopConditions.max_time(5)],
    [StopConditions.max_iterations(20000)]
]
experiment_step_sizes(objective_function_1, domain_1, step_sizes_1,
↳ conditions_1, n_starting_points=10)
```

Generating starting points...

```
Starting points: [array([-0.52631579]), array([-2.63157895]),
array([7.89473684]), array([-3.68421053]), array([0.52631579]),
array([3.68421053]), array([7.89473684]), array([-3.68421053]),
array([-3.68421053]), array([-10.])]
```

```
(1/320) [00:00] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0009, x=array([-0.52631579]),
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))
GradientDescentIteration(n_iter=2501, time_running=1.0002,
x=array([-0.18687333]), f_value=0.00030488023123328376,
gradient_value=array([-0.00652592]))
GradientDescentIteration(n_iter=23484, time_running=2.0001,
x=array([-0.06475491]), f_value=4.3957283469953885e-06,
gradient_value=array([-0.00027153]))
GradientDescentIteration(n_iter=44719, time_running=3.0001,
x=array([-0.04709671]), f_value=1.2299923786695954e-06,
gradient_value=array([-0.00010447]))
GradientDescentIteration(n_iter=65646, time_running=4.0001,
x=array([-0.03892173]), f_value=5.737313791866433e-07,
gradient_value=array([-5.89625756e-05]))
GradientDescentResult(n_iter=85686, time_running=5.0, x=array([-0.03408959]),
stop_condition='max_time(5)', f_value=3.3761916633196155e-07,
gradient_value=array([-3.96155157e-05]))
```

```
(2/320) [10:21] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[-2.63157895]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),
f_value=11.989625616746345, gradient_value=array([-18.22423094]))
GradientDescentIteration(n_iter=22316, time_running=1.0001,
x=array([-0.06691101]), f_value=5.011068700190828e-06,
gradient_value=array([-0.00029957]))
GradientDescentIteration(n_iter=43998, time_running=2.0001,
x=array([-0.04766312]), f_value=1.2902383307456737e-06,
gradient_value=array([-0.00010828]))
GradientDescentIteration(n_iter=64644, time_running=3.0001,
x=array([-0.03932477]), f_value=5.978669758915949e-07,
gradient_value=array([-6.08132785e-05]))
GradientDescentIteration(n_iter=85588, time_running=4.0001,
x=array([-0.0341775]), f_value=3.4111544389489413e-07,
gradient_value=array([-3.99228034e-05]))
GradientDescentResult(n_iter=106635, time_running=5.0, x=array([-0.03062013]),
stop_condition='max_time(5)', f_value=2.1976992510325036e-07,
gradient_value=array([-2.87092065e-05]))
```

```
(3/320) [15:21] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=20116, time_running=1.0001,
x=array([0.07049071]), f_value=6.172592131835737e-06,
gradient_value=array([0.00035026]))
GradientDescentIteration(n_iter=40917, time_running=2.0001,
x=array([0.04943078]), f_value=1.4925530998207602e-06,
gradient_value=array([0.00012078]))
GradientDescentIteration(n_iter=61581, time_running=3.0001,
x=array([0.04029414]), f_value=6.59033600065926e-07,
gradient_value=array([6.54222779e-05]))
GradientDescentIteration(n_iter=83435, time_running=4.0001,
x=array([0.03461781]), f_value=3.5903643244321903e-07,
gradient_value=array([4.1485744e-05]))
GradientDescentResult(n_iter=104198, time_running=5.0, x=array([0.03097768]),
stop_condition='max_time(5)', f_value=2.302161658616435e-07,
gradient_value=array([2.97267107e-05]))
```

```
(4/320) [18:02] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=19954, time_running=1.0001,
x=array([-0.07076848]), f_value=6.270461503261585e-06,
gradient_value=array([-0.00035442]))
GradientDescentIteration(n_iter=41177, time_running=2.0001,
x=array([-0.04927191]), f_value=1.4734580154194427e-06,
gradient_value=array([-0.00011962]))
GradientDescentIteration(n_iter=61760, time_running=3.0001,
x=array([-0.0402343]), f_value=6.551276151212687e-07,
gradient_value=array([-6.51312515e-05]))
GradientDescentIteration(n_iter=82950, time_running=4.0001,
x=array([-0.03471795]), f_value=3.6320879012484036e-07,
gradient_value=array([-4.18468003e-05]))
GradientDescentResult(n_iter=103830, time_running=5.0, x=array([-0.03103188]),
stop_condition='max_time(5)', f_value=2.3183145129415302e-07,
gradient_value=array([-2.98830044e-05]))
```

```
(5/320) [19:23] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([0.52631579]),
f_value=0.019183400986793997, gradient_value=array([0.14579385]))
GradientDescentIteration(n_iter=20382, time_running=1.0001,
x=array([0.06942764]), f_value=5.808575211656174e-06,
```



```

gradient_value=array([0.00033465]))
GradientDescentIteration(n_iter=40926, time_running=2.0001,
x=array([0.04921241]), f_value=1.4663530859381587e-06,
gradient_value=array([0.00011919]))
GradientDescentIteration(n_iter=60903, time_running=3.0001,
x=array([0.04040021]), f_value=6.660007329462747e-07,
gradient_value=array([6.5940316e-05]))
GradientDescentIteration(n_iter=82112, time_running=4.0001,
x=array([0.03482037]), f_value=3.6751368359695256e-07,
gradient_value=array([4.22182404e-05]))
GradientDescentResult(n_iter=102260, time_running=5.0, x=array([0.03121569]),
stop_condition='max_time(5)', f_value=2.373731818084378e-07,
gradient_value=array([3.04171649e-05]))

```

```

(6/320) [20:16] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([3.68421053]),
f_value=46.05934576929274, gradient_value=array([50.00728969]))
GradientDescentIteration(n_iter=20860, time_running=1.0001,
x=array([0.06921555]), f_value=5.737922206214047e-06,
gradient_value=array([0.0003316]))
GradientDescentIteration(n_iter=43348, time_running=2.0001,
x=array([0.04802261]), f_value=1.3296058988297977e-06,
gradient_value=array([0.00011075]))
GradientDescentIteration(n_iter=65218, time_running=3.0001,
x=array([0.03915335]), f_value=5.875104048515803e-07,
gradient_value=array([6.00214752e-05]))
GradientDescentIteration(n_iter=85916, time_running=4.0001,
x=array([0.03411352]), f_value=3.3856805171749923e-07,
gradient_value=array([3.96989915e-05]))
GradientDescentResult(n_iter=106685, time_running=5.0, x=array([0.0306139]),
stop_condition='max_time(5)', f_value=2.1959094945409902e-07,
gradient_value=array([2.86916696e-05]))

```

```

(7/320) [20:56] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=20740, time_running=1.0001,
x=array([0.06942262]), f_value=5.806896406545021e-06,
gradient_value=array([0.00033458]))
GradientDescentIteration(n_iter=42527, time_running=2.0001,
x=array([0.04848627]), f_value=1.3817037770313786e-06,
gradient_value=array([0.00011399]))
GradientDescentIteration(n_iter=64044, time_running=3.0001,
x=array([0.03951184]), f_value=6.093251149784222e-07,

```

```

gradient_value=array([6.16853173e-05]))
GradientDescentIteration(n_iter=85502, time_running=4.0001,
x=array([0.03419686]), f_value=3.418887921538008e-07,
gradient_value=array([3.99906664e-05]))
GradientDescentResult(n_iter=105736, time_running=5.0, x=array([0.03075158]),
stop_condition='max_time(5)', f_value=2.23568174737687e-07,
gradient_value=array([2.90805408e-05]))

(8/320) [21:22] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=21778, time_running=1.0001,
x=array([-0.0677419]), f_value=5.26464936164907e-06,
gradient_value=array([-0.00031087]))
GradientDescentIteration(n_iter=44563, time_running=2.0001,
x=array([-0.04736362]), f_value=1.2581123422887757e-06,
gradient_value=array([-0.00010625]))
GradientDescentIteration(n_iter=66233, time_running=3.0001,
x=array([-0.03885224]), f_value=5.69645162491286e-07,
gradient_value=array([-5.86473379e-05]))
GradientDescentIteration(n_iter=87574, time_running=4.0001,
x=array([-0.0337891]), f_value=3.2587145516224697e-07,
gradient_value=array([-3.85771139e-05]))
GradientDescentResult(n_iter=108294, time_running=5.0, x=array([-0.03038565]),
stop_condition='max_time(5)', f_value=2.131150128687146e-07,
gradient_value=array([-2.80546934e-05]))

(9/320) [21:43] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=21487, time_running=1.0001,
x=array([-0.0681988]), f_value=5.408128979091629e-06,
gradient_value=array([-0.0003172]))
GradientDescentIteration(n_iter=42598, time_running=2.0001,
x=array([-0.04844339]), f_value=1.3768226453716922e-06,
gradient_value=array([-0.00011369]))
GradientDescentIteration(n_iter=64041, time_running=3.0001,
x=array([-0.03951143]), f_value=6.092997620893147e-07,
gradient_value=array([-6.16833923e-05]))
GradientDescentIteration(n_iter=85251, time_running=4.0001,
x=array([-0.03424629]), f_value=3.4386979533128625e-07,
gradient_value=array([-4.0164329e-05]))
GradientDescentResult(n_iter=106375, time_running=5.0, x=array([-0.03065847]),
stop_condition='max_time(5)', f_value=2.2087251933720704e-07,

```

```
gradient_value=array([-2.88171653e-05]))
```

```
(10/320) [21:56] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[-10.]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-10.]),
f_value=2500.0, gradient_value=array([-1000.]))
GradientDescentIteration(n_iter=19959, time_running=1.0001,
x=array([-0.07076647]), f_value=6.26974799319104e-06,
gradient_value=array([-0.00035439]))
GradientDescentIteration(n_iter=41683, time_running=2.0001,
x=array([-0.0489743]), f_value=1.4381794920763384e-06,
gradient_value=array([-0.00011746]))
GradientDescentIteration(n_iter=62661, time_running=3.0001,
x=array([-0.03994529]), f_value=6.365054210489381e-07,
gradient_value=array([-6.37377273e-05]))
GradientDescentIteration(n_iter=84077, time_running=4.0001,
x=array([-0.03448531]), f_value=3.535709268337884e-07,
gradient_value=array([-4.10111931e-05]))
GradientDescentResult(n_iter=105689, time_running=5.0, x=array([-0.03075835]),
stop_condition='max_time(5)', f_value=2.2376505854875905e-07,
gradient_value=array([-2.90997459e-05]))
```

```
(11/320) [22:07] step_size=0.005 min_gradient(0.0001)
starting_point=[-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-0.52631579]),
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))
GradientDescentIteration(n_iter=20843, time_running=1.0001,
x=array([-0.06866886]), f_value=5.558777002285805e-06,
gradient_value=array([-0.0003238]))
GradientDescentIteration(n_iter=42106, time_running=2.0001,
x=array([-0.04852392]), f_value=1.386001479007911e-06,
gradient_value=array([-0.00011425]))
GradientDescentResult(n_iter=46052, time_running=2.1839, x=array([-0.04641551]),
stop_condition='min_gradient(0.0001)', f_value=1.160359463043665e-06,
gradient_value=array([-9.99975604e-05]))
```

```
(12/320) [21:07] step_size=0.005 min_gradient(0.0001)
starting_point=[-2.63157895]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),
f_value=11.989625616746345, gradient_value=array([-18.22423094]))
GradientDescentIteration(n_iter=21937, time_running=1.0001,
x=array([-0.06748604]), f_value=5.1855602257944805e-06,
gradient_value=array([-0.00030736]))
GradientDescentIteration(n_iter=43704, time_running=2.0001,
x=array([-0.0478231]), f_value=1.307647889019999e-06,
```

```

gradient_value=array([-0.00010937]))
GradientDescentResult(n_iter=46396, time_running=2.1266, x=array([-0.04641557]),
stop_condition='min_gradient(0.0001)', f_value=1.1603656745293682e-06,
gradient_value=array([-9.99979619e-05]))

```

```

(13/320) [20:14] step_size=0.005 min_gradient(0.0001)
starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=21750, time_running=1.0001,
x=array([0.06779221]), f_value=5.280307397957014e-06,
gradient_value=array([0.00031156]))
GradientDescentIteration(n_iter=42060, time_running=2.0001,
x=array([0.04875464]), f_value=1.412550236495775e-06,
gradient_value=array([0.00011589]))
GradientDescentResult(n_iter=46407, time_running=2.2039, x=array([0.04641551]),
stop_condition='min_gradient(0.0001)', f_value=1.160359534497451e-06,
gradient_value=array([9.9997565e-05]))

```

```

(14/320) [19:29] step_size=0.005 min_gradient(0.0001)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=21167, time_running=1.0001,
x=array([-0.06871208]), f_value=5.572784737053009e-06,
gradient_value=array([-0.00032441]))
GradientDescentIteration(n_iter=42409, time_running=2.0001,
x=array([-0.04855118]), f_value=1.3891180662468939e-06,
gradient_value=array([-0.00011445]))
GradientDescentResult(n_iter=46402, time_running=2.1888, x=array([-0.04641585]),
stop_condition='min_gradient(0.0001)', f_value=1.160392903746844e-06,
gradient_value=array([-9.9997218e-05]))

```

```

(15/320) [18:50] step_size=0.005 min_gradient(0.0001)
starting_point=[0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([0.52631579]),
f_value=0.019183400986793997, gradient_value=array([0.14579385]))
GradientDescentIteration(n_iter=22547, time_running=1.0001,
x=array([0.06606583]), f_value=4.7626383692340315e-06,
gradient_value=array([0.00028836]))
GradientDescentIteration(n_iter=44259, time_running=2.0001,
x=array([0.04733884]), f_value=1.255482124837781e-06,
gradient_value=array([0.00010608]))
GradientDescentResult(n_iter=46052, time_running=2.0854, x=array([0.04641551]),
stop_condition='min_gradient(0.0001)', f_value=1.160359463043665e-06,

```

```
gradient_value=array([9.99975604e-05]))
```

```
(16/320) [18:14] step_size=0.005 min_gradient(0.0001)
starting_point=[3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([3.68421053]),
f_value=46.05934576929274, gradient_value=array([50.00728969]))
GradientDescentIteration(n_iter=21816, time_running=1.0001,
x=array([0.06768291]), f_value=5.246335588658161e-06,
gradient_value=array([0.00031005]))
GradientDescentIteration(n_iter=45792, time_running=2.0001,
x=array([0.04672389]), f_value=1.1915052058893023e-06,
gradient_value=array([0.000102]))
GradientDescentResult(n_iter=46402, time_running=2.0281, x=array([0.04641585]),
stop_condition='min_gradient(0.0001)', f_value=1.160392903746844e-06,
gradient_value=array([9.9997218e-05]))
```

```
(17/320) [17:42] step_size=0.005 min_gradient(0.0001)
starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=20976, time_running=1.0001,
x=array([0.06903114]), f_value=5.677016277280055e-06,
gradient_value=array([0.00032895]))
GradientDescentIteration(n_iter=42934, time_running=2.0001,
x=array([0.04825595]), f_value=1.355637096057645e-06,
gradient_value=array([0.00011237]))
GradientDescentResult(n_iter=46407, time_running=2.1619, x=array([0.04641551]),
stop_condition='min_gradient(0.0001)', f_value=1.160359534497451e-06,
gradient_value=array([9.9997565e-05]))
```

```
(18/320) [17:14] step_size=0.005 min_gradient(0.0001)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=21787, time_running=1.0001,
x=array([-0.06772791]), f_value=5.260303232493764e-06,
gradient_value=array([-0.00031067]))
GradientDescentIteration(n_iter=42988, time_running=2.0001,
x=array([-0.04822321]), f_value=1.3519616510795544e-06,
gradient_value=array([-0.00011214]))
GradientDescentResult(n_iter=46402, time_running=2.1949, x=array([-0.04641585]),
stop_condition='min_gradient(0.0001)', f_value=1.160392903746844e-06,
gradient_value=array([-9.9997218e-05]))
```

```
(19/320) [16:50] step_size=0.005 min_gradient(0.0001)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=21277, time_running=1.0001,
x=array([-0.06853433]), f_value=5.515346214825749e-06,
gradient_value=array([-0.0003219]))
GradientDescentIteration(n_iter=41664, time_running=2.0001,
x=array([-0.0489832]), f_value=1.4392243027744047e-06,
gradient_value=array([-0.00011753]))
GradientDescentResult(n_iter=46402, time_running=2.2284, x=array([-0.04641585]),
stop_condition='min_gradient(0.0001)', f_value=1.160392903746844e-06,
gradient_value=array([-9.99997218e-05]))
```

```
(20/320) [16:28] step_size=0.005 min_gradient(0.0001) starting_point=[-10.]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-10.]),
f_value=2500.0, gradient_value=array([-1000.]))
GradientDescentIteration(n_iter=21153, time_running=1.0001,
x=array([-0.06874107]), f_value=5.582197347809822e-06,
gradient_value=array([-0.00032482]))
GradientDescentIteration(n_iter=42163, time_running=2.0001,
x=array([-0.0486948]), f_value=1.4056274232617145e-06,
gradient_value=array([-0.00011546]))
GradientDescentResult(n_iter=46406, time_running=2.2164, x=array([-0.04641578]),
stop_condition='min_gradient(0.0001)', f_value=1.1603860993521283e-06,
gradient_value=array([-9.9999282e-05]))
```

```
(21/320) [16:08] step_size=0.005 max_time(5) starting_point=[-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-0.52631579]),
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))
GradientDescentIteration(n_iter=21283, time_running=1.0001,
x=array([-0.06796736]), f_value=5.3350896170657135e-06,
gradient_value=array([-0.00031398]))
GradientDescentIteration(n_iter=44414, time_running=2.0001,
x=array([-0.04725684]), f_value=1.2468053858843763e-06,
gradient_value=array([-0.00010553]))
GradientDescentIteration(n_iter=66555, time_running=3.0001,
x=array([-0.03865648]), f_value=5.58250618679631e-07,
gradient_value=array([-5.77652827e-05]))
GradientDescentIteration(n_iter=88731, time_running=4.0001,
x=array([-0.03350199]), f_value=3.1493599875973347e-07,
gradient_value=array([-3.76020673e-05]))
GradientDescentResult(n_iter=111064, time_running=5.0, x=array([-0.02995714]),
stop_condition='max_time(5)', f_value=2.0134516428351778e-07,
gradient_value=array([-2.68844339e-05]))
```

```
(22/320) [16:26] step_size=0.005 max_time(5) starting_point=[-2.63157895]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),
f_value=11.989625616746345, gradient_value=array([-18.22423094]))
GradientDescentIteration(n_iter=21961, time_running=1.0001,
x=array([-0.06744918]), f_value=5.174242270962022e-06,
gradient_value=array([-0.00030685]))
GradientDescentIteration(n_iter=45074, time_running=2.0001,
x=array([-0.04709103]), f_value=1.229399147995724e-06,
gradient_value=array([-0.00010443]))
GradientDescentIteration(n_iter=67249, time_running=3.0001,
x=array([-0.03855582]), f_value=5.524586404209653e-07,
gradient_value=array([-5.73152012e-05]))
GradientDescentIteration(n_iter=88579, time_running=4.0001,
x=array([-0.03359566]), f_value=3.184729324278617e-07,
gradient_value=array([-3.79183463e-05]))
GradientDescentResult(n_iter=110246, time_running=5.0, x=array([-0.03011459]),
stop_condition='max_time(5)', f_value=2.0561157864470428e-07,
gradient_value=array([-2.7310564e-05]))
```

```
(23/320) [16:42] step_size=0.005 max_time(5) starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=22522, time_running=1.0001,
x=array([0.06662064]), f_value=4.924648387022042e-06,
gradient_value=array([0.00029568]))
GradientDescentIteration(n_iter=45262, time_running=2.0001,
x=array([0.04699882]), f_value=1.2197978908078574e-06,
gradient_value=array([0.00010382]))
GradientDescentIteration(n_iter=67015, time_running=3.0001,
x=array([0.03862619]), f_value=5.565029026819697e-07,
gradient_value=array([5.76295952e-05]))
GradientDescentIteration(n_iter=88683, time_running=4.0001,
x=array([0.03357801]), f_value=3.1780452045796174e-07,
gradient_value=array([3.78586433e-05]))
GradientDescentResult(n_iter=111390, time_running=5.0, x=array([0.02996104]),
stop_condition='max_time(5)', f_value=2.0145001313936677e-07,
gradient_value=array([2.68949331e-05]))
```

```
(24/320) [16:56] step_size=0.005 max_time(5) starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=22869, time_running=1.0001,
x=array([-0.06610721]), f_value=4.774580209070549e-06,
gradient_value=array([-0.0002889]))
GradientDescentIteration(n_iter=45608, time_running=2.0001,
```

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x=array([-0.04681802]), f_value=1.2011357728117083e-06,
gradient_value=array([-0.00010262]))
GradientDescentIteration(n_iter=67542, time_running=3.0001,
x=array([-0.03847399]), f_value=5.477833961064271e-07,
gradient_value=array([-5.69510377e-05]))
GradientDescentIteration(n_iter=90462, time_running=4.0001,
x=array([-0.03324545]), f_value=3.053997411256144e-07,
gradient_value=array([-3.67448511e-05]))
GradientDescentResult(n_iter=113608, time_running=5.0, x=array([-0.02966658]),
stop_condition='max_time(5)', f_value=1.936467498578375e-07,
gradient_value=array([-2.61097463e-05]))

```

```

(25/320) [17:09] step_size=0.005 max_time(5) starting_point=[0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([0.52631579]),
f_value=0.019183400986793997, gradient_value=array([0.14579385]))
GradientDescentIteration(n_iter=22206, time_running=1.0001,
x=array([0.06656305]), f_value=4.907642814795527e-06,
gradient_value=array([0.00029492]))
GradientDescentIteration(n_iter=45509, time_running=2.0001,
x=array([0.04668941]), f_value=1.1879925300909462e-06,
gradient_value=array([0.00010178]))
GradientDescentIteration(n_iter=67803, time_running=3.0001,
x=array([0.03830098]), f_value=5.379969045385482e-07,
gradient_value=array([5.61862214e-05]))
GradientDescentIteration(n_iter=92173, time_running=4.0001,
x=array([0.03287302]), f_value=2.9194314486023663e-07,
gradient_value=array([3.55237418e-05]))
GradientDescentResult(n_iter=115428, time_running=5.0, x=array([0.0293872]),
stop_condition='max_time(5)', f_value=1.8645452599094986e-07,
gradient_value=array([2.53790108e-05]))

```

```

(26/320) [17:21] step_size=0.005 max_time(5) starting_point=[3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([3.68421053]),
f_value=46.05934576929274, gradient_value=array([50.00728969]))
GradientDescentIteration(n_iter=23585, time_running=1.0001,
x=array([0.06509657]), f_value=4.489235858166649e-06,
gradient_value=array([0.00027585]))
GradientDescentIteration(n_iter=45270, time_running=2.0001,
x=array([0.04699242]), f_value=1.219133572100483e-06,
gradient_value=array([0.00010377]))
GradientDescentIteration(n_iter=68067, time_running=3.0001,
x=array([0.03832536]), f_value=5.393675533595941e-07,
gradient_value=array([5.6293546e-05]))
GradientDescentIteration(n_iter=90939, time_running=4.0001,
x=array([0.03315815]), f_value=3.0220481372641364e-07,
gradient_value=array([3.64561688e-05]))

```



```
GradientDescentResult(n_iter=112764, time_running=5.0, x=array([0.02977739]),
stop_condition='max_time(5)', f_value=1.9655599767813854e-07,
gradient_value=array([2.64033917e-05]))
```

```
(27/320) [17:32] step_size=0.005 max_time(5) starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=22163, time_running=1.0001,
x=array([0.06715784]), f_value=5.085420072505054e-06,
gradient_value=array([0.00030289]))
GradientDescentIteration(n_iter=45738, time_running=2.0001,
x=array([0.04675367]), f_value=1.1945457593077248e-06,
gradient_value=array([0.0001022]))
GradientDescentIteration(n_iter=68092, time_running=3.0001,
x=array([0.03831954]), f_value=5.390402277697863e-07,
gradient_value=array([5.62679219e-05]))
GradientDescentIteration(n_iter=90466, time_running=4.0001,
x=array([0.03324551]), f_value=3.0540198656462705e-07,
gradient_value=array([3.67450537e-05]))
GradientDescentResult(n_iter=111666, time_running=5.0, x=array([0.02992399]),
stop_condition='max_time(5)', f_value=2.0045549826992516e-07,
gradient_value=array([2.67952907e-05]))
```

```
(28/320) [17:41] step_size=0.005 max_time(5) starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=23266, time_running=1.0001,
x=array([-0.06554108]), f_value=4.6131154273718255e-06,
gradient_value=array([-0.00028154]))
GradientDescentIteration(n_iter=45710, time_running=2.0001,
x=array([-0.04676577]), f_value=1.1957827283705624e-06,
gradient_value=array([-0.00010228]))
GradientDescentIteration(n_iter=67941, time_running=3.0001,
x=array([-0.03836087]), f_value=5.413695784223641e-07,
gradient_value=array([-5.64501862e-05]))
GradientDescentIteration(n_iter=90606, time_running=4.0001,
x=array([-0.03321902]), f_value=3.044299164880475e-07,
gradient_value=array([-3.66573014e-05]))
GradientDescentResult(n_iter=112726, time_running=5.0, x=array([-0.0297824]),
stop_condition='max_time(5)', f_value=1.966885219633932e-07,
gradient_value=array([-2.64167421e-05]))
```

```
(29/320) [17:49] step_size=0.005 max_time(5) starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
```

```

GradientDescentIteration(n_iter=21172, time_running=1.0001,
x=array([-0.06870397]), f_value=5.570154469025629e-06,
gradient_value=array([-0.0003243]))
GradientDescentIteration(n_iter=43224, time_running=2.0001,
x=array([-0.04809142]), f_value=1.3372431951581237e-06,
gradient_value=array([-0.00011123]))
GradientDescentIteration(n_iter=65782, time_running=3.0001,
x=array([-0.03898517]), f_value=5.774812764127918e-07,
gradient_value=array([-5.92513738e-05]))
GradientDescentIteration(n_iter=87868, time_running=4.0001,
x=array([-0.03373253]), f_value=3.2369475738873164e-07,
gradient_value=array([-3.83836917e-05]))
GradientDescentResult(n_iter=109221, time_running=5.0, x=array([-0.03025644]),
stop_condition='max_time(5)', f_value=2.0951324426607572e-07,
gradient_value=array([-2.76983313e-05]))

```

```

(30/320) [17:56] step_size=0.005 max_time(5) starting_point=[-10.]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-10.]),
f_value=2500.0, gradient_value=array([-1000.]))
GradientDescentIteration(n_iter=23643, time_running=1.0001,
x=array([-0.06502203]), f_value=4.468708796642479e-06,
gradient_value=array([-0.0002749]))
GradientDescentIteration(n_iter=45766, time_running=2.0001,
x=array([-0.04673913]), f_value=1.1930603928921855e-06,
gradient_value=array([-0.0001021]))
GradientDescentIteration(n_iter=68183, time_running=3.0001,
x=array([-0.03829383]), f_value=5.375951383118349e-07,
gradient_value=array([-5.61547493e-05]))
GradientDescentIteration(n_iter=90363, time_running=4.0001,
x=array([-0.03326436]), f_value=3.0609535904612584e-07,
gradient_value=array([-3.68076044e-05]))
GradientDescentResult(n_iter=113184, time_running=5.0, x=array([-0.0297226]),
stop_condition='max_time(5)', f_value=1.95113453484711e-07,
gradient_value=array([-2.62579251e-05]))

```

```

(31/320) [18:03] step_size=0.005 max_iterations(20000)
starting_point=[-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-0.52631579]),
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))
GradientDescentResult(n_iter=20000, time_running=0.9143, x=array([-0.07007582]),
stop_condition='max_iterations(20000)', f_value=6.028548389726122e-06,
gradient_value=array([-0.00034412]))

```

```

(32/320) [17:33] step_size=0.005 max_iterations(20000)
starting_point=[-2.63157895]

```

```
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),  
f_value=11.989625616746345, gradient_value=array([-18.22423094]))  
GradientDescentResult(n_iter=20000, time_running=0.8835, x=array([-0.07067555]),  
stop_condition='max_iterations(20000)', f_value=6.237588316213895e-06,  
gradient_value=array([-0.00035303]))
```

```
(33/320) [17:05] step_size=0.005 max_iterations(20000)  
starting_point=[7.89473684]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),  
f_value=971.159674956454, gradient_value=array([492.05423531]))  
GradientDescentResult(n_iter=20000, time_running=0.9019, x=array([0.07069475]),  
stop_condition='max_iterations(20000)', f_value=6.244372105392035e-06,  
gradient_value=array([0.00035331]))
```

```
(34/320) [16:39] step_size=0.005 max_iterations(20000)  
starting_point=[-3.68421053]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),  
f_value=46.05934576929274, gradient_value=array([-50.00728969]))  
GradientDescentResult(n_iter=20000, time_running=0.8437, x=array([-0.0706871]),  
stop_condition='max_iterations(20000)', f_value=6.241668659066277e-06,  
gradient_value=array([-0.0003532]))
```

```
(35/320) [16:14] step_size=0.005 max_iterations(20000)  
starting_point=[0.52631579]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([0.52631579]),  
f_value=0.019183400986793997, gradient_value=array([0.14579385]))  
GradientDescentResult(n_iter=20000, time_running=0.8913, x=array([0.07007582]),  
stop_condition='max_iterations(20000)', f_value=6.028548389726122e-06,  
gradient_value=array([0.00034412]))
```

```
(36/320) [15:50] step_size=0.005 max_iterations(20000)  
starting_point=[3.68421053]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([3.68421053]),  
f_value=46.05934576929274, gradient_value=array([50.00728969]))  
GradientDescentResult(n_iter=20000, time_running=0.9015, x=array([0.0706871]),  
stop_condition='max_iterations(20000)', f_value=6.241668659066277e-06,  
gradient_value=array([0.0003532]))
```

```
(37/320) [15:28] step_size=0.005 max_iterations(20000)  
starting_point=[7.89473684]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),  
f_value=971.159674956454, gradient_value=array([492.05423531]))  
GradientDescentResult(n_iter=20000, time_running=0.8777, x=array([0.07069475]),
```

```
stop_condition='max_iterations(20000)', f_value=6.244372105392035e-06,  
gradient_value=array([0.00035331]))
```

```
(38/320) [15:07] step_size=0.005 max_iterations(20000)  
starting_point=[-3.68421053]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),  
f_value=46.05934576929274, gradient_value=array([-50.00728969]))  
GradientDescentResult(n_iter=20000, time_running=0.8646, x=array([-0.0706871]),  
stop_condition='max_iterations(20000)', f_value=6.241668659066277e-06,  
gradient_value=array([-0.0003532]))
```

```
(39/320) [14:46] step_size=0.005 max_iterations(20000)  
starting_point=[-3.68421053]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),  
f_value=46.05934576929274, gradient_value=array([-50.00728969]))  
GradientDescentResult(n_iter=20000, time_running=0.8767, x=array([-0.0706871]),  
stop_condition='max_iterations(20000)', f_value=6.241668659066277e-06,  
gradient_value=array([-0.0003532]))
```

```
(40/320) [14:27] step_size=0.005 max_iterations(20000) starting_point=[-10.]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-10.]),  
f_value=2500.0, gradient_value=array([-1000.]))  
GradientDescentResult(n_iter=20000, time_running=0.9307, x=array([-0.07069393]),  
stop_condition='max_iterations(20000)', f_value=6.244079566069144e-06,  
gradient_value=array([-0.0003533]))
```

```
(41/320) [14:09] step_size=0.01 max_time(5) min_gradient(1e-06)  
starting_point=[-0.52631579]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-0.52631579]),  
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))  
GradientDescentIteration(n_iter=21122, time_running=1.0001,  
x=array([-0.04844324]), f_value=1.3768065113120856e-06,  
gradient_value=array([-0.00011368]))  
GradientDescentIteration(n_iter=41563, time_running=2.0001,  
x=array([-0.03460743]), f_value=3.586058028794127e-07,  
gradient_value=array([-4.14484198e-05]))  
GradientDescentIteration(n_iter=60966, time_running=3.0001,  
x=array([-0.02859458]), f_value=1.6713781727952717e-07,  
gradient_value=array([-2.33803516e-05]))  
GradientDescentIteration(n_iter=79976, time_running=4.0001,  
x=array([-0.02497487]), f_value=9.72641879449877e-08,  
gradient_value=array([-1.55779289e-05]))  
GradientDescentResult(n_iter=100325, time_running=5.0, x=array([-0.02230385]),  
stop_condition='max_time(5)', f_value=6.186704422433083e-08,
```

```
gradient_value=array([-1.1095312e-05]))
```

```
(42/320) [14:17] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[-2.63157895]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),
f_value=11.989625616746345, gradient_value=array([-18.22423094]))
GradientDescentIteration(n_iter=20069, time_running=1.0001,
x=array([-0.04989755]), f_value=1.5497327026656843e-06,
gradient_value=array([-0.00012423]))
GradientDescentIteration(n_iter=39540, time_running=2.0001,
x=array([-0.03555423]), f_value=3.994894144676448e-07,
gradient_value=array([-4.49442306e-05]))
GradientDescentIteration(n_iter=59299, time_running=3.0001,
x=array([-0.02903421]), f_value=1.7765598309417493e-07,
gradient_value=array([-2.44754042e-05]))
GradientDescentIteration(n_iter=79140, time_running=4.0001,
x=array([-0.02513321]), f_value=9.975429678645538e-08,
gradient_value=array([-1.58760955e-05]))
GradientDescentResult(n_iter=98551, time_running=5.0, x=array([-0.02252282]),
stop_condition='max_time(5)', f_value=6.433258002060563e-08,
gradient_value=array([-1.1425316e-05]))
```

```
(43/320) [14:24] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=19895, time_running=1.0001,
x=array([0.05011812]), f_value=1.5773172589902135e-06,
gradient_value=array([0.00012589]))
GradientDescentIteration(n_iter=40165, time_running=2.0001,
x=array([0.03527765]), f_value=3.872030114090201e-07,
gradient_value=array([4.39034893e-05]))
GradientDescentIteration(n_iter=61291, time_running=3.0001,
x=array([0.02855915]), f_value=1.6631103329147222e-07,
gradient_value=array([2.32935558e-05]))
GradientDescentIteration(n_iter=82227, time_running=4.0001,
x=array([0.02465735]), f_value=9.241136889000445e-08,
gradient_value=array([1.49912914e-05]))
GradientDescentResult(n_iter=103115, time_running=5.0, x=array([0.02201905]),
stop_condition='max_time(5)', f_value=5.876715127420001e-08,
gradient_value=array([1.06756905e-05]))
```

```
(44/320) [14:32] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
```

```

f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=20778, time_running=1.0001,
x=array([-0.04904287]), f_value=1.4462508050718326e-06,
gradient_value=array([-0.00011796]))
GradientDescentIteration(n_iter=41567, time_running=2.0001,
x=array([-0.03467802]), f_value=3.61540840959845e-07,
gradient_value=array([-4.17025889e-05]))
GradientDescentIteration(n_iter=63359, time_running=3.0001,
x=array([-0.02808942]), f_value=1.556364252698388e-07,
gradient_value=array([-2.2162994e-05]))
GradientDescentIteration(n_iter=84113, time_running=4.0001,
x=array([-0.02437947]), f_value=8.831550825745812e-08,
gradient_value=array([-1.44901447e-05]))
GradientDescentResult(n_iter=105082, time_running=5.0, x=array([-0.02181208]),
stop_condition='max_time(5)', f_value=5.658853942653078e-08,
gradient_value=array([-1.03774669e-05]))

```

```

(45/320) [14:38] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([0.52631579]),
f_value=0.019183400986793997, gradient_value=array([0.14579385]))
GradientDescentIteration(n_iter=20620, time_running=1.0001,
x=array([0.04902425]), f_value=1.444054999194768e-06,
gradient_value=array([0.00011782]))
GradientDescentIteration(n_iter=42241, time_running=2.0001,
x=array([0.03432978]), f_value=3.47235485927156e-07,
gradient_value=array([4.04588067e-05]))
GradientDescentIteration(n_iter=63553, time_running=3.0001,
x=array([0.02800826]), f_value=1.5384539152099186e-07,
gradient_value=array([2.19714319e-05]))
GradientDescentIteration(n_iter=84046, time_running=4.0001,
x=array([0.02436401]), f_value=8.809169370196753e-08,
gradient_value=array([1.44625946e-05]))
GradientDescentResult(n_iter=104813, time_running=5.0, x=array([0.02182197]),
stop_condition='max_time(5)', f_value=5.66912037930446e-08,
gradient_value=array([1.0391584e-05]))

```

```

(46/320) [14:44] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([3.68421053]),
f_value=46.05934576929274, gradient_value=array([50.00728969]))
GradientDescentIteration(n_iter=19664, time_running=1.0001,
x=array([0.05041227]), f_value=1.6146741687486116e-06,
gradient_value=array([0.00012812]))
GradientDescentIteration(n_iter=40089, time_running=2.0001,
x=array([0.03531134]), f_value=3.8868391069092616e-07,

```

```

gradient_value=array([4.40293644e-05]))
GradientDescentIteration(n_iter=60502, time_running=3.0001,
x=array([0.02874488]), f_value=1.706796430118659e-07,
gradient_value=array([2.37509665e-05]))
GradientDescentIteration(n_iter=80853, time_running=4.0001,
x=array([0.02486604]), f_value=9.55798952831147e-08,
gradient_value=array([1.53751693e-05]))
GradientDescentResult(n_iter=101531, time_running=5.0, x=array([0.0221902]),
stop_condition='max_time(5)', f_value=6.061564350333447e-08,
gradient_value=array([1.0926562e-05]))

```

```

(47/320) [14:50] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=19958, time_running=1.0001,
x=array([0.05003899]), f_value=1.5673799812169384e-06,
gradient_value=array([0.00012529]))
GradientDescentIteration(n_iter=41425, time_running=2.0001,
x=array([0.03473714]), f_value=3.64012437671393e-07,
gradient_value=array([4.19162248e-05]))
GradientDescentIteration(n_iter=61364, time_running=3.0001,
x=array([0.02854216]), f_value=1.659156443357876e-07,
gradient_value=array([2.32520098e-05]))
GradientDescentIteration(n_iter=81873, time_running=4.0001,
x=array([0.02471059]), f_value=9.321211818320078e-08,
gradient_value=array([1.50886115e-05]))
GradientDescentResult(n_iter=102418, time_running=5.0, x=array([0.02209384]),
stop_condition='max_time(5)', f_value=5.956965717730122e-08,
gradient_value=array([1.07848428e-05]))

```

```

(48/320) [14:55] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=20891, time_running=1.0001,
x=array([-0.04891012]), f_value=1.4306545813858256e-06,
gradient_value=array([-0.000117]))
GradientDescentIteration(n_iter=40655, time_running=2.0001,
x=array([-0.03506473]), f_value=3.7793933806718246e-07,
gradient_value=array([-4.31133299e-05]))
GradientDescentIteration(n_iter=61070, time_running=3.0001,
x=array([-0.02861091]), f_value=1.675200237148961e-07,
gradient_value=array([-2.34204394e-05]))
GradientDescentIteration(n_iter=82850, time_running=4.0001,
x=array([-0.02456457]), f_value=9.10283208322915e-08,

```

```

gradient_value=array([-1.48227025e-05]))
GradientDescentResult(n_iter=104290, time_running=5.0, x=array([-0.02189474]),
stop_condition='max_time(5)', f_value=5.745120427579896e-08,
gradient_value=array([-1.04958918e-05]))

```

```

(49/320) [15:00] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=21648, time_running=1.0001,
x=array([-0.04804773]), f_value=1.3323905365051204e-06,
gradient_value=array([-0.00011092]))
GradientDescentIteration(n_iter=42914, time_running=2.0001,
x=array([-0.03412957]), f_value=3.392058392390346e-07,
gradient_value=array([-3.97550664e-05]))
GradientDescentIteration(n_iter=63860, time_running=3.0001,
x=array([-0.02797904]), f_value=1.5320437243890441e-07,
gradient_value=array([-2.19027358e-05]))
GradientDescentIteration(n_iter=84981, time_running=4.0001,
x=array([-0.02425466]), f_value=8.652082625378261e-08,
gradient_value=array([-1.42687355e-05]))
GradientDescentResult(n_iter=106885, time_running=5.0, x=array([-0.02162735]),
stop_condition='max_time(5)', f_value=5.46956916134797e-08,
gradient_value=array([-1.01160236e-05]))

```

```

(50/320) [15:05] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[-10.]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-10.]),
f_value=2500.0, gradient_value=array([-1000.]))
GradientDescentResult(n_iter=1, time_running=0.0003, x=array([0.]),
stop_condition='min_gradient(1e-06)', f_value=0.0, gradient_value=array([0.]))

```

```

(51/320) [14:44] step_size=0.01 min_gradient(0.0001)
starting_point=[-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-0.52631579]),
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))
GradientDescentIteration(n_iter=21135, time_running=1.0001,
x=array([-0.04842847]), f_value=1.3751278594478955e-06,
gradient_value=array([-0.00011358]))
GradientDescentResult(n_iter=23024, time_running=1.0842, x=array([-0.04641569]),
stop_condition='min_gradient(0.0001)', f_value=1.1603772135966184e-06,
gradient_value=array([-9.99987077e-05]))

```

```

(52/320) [14:29] step_size=0.01 min_gradient(0.0001)

```



```

starting_point=[-2.63157895]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),
f_value=11.989625616746345, gradient_value=array([-18.22423094]))
GradientDescentIteration(n_iter=20943, time_running=1.0001,
x=array([-0.04884592]), f_value=1.423157634427676e-06,
gradient_value=array([-0.00011654]))
GradientDescentResult(n_iter=23195, time_running=1.1306, x=array([-0.04641551]),
stop_condition='min_gradient(0.0001)', f_value=1.1603592866875306e-06,
gradient_value=array([-9.9997549e-05]))

```

```

(53/320) [14:15] step_size=0.01 min_gradient(0.0001) starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=21861, time_running=1.0001,
x=array([0.04781252]), f_value=1.3064916017129483e-06,
gradient_value=array([0.0001093]))
GradientDescentResult(n_iter=23197, time_running=1.062, x=array([0.04641588]),
stop_condition='min_gradient(0.0001)', f_value=1.1603962420858358e-06,
gradient_value=array([9.9999375e-05]))

```

```

(54/320) [14:01] step_size=0.01 min_gradient(0.0001)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=21704, time_running=1.0001,
x=array([-0.04798573]), f_value=1.3255268341217326e-06,
gradient_value=array([-0.00011049]))
GradientDescentResult(n_iter=23198, time_running=1.0694, x=array([-0.04641549]),
stop_condition='min_gradient(0.0001)', f_value=1.160357070626039e-06,
gradient_value=array([-9.99974058e-05]))

```

```

(55/320) [13:48] step_size=0.01 min_gradient(0.0001) starting_point=[0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([0.52631579]),
f_value=0.019183400986793997, gradient_value=array([0.14579385]))
GradientDescentIteration(n_iter=21386, time_running=1.0001,
x=array([0.04814587]), f_value=1.3433096491949534e-06,
gradient_value=array([0.0001116]))
GradientDescentResult(n_iter=23024, time_running=1.0756, x=array([0.04641569]),
stop_condition='min_gradient(0.0001)', f_value=1.1603772135966184e-06,
gradient_value=array([9.99987077e-05]))

```

```

(56/320) [13:35] step_size=0.01 min_gradient(0.0001) starting_point=[3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([3.68421053]),
f_value=46.05934576929274, gradient_value=array([50.00728969]))

```

```

GradientDescentIteration(n_iter=21533, time_running=1.0001,
x=array([0.04817581]), f_value=1.3466538385940138e-06,
gradient_value=array([0.00011181]))
GradientDescentResult(n_iter=23198, time_running=1.0772, x=array([0.04641549]),
stop_condition='min_gradient(0.0001)', f_value=1.160357070626039e-06,
gradient_value=array([9.99974058e-05]))

```

```

(57/320) [13:22] step_size=0.01 min_gradient(0.0001) starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=21255, time_running=1.0001, x=array([0.048489]),
f_value=1.3820156501605925e-06, gradient_value=array([0.00011401]))
GradientDescentResult(n_iter=23197, time_running=1.0894, x=array([0.04641588]),
stop_condition='min_gradient(0.0001)', f_value=1.1603962420858358e-06,
gradient_value=array([9.9999375e-05]))

```

```

(58/320) [13:10] step_size=0.01 min_gradient(0.0001)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=21736, time_running=1.0001,
x=array([-0.04795041]), f_value=1.3216285048979314e-06,
gradient_value=array([-0.00011025]))
GradientDescentResult(n_iter=23198, time_running=1.0674, x=array([-0.04641549]),
stop_condition='min_gradient(0.0001)', f_value=1.160357070626039e-06,
gradient_value=array([-9.99974058e-05]))

```

```

(59/320) [12:58] step_size=0.01 min_gradient(0.0001)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=20895, time_running=1.0001,
x=array([-0.04890544]), f_value=1.4301071331188665e-06,
gradient_value=array([-0.00011697]))
GradientDescentResult(n_iter=23198, time_running=1.1107, x=array([-0.04641549]),
stop_condition='min_gradient(0.0001)', f_value=1.160357070626039e-06,
gradient_value=array([-9.99974058e-05]))

```

```

(60/320) [12:47] step_size=0.01 min_gradient(0.0001) starting_point=[-10.]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-10.]),
f_value=2500.0, gradient_value=array([-1000.]))
GradientDescentResult(n_iter=1, time_running=0.0002, x=array([0.]),
stop_condition='min_gradient(0.0001)', f_value=0.0, gradient_value=array([0.]))

```

```
(61/320) [12:31] step_size=0.01 max_time(5) starting_point=[-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-0.52631579]),
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))
GradientDescentIteration(n_iter=22802, time_running=1.0001,
x=array([-0.0466393]), f_value=1.1829000199610918e-06,
gradient_value=array([-0.00010145]))
GradientDescentIteration(n_iter=45994, time_running=2.0001,
x=array([-0.03290519]), f_value=2.930875440622433e-07,
gradient_value=array([-3.56281289e-05]))
GradientDescentIteration(n_iter=68823, time_running=3.0001,
x=array([-0.02691754]), f_value=1.3124465372747022e-07,
gradient_value=array([-1.95032151e-05]))
GradientDescentIteration(n_iter=90491, time_running=4.0001,
x=array([-0.02348218]), f_value=7.601402292374146e-08,
gradient_value=array([-1.29483754e-05]))
GradientDescentResult(n_iter=112948, time_running=5.0, x=array([-0.02102276]),
stop_condition='max_time(5)', f_value=4.8831364466876414e-08,
gradient_value=array([-9.29114287e-06]))
```

```
(62/320) [12:36] step_size=0.01 max_time(5) starting_point=[-2.63157895]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),
f_value=11.989625616746345, gradient_value=array([-18.22423094]))
GradientDescentIteration(n_iter=24215, time_running=1.0001,
x=array([-0.04542794]), f_value=1.064711970918644e-06,
gradient_value=array([-9.37495294e-05]))
GradientDescentIteration(n_iter=46218, time_running=2.0001,
x=array([-0.03288625]), f_value=2.924136591347729e-07,
gradient_value=array([-3.55666724e-05]))
GradientDescentIteration(n_iter=68831, time_running=3.0001,
x=array([-0.02694935]), f_value=1.3186618934509157e-07,
gradient_value=array([-1.95724452e-05]))
GradientDescentIteration(n_iter=91527, time_running=4.0001,
x=array([-0.02337095]), f_value=7.458394772167625e-08,
gradient_value=array([-1.2765241e-05]))
GradientDescentResult(n_iter=113256, time_running=5.0, x=array([-0.02101003]),
stop_condition='max_time(5)', f_value=4.871315825716721e-08,
gradient_value=array([-9.27426944e-06]))
```

```
(63/320) [12:41] step_size=0.01 max_time(5) starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentIteration(n_iter=22362, time_running=1.0001,
x=array([0.04727414]), f_value=1.2486319155170326e-06,
gradient_value=array([0.00010565]))
GradientDescentIteration(n_iter=44861, time_running=2.0001,
```

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x=array([0.03338068]), f_value=3.1039912258800976e-07,
gradient_value=array([3.7195068e-05]))
GradientDescentIteration(n_iter=67453, time_running=3.0001,
x=array([0.02722366]), f_value=1.3731757741171536e-07,
gradient_value=array([2.01762102e-05]))
GradientDescentIteration(n_iter=89676, time_running=4.0001,
x=array([0.02361119]), f_value=7.769832398383548e-08,
gradient_value=array([1.31629652e-05]))
GradientDescentResult(n_iter=112487, time_running=5.0, x=array([0.02108193]),
stop_condition='max_time(5)', f_value=4.93834731018776e-08,
gradient_value=array([9.36981932e-06]))

```

```

(64/320) [12:45] step_size=0.01 max_time(5) starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=24274, time_running=1.0001,
x=array([-0.04537551]), f_value=1.0598050714480341e-06,
gradient_value=array([-9.34252973e-05]))
GradientDescentIteration(n_iter=47398, time_running=2.0001,
x=array([-0.03247545]), f_value=2.7807413994878805e-07,
gradient_value=array([-3.4250384e-05]))
GradientDescentIteration(n_iter=69644, time_running=3.0001,
x=array([-0.0267922]), f_value=1.288170372289153e-07,
gradient_value=array([-1.92320232e-05]))
GradientDescentIteration(n_iter=92845, time_running=4.0001,
x=array([-0.02320487]), f_value=7.248638955830927e-08,
gradient_value=array([-1.24950312e-05]))
GradientDescentResult(n_iter=115619, time_running=5.0, x=array([-0.02079451]),
stop_condition='max_time(5)', f_value=4.674496454414311e-08,
gradient_value=array([-8.99178925e-06]))

```

```

(65/320) [12:50] step_size=0.01 max_time(5) starting_point=[0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([0.52631579]),
f_value=0.019183400986793997, gradient_value=array([0.14579385]))
GradientDescentIteration(n_iter=22466, time_running=1.0001,
x=array([0.04698397]), f_value=1.2182566142148923e-06,
gradient_value=array([0.00010372]))
GradientDescentIteration(n_iter=45598, time_running=2.0001,
x=array([0.03304719]), f_value=2.981797138421951e-07,
gradient_value=array([3.60913863e-05]))
GradientDescentIteration(n_iter=68682, time_running=3.0001,
x=array([0.02694508]), f_value=1.3178263717464055e-07,
gradient_value=array([1.95631434e-05]))
GradientDescentIteration(n_iter=91051, time_running=4.0001, x=array([0.02341]),
f_value=7.508374357647664e-08, gradient_value=array([1.28293435e-05]))
GradientDescentResult(n_iter=114761, time_running=5.0, x=array([0.02085631]),

```

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stop_condition='max_time(5)', f_value=4.73031096193513e-08,  
gradient_value=array([9.07219261e-06]))
```

```
(66/320) [12:54] step_size=0.01 max_time(5) starting_point=[3.68421053]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([3.68421053]),  
f_value=46.05934576929274, gradient_value=array([50.00728969]))  
GradientDescentIteration(n_iter=22800, time_running=1.0001,  
x=array([0.04681868]), f_value=1.2012040350005107e-06,  
gradient_value=array([0.00010263]))  
GradientDescentIteration(n_iter=45936, time_running=2.0001,  
x=array([0.03298808]), f_value=2.960522627679593e-07,  
gradient_value=array([3.5898085e-05]))  
GradientDescentIteration(n_iter=68505, time_running=3.0001,  
x=array([0.02701398]), f_value=1.3313554471728552e-07,  
gradient_value=array([1.97135804e-05]))  
GradientDescentIteration(n_iter=91506, time_running=4.0001,  
x=array([0.02337401]), f_value=7.462303561802015e-08,  
gradient_value=array([1.27702582e-05]))  
GradientDescentResult(n_iter=114510, time_running=5.0, x=array([0.02089495]),  
stop_condition='max_time(5)', f_value=4.765468876940146e-08,  
gradient_value=array([9.1227174e-06]))
```

```
(67/320) [12:57] step_size=0.01 max_time(5) starting_point=[7.89473684]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),  
f_value=971.159674956454, gradient_value=array([492.05423531]))  
GradientDescentIteration(n_iter=22962, time_running=1.0001,  
x=array([0.04665269]), f_value=1.1842588158903397e-06,  
gradient_value=array([0.00010154]))  
GradientDescentIteration(n_iter=45414, time_running=2.0001,  
x=array([0.03317687]), f_value=3.0288753234811516e-07,  
gradient_value=array([3.65179207e-05]))  
GradientDescentIteration(n_iter=68471, time_running=3.0001,  
x=array([0.02702056]), f_value=1.3326540969317735e-07,  
gradient_value=array([1.97280006e-05]))  
GradientDescentIteration(n_iter=91205, time_running=4.0001,  
x=array([0.02341247]), f_value=7.511533948264272e-08,  
gradient_value=array([1.28333923e-05]))  
GradientDescentResult(n_iter=113093, time_running=5.0, x=array([0.02102538]),  
stop_condition='max_time(5)', f_value=4.8855709082122836e-08,  
gradient_value=array([9.29461669e-06]))
```

```
(68/320) [13:01] step_size=0.01 max_time(5) starting_point=[-3.68421053]  
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),  
f_value=46.05934576929274, gradient_value=array([-50.00728969]))  
GradientDescentIteration(n_iter=23596, time_running=1.0001,
```

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x=array([-0.04602253]), f_value=1.1215587496037214e-06,
gradient_value=array([-9.74791008e-05]))
GradientDescentIteration(n_iter=47644, time_running=2.0001,
x=array([-0.03239152]), f_value=2.7521060343700675e-07,
gradient_value=array([-3.39855158e-05]))
GradientDescentIteration(n_iter=70620, time_running=3.0001,
x=array([-0.02660644]), f_value=1.252815419903958e-07,
gradient_value=array([-1.88347696e-05]))
GradientDescentIteration(n_iter=92412, time_running=4.0001,
x=array([-0.02325916]), f_value=7.316717798715117e-08,
gradient_value=array([-1.25829428e-05]))
GradientDescentResult(n_iter=115606, time_running=5.0, x=array([-0.02079568]),
stop_condition='max_time(5)', f_value=4.675547718077629e-08,
gradient_value=array([-8.99330586e-06]))

```

```

(69/320) [13:04] step_size=0.01 max_time(5) starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=22697, time_running=1.0001,
x=array([-0.04692475]), f_value=1.2121263865792924e-06,
gradient_value=array([-0.00010333]))
GradientDescentIteration(n_iter=45431, time_running=2.0001,
x=array([-0.03317088]), f_value=3.0266905392648686e-07,
gradient_value=array([-3.64981631e-05]))
GradientDescentIteration(n_iter=66393, time_running=3.0001,
x=array([-0.02744021]), f_value=1.4173913654509962e-07,
gradient_value=array([-2.0661523e-05]))
GradientDescentIteration(n_iter=87284, time_running=4.0001,
x=array([-0.02393258]), f_value=8.201587151053689e-08,
gradient_value=array([-1.37078209e-05]))
GradientDescentResult(n_iter=109281, time_running=5.0, x=array([-0.02138897]),
stop_condition='max_time(5)', f_value=5.232379500665074e-08,
gradient_value=array([-9.78519354e-06]))

```

```

(70/320) [13:07] step_size=0.01 max_time(5) starting_point=[-10.]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-10.]),
f_value=2500.0, gradient_value=array([-1000.]))
GradientDescentIteration(n_iter=22059, time_running=1.0001, x=array([0.]),
f_value=0.0, gradient_value=array([0.]))
GradientDescentIteration(n_iter=44512, time_running=2.0001, x=array([0.]),
f_value=0.0, gradient_value=array([0.]))
GradientDescentIteration(n_iter=67319, time_running=3.0001, x=array([0.]),
f_value=0.0, gradient_value=array([0.]))
GradientDescentIteration(n_iter=89080, time_running=4.0001, x=array([0.]),
f_value=0.0, gradient_value=array([0.]))
GradientDescentResult(n_iter=111521, time_running=5.0, x=array([0.]),

```

```
stop_condition='max_time(5)', f_value=0.0, gradient_value=array([0.]))
```

```
(71/320) [13:09] step_size=0.01 max_iterations(20000)
starting_point=[-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-0.52631579]),
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))
GradientDescentResult(n_iter=20000, time_running=0.9093, x=array([-0.04977153]),
stop_condition='max_iterations(20000)', f_value=1.5341359478844374e-06,
gradient_value=array([-0.00012329]))
```

```
(72/320) [12:58] step_size=0.01 max_iterations(20000)
starting_point=[-2.63157895]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),
f_value=11.989625616746345, gradient_value=array([-18.22423094]))
GradientDescentResult(n_iter=20000, time_running=0.9217, x=array([-0.04998349]),
stop_condition='max_iterations(20000)', f_value=1.5604376214402326e-06,
gradient_value=array([-0.00012488]))
```

```
(73/320) [12:47] step_size=0.01 max_iterations(20000)
starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentResult(n_iter=20000, time_running=0.8943, x=array([0.04998645]),
stop_condition='max_iterations(20000)', f_value=1.5608072105294452e-06,
gradient_value=array([0.0001249]))
```

```
(74/320) [12:37] step_size=0.01 max_iterations(20000)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentResult(n_iter=20000, time_running=0.9425, x=array([-0.04998721]),
stop_condition='max_iterations(20000)', f_value=1.560902108945783e-06,
gradient_value=array([-0.0001249]))
```

```
(75/320) [12:27] step_size=0.01 max_iterations(20000)
starting_point=[0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([0.52631579]),
f_value=0.019183400986793997, gradient_value=array([0.14579385]))
GradientDescentResult(n_iter=20000, time_running=0.886, x=array([0.04977153]),
stop_condition='max_iterations(20000)', f_value=1.5341359478844374e-06,
gradient_value=array([0.00012329]))
```

```
(76/320) [12:17] step_size=0.01 max_iterations(20000)
starting_point=[3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([3.68421053]),
f_value=46.05934576929274, gradient_value=array([50.00728969]))
GradientDescentResult(n_iter=20000, time_running=0.9294, x=array([0.04998721]),
stop_condition='max_iterations(20000)', f_value=1.560902108945783e-06,
gradient_value=array([0.0001249]))
```

```
(77/320) [12:07] step_size=0.01 max_iterations(20000)
starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentResult(n_iter=20000, time_running=0.9017, x=array([0.04998645]),
stop_condition='max_iterations(20000)', f_value=1.5608072105294452e-06,
gradient_value=array([0.0001249]))
```

```
(78/320) [11:57] step_size=0.01 max_iterations(20000)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentResult(n_iter=20000, time_running=0.8906, x=array([-0.04998721]),
stop_condition='max_iterations(20000)', f_value=1.560902108945783e-06,
gradient_value=array([-0.0001249]))
```

```
(79/320) [11:48] step_size=0.01 max_iterations(20000)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentResult(n_iter=20000, time_running=0.912, x=array([-0.04998721]),
stop_condition='max_iterations(20000)', f_value=1.560902108945783e-06,
gradient_value=array([-0.0001249]))
```

```
(80/320) [11:39] step_size=0.01 max_iterations(20000) starting_point=[-10.]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-10.]),
f_value=2500.0, gradient_value=array([-1000.]))
GradientDescentResult(n_iter=20000, time_running=0.871, x=array([0.]),
stop_condition='max_iterations(20000)', f_value=0.0, gradient_value=array([0.]))
```

```
(81/320) [11:30] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-0.52631579]),
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))
GradientDescentIteration(n_iter=20877, time_running=1.0001,
```



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x=array([-0.02186458]), f_value=5.7135319576938434e-08,
gradient_value=array([-1.04525797e-05]))
GradientDescentIteration(n_iter=41712, time_running=2.0001,
x=array([-0.01547583]), f_value=1.4340237475803822e-08,
gradient_value=array([-3.70648504e-06]))
GradientDescentIteration(n_iter=62562, time_running=3.0001,
x=array([-0.01263863]), f_value=6.378814471298971e-09,
gradient_value=array([-2.01883103e-06]))
GradientDescentIteration(n_iter=84004, time_running=4.0001,
x=array([-0.01090791]), f_value=3.539213016239662e-09,
gradient_value=array([-1.29785155e-06]))
GradientDescentResult(n_iter=99958, time_running=4.7503, x=array([-0.01]),
stop_condition='min_gradient(1e-06)', f_value=2.4999973121463966e-09,
gradient_value=array([-9.99999194e-07]))

(82/320) [11:31] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[-2.63157895]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),
f_value=11.989625616746345, gradient_value=array([-18.22423094]))
GradientDescentIteration(n_iter=20461, time_running=1.0001,
x=array([-0.02210247]), f_value=5.966276412937081e-08,
gradient_value=array([-1.07974828e-05]))
GradientDescentIteration(n_iter=42064, time_running=2.0001,
x=array([-0.01541684]), f_value=1.412282015885113e-08,
gradient_value=array([-3.66425817e-06]))
GradientDescentIteration(n_iter=63898, time_running=3.0001,
x=array([-0.01250901]), f_value=6.121124830775709e-09,
gradient_value=array([-1.95734969e-06]))
GradientDescentIteration(n_iter=85072, time_running=4.0001,
x=array([-0.01084129]), f_value=3.4535337105067622e-09,
gradient_value=array([-1.27421516e-06]))
GradientDescentResult(n_iter=99990, time_running=4.71, x=array([-0.00999999]),
stop_condition='min_gradient(1e-06)', f_value=2.499989393166294e-09,
gradient_value=array([-9.99996818e-07]))

(83/320) [11:33] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentResult(n_iter=1, time_running=0.0003, x=array([-16.70797492]),
stop_condition='X_OUT_OF_DOMAIN', f_value=19482.07755026256,
gradient_value=array([-4664.13856605]))

(84/320) [11:22] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[-3.68421053]

```

```

GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=20968, time_running=1.0001,
x=array([-0.0218321]), f_value=5.67965430676914e-08,
gradient_value=array([-1.04060623e-05]))
GradientDescentIteration(n_iter=42253, time_running=2.0001,
x=array([-0.01538176]), f_value=1.3994710644743767e-08,
gradient_value=array([-3.63930066e-06]))
GradientDescentIteration(n_iter=63107, time_running=3.0001,
x=array([-0.01258683]), f_value=6.274889014032713e-09,
gradient_value=array([-1.99411192e-06]))
GradientDescentIteration(n_iter=84937, time_running=4.0001,
x=array([-0.0108497]), f_value=3.4642639915234317e-09,
gradient_value=array([-1.27718329e-06]))
GradientDescentResult(n_iter=99987, time_running=4.7263, x=array([-0.00999998]),
stop_condition='min_gradient(1e-06)', f_value=2.499982819190097e-09,
gradient_value=array([-9.99994846e-07]))

```

```

(85/320) [11:24] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([0.52631579]),
f_value=0.019183400986793997, gradient_value=array([0.14579385]))
GradientDescentIteration(n_iter=21251, time_running=1.0001,
x=array([0.02167169]), f_value=5.514567203975957e-08,
gradient_value=array([1.01783778e-05]))
GradientDescentIteration(n_iter=42061, time_running=2.0001,
x=array([0.01541156]), f_value=1.410347718953555e-08,
gradient_value=array([3.66049353e-06]))
GradientDescentIteration(n_iter=62705, time_running=3.0001,
x=array([0.01262422]), f_value=6.3497725653839514e-09,
gradient_value=array([2.0119335e-06]))
GradientDescentIteration(n_iter=83163, time_running=4.0001,
x=array([0.0109629]), f_value=3.6111209695212595e-09,
gradient_value=array([1.31757857e-06]))
GradientDescentResult(n_iter=99958, time_running=4.8011, x=array([0.01]),
stop_condition='min_gradient(1e-06)', f_value=2.4999973121463966e-09,
gradient_value=array([9.99999194e-07]))

```

```

(86/320) [11:25] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([3.68421053]),
f_value=46.05934576929274, gradient_value=array([50.00728969]))
GradientDescentIteration(n_iter=21822, time_running=1.0201,
x=array([0.02140087]), f_value=5.2440321741592226e-08,
gradient_value=array([9.80153295e-06]))
GradientDescentIteration(n_iter=42091, time_running=2.0001,

```

```

x=array([0.01541132]), f_value=1.4102613059298798e-08,
gradient_value=array([3.66032532e-06]))
GradientDescentIteration(n_iter=62454, time_running=3.0001,
x=array([0.01265245]), f_value=6.406765661093782e-09,
gradient_value=array([2.02546211e-06]))
GradientDescentIteration(n_iter=83226, time_running=4.0001,
x=array([0.01096064]), f_value=3.6081463991313345e-09,
gradient_value=array([1.31676449e-06]))
GradientDescentResult(n_iter=99987, time_running=4.8359, x=array([0.00999998]),
stop_condition='min_gradient(1e-06)', f_value=2.499982819190097e-09,
gradient_value=array([9.99994846e-07]))

```

```

(87/320) [11:27] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentResult(n_iter=1, time_running=0.0003, x=array([-16.70797492]),
stop_condition='X_OUT_OF_DOMAIN', f_value=19482.07755026256,
gradient_value=array([-4664.13856605]))

```

```

(88/320) [11:16] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=21674, time_running=1.0001,
x=array([-0.02147377]), f_value=5.315855928006055e-08,
gradient_value=array([-9.90204499e-06]))
GradientDescentIteration(n_iter=43120, time_running=2.0001,
x=array([-0.01522638]), f_value=1.3437746949253508e-08,
gradient_value=array([-3.530123e-06]))
GradientDescentIteration(n_iter=63927, time_running=3.0001,
x=array([-0.01250586]), f_value=6.114974262725207e-09,
gradient_value=array([-1.95587443e-06]))
GradientDescentIteration(n_iter=84571, time_running=4.0001,
x=array([-0.01087315]), f_value=3.4943091960332457e-09,
gradient_value=array([-1.28548198e-06]))
GradientDescentResult(n_iter=99987, time_running=4.7441, x=array([-0.00999998]),
stop_condition='min_gradient(1e-06)', f_value=2.499982819190097e-09,
gradient_value=array([-9.99994846e-07]))

```

```

(89/320) [11:17] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentIteration(n_iter=20597, time_running=1.0001,

```

```

x=array([-0.02202774]), f_value=5.885989294047888e-08,
gradient_value=array([-1.06883236e-05]))
GradientDescentIteration(n_iter=41590, time_running=2.0001,
x=array([-0.01550384]), f_value=1.4444325605124076e-08,
gradient_value=array([-3.72664433e-06]))
GradientDescentIteration(n_iter=63395, time_running=3.0001,
x=array([-0.01255822]), f_value=6.218016530508443e-09,
gradient_value=array([-1.98054128e-06]))
GradientDescentIteration(n_iter=84944, time_running=4.0001,
x=array([-0.01084925]), f_value=3.4636931380373816e-09,
gradient_value=array([-1.27702544e-06]))
GradientDescentResult(n_iter=99987, time_running=4.6973, x=array([-0.00999998]),
stop_condition='min_gradient(1e-06)', f_value=2.499982819190097e-09,
gradient_value=array([-9.99994846e-07]))

```

```

(90/320) [11:18] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[-10.]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-10.]),
f_value=2500.0, gradient_value=array([-1000.]))
GradientDescentResult(n_iter=1, time_running=0.0002, x=array([40.]),
stop_condition='X_OUT_OF_DOMAIN', f_value=640000.0,
gradient_value=array([64000.]))

```

```

(91/320) [11:08] step_size=0.05 min_gradient(0.0001)
starting_point=[-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-0.52631579]),
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))
GradientDescentResult(n_iter=4602, time_running=0.1967, x=array([-0.04641508]),
stop_condition='min_gradient(0.0001)', f_value=1.1603160626472108e-06,
gradient_value=array([-9.99947553e-05]))

```

```

(92/320) [10:58] step_size=0.05 min_gradient(0.0001)
starting_point=[-2.63157895]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),
f_value=11.989625616746345, gradient_value=array([-18.22423094]))
GradientDescentResult(n_iter=4634, time_running=0.2106, x=array([-0.04641428]),
stop_condition='min_gradient(0.0001)', f_value=1.1602368726872976e-06,
gradient_value=array([-9.99896368e-05]))

```

```

(93/320) [10:49] step_size=0.05 min_gradient(0.0001) starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentResult(n_iter=1, time_running=0.0002, x=array([-16.70797492]),
stop_condition='X_OUT_OF_DOMAIN', f_value=19482.07755026256,

```

```
gradient_value=array([-4664.13856605]))
```

```
(94/320) [10:39] step_size=0.05 min_gradient(0.0001)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentResult(n_iter=4631, time_running=0.2129, x=array([-0.04641363]),
stop_condition='min_gradient(0.0001)', f_value=1.1601711386660155e-06,
gradient_value=array([-9.99853881e-05]))
```

```
(95/320) [10:30] step_size=0.05 min_gradient(0.0001) starting_point=[0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([0.52631579]),
f_value=0.019183400986793997, gradient_value=array([0.14579385]))
GradientDescentResult(n_iter=4602, time_running=0.2123, x=array([0.04641508]),
stop_condition='min_gradient(0.0001)', f_value=1.1603160626472108e-06,
gradient_value=array([9.99947553e-05]))
```

```
(96/320) [10:21] step_size=0.05 min_gradient(0.0001) starting_point=[3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([3.68421053]),
f_value=46.05934576929274, gradient_value=array([50.00728969]))
GradientDescentResult(n_iter=4631, time_running=0.213, x=array([0.04641363]),
stop_condition='min_gradient(0.0001)', f_value=1.1601711386660155e-06,
gradient_value=array([9.99853881e-05]))
```

```
(97/320) [10:13] step_size=0.05 min_gradient(0.0001) starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentResult(n_iter=1, time_running=0.0002, x=array([-16.70797492]),
stop_condition='X_OUT_OF_DOMAIN', f_value=19482.07755026256,
gradient_value=array([-4664.13856605]))
```

```
(98/320) [10:04] step_size=0.05 min_gradient(0.0001)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentResult(n_iter=4631, time_running=0.2183, x=array([-0.04641363]),
stop_condition='min_gradient(0.0001)', f_value=1.1601711386660155e-06,
gradient_value=array([-9.99853881e-05]))
```

```
(99/320) [09:55] step_size=0.05 min_gradient(0.0001)
starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
```

```
f_value=46.05934576929274, gradient_value=array([-50.00728969]))
GradientDescentResult(n_iter=4631, time_running=0.2127, x=array([-0.04641363]),
stop_condition='min_gradient(0.0001)', f_value=1.1601711386660155e-06,
gradient_value=array([-9.99853881e-05]))
```

```
(100/320) [09:47] step_size=0.05 min_gradient(0.0001) starting_point=[-10.]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-10.]),
f_value=2500.0, gradient_value=array([-1000.]))
GradientDescentResult(n_iter=1, time_running=0.0002, x=array([40.]),
stop_condition='X_OUT_OF_DOMAIN', f_value=640000.0,
gradient_value=array([64000.]))
```

```
(101/320) [09:39] step_size=0.05 max_time(5) starting_point=[-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-0.52631579]),
f_value=0.019183400986794257, gradient_value=array([-0.14579385]))
GradientDescentIteration(n_iter=22548, time_running=1.0001,
x=array([-0.02104031]), f_value=4.8994618827249103e-08,
gradient_value=array([-9.31442995e-06]))
GradientDescentIteration(n_iter=44958, time_running=2.0001,
x=array([-0.01490721]), f_value=1.234597813271398e-08,
gradient_value=array([-3.3127535e-06]))
GradientDescentIteration(n_iter=67458, time_running=3.0001,
x=array([-0.01217163]), f_value=5.487007635941992e-09,
gradient_value=array([-1.80321149e-06]))
GradientDescentIteration(n_iter=90058, time_running=4.0001,
x=array([-0.01053508]), f_value=3.0795741268997183e-09,
gradient_value=array([-1.16926511e-06]))
GradientDescentResult(n_iter=113627, time_running=5.0, x=array([-0.00937948]),
stop_condition='max_time(5)', f_value=1.9348833390125708e-09,
gradient_value=array([-8.25156039e-07]))
```

```
(102/320) [09:41] step_size=0.05 max_time(5) starting_point=[-2.63157895]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-2.63157895]),
f_value=11.989625616746345, gradient_value=array([-18.22423094]))
GradientDescentIteration(n_iter=22390, time_running=1.0001,
x=array([-0.02112929]), f_value=4.982866578800058e-08,
gradient_value=array([-9.43309998e-06]))
GradientDescentIteration(n_iter=45092, time_running=2.0001,
x=array([-0.01489032]), f_value=1.2290111500869584e-08,
gradient_value=array([-3.30150425e-06]))
GradientDescentIteration(n_iter=67752, time_running=3.0001,
x=array([-0.01214807]), f_value=5.44463285273679e-09,
gradient_value=array([-1.79275707e-06]))
GradientDescentIteration(n_iter=88947, time_running=4.0001,
x=array([-0.01060253]), f_value=3.1592103077733764e-09,
```

```

gradient_value=array([-1.19187002e-06]))
GradientDescentResult(n_iter=111332, time_running=5.0, x=array([-0.00947698]),
stop_condition='max_time(5)', f_value=2.016599656674015e-09,
gradient_value=array([-8.51157167e-07]))

```

```

(103/320) [09:42] step_size=0.05 max_time(5) starting_point=[7.89473684]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([7.89473684]),
f_value=971.159674956454, gradient_value=array([492.05423531]))
GradientDescentResult(n_iter=1, time_running=0.0003, x=array([-16.70797492]),
stop_condition='X_OUT_OF_DOMAIN', f_value=19482.07755026256,
gradient_value=array([-4664.13856605]))

```

```

(104/320) [09:34] step_size=0.05 max_time(5) starting_point=[-3.68421053]
GradientDescentIteration(n_iter=0, time_running=0.0001, x=array([-3.68421053]),
f_value=46.05934576929274, gradient_value=array([-50.00728969]))

```

```

[ ]: step_sizes_2 = [0.005, 0.01, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1]
conditions_2 = [
    [StopConditions.max_time(5), StopConditions.min_gradient(1e-6)],
    [StopConditions.min_gradient(1e-4)],
    [StopConditions.max_time(5)],
    [StopConditions.max_iterations(20000)]
]
experiment_step_sizes(objective_function_2, domain_2, step_sizes_2,
↳conditions_2, n_starting_points=10)

```

Generating starting points...

```

Starting points: [array([ 0.63157895, -0.21052632]), array([2.47368421,
0.10526316]), array([-0.68421053, 0.10526316]), array([-1.47368421,
-3.36842105]), array([0.10526316, 1.05263158]), array([2.47368421, 0.73684211]),
array([ 2.73684211, -2.42105263]), array([-0.15789474, -0.52631579]),
array([-0.68421053, -3.36842105]), array([2.21052632, 2.
    ])]

```

```

(1/320) [00:00] step_size=0.005 max_time(5) min_gradient(1e-06) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0013, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=1455, time_running=0.6271, x=array([ 0.00347644,
-0.00695231]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367659573, gradient_value=array([ 8.41614847e-07,
-5.34155712e-07]))

```

```

(2/320) [00:48] step_size=0.005 max_time(5) min_gradient(1e-06)

```

```

starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5050, time_running=1.0002, x=array([ 0.00364875,
-0.0069938 ]), f_value=0.4965725149125161, gradient_value=array([
3.43101198e-04, -7.67113741e-05]))
GradientDescentResult(n_iter=5641, time_running=1.1237, x=array([ 0.00347649,
-0.0069522 ]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367659296, gradient_value=array([ 9.44046134e-07,
-3.15989309e-07]))

```

```

(3/320) [02:29] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=1451, time_running=0.2981, x=array([ 0.00347555,
-0.00695183]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765924,
gradient_value=array([-9.23007845e-07, 3.67088582e-07]))

```

```

(4/320) [02:15] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=4993, time_running=1.0001, x=array([-1.45123626,
-3.35581894]), f_value=1.4998029352098277, gradient_value=array([-0.00096297,
-0.00054063]))
GradientDescentIteration(n_iter=10047, time_running=2.0002, x=array([-1.4249342
, -3.34105187]), f_value=1.499766854600294, gradient_value=array([-0.001127
, -0.00063277]))
GradientDescentIteration(n_iter=15090, time_running=3.0002,
x=array([-1.39382282, -3.32358279]), f_value=1.4997162224668399,
gradient_value=array([-0.00135406, -0.00076035]))
GradientDescentIteration(n_iter=20160, time_running=4.0001,
x=array([-1.35553082, -3.30207897]), f_value=1.499639824688784,
gradient_value=array([-0.00169095, -0.00094967]))
GradientDescentIteration(n_iter=25202, time_running=5.0001,
x=array([-1.30668983, -3.27464648]), f_value=1.49951454678768,
gradient_value=array([-0.00223159, -0.00125354]))
GradientDescentResult(n_iter=25202, time_running=5.0001, x=array([-1.30668983,
-3.27464648]), stop_condition='max_time(5)', f_value=1.49951454678768,
gradient_value=array([-0.00223159, -0.00125354]))

```



```
(5/320) [06:59] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=1558, time_running=0.3092, x=array([ 0.00347597,
-0.00695152]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765959,
gradient_value=array([-8.02642388e-08,  9.91834010e-07]))
```

```
(6/320) [06:04] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=5046, time_running=1.0001, x=array([2.16206077,
0.63768379]), f_value=1.493663925690448, gradient_value=array([0.02715136,
0.00857401]))
GradientDescentResult(n_iter=8568, time_running=1.7042, x=array([ 0.00347651,
-0.00695195]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367659107, gradient_value=array([9.84641409e-07,
1.69676873e-07]))
```

```
(7/320) [06:26] step_size=0.005 max_time(5) min_gradient(1e-06) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=4124, time_running=0.8204, x=array([ 0.9845107 ,
-1.96902109]), stop_condition='min_gradient(1e-06)', f_value=0.992742092023255,
gradient_value=array([ 5.72314642e-07, -8.19254062e-07]))
```

```
(8/320) [06:08] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=1427, time_running=0.3151, x=array([ 0.00347599,
-0.00695254]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765964,
gradient_value=array([-6.10699843e-08, -9.96070279e-07]))
```

```
(9/320) [05:37] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
```

```

-0.01238275]))
GradientDescentIteration(n_iter=4991, time_running=1.0001, x=array([ 0.9844635 ,
-1.96914943]), f_value=0.9927421009082241,
gradient_value=array([-5.38012554e-05, -1.18061727e-04]))
GradientDescentResult(n_iter=6043, time_running=1.2029, x=array([ 0.9845101 ,
-1.96902125]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232365,
gradient_value=array([-3.64474773e-08, -9.98664023e-07]))

```

```

(10/320) [05:39] step_size=0.005 max_time(5) min_gradient(1e-06)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5103, time_running=1.0002, x=array([2.19393056,
1.98498271]), f_value=1.4998420751188684, gradient_value=array([0.00069292,
0.00062702]))
GradientDescentIteration(n_iter=10234, time_running=2.0002, x=array([2.17485166,
1.96771839]), f_value=1.4998162245333935, gradient_value=array([0.00079933,
0.00072331]))
GradientDescentIteration(n_iter=15254, time_running=3.0001, x=array([2.15312749,
1.94806029]), f_value=1.4997819516480277, gradient_value=array([0.00093893,
0.00084963]))
GradientDescentIteration(n_iter=20457, time_running=4.0002, x=array([2.12621188,
1.92370434]), f_value=1.4997311367354629, gradient_value=array([0.00114326,
0.00103454]))
GradientDescentIteration(n_iter=25459, time_running=5.0001, x=array([2.09418485,
1.89472283]), f_value=1.49965620715825, gradient_value=array([0.00143985,
0.00130294]))
GradientDescentResult(n_iter=25459, time_running=5.0001, x=array([2.09418485,
1.89472283]), stop_condition='max_time(5)', f_value=1.49965620715825,
gradient_value=array([0.00143985, 0.00130294]))

```

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(11/320) [07:26] step_size=0.005 min_gradient(0.0001) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=987, time_running=0.1882, x=array([ 0.00352046,
-0.00697671]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248620846994, gradient_value=array([ 8.78898028e-05,
-4.69208810e-05]))

```

```

(12/320) [06:53] step_size=0.005 min_gradient(0.0001) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,

```

```

0.00331099]))
GradientDescentIteration(n_iter=5148, time_running=1.0002, x=array([ 0.00354098,
-0.00696891]), f_value=0.49657248812915555, gradient_value=array([
1.28996123e-04, -3.11401163e-05]))
GradientDescentResult(n_iter=5177, time_running=1.0062, x=array([ 0.00352465,
-0.00696493]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248617885497, gradient_value=array([ 9.65778071e-05,
-2.38281300e-05]))

```

```

(13/320) [06:43] step_size=0.005 min_gradient(0.0001)
starting_point=[-0.68421053  0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=986, time_running=0.1889, x=array([ 0.00342813,
-0.00693644]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724861770839, gradient_value=array([-9.49918845e-05,
2.90774566e-05]))

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```

(14/320) [06:17] step_size=0.005 min_gradient(0.0001)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5064, time_running=1.0001, x=array([-1.45089406,
-3.35562683]), f_value=1.4998025013659233, gradient_value=array([-0.00096496,
-0.00054174]))
GradientDescentIteration(n_iter=10224, time_running=2.0001,
x=array([-1.42393386, -3.3404902 ]), f_value=1.4997653673936833,
gradient_value=array([-0.00113372, -0.00063655]))
GradientDescentIteration(n_iter=15279, time_running=3.0002,
x=array([-1.39253844, -3.32286157]), f_value=1.4997139263505195,
gradient_value=array([-0.00136428, -0.00076609]))
GradientDescentIteration(n_iter=19866, time_running=4.0001,
x=array([-1.35799886, -3.30346506]), f_value=1.4996452755616259,
gradient_value=array([-0.00166712, -0.00093627]))
GradientDescentIteration(n_iter=24899, time_running=5.0002, x=array([-1.3100389
, -3.27652772]), f_value=1.4995242868776046, gradient_value=array([-0.00219001,
-0.00123016]))
GradientDescentIteration(n_iter=29936, time_running=6.0002, x=array([-1.2441688
, -3.23952249]), f_value=1.4992951091072233, gradient_value=array([-0.00315201,
-0.00177101]))
GradientDescentIteration(n_iter=34735, time_running=7.0002,
x=array([-1.14725776, -3.18506007]), f_value=1.4987686424684483,
gradient_value=array([-0.00526701, -0.00296061]))
GradientDescentIteration(n_iter=39768, time_running=8.0001, x=array([-0.9308504

```

```
, -3.06335429]], f_value=1.4960851792704035, gradient_value=array([-0.0150472 ,
-0.00846702]))
GradientDescentIteration(n_iter=44819, time_running=9.0001, x=array([
0.98415618, -1.96929288]), f_value=0.9927421927180189,
gradient_value=array([-0.00036495, -0.00026486]))
GradientDescentResult(n_iter=45117, time_running=9.0657, x=array([ 0.98443451,
-1.96908444]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420969729912, gradient_value=array([-7.82940554e-05,
-6.19633224e-05]))
```

```
(15/320) [08:52] step_size=0.005 min_gradient(0.0001) starting_point=[0.10526316
1.05263158]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
```

```
GradientDescentResult(n_iter=1088, time_running=0.2109, x=array([ 0.0034743,
-0.0069012]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248619703576, gradient_value=array([-2.00051399e-06,
9.91241709e-05]))
```

```
(16/320) [08:22] step_size=0.005 min_gradient(0.0001) starting_point=[2.47368421
0.73684211]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
```

```
GradientDescentIteration(n_iter=4846, time_running=1.0002, x=array([2.18774244,
0.64579874]), f_value=1.4943902561224853, gradient_value=array([0.02432294,
0.00769035]))
```

```
GradientDescentResult(n_iter=8111, time_running=1.6567, x=array([ 0.0035245 ,
-0.00694202]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724861299141, gradient_value=array([9.69048304e-05,
2.08696441e-05]))
```

```
(17/320) [08:20] step_size=0.005 min_gradient(0.0001) starting_point=[
2.73684211 -2.42105263]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
```

```
GradientDescentResult(n_iter=3065, time_running=0.6246, x=array([ 0.98458734,
-1.96910468]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420977395907, gradient_value=array([ 7.07737643e-05,
-7.05421350e-05]))
```

```
(18/320) [08:01] step_size=0.005 min_gradient(0.0001)
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```

starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=959, time_running=0.2089, x=array([ 0.00346999,
-0.00700246]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248620257405, gradient_value=array([-1.34103584e-05,
-9.85769066e-05]))

(19/320) [07:37] step_size=0.005 min_gradient(0.0001)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=4979, time_running=1.0002, x=array([ 0.98446015,
-1.96915673]), f_value=0.9927421019809839,
gradient_value=array([-5.75464300e-05, -1.24771616e-04]))
GradientDescentResult(n_iter=5046, time_running=1.0142, x=array([ 0.98447626,
-1.96912068]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420972983745, gradient_value=array([-3.94707263e-05,
-9.16627686e-05]))

(20/320) [07:28] step_size=0.005 min_gradient(0.0001) starting_point=[2.21052632
2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5047, time_running=1.0001, x=array([2.19412443,
1.98515814]), f_value=1.4998423192808517, gradient_value=array([0.00069191,
0.0006261 ]))
GradientDescentIteration(n_iter=10047, time_running=2.0002, x=array([2.17559695,
1.9683928 ]), f_value=1.4998173050712755, gradient_value=array([0.0007949,
0.0007193]))
GradientDescentIteration(n_iter=15195, time_running=3.0002, x=array([2.15340418,
1.94831068]), f_value=1.4997824236969788, gradient_value=array([0.00093701,
0.0008479 ]))
GradientDescentIteration(n_iter=20298, time_running=4.0001, x=array([2.12711777,
1.92452409]), f_value=1.4997330142748126, gradient_value=array([0.00113576,
0.00102776]))
GradientDescentIteration(n_iter=25298, time_running=5.0001, x=array([2.09533913,
1.89576735]), f_value=1.4996592176802543, gradient_value=array([0.00142803,
0.00129224]))
GradientDescentIteration(n_iter=30450, time_running=6.0002, x=array([2.05271549,
1.8571965 ]), f_value=1.4995299626301888, gradient_value=array([0.00192959,
0.00174613]))
GradientDescentIteration(n_iter=35469, time_running=7.0002, x=array([1.9937723 ,
1.80385696]), f_value=1.4992746776377186, gradient_value=array([0.00289206,

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0.00261714]))
GradientDescentIteration(n_iter=40662, time_running=8.0001, x=array([1.88991979,
1.7098748 ]), f_value=1.4984894940853366, gradient_value=array([0.00570899,
0.00516651]))
GradientDescentIteration(n_iter=45845, time_running=9.0002, x=array([1.53131743,
1.38531456]), f_value=1.4859305152002389, gradient_value=array([0.04308175,
0.03899722]))
GradientDescentResult(n_iter=47984, time_running=9.4232, x=array([ 0.00350929,
-0.0069148 ]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248616677085, gradient_value=array([6.73605529e-05,
7.35713253e-05]))

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(21/320) [09:18] step_size=0.005 max_time(5) starting_point=[ 0.63157895
-0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=5131, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=10271, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=15325, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=20309, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentResult(n_iter=25393, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 4.33680869e-17, -8.67361738e-17]))

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(22/320) [09:58] step_size=0.005 max_time(5) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5074, time_running=1.0002, x=array([ 0.00361196,
-0.0069855 ]), f_value=0.49657250305941214, gradient_value=array([
2.70003080e-04, -6.15401143e-05]))
GradientDescentIteration(n_iter=9930, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=14712, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([

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4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=19638, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentResult(n_iter=24731, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 4.33680869e-17, -8.67361738e-17]))

(23/320) [10:33] step_size=0.005 max_time(5) starting_point=[-0.68421053
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=5066, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentIteration(n_iter=10076, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentIteration(n_iter=15152, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentIteration(n_iter=20043, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentResult(n_iter=25043, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))

(24/320) [11:05] step_size=0.005 max_time(5) starting_point=[-1.47368421
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5065, time_running=1.0002, x=array([-1.45088923,
-3.35562412]), f_value=1.4998024952426736, gradient_value=array([-0.00096499,
-0.00054176]))
GradientDescentIteration(n_iter=10242, time_running=2.0002,
x=array([-1.42383179, -3.3404329 ]), f_value=1.4997652151583618,
gradient_value=array([-0.0011344 , -0.00063694]))
GradientDescentIteration(n_iter=15448, time_running=3.0001,
x=array([-1.39138175, -3.32221204]), f_value=1.4997118436619863,
gradient_value=array([-0.00137354, -0.0007713 ]))
GradientDescentIteration(n_iter=20586, time_running=4.0002, x=array([-1.3518914,
-3.300035 ]), f_value=1.499631644262881, gradient_value=array([-0.00172667,
-0.00096974]))

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GradientDescentIteration(n_iter=25509, time_running=5.0001,
x=array([-1.30323111, -3.27270363]), f_value=1.4995042937688046,
gradient_value=array([-0.00227529, -0.0012781 ]))
GradientDescentResult(n_iter=25509, time_running=5.0001, x=array([-1.30323111,
-3.27270363]), stop_condition='max_time(5)', f_value=1.4995042937688046,
gradient_value=array([-0.00227529, -0.0012781 ]))

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(25/320) [11:35] step_size=0.005 max_time(5) starting_point=[0.10526316
1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5016, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentIteration(n_iter=10096, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentIteration(n_iter=15169, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentIteration(n_iter=20202, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentResult(n_iter=25245, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))

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(26/320) [12:02] step_size=0.005 max_time(5) starting_point=[2.47368421
0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=4933, time_running=1.0001, x=array([2.17691633,
0.64237666]), f_value=1.4940937665587177, gradient_value=array([0.02548241,
0.00805274]))
GradientDescentIteration(n_iter=9973, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([7.48956452e-13,
3.58914287e-15]))
GradientDescentIteration(n_iter=15009, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=20118, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentResult(n_iter=25113, time_running=5.0, x=array([ 0.00347601,

```



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-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([ 4.33680869e-17, -8.67361738e-17]))
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(27/320) [12:26] step_size=0.005 max_time(5) starting_point=[ 2.73684211  
-2.42105263]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,  
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,  
-0.01727565]))  
GradientDescentIteration(n_iter=4982, time_running=1.0002, x=array([ 0.98451008,  
-1.96902015]), f_value=0.9927420920226764, gradient_value=array([  
1.26277056e-08, -2.11115410e-08]))  
GradientDescentIteration(n_iter=10020, time_running=2.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([  
1.10866177e-14, -2.21732355e-14]))  
GradientDescentIteration(n_iter=15130, time_running=3.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([  
1.10866177e-14, -2.21732355e-14]))  
GradientDescentIteration(n_iter=20211, time_running=4.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([  
1.10866177e-14, -2.21732355e-14]))  
GradientDescentIteration(n_iter=25018, time_running=5.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([  
1.10866177e-14, -2.21732355e-14]))  
GradientDescentResult(n_iter=25018, time_running=5.0001, x=array([ 0.98451006,  
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226762,  
gradient_value=array([ 1.10866177e-14, -2.21732355e-14]))
```

```
(28/320) [12:49] step_size=0.005 max_time(5) starting_point=[-0.15789474  
-0.52631579]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,  
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,  
-0.7343479 ]))  
GradientDescentIteration(n_iter=4975, time_running=1.0002, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
4.33680869e-17, -8.67361738e-17]))  
GradientDescentIteration(n_iter=9775, time_running=2.0001, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
4.33680869e-17, -8.67361738e-17]))  
GradientDescentIteration(n_iter=14641, time_running=3.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
4.33680869e-17, -8.67361738e-17]))  
GradientDescentIteration(n_iter=19770, time_running=4.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
4.33680869e-17, -8.67361738e-17]))  
GradientDescentResult(n_iter=24815, time_running=5.0, x=array([ 0.00347601,  
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
```

```
gradient_value=array([ 4.33680869e-17, -8.67361738e-17]))
```

```
(29/320) [13:09] step_size=0.005 max_time(5) starting_point=[-0.68421053  
-3.36842105]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,  
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,  
-0.01238275]))  
GradientDescentIteration(n_iter=5035, time_running=1.0001, x=array([ 0.98447401,  
-1.96912586]), f_value=0.9927420978772863,  
gradient_value=array([-4.20005891e-05, -9.64186922e-05]))  
GradientDescentIteration(n_iter=9825, time_running=2.0001, x=array([ 0.98451006,  
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 3.15372728e-14,  
-6.57772448e-14]))  
GradientDescentIteration(n_iter=14892, time_running=3.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([  
1.10866177e-14, -2.21732355e-14]))  
GradientDescentIteration(n_iter=20014, time_running=4.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([  
1.10866177e-14, -2.21732355e-14]))  
GradientDescentIteration(n_iter=25090, time_running=5.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([  
1.10866177e-14, -2.21732355e-14]))  
GradientDescentResult(n_iter=25090, time_running=5.0001, x=array([ 0.98451006,  
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226762,  
gradient_value=array([ 1.10866177e-14, -2.21732355e-14]))
```

```
(30/320) [13:27] step_size=0.005 max_time(5) starting_point=[2.21052632 2.  
]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.  
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))  
GradientDescentIteration(n_iter=4852, time_running=1.0001, x=array([2.19479733,  
1.98576704]), f_value=1.4998431639596892, gradient_value=array([0.00068842,  
0.00062294]))  
GradientDescentIteration(n_iter=9926, time_running=2.0002, x=array([2.176077 ,  
1.9688272]), f_value=1.4998179978861574, gradient_value=array([0.00079207,  
0.00071673]))  
GradientDescentIteration(n_iter=14989, time_running=3.0002, x=array([2.15436588,  
1.94918091]), f_value=1.499784056894671, gradient_value=array([0.0009304 ,  
0.00084191]))  
GradientDescentIteration(n_iter=19900, time_running=4.0002, x=array([2.12935953,  
1.92655266]), f_value=1.4997376077173832, gradient_value=array([0.0011174 ,  
0.00101114]))  
GradientDescentResult(n_iter=24696, time_running=5.0, x=array([2.09957267,  
1.89959833]), stop_condition='max_time(5)', f_value=1.499670049105489,  
gradient_value=array([0.00138544, 0.0012537 ]))
```

```
(31/320) [13:45] step_size=0.005 max_iterations(20000) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=5045, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=10141, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=15369, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentResult(n_iter=20000, time_running=3.9017, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 4.33680869e-17,
-8.67361738e-17]))
```

```
(32/320) [13:51] step_size=0.005 max_iterations(20000)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5132, time_running=1.0001, x=array([ 0.00355222,
-0.00697161]), f_value=0.4965724897959946, gradient_value=array([
1.51332427e-04, -3.60892979e-05]))
GradientDescentIteration(n_iter=10216, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=15337, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentResult(n_iter=20000, time_running=3.9376, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 4.33680869e-17,
-8.67361738e-17]))
```

```
(33/320) [13:57] step_size=0.005 max_iterations(20000)
starting_point=[-0.68421053 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=4928, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
```

```

gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentIteration(n_iter=10086, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentIteration(n_iter=15053, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentResult(n_iter=20000, time_running=3.9495, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-4.33680869e-17,
8.67361738e-17]))

```

```

(34/320) [14:02] step_size=0.005 max_iterations(20000)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5114, time_running=1.0002, x=array([-1.45065265,
-3.35549129]), f_value=1.4998021947673406, gradient_value=array([-0.00096636,
-0.00054253]))
GradientDescentIteration(n_iter=9945, time_running=2.0001, x=array([-1.42550798,
-3.34137402]), f_value=1.4997677036535781, gradient_value=array([-0.00112316,
-0.00063062]))
GradientDescentIteration(n_iter=15097, time_running=3.0001,
x=array([-1.39377542, -3.32355617]), f_value=1.4997161380392579,
gradient_value=array([-0.00135443, -0.00076056]))
GradientDescentResult(n_iter=20000, time_running=3.9663, x=array([-1.35687832,
-3.30283574]), stop_condition='max_iterations(20000)',
f_value=1.4996428103358603, gradient_value=array([-0.0016779 , -0.00094233]))

```

```

(35/320) [14:07] step_size=0.005 max_iterations(20000)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5159, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentIteration(n_iter=10212, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentIteration(n_iter=15295, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-4.33680869e-17,  8.67361738e-17]))
GradientDescentResult(n_iter=20000, time_running=3.9201, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',

```

```
f_value=0.4965724836763416, gradient_value=array([-4.33680869e-17,  
8.67361738e-17]))
```

```
(36/320) [14:11] step_size=0.005 max_iterations(20000)  
starting_point=[2.47368421 0.73684211]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,  
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,  
0.00205835]))  
GradientDescentIteration(n_iter=5053, time_running=1.0001, x=array([2.16110882,  
0.63738318]), f_value=1.4936354441078377, gradient_value=array([0.02726146,  
0.00860839]))  
GradientDescentIteration(n_iter=10014, time_running=2.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
4.96743272e-13, -4.71844785e-16]))  
GradientDescentIteration(n_iter=14865, time_running=3.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
4.33680869e-17, -8.67361738e-17]))  
GradientDescentIteration(n_iter=19930, time_running=4.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
4.33680869e-17, -8.67361738e-17]))  
GradientDescentResult(n_iter=20000, time_running=4.0153, x=array([ 0.00347601,  
-0.00695203]), stop_condition='max_iterations(20000)',  
f_value=0.4965724836763416, gradient_value=array([ 4.33680869e-17,  
-8.67361738e-17]))
```

```
(37/320) [14:16] step_size=0.005 max_iterations(20000) starting_point=[  
2.73684211 -2.42105263]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,  
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,  
-0.01727565]))  
GradientDescentIteration(n_iter=5014, time_running=1.0001, x=array([ 0.98451007,  
-1.96902014]), f_value=0.9927420920226764, gradient_value=array([  
1.09659914e-08, -1.84104567e-08]))  
GradientDescentIteration(n_iter=10009, time_running=2.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([  
1.10866177e-14, -2.21732355e-14]))  
GradientDescentIteration(n_iter=15168, time_running=3.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([  
1.10866177e-14, -2.21732355e-14]))  
GradientDescentResult(n_iter=20000, time_running=3.9758, x=array([ 0.98451006,  
-1.96902012]), stop_condition='max_iterations(20000)',  
f_value=0.9927420920226762, gradient_value=array([ 1.10866177e-14,  
-2.21732355e-14]))
```

```
(38/320) [14:20] step_size=0.005 max_iterations(20000)
```

```

starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0004, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentIteration(n_iter=4952, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=10101, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=14889, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentIteration(n_iter=19944, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
4.33680869e-17, -8.67361738e-17]))
GradientDescentResult(n_iter=20000, time_running=4.0118, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 4.33680869e-17,
-8.67361738e-17]))

```

```

(39/320) [14:23] step_size=0.005 max_iterations(20000)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=4961, time_running=1.0002, x=array([ 0.98445468,
-1.96916846]), f_value=0.9927421038396704,
gradient_value=array([-6.36471236e-05, -1.35563464e-04]))
GradientDescentIteration(n_iter=10018, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.31197136e-14, -2.89421265e-14]))
GradientDescentIteration(n_iter=15041, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.10866177e-14, -2.21732355e-14]))
GradientDescentResult(n_iter=20000, time_running=3.9812, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.9927420920226762, gradient_value=array([ 1.10866177e-14,
-2.21732355e-14]))

```

```

(40/320) [14:26] step_size=0.005 max_iterations(20000)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5070, time_running=1.0001, x=array([2.19404484,
1.98508612]), f_value=1.4998422190862377, gradient_value=array([0.00069233,

```

```

0.00062648]))
GradientDescentIteration(n_iter=10106, time_running=2.0002, x=array([2.17536225,
1.96818043]), f_value=1.4998169654477496, gradient_value=array([0.0007963 ,
0.00072056]))
GradientDescentIteration(n_iter=15144, time_running=3.0002, x=array([2.15364291,
1.9485267 ]), f_value=1.4997828301925278, gradient_value=array([0.00093537,
0.00084641]))
GradientDescentResult(n_iter=20000, time_running=3.9899, x=array([2.1287997 ,
1.92604607]), stop_condition='max_iterations(20000)',
f_value=1.4997364676168319, gradient_value=array([0.00112196, 0.00101526]))

```

```

(41/320) [14:29] step_size=0.01 max_time(5) min_gradient(1e-06) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=724, time_running=0.1417, x=array([ 0.00347644,
-0.00695231]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765967,
gradient_value=array([ 8.42544186e-07, -5.36336992e-07]))

```

```

(42/320) [14:06] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentResult(n_iter=2818, time_running=0.5527, x=array([ 0.00347649,
-0.0069522 ]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765944,
gradient_value=array([ 9.46234362e-07, -3.18452134e-07]))

```

```

(43/320) [13:47] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=722, time_running=0.1396, x=array([ 0.00347555,
-0.00695183]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765932,
gradient_value=array([-9.24049558e-07, 3.68686210e-07]))

```

```

(44/320) [13:26] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5181, time_running=1.0001, x=array([-1.42315065,

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-3.34005045]), f_value=1.4997641968134843, gradient_value=array([-0.001139 ,
-0.00063952]))
GradientDescentIteration(n_iter=10178, time_running=2.0002,
x=array([-1.35386881, -3.30114556]), f_value=1.4996361101626026,
gradient_value=array([-0.00170718, -0.00095878]))
GradientDescentIteration(n_iter=15279, time_running=3.0003,
x=array([-1.23410764, -3.23386934]), f_value=1.499252224354949,
gradient_value=array([-0.0033287 , -0.00187036]))
GradientDescentIteration(n_iter=20359, time_running=4.0001,
x=array([-0.84498908, -3.01502905]), f_value=1.4940129227611132,
gradient_value=array([-0.02198177, -0.01237495]))
GradientDescentResult(n_iter=23016, time_running=4.5356, x=array([ 0.98450946,
-1.96902093]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231782,
gradient_value=array([-6.43261968e-07, -7.59758148e-07]))

```

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(45/320) [13:33] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=776, time_running=0.1594, x=array([ 0.00347597,
-0.00695152]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367659235, gradient_value=array([-8.05481523e-08,
9.84563323e-07]))

```

```

(46/320) [13:13] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentResult(n_iter=4282, time_running=0.8546, x=array([ 0.0034765 ,
-0.00695195]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367658896, gradient_value=array([9.80459353e-07,
1.68181268e-07]))

```

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(47/320) [12:58] step_size=0.01 max_time(5) min_gradient(1e-06) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=2060, time_running=0.4227, x=array([ 0.9845107 ,
-1.96902108]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232478,
gradient_value=array([ 5.68029475e-07, -8.14714049e-07]))

```



```
(48/320) [12:42] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=711, time_running=0.1442, x=array([ 0.00347599,
-0.00695253]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765876,
gradient_value=array([-5.84774061e-08, -9.78812289e-07]))
```

```
(49/320) [12:24] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=3020, time_running=0.6168, x=array([ 0.9845101 ,
-1.96902124]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232289,
gradient_value=array([-3.35914886e-08, -9.91817221e-07]))
```

```
(50/320) [12:10] step_size=0.01 max_time(5) min_gradient(1e-06)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=4959, time_running=1.0001, x=array([2.17610919,
1.96885633]), f_value=1.4998180442558404, gradient_value=array([0.00079187,
0.00071656]))
GradientDescentIteration(n_iter=9813, time_running=2.0002, x=array([2.1308835 ,
1.92793171]), f_value=1.4997406879041328, gradient_value=array([0.00110507,
0.00099998]))
GradientDescentIteration(n_iter=14681, time_running=3.0001, x=array([2.06284962,
1.86636709]), f_value=1.4995642994059843, gradient_value=array([0.00179747,
0.00162656]))
GradientDescentIteration(n_iter=19727, time_running=4.0001, x=array([1.92110557,
1.73809703]), f_value=1.4987831353148118, gradient_value=array([0.00467509,
0.00423079]))
GradientDescentResult(n_iter=24220, time_running=4.8919, x=array([ 0.00347632,
-0.00695164]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765891,
gradient_value=array([6.27383820e-07, 7.70492177e-07]))
```

```
(51/320) [12:18] step_size=0.01 min_gradient(0.0001) starting_point=[ 0.63157895
-0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=492, time_running=0.1047, x=array([ 0.00351977,
-0.0069764 ]), stop_condition='min_gradient(0.0001)',
```

```
f_value=0.4965724861341834, gradient_value=array([ 8.65276577e-05,
-4.63413257e-05]))
```

```
(52/320) [12:02] step_size=0.01 min_gradient(0.0001) starting_point=[2.47368421
0.10526316]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
```

```
GradientDescentResult(n_iter=2588, time_running=0.5464, x=array([ 0.00352395,
-0.00696482]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248610890986, gradient_value=array([ 9.51803694e-05,
-2.36363182e-05]))
```

```
(53/320) [11:48] step_size=0.01 min_gradient(0.0001) starting_point=[-0.68421053
0.10526316]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
```

```
GradientDescentResult(n_iter=491, time_running=0.0983, x=array([ 0.0034284 ,
-0.00693649]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248615024685, gradient_value=array([-9.44556147e-05,
2.89982853e-05]))
```

```
(54/320) [11:33] step_size=0.01 min_gradient(0.0001) starting_point=[-1.47368421
-3.36842105]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
```

```
GradientDescentIteration(n_iter=5176, time_running=1.0001, x=array([-1.42320759,
-3.34008242]), f_value=1.499764282097251, gradient_value=array([-0.00113862,
-0.0006393 ]))
```

```
GradientDescentIteration(n_iter=10138, time_running=2.0002,
x=array([-1.35455031, -3.3015283 ]), f_value=1.4996376375864726,
gradient_value=array([-0.00170051, -0.00095504]))
```

```
GradientDescentIteration(n_iter=15338, time_running=3.0002,
x=array([-1.23213339, -3.23276002]), f_value=1.499243531541995,
gradient_value=array([-0.0033644 , -0.00189044]))
```

```
GradientDescentIteration(n_iter=20556, time_running=4.0001,
x=array([-0.79716735, -2.98810344]), f_value=1.492478053110103,
gradient_value=array([-0.02689601, -0.0151457 ]))
```

```
GradientDescentResult(n_iter=22558, time_running=4.3975, x=array([ 0.9844346 ,
-1.96908445]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420969666898, gradient_value=array([-7.82052689e-05,
-6.19695869e-05]))
```

(55/320) [11:39] step\_size=0.01 min\_gradient(0.0001) starting\_point=[0.10526316  
1.05263158]

GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([0.10526316,  
1.05263158]), f\_value=1.1734107031477512, gradient\_value=array([0.06871534,  
0.68763702]))

GradientDescentResult(n\_iter=542, time\_running=0.1047, x=array([ 0.00347428,  
-0.00690142]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.49657248617547767, gradient\_value=array([-2.05277462e-06,  
9.86973160e-05]))

(56/320) [11:25] step\_size=0.01 min\_gradient(0.0001) starting\_point=[2.47368421  
0.73684211]

GradientDescentIteration(n\_iter=0, time\_running=0.0003, x=array([2.47368421,  
0.73684211]), f\_value=1.4986896501592173, gradient\_value=array([0.00641913,  
0.00205835]))

GradientDescentResult(n\_iter=4055, time\_running=0.7776, x=array([ 0.00352395,  
-0.00694216]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.4965724860745589, gradient\_value=array([9.58160654e-05,  
2.05813916e-05]))

(57/320) [11:14] step\_size=0.01 min\_gradient(0.0001) starting\_point=[ 2.73684211  
-2.42105263]

GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([ 2.73684211,  
-2.42105263]), f\_value=1.4794927226497707, gradient\_value=array([ 0.07123899,  
-0.01727565]))

GradientDescentResult(n\_iter=1531, time\_running=0.3002, x=array([ 0.9845872 ,  
-1.96910468]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.9927420977305036, gradient\_value=array([ 7.06444625e-05,  
-7.05527313e-05]))

(58/320) [11:01] step\_size=0.01 min\_gradient(0.0001) starting\_point=[-0.15789474  
-0.52631579]

GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([-0.15789474,  
-0.52631579]), f\_value=0.7457052280513097, gradient\_value=array([-0.26802165,  
-0.7343479 ]))

GradientDescentResult(n\_iter=477, time\_running=0.1026, x=array([ 0.00347 ,  
-0.00700273]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.49657248622854977, gradient\_value=array([-1.33888803e-05,  
-9.90931531e-05]))

(59/320) [10:48] step\_size=0.01 min\_gradient(0.0001) starting\_point=[-0.68421053  
-3.36842105]

GradientDescentIteration(n\_iter=0, time\_running=0.0004, x=array([-0.68421053,

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-3.36842105]], f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=2522, time_running=0.4957, x=array([ 0.9844765 ,
-1.96912051]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420972731882, gradient_value=array([-3.92193973e-05,
-9.14963396e-05]))
```

```
(60/320) [10:36] step_size=0.01 min_gradient(0.0001) starting_point=[2.21052632
2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5182, time_running=1.0002, x=array([2.17433164,
1.96724783]), f_value=1.4998154670351123, gradient_value=array([0.00080243,
0.00072612]))
GradientDescentIteration(n_iter=10304, time_running=2.0001, x=array([2.12534784,
1.92292247]), f_value=1.4997293343736442, gradient_value=array([0.00115046,
0.00104105]))
GradientDescentIteration(n_iter=15486, time_running=3.0001, x=array([2.04759481,
1.85256267]), f_value=1.4995116657952852, gradient_value=array([0.0019997 ,
0.00180958]))
GradientDescentIteration(n_iter=20632, time_running=4.0001, x=array([1.87176808,
1.69344786]), f_value=1.4982897550763588, gradient_value=array([0.00640181,
0.00579353]))
GradientDescentResult(n_iter=23992, time_running=4.65, x=array([ 0.00350891,
-0.00691518]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248611342064, gradient_value=array([6.65883629e-05,
7.28208217e-05]))
```

```
(61/320) [10:43] step_size=0.01 max_time(5) starting_point=[ 0.63157895
-0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=5233, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=10475, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=15754, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=20722, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentResult(n_iter=25894, time_running=5.0, x=array([ 0.00347601,
```

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-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([ 2.08166817e-17, -4.16333634e-17]))
```

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(62/320) [10:51] step_size=0.01 max_time(5) starting_point=[2.47368421  
0.10526316]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,  
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,  
0.00331099]))  
GradientDescentIteration(n_iter=5269, time_running=1.0001, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.08166817e-17, -4.16333634e-17]))  
GradientDescentIteration(n_iter=10546, time_running=2.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.08166817e-17, -4.16333634e-17]))  
GradientDescentIteration(n_iter=15692, time_running=3.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.08166817e-17, -4.16333634e-17]))  
GradientDescentIteration(n_iter=20451, time_running=4.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.08166817e-17, -4.16333634e-17]))  
GradientDescentResult(n_iter=25524, time_running=5.0, x=array([ 0.00347601,  
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([ 2.08166817e-17, -4.16333634e-17]))
```

```
(63/320) [10:59] step_size=0.01 max_time(5) starting_point=[-0.68421053  
0.10526316]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,  
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,  
0.13183868]))  
GradientDescentIteration(n_iter=5165, time_running=1.0001, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416,  
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))  
GradientDescentIteration(n_iter=10381, time_running=2.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416,  
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))  
GradientDescentIteration(n_iter=15547, time_running=3.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416,  
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))  
GradientDescentIteration(n_iter=20720, time_running=4.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416,  
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))  
GradientDescentIteration(n_iter=25960, time_running=5.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416,  
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))  
GradientDescentResult(n_iter=25960, time_running=5.0001, x=array([ 0.00347601,  
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
```

```
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
```

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(64/320) [11:06] step_size=0.01 max_time(5) starting_point=[-1.47368421
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5064, time_running=1.0002, x=array([-1.42447799,
-3.34079572]), f_value=1.4997661774500837, gradient_value=array([-0.00113006,
-0.00063449]))
GradientDescentIteration(n_iter=10162, time_running=2.0161,
x=array([-1.35414173, -3.30129884]), f_value=1.4996367225681575,
gradient_value=array([-0.0017045 , -0.00095728]))
GradientDescentIteration(n_iter=15246, time_running=3.0001,
x=array([-1.23520276, -3.23448468]), f_value=1.499257006416411,
gradient_value=array([-0.00330904, -0.00185931]))
GradientDescentIteration(n_iter=20494, time_running=4.0002,
x=array([-0.81328086, -2.99717685]), f_value=1.4930301277547995,
gradient_value=array([-0.02514711, -0.01415952]))
GradientDescentResult(n_iter=25718, time_running=5.0, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 5.54764568e-15, -1.10952914e-14]))
```

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(65/320) [11:12] step_size=0.01 max_time(5) starting_point=[0.10526316
1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5221, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentIteration(n_iter=10459, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentIteration(n_iter=15614, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentIteration(n_iter=20851, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentIteration(n_iter=26056, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentResult(n_iter=26056, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
```

```
(66/320) [11:19] step_size=0.01 max_time(5) starting_point=[2.47368421
0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=5092, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.49657248367634166, gradient_value=array([
7.98233007e-14, -2.35401976e-15]))
GradientDescentIteration(n_iter=10307, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=15472, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=20643, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=25684, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentResult(n_iter=25684, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 2.08166817e-17, -4.16333634e-17]))
```

```
(67/320) [11:25] step_size=0.01 max_time(5) starting_point=[ 2.73684211
-2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=4937, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.54764568e-15,
-1.10952914e-14]))
GradientDescentIteration(n_iter=10139, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
GradientDescentIteration(n_iter=15230, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
GradientDescentIteration(n_iter=20215, time_running=4.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
GradientDescentResult(n_iter=24840, time_running=5.0, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 5.54764568e-15, -1.10952914e-14]))
```

```

(68/320) [11:30] step_size=0.01 max_time(5) starting_point=[-0.15789474
-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentIteration(n_iter=5191, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=10324, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=15279, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=20361, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=25080, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentResult(n_iter=25080, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 2.08166817e-17, -4.16333634e-17]))

```

```

(69/320) [11:35] step_size=0.01 max_time(5) starting_point=[-0.68421053
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=4950, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([
2.17898616e-14, -4.65079364e-14]))
GradientDescentIteration(n_iter=10066, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
GradientDescentIteration(n_iter=14861, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
GradientDescentIteration(n_iter=19790, time_running=4.0069, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
GradientDescentResult(n_iter=24812, time_running=5.0, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 5.54764568e-15, -1.10952914e-14]))

```



```
(70/320) [11:40] step_size=0.01 max_time(5) starting_point=[2.21052632 2.
]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5122, time_running=1.0002, x=array([2.17481223,
1.96768271]), f_value=1.499816167192165, gradient_value=array([0.00079957,
0.00072352]))
GradientDescentIteration(n_iter=10161, time_running=2.0001, x=array([2.12698318,
1.92440229]), f_value=1.4997327360996127, gradient_value=array([0.00113687,
0.00102876]))
GradientDescentIteration(n_iter=15061, time_running=3.0001, x=array([2.05585078,
1.86003369]), f_value=1.499540846970952, gradient_value=array([0.00188779,
0.0017083 ]))
GradientDescentIteration(n_iter=20164, time_running=4.0001, x=array([1.8992045 ,
1.71827723]), f_value=1.4985831210033398, gradient_value=array([0.00538144,
0.00487006]))
GradientDescentIteration(n_iter=25211, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.49657248367634166,
gradient_value=array([8.35269354e-16, 2.14238349e-15]))
GradientDescentResult(n_iter=25211, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.49657248367634166,
gradient_value=array([8.35269354e-16, 2.14238349e-15]))
```

```
(71/320) [11:45] step_size=0.01 max_iterations(20000) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=4825, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=9728, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=14675, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=19652, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentResult(n_iter=20000, time_running=4.0699, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 2.08166817e-17,
-4.16333634e-17]))
```

```
(72/320) [11:46] step_size=0.01 max_iterations(20000) starting_point=[2.47368421
```

```

0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5111, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=10129, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=14939, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=19752, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentResult(n_iter=20000, time_running=4.0536, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 2.08166817e-17,
-4.16333634e-17]))

```

```

(73/320) [11:47] step_size=0.01 max_iterations(20000)
starting_point=[-0.68421053  0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=4782, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentIteration(n_iter=9991, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentIteration(n_iter=15061, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentResult(n_iter=20000, time_running=3.963, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-2.08166817e-17,
4.16333634e-17]))

```

```

(74/320) [11:48] step_size=0.01 max_iterations(20000)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5135, time_running=1.0002, x=array([-1.42367376,

```

```

-3.34034417]), f_value=1.4997649792546783, gradient_value=array([-0.00113547,
-0.00063753]))
GradientDescentIteration(n_iter=10165, time_running=2.0002,
x=array([-1.35409059, -3.30127011]), f_value=1.4996366078883352,
gradient_value=array([-0.00170501, -0.00095756]))
GradientDescentIteration(n_iter=14850, time_running=3.0001,
x=array([-1.24786095, -3.24159698]), f_value=1.499310268009797,
gradient_value=array([-0.00308933, -0.00173577]))
GradientDescentIteration(n_iter=19871, time_running=4.0001,
x=array([-0.93290474, -3.06451027]), f_value=1.4961256897251334,
gradient_value=array([-0.01490744, -0.00838829]))
GradientDescentResult(n_iter=20000, time_running=4.0278, x=array([-0.91279273,
-3.05319281]), stop_condition='max_iterations(20000)',
f_value=1.4957124090535432, gradient_value=array([-0.0163248 , -0.00918682]))

```

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(75/320) [11:49] step_size=0.01 max_iterations(20000) starting_point=[0.10526316
1.05263158]

```

```

GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5164, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentIteration(n_iter=10339, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentIteration(n_iter=15156, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.08166817e-17,  4.16333634e-17]))
GradientDescentResult(n_iter=20000, time_running=3.998, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-2.08166817e-17,
4.16333634e-17]))

```

```

(76/320) [11:49] step_size=0.01 max_iterations(20000) starting_point=[2.47368421
0.73684211]

```

```

GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=4939, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([1.73624309e-12, 2.50997140e-14]))
GradientDescentIteration(n_iter=10182, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=15386, time_running=3.0001, x=array([

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```
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentResult(n_iter=20000, time_running=3.8873, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 2.08166817e-17,
-4.16333634e-17]))
```

```
(77/320) [11:49] step_size=0.01 max_iterations(20000) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=5051, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.54764568e-15,
-1.10952914e-14]))
GradientDescentIteration(n_iter=10267, time_running=2.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
GradientDescentIteration(n_iter=15524, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
GradientDescentResult(n_iter=20000, time_running=3.8706, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 5.54764568e-15,
-1.10952914e-14]))
```

```
(78/320) [11:49] step_size=0.01 max_iterations(20000)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentIteration(n_iter=4756, time_running=1.0003, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=9353, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=13992, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentIteration(n_iter=18575, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.08166817e-17, -4.16333634e-17]))
GradientDescentResult(n_iter=20000, time_running=4.3123, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 2.08166817e-17,
```

-4.16333634e-17]))

```
(79/320) [11:50] step_size=0.01 max_iterations(20000)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=4767, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.01701633e-13, -2.23227686e-13]))
GradientDescentIteration(n_iter=9608, time_running=2.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.54764568e-15,
-1.10952914e-14]))
GradientDescentIteration(n_iter=14652, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
GradientDescentIteration(n_iter=19901, time_running=4.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.54764568e-15, -1.10952914e-14]))
GradientDescentResult(n_iter=20000, time_running=4.021, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 5.54764568e-15,
-1.10952914e-14]))
```

```
(80/320) [11:50] step_size=0.01 max_iterations(20000) starting_point=[2.21052632
2.
]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5271, time_running=1.0001, x=array([2.17361559,
1.96659988]), f_value=1.4998144191701444, gradient_value=array([0.00080673,
0.00073
]))
GradientDescentIteration(n_iter=10533, time_running=2.0001, x=array([2.12268789,
1.92051547]), f_value=1.4997237143271156, gradient_value=array([0.00117288,
0.00106134]))
GradientDescentIteration(n_iter=15729, time_running=3.0002, x=array([2.0426519 ,
1.84808971]), f_value=1.4994933747369952, gradient_value=array([0.00206959,
0.00187283]))
GradientDescentResult(n_iter=20000, time_running=3.8411, x=array([1.90778867,
1.72604566]), stop_condition='max_iterations(20000)',
f_value=1.4986648848561182, gradient_value=array([0.00509383, 0.00460976]))
```

```
(81/320) [11:50] step_size=0.05 max_time(5) min_gradient(1e-06) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
```

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-0.20675964]))
GradientDescentResult(n_iter=140, time_running=0.0286, x=array([ 0.00347641,
-0.00695229]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367656187, gradient_value=array([ 7.76195908e-07,
-5.08398978e-07]))

(82/320) [11:38] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentResult(n_iter=560, time_running=0.1074, x=array([ 0.00347648,
-0.0069522 ]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367658186, gradient_value=array([ 9.17381117e-07,
-3.24088027e-07]))

(83/320) [11:27] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=139, time_running=0.027, x=array([ 0.00347556,
-0.00695183]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765856,
gradient_value=array([-9.06114304e-07, 3.72139732e-07]))

(84/320) [11:16] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentResult(n_iter=4601, time_running=0.8967, x=array([ 0.98450949,
-1.96902092]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231532,
gradient_value=array([-6.15086888e-07, -7.49490931e-07]))

(85/320) [11:08] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=150, time_running=0.0302, x=array([ 0.00347596,
-0.00695154]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367657886, gradient_value=array([-8.51706081e-08,
9.57099951e-07]))

```

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(86/320) [10:57] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentResult(n_iter=853, time_running=0.1635, x=array([ 0.00347649,
-0.00695195]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367657714, gradient_value=array([9.57742105e-07,
1.57827531e-07]))

(87/320) [10:47] step_size=0.05 max_time(5) min_gradient(1e-06) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=408, time_running=0.0845, x=array([ 0.98451068,
-1.96902107]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232295,
gradient_value=array([ 5.53374118e-07, -8.04921716e-07]))

(88/320) [10:38] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=137, time_running=0.0298, x=array([ 0.003476 ,
-0.00695251]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367657264, gradient_value=array([-4.56710180e-08,
-9.48897513e-07]))

(89/320) [10:28] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=601, time_running=0.1266, x=array([ 0.98451012,
-1.96902121]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231974,
gradient_value=array([-1.37512547e-08, -9.62424963e-07]))

(90/320) [10:18] step_size=0.05 max_time(5) min_gradient(1e-06)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentResult(n_iter=4842, time_running=0.9503, x=array([ 0.0034763 ,

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-0.00695167]), stop_condition='min_gradient(1e-06)',  
f_value=0.49657248367655127, gradient_value=array([5.71238187e-07,  
7.14173343e-07]))
```

```
(91/320) [10:11] step_size=0.05 min_gradient(0.0001) starting_point=[ 0.63157895  
-0.21052632]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,  
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,  
-0.20675964]))  
GradientDescentResult(n_iter=95, time_running=0.0207, x=array([ 0.0035184 ,  
-0.00697618]), stop_condition='min_gradient(0.0001)',  
f_value=0.4965724860074251, gradient_value=array([ 8.38111278e-05,  
-4.59395752e-05]))
```

```
(92/320) [10:02] step_size=0.05 min_gradient(0.0001) starting_point=[2.47368421  
0.10526316]  
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,  
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,  
0.00331099]))  
GradientDescentResult(n_iter=516, time_running=0.1151, x=array([ 0.00352189,  
-0.00696485]), stop_condition='min_gradient(0.0001)',  
f_value=0.49657248591750025, gradient_value=array([ 9.10724561e-05,  
-2.37361699e-05]))
```

```
(93/320) [09:53] step_size=0.05 min_gradient(0.0001) starting_point=[-0.68421053  
0.10526316]  
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,  
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,  
0.13183868]))  
GradientDescentResult(n_iter=95, time_running=0.0205, x=array([ 0.00343084,  
-0.00693692]), stop_condition='min_gradient(0.0001)',  
f_value=0.4965724859136885, gradient_value=array([-8.96136857e-05,  
2.82201809e-05]))
```

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(94/320) [09:44] step_size=0.05 min_gradient(0.0001) starting_point=[-1.47368421  
-3.36842105]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,  
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,  
-0.00047196]))  
GradientDescentResult(n_iter=4511, time_running=0.9311, x=array([ 0.98443624,  
-1.96908386]), stop_condition='min_gradient(0.0001)',  
f_value=0.9927420968034846, gradient_value=array([-7.65579384e-05,  
-6.13411503e-05]))
```



(95/320) [09:38] step\_size=0.05 min\_gradient(0.0001) starting\_point=[0.10526316  
1.05263158]

GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([0.10526316,  
1.05263158]), f\_value=1.1734107031477512, gradient\_value=array([0.06871534,  
0.68763702]))

GradientDescentResult(n\_iter=105, time\_running=0.0217, x=array([ 0.00347407,  
-0.00690244]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.49657248607656024, gradient\_value=array([-2.50276983e-06,  
9.67062757e-05]))

(96/320) [09:29] step\_size=0.05 min\_gradient(0.0001) starting\_point=[2.47368421  
0.73684211]

GradientDescentIteration(n\_iter=0, time\_running=0.0003, x=array([2.47368421,  
0.73684211]), f\_value=1.4986896501592173, gradient\_value=array([0.00641913,  
0.00205835]))

GradientDescentResult(n\_iter=810, time\_running=0.1624, x=array([ 0.0035203 ,  
-0.00694313]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.4965724857185452, gradient\_value=array([8.85001764e-05,  
1.85911674e-05]))

(97/320) [09:21] step\_size=0.05 min\_gradient(0.0001) starting\_point=[ 2.73684211  
-2.42105263]

GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([ 2.73684211,  
-2.42105263]), f\_value=1.4794927226497707, gradient\_value=array([ 0.07123899,  
-0.01727565]))

GradientDescentResult(n\_iter=304, time\_running=0.0631, x=array([ 0.98458536,  
-1.96910392]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.9927420975480015, gradient\_value=array([ 6.88765369e-05,  
-6.99854135e-05]))

(98/320) [09:13] step\_size=0.05 min\_gradient(0.0001) starting\_point=[-0.15789474  
-0.52631579]

GradientDescentIteration(n\_iter=0, time\_running=0.0003, x=array([-0.15789474,  
-0.52631579]), f\_value=0.7457052280513097, gradient\_value=array([-0.26802165,  
-0.7343479 ]))

GradientDescentResult(n\_iter=92, time\_running=0.0208, x=array([ 0.00347058,  
-0.00700158]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.4965724861089497, gradient\_value=array([-1.22012403e-05,  
-9.68427349e-05]))

(99/320) [09:05] step\_size=0.05 min\_gradient(0.0001) starting\_point=[-0.68421053  
-3.36842105]

GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([-0.68421053,

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-3.36842105]], f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=503, time_running=0.1088, x=array([ 0.98447891,
-1.96911812]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420969664856, gradient_value=array([-3.67062893e-05,
-8.92241250e-05]))
```

```
(100/320) [08:58] step_size=0.05 min_gradient(0.0001) starting_point=[2.21052632
2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentResult(n_iter=4798, time_running=0.9993, x=array([ 0.00350699,
-0.00691696]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724858620029, gradient_value=array([6.27048658e-05,
6.92778756e-05]))
```

```
(101/320) [08:52] step_size=0.05 max_time(5) starting_point=[ 0.63157895
-0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=4814, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=9787, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15014, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=20107, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=25053, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=25053, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 3.46944695e-18, -6.93889390e-18]))
```

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(102/320) [08:55] step_size=0.05 max_time(5) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
```

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GradientDescentIteration(n_iter=4974, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=9963, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=14893, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=20060, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=25206, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=25206, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 3.46944695e-18, -6.93889390e-18]))

```

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(103/320) [08:58] step_size=0.05 max_time(5) starting_point=[-0.68421053
0.10526316]

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GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=5129, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=10203, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=15427, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=20651, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentResult(n_iter=25705, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))

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(104/320) [09:00] step_size=0.05 max_time(5) starting_point=[-1.47368421
-3.36842105]

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GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5222, time_running=1.0002, x=array([ 0.98451006,

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-1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=10421, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=15607, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=20601, time_running=4.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=25712, time_running=5.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentResult(n_iter=25712, time_running=5.0001, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226762,
gradient_value=array([ 1.05991604e-15, -2.11983209e-15]))

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(105/320) [09:03] step_size=0.05 max_time(5) starting_point=[0.10526316
1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=4970, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=10061, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=15013, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=19955, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=24818, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentResult(n_iter=24818, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))

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(106/320) [09:05] step_size=0.05 max_time(5) starting_point=[2.47368421
0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,

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0.00205835]))
GradientDescentIteration(n_iter=4801, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=9837, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=14829, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=20023, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=24917, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 3.46944695e-18, -6.93889390e-18]))

```

```

(107/320) [09:07] step_size=0.05 max_time(5) starting_point=[ 2.73684211
-2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=5137, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=10317, time_running=2.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=15307, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=20525, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=25688, time_running=5.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentResult(n_iter=25688, time_running=5.0001, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226762,
gradient_value=array([ 1.05991604e-15, -2.11983209e-15]))

```

```

(108/320) [09:09] step_size=0.05 max_time(5) starting_point=[-0.15789474
-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))

```

```

GradientDescentIteration(n_iter=5171, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10290, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15564, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=20732, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=25981, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 3.46944695e-18, -6.93889390e-18]))

```

```

(109/320) [09:11] step_size=0.05 max_time(5) starting_point=[-0.68421053
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=5209, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=10469, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=15693, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=20920, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=26074, time_running=5.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentResult(n_iter=26074, time_running=5.0001, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226762,
gradient_value=array([ 1.05991604e-15, -2.11983209e-15]))

```

```

(110/320) [09:13] step_size=0.05 max_time(5) starting_point=[2.21052632 2.
]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5045, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([1.73472348e-16,

```

```

5.39499001e-16]))
GradientDescentIteration(n_iter=10262, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([3.46944695e-18, 6.93889390e-18]))
GradientDescentIteration(n_iter=15465, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([3.46944695e-18, 6.93889390e-18]))
GradientDescentIteration(n_iter=20636, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([3.46944695e-18, 6.93889390e-18]))
GradientDescentResult(n_iter=25904, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([3.46944695e-18, 6.93889390e-18]))

(111/320) [09:15] step_size=0.05 max_iterations(20000) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=5060, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10283, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15354, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.8891, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 3.46944695e-18,
-6.93889390e-18]))

(112/320) [09:15] step_size=0.05 max_iterations(20000)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5159, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10303, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15338, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([

```

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3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9149, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 3.46944695e-18,
-6.93889390e-18]))

(113/320) [09:14] step_size=0.05 max_iterations(20000)
starting_point=[-0.68421053  0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=4807, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=9885, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=15072, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9604, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-3.46944695e-18,
6.93889390e-18]))

(114/320) [09:14] step_size=0.05 max_iterations(20000)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5177, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=10329, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=15561, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentResult(n_iter=20000, time_running=3.8724, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.9927420920226762, gradient_value=array([ 1.05991604e-15,
-2.11983209e-15]))

(115/320) [09:13] step_size=0.05 max_iterations(20000)

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starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5129, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=10366, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=15573, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.8738, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-3.46944695e-18,
6.93889390e-18]))

```

```

(116/320) [09:12] step_size=0.05 max_iterations(20000)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=5043, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10107, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15052, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=19975, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0059, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 3.46944695e-18,
-6.93889390e-18]))

```

```

(117/320) [09:12] step_size=0.05 max_iterations(20000) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=4867, time_running=1.0002, x=array([ 0.98451006,

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-1.96902012]], f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=10025, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=14829, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=19769, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentResult(n_iter=20000, time_running=4.0467, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.9927420920226762, gradient_value=array([ 1.05991604e-15,
-2.11983209e-15]))

```

```

(118/320) [09:11] step_size=0.05 max_iterations(20000)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentIteration(n_iter=4860, time_running=1.0003, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=9917, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15191, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9319, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 3.46944695e-18,
-6.93889390e-18]))

```

```

(119/320) [09:11] step_size=0.05 max_iterations(20000)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=5168, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=10308, time_running=2.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))

```

```

GradientDescentIteration(n_iter=15136, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentIteration(n_iter=19781, time_running=4.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226762, gradient_value=array([
1.05991604e-15, -2.11983209e-15]))
GradientDescentResult(n_iter=20000, time_running=4.0472, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.9927420920226762, gradient_value=array([ 1.05991604e-15,
-2.11983209e-15]))

```

```

(120/320) [09:10] step_size=0.05 max_iterations(20000)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0004, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5096, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([3.46944695e-18,
6.93889390e-18]))
GradientDescentIteration(n_iter=9752, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([3.46944695e-18,
6.93889390e-18]))
GradientDescentIteration(n_iter=14684, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([3.46944695e-18, 6.93889390e-18]))
GradientDescentIteration(n_iter=19635, time_running=4.0003, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([3.46944695e-18, 6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=4.078, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([3.46944695e-18,
6.93889390e-18]))

```

```

(121/320) [09:09] step_size=0.06 max_time(5) min_gradient(1e-06)
starting_point=[ 0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=115, time_running=0.025, x=array([ 0.00347643,
-0.00695231]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765907,
gradient_value=array([ 8.24232611e-07, -5.42442383e-07]))

```

```

(122/320) [09:02] step_size=0.06 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,

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```
0.00331099]))
GradientDescentResult(n_iter=466, time_running=0.0954, x=array([ 0.00347647,
-0.0069522 ]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765726,
gradient_value=array([ 8.98077245e-07, -3.21591940e-07]))
```

```
(123/320) [08:55] step_size=0.06 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053  0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=115, time_running=0.0246, x=array([ 0.00347558,
-0.00695184]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765655,
gradient_value=array([-8.66660827e-07,  3.59453718e-07]))
```

```
(124/320) [08:48] step_size=0.06 max_time(5) min_gradient(1e-06)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentResult(n_iter=3834, time_running=0.8281, x=array([ 0.9845095 ,
-1.96902091]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231302,
gradient_value=array([-5.96068452e-07, -7.34016046e-07]))
```

```
(125/320) [08:42] step_size=0.06 max_time(5) min_gradient(1e-06)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0004, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=124, time_running=0.0312, x=array([ 0.00347596,
-0.00695155]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765696,
gradient_value=array([-8.54530461e-08,  9.38054997e-07]))
```

```
(126/320) [08:36] step_size=0.06 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentResult(n_iter=710, time_running=0.1447, x=array([ 0.0034765 ,
-0.00695195]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765804,
gradient_value=array([9.64631677e-07, 1.57315950e-07]))
```

```
(127/320) [08:29] step_size=0.06 max_time(5) min_gradient(1e-06)
starting_point=[ 2.73684211 -2.42105263]
```

```

GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=339, time_running=0.0686, x=array([ 0.98451069,
-1.96902107]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232342,
gradient_value=array([ 5.54537079e-07, -8.09130817e-07]))

```

```

(128/320) [08:23] step_size=0.06 max_time(5) min_gradient(1e-06)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=113, time_running=0.0262, x=array([ 0.003476 ,
-0.00695251]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765725,
gradient_value=array([-4.28062898e-08, -9.48758396e-07]))

```

```

(129/320) [08:16] step_size=0.06 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=500, time_running=0.1136, x=array([ 0.98451012,
-1.96902121]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232012,
gradient_value=array([-9.72972708e-09, -9.65687031e-07]))

```

```

(130/320) [08:10] step_size=0.06 max_time(5) min_gradient(1e-06)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentResult(n_iter=4034, time_running=0.8371, x=array([ 0.00347631,
-0.00695165]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765733,
gradient_value=array([5.99412684e-07, 7.51533994e-07]))

```

```

(131/320) [08:05] step_size=0.06 min_gradient(0.0001) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=79, time_running=0.0172, x=array([ 0.00351521,
-0.00697456]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248567807283, gradient_value=array([ 7.74954102e-05,
-4.28672189e-05]))

```

```
(132/320) [07:59] step_size=0.06 min_gradient(0.0001) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentResult(n_iter=430, time_running=0.0921, x=array([ 0.00351941,
-0.00696434]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724856861674, gradient_value=array([ 8.61494450e-05,
-2.28143969e-05]))
```

```
(133/320) [07:53] step_size=0.06 min_gradient(0.0001)
starting_point=[-0.68421053  0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=78, time_running=0.0168, x=array([ 0.00342862,
-0.00693614]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248614029703, gradient_value=array([-9.40169522e-05,
2.96911153e-05]))
```

```
(134/320) [07:47] step_size=0.06 min_gradient(0.0001)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentResult(n_iter=3759, time_running=0.7764, x=array([ 0.98443648,
-1.96908385]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420967837799, gradient_value=array([-7.63133751e-05,
-6.13147917e-05]))
```

```
(135/320) [07:42] step_size=0.06 min_gradient(0.0001) starting_point=[0.10526316
1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=87, time_running=0.0181, x=array([ 0.00347405,
-0.00690403]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248592485803, gradient_value=array([-2.58860164e-06,
9.35936327e-05]))
```

```
(136/320) [07:36] step_size=0.06 min_gradient(0.0001) starting_point=[2.47368421
0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
```

```

0.00205835]))
GradientDescentResult(n_iter=674, time_running=0.1299, x=array([ 0.00352391,
-0.00694242]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248606533017, gradient_value=array([9.57262591e-05,
2.00754700e-05]))

(137/320) [07:30] step_size=0.06 min_gradient(0.0001) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=253, time_running=0.0486, x=array([ 0.98458435,
-1.96910317]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420974265787, gradient_value=array([ 6.79281205e-05,
-6.93801387e-05]))

(138/320) [07:25] step_size=0.06 min_gradient(0.0001)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=76, time_running=0.0149, x=array([ 0.00347078,
-0.00700096]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248604645415, gradient_value=array([-1.17987390e-05,
-9.56171722e-05]))

(139/320) [07:19] step_size=0.06 min_gradient(0.0001)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=419, time_running=0.0865, x=array([ 0.9844798 ,
-1.96911682]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420968182923, gradient_value=array([-3.57463829e-05,
-8.80045990e-05]))

(140/320) [07:13] step_size=0.06 min_gradient(0.0001) starting_point=[2.21052632
2.
]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentResult(n_iter=3998, time_running=0.8314, x=array([ 0.00350745,
-0.00691637]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724859324328, gradient_value=array([6.36347710e-05,
7.04490836e-05]))

```

```
(141/320) [07:09] step_size=0.06 max_time(5) starting_point=[ 0.63157895
-0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=5073, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10205, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15341, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=20474, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=25636, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 3.46944695e-18, -6.93889390e-18]))
```

```
(142/320) [07:10] step_size=0.06 max_time(5) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5068, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10293, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15217, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=20366, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=25504, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=25504, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 3.46944695e-18, -6.93889390e-18]))
```



```
(143/320) [07:11] step_size=0.06 max_time(5) starting_point=[-0.68421053
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=5196, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=10438, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=15694, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=20865, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=26137, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentResult(n_iter=26137, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
```

```
(144/320) [07:11] step_size=0.06 max_time(5) starting_point=[-1.47368421
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5019, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=10200, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=15391, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=20545, time_running=4.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=25754, time_running=5.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentResult(n_iter=25754, time_running=5.0001, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226759,
```

```
gradient_value=array([ 8.63892291e-16, -1.72778458e-15]))
```

```
(145/320) [07:12] step_size=0.06 max_time(5) starting_point=[0.10526316
1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5131, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=9968, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=15043, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=20192, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentIteration(n_iter=25411, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
GradientDescentResult(n_iter=25411, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18,  6.93889390e-18]))
```

```
(146/320) [07:12] step_size=0.06 max_time(5) starting_point=[2.47368421
0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=5072, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10161, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15272, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=20524, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=25681, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 3.46944695e-18, -6.93889390e-18]))
```

```
(147/320) [07:13] step_size=0.06 max_time(5) starting_point=[ 2.73684211
-2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=5096, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=10251, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=15487, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=20665, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=25881, time_running=5.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentResult(n_iter=25881, time_running=5.0001, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226759,
gradient_value=array([ 8.63892291e-16, -1.72778458e-15]))
```

```
(148/320) [07:13] step_size=0.06 max_time(5) starting_point=[-0.15789474
-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentIteration(n_iter=4873, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=9951, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15200, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=20350, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=25465, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=25465, time_running=5.0001, x=array([ 0.00347601,
```

```
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([ 3.46944695e-18, -6.93889390e-18]))
```

```
(149/320) [07:13] step_size=0.06 max_time(5) starting_point=[-0.68421053  
-3.36842105]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,  
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,  
-0.01238275]))  
GradientDescentIteration(n_iter=5169, time_running=1.0001, x=array([ 0.98451006,  
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
8.63892291e-16, -1.72778458e-15]))  
GradientDescentIteration(n_iter=10276, time_running=2.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
8.63892291e-16, -1.72778458e-15]))  
GradientDescentIteration(n_iter=15378, time_running=3.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
8.63892291e-16, -1.72778458e-15]))  
GradientDescentIteration(n_iter=20527, time_running=4.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
8.63892291e-16, -1.72778458e-15]))  
GradientDescentResult(n_iter=25577, time_running=5.0, x=array([ 0.98451006,  
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226759,  
gradient_value=array([ 8.63892291e-16, -1.72778458e-15]))
```

```
(150/320) [07:13] step_size=0.06 max_time(5) starting_point=[2.21052632 2.  
]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.  
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))  
GradientDescentIteration(n_iter=5162, time_running=1.0002, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([3.46944695e-18,  
6.93889390e-18]))  
GradientDescentIteration(n_iter=10286, time_running=2.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416,  
gradient_value=array([3.46944695e-18, 6.93889390e-18]))  
GradientDescentIteration(n_iter=15450, time_running=3.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416,  
gradient_value=array([3.46944695e-18, 6.93889390e-18]))  
GradientDescentIteration(n_iter=20318, time_running=4.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416,  
gradient_value=array([3.46944695e-18, 6.93889390e-18]))  
GradientDescentResult(n_iter=25428, time_running=5.0, x=array([ 0.00347601,  
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([3.46944695e-18, 6.93889390e-18]))
```

```
(151/320) [07:14] step_size=0.06 max_iterations(20000) starting_point=[
```

```

0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=5070, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10112, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15194, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9612, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 3.46944695e-18,
-6.93889390e-18]))

```

```

(152/320) [07:12] step_size=0.06 max_iterations(20000)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5075, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10188, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15235, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9379, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 3.46944695e-18,
-6.93889390e-18]))

```

```

(153/320) [07:11] step_size=0.06 max_iterations(20000)
starting_point=[-0.68421053 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=5029, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18, 6.93889390e-18]))
GradientDescentIteration(n_iter=10069, time_running=2.0001, x=array([

```

```

0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18, 6.93889390e-18]))
GradientDescentIteration(n_iter=15171, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18, 6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9638, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-3.46944695e-18,
6.93889390e-18]))

```

```

(154/320) [07:10] step_size=0.06 max_iterations(20000)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=4988, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=10041, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=15127, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentResult(n_iter=20000, time_running=3.9679, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.9927420920226759, gradient_value=array([ 8.63892291e-16,
-1.72778458e-15]))

```

```

(155/320) [07:09] step_size=0.06 max_iterations(20000)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5052, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18, 6.93889390e-18]))
GradientDescentIteration(n_iter=10086, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18, 6.93889390e-18]))
GradientDescentIteration(n_iter=15125, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-3.46944695e-18, 6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9674, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-3.46944695e-18,

```

6.93889390e-18]))

```
(156/320) [07:08] step_size=0.06 max_iterations(20000)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=5040, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10103, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15092, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9646, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 3.46944695e-18,
-6.93889390e-18]))
```

```
(157/320) [07:06] step_size=0.06 max_iterations(20000) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=5023, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=9900, time_running=2.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=14929, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentResult(n_iter=20000, time_running=3.9888, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.9927420920226759, gradient_value=array([ 8.63892291e-16,
-1.72778458e-15]))
```

```
(158/320) [07:05] step_size=0.06 max_iterations(20000)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
```

```

GradientDescentIteration(n_iter=5185, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=10221, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentIteration(n_iter=15314, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
3.46944695e-18, -6.93889390e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9292, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 3.46944695e-18,
-6.93889390e-18]))

```

```

(159/320) [07:04] step_size=0.06 max_iterations(20000)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=5064, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=10145, time_running=2.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentIteration(n_iter=15215, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
8.63892291e-16, -1.72778458e-15]))
GradientDescentResult(n_iter=20000, time_running=3.9201, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.9927420920226759, gradient_value=array([ 8.63892291e-16,
-1.72778458e-15]))

```

```

(160/320) [07:02] step_size=0.06 max_iterations(20000)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5053, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([3.46944695e-18,
6.93889390e-18]))
GradientDescentIteration(n_iter=10103, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([3.46944695e-18, 6.93889390e-18]))
GradientDescentIteration(n_iter=14969, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([3.46944695e-18, 6.93889390e-18]))

```



```
GradientDescentResult(n_iter=20000, time_running=3.9979, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([3.46944695e-18,
6.93889390e-18]))
```

```
(161/320) [07:01] step_size=0.07 max_time(5) min_gradient(1e-06)
starting_point=[ 0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=98, time_running=0.0202, x=array([ 0.0034764,
-0.0069523]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765607,
gradient_value=array([ 7.70687907e-07, -5.12033929e-07]))
```

```
(162/320) [06:56] step_size=0.07 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentResult(n_iter=399, time_running=0.0914, x=array([ 0.00347645,
-0.00695219]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765539,
gradient_value=array([ 8.59261742e-07, -3.12390328e-07]))
```

```
(163/320) [06:50] step_size=0.07 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=98, time_running=0.0204, x=array([ 0.0034756 ,
-0.00695185]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765377,
gradient_value=array([-8.09777725e-07, 3.39681485e-07]))
```

```
(164/320) [06:45] step_size=0.07 max_time(5) min_gradient(1e-06)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentResult(n_iter=3286, time_running=0.6687, x=array([ 0.98450952,
-1.9690209 ]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231168,
gradient_value=array([-5.83632450e-07, -7.25626111e-07]))
```

```
(165/320) [06:41] step_size=0.07 max_time(5) min_gradient(1e-06)
starting_point=[0.10526316 1.05263158]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=105, time_running=0.0214, x=array([ 0.00347596,
-0.00695153]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765893,
gradient_value=array([-9.03961716e-08,  9.77632198e-07]))
```

```
(166/320) [06:36] step_size=0.07 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentResult(n_iter=608, time_running=0.1221, x=array([ 0.00347649,
-0.00695196]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765731,
gradient_value=array([9.49976634e-07, 1.53004133e-07]))
```

```
(167/320) [06:31] step_size=0.07 max_time(5) min_gradient(1e-06)
starting_point=[ 2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=290, time_running=0.0588, x=array([ 0.98451068,
-1.96902106]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232191,
gradient_value=array([ 5.45465416e-07, -7.99164546e-07]))
```

```
(168/320) [06:26] step_size=0.07 max_time(5) min_gradient(1e-06)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=96, time_running=0.0199, x=array([ 0.003476 ,
-0.0069525]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765624,
gradient_value=array([-3.86461757e-08, -9.27966279e-07]))
```

```
(169/320) [06:22] step_size=0.07 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=428, time_running=0.087, x=array([ 0.98451012,
-1.96902121]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231953,
gradient_value=array([-4.93334105e-09, -9.59960850e-07]))
```

```
(170/320) [06:17] step_size=0.07 max_time(5) min_gradient(1e-06)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentResult(n_iter=3457, time_running=0.6949, x=array([ 0.00347632,
-0.00695164]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367658685, gradient_value=array([6.15315203e-07,
7.74265049e-07]))
```

```
(171/320) [06:13] step_size=0.07 min_gradient(0.0001) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=67, time_running=0.0138, x=array([ 0.00351539,
-0.00697479]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724857015443, gradient_value=array([ 7.78369275e-05,
-4.33112643e-05]))
```

```
(172/320) [06:08] step_size=0.07 min_gradient(0.0001) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentResult(n_iter=368, time_running=0.0748, x=array([ 0.00352065,
-0.00696481]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248580486774, gradient_value=array([ 8.85947774e-05,
-2.36965643e-05]))
```

```
(173/320) [06:04] step_size=0.07 min_gradient(0.0001)
starting_point=[-0.68421053  0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=67, time_running=0.0137, x=array([ 0.00343408,
-0.00693778]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724856100892, gradient_value=array([-8.31755408e-05,
2.66404992e-05]))
```

```
(174/320) [05:59] step_size=0.07 min_gradient(0.0001)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
```

```
GradientDescentResult(n_iter=3222, time_running=0.645, x=array([ 0.98443753,
-1.96908319]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420966646585, gradient_value=array([-7.52429598e-05,
-6.06693002e-05]))
```

```
(175/320) [05:55] step_size=0.07 min_gradient(0.0001) starting_point=[0.10526316
1.05263158]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
```

```
GradientDescentResult(n_iter=74, time_running=0.0144, x=array([ 0.003474 ,
-0.00690466]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248586710395, gradient_value=array([-2.69755668e-06,
9.23778658e-05]))
```

```
(176/320) [05:51] step_size=0.07 min_gradient(0.0001) starting_point=[2.47368421
0.73684211]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
```

```
GradientDescentResult(n_iter=578, time_running=0.1101, x=array([ 0.00351967,
-0.00694336]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724856593923, gradient_value=array([8.72487720e-05,
1.81210009e-05]))
```

```
(177/320) [05:46] step_size=0.07 min_gradient(0.0001) starting_point=[
2.73684211 -2.42105263]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
```

```
GradientDescentResult(n_iter=216, time_running=0.0439, x=array([ 0.98458616,
-1.9691053 ]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420977001715, gradient_value=array([ 6.95744501e-05,
-7.11613369e-05]))
```

```
(178/320) [05:42] step_size=0.07 min_gradient(0.0001)
starting_point=[-0.15789474 -0.52631579]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
```

```
GradientDescentResult(n_iter=65, time_running=0.0135, x=array([ 0.00347133,
-0.00699735]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724857077221, gradient_value=array([-1.05886688e-05,
-8.85563335e-05]))
```

```

(179/320) [05:38] step_size=0.07 min_gradient(0.0001)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=359, time_running=0.0725, x=array([ 0.98448069,
-1.96911551]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420966723033, gradient_value=array([-3.47900583e-05,
-8.67813737e-05]))

(180/320) [05:33] step_size=0.07 min_gradient(0.0001) starting_point=[2.21052632
2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentResult(n_iter=3427, time_running=0.6662, x=array([ 0.00350588,
-0.00691802]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248572157564, gradient_value=array([6.04601614e-05,
6.71888142e-05]))

(181/320) [05:30] step_size=0.07 max_time(5) starting_point=[ 0.63157895
-0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=5121, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=10158, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=15246, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=20232, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=25291, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentResult(n_iter=25291, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 2.60208521e-18, -5.20417043e-18]))

```

```
(182/320) [05:29] step_size=0.07 max_time(5) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5087, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=10121, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=15111, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=20167, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=25260, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentResult(n_iter=25260, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 2.60208521e-18, -5.20417043e-18]))
```

```
(183/320) [05:29] step_size=0.07 max_time(5) starting_point=[-0.68421053
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=4899, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=9879, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=14976, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=20018, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=25013, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentResult(n_iter=25013, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
```

```
(184/320) [05:28] step_size=0.07 max_time(5) starting_point=[-1.47368421
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5102, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentIteration(n_iter=10114, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentIteration(n_iter=15156, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentIteration(n_iter=20313, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentIteration(n_iter=25457, time_running=5.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentResult(n_iter=25457, time_running=5.0001, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226759,
gradient_value=array([ 7.78890841e-16, -1.55778168e-15]))
```

```
(185/320) [05:28] step_size=0.07 max_time(5) starting_point=[0.10526316
1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5068, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=10123, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=15192, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=20090, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=25084, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentResult(n_iter=25084, time_running=5.0001, x=array([ 0.00347601,
```

```
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([-2.60208521e-18, 5.20417043e-18]))
```

```
(186/320) [05:27] step_size=0.07 max_time(5) starting_point=[2.47368421  
0.73684211]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,  
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,  
0.00205835]))  
GradientDescentIteration(n_iter=4948, time_running=1.0001, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=9968, time_running=2.0001, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=15019, time_running=3.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=19965, time_running=4.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentResult(n_iter=24955, time_running=5.0, x=array([ 0.00347601,  
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([ 2.60208521e-18, -5.20417043e-18]))
```

```
(187/320) [05:26] step_size=0.07 max_time(5) starting_point=[ 2.73684211  
-2.42105263]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,  
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,  
-0.01727565]))  
GradientDescentIteration(n_iter=5013, time_running=1.0001, x=array([ 0.98451006,  
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=10006, time_running=2.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=15034, time_running=3.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=20013, time_running=4.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=25042, time_running=5.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentResult(n_iter=25042, time_running=5.0001, x=array([ 0.98451006,  
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226759,
```



```
gradient_value=array([ 7.78890841e-16, -1.55778168e-15]))
```

```
(188/320) [05:25] step_size=0.07 max_time(5) starting_point=[-0.15789474  
-0.52631579]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,  
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,  
-0.7343479 ]))  
GradientDescentIteration(n_iter=5109, time_running=1.0002, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=10219, time_running=2.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=15132, time_running=3.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=20051, time_running=4.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentResult(n_iter=25011, time_running=5.0, x=array([ 0.00347601,  
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([ 2.60208521e-18, -5.20417043e-18]))
```

```
(189/320) [05:25] step_size=0.07 max_time(5) starting_point=[-0.68421053  
-3.36842105]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,  
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,  
-0.01238275]))  
GradientDescentIteration(n_iter=4992, time_running=1.0001, x=array([ 0.98451006,  
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=10023, time_running=2.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=15074, time_running=3.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=19961, time_running=4.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=24815, time_running=5.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentResult(n_iter=24815, time_running=5.0001, x=array([ 0.98451006,  
-1.96902012]), stop_condition='max_time(5)', f_value=0.9927420920226759,  
gradient_value=array([ 7.78890841e-16, -1.55778168e-15]))
```

```
(190/320) [05:24] step_size=0.07 max_time(5) starting_point=[2.21052632 2.
]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=4863, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([2.60208521e-18,
5.20417043e-18]))
GradientDescentIteration(n_iter=9864, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([2.60208521e-18,
5.20417043e-18]))
GradientDescentIteration(n_iter=14676, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([2.60208521e-18, 5.20417043e-18]))
GradientDescentIteration(n_iter=19582, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([2.60208521e-18, 5.20417043e-18]))
GradientDescentResult(n_iter=24644, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([2.60208521e-18, 5.20417043e-18]))
```

```
(191/320) [05:23] step_size=0.07 max_iterations(20000) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=5080, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=9890, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=14940, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9904, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 2.60208521e-18,
-5.20417043e-18]))
```

```
(192/320) [05:21] step_size=0.07 max_iterations(20000)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
```

```

GradientDescentIteration(n_iter=5025, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=10047, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=14990, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9978, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 2.60208521e-18,
-5.20417043e-18]))

```

```

(193/320) [05:20] step_size=0.07 max_iterations(20000)
starting_point=[-0.68421053  0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=4996, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=9993, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=14909, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=19967, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0071, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-2.60208521e-18,
5.20417043e-18]))

```

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(194/320) [05:18] step_size=0.07 max_iterations(20000)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5042, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentIteration(n_iter=10028, time_running=2.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([

```

```

7.78890841e-16, -1.55778168e-15]))
GradientDescentIteration(n_iter=15019, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentIteration(n_iter=19916, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentResult(n_iter=20000, time_running=4.0169, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.9927420920226759, gradient_value=array([ 7.78890841e-16,
-1.55778168e-15]))

(195/320) [05:16] step_size=0.07 max_iterations(20000)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=4982, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=9958, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=14996, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9947, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-2.60208521e-18,
5.20417043e-18]))

(196/320) [05:15] step_size=0.07 max_iterations(20000)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=5034, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=10054, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=15069, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9752, x=array([ 0.00347601,

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-0.00695203]), stop_condition='max_iterations(20000)',  
f_value=0.4965724836763416, gradient_value=array([ 2.60208521e-18,  
-5.20417043e-18]))
```

```
(197/320) [05:13] step_size=0.07 max_iterations(20000) starting_point=[  
2.73684211 -2.42105263]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,  
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,  
-0.01727565]))  
GradientDescentIteration(n_iter=5022, time_running=1.0001, x=array([ 0.98451006,  
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=9995, time_running=2.0001, x=array([ 0.98451006,  
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=14957, time_running=3.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentIteration(n_iter=19956, time_running=4.0001, x=array([  
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([  
7.78890841e-16, -1.55778168e-15]))  
GradientDescentResult(n_iter=20000, time_running=4.0091, x=array([ 0.98451006,  
-1.96902012]), stop_condition='max_iterations(20000)',  
f_value=0.9927420920226759, gradient_value=array([ 7.78890841e-16,  
-1.55778168e-15]))
```

```
(198/320) [05:11] step_size=0.07 max_iterations(20000)  
starting_point=[-0.15789474 -0.52631579]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,  
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,  
-0.7343479 ]))  
GradientDescentIteration(n_iter=5009, time_running=1.0001, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=9917, time_running=2.0281, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=14835, time_running=3.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=19897, time_running=4.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentResult(n_iter=20000, time_running=4.0208, x=array([ 0.00347601,  
-0.00695203]), stop_condition='max_iterations(20000)',  
f_value=0.4965724836763416, gradient_value=array([ 2.60208521e-18,
```

-5.20417043e-18]))

```
(199/320) [05:10] step_size=0.07 max_iterations(20000)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=5022, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentIteration(n_iter=10050, time_running=2.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentIteration(n_iter=15143, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.9927420920226759, gradient_value=array([
7.78890841e-16, -1.55778168e-15]))
GradientDescentResult(n_iter=20000, time_running=3.9724, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.9927420920226759, gradient_value=array([ 7.78890841e-16,
-1.55778168e-15]))
```

```
(200/320) [05:08] step_size=0.07 max_iterations(20000)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5027, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([2.60208521e-18,
5.20417043e-18]))
GradientDescentIteration(n_iter=9888, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([2.60208521e-18,
5.20417043e-18]))
GradientDescentIteration(n_iter=14762, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([2.60208521e-18, 5.20417043e-18]))
GradientDescentIteration(n_iter=19774, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([2.60208521e-18, 5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0442, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([2.60208521e-18,
5.20417043e-18]))
```

```
(201/320) [05:06] step_size=0.08 max_time(5) min_gradient(1e-06)
starting_point=[ 0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
```

```
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=85, time_running=0.0166, x=array([ 0.00347639,
-0.00695229]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765504,
gradient_value=array([ 7.50317064e-07, -5.02641554e-07]))
```

```
(202/320) [05:02] step_size=0.08 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentResult(n_iter=348, time_running=0.0669, x=array([ 0.00347648,
-0.00695221]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765928,
gradient_value=array([ 9.33817687e-07, -3.42200285e-07]))
```

```
(203/320) [04:58] step_size=0.08 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=85, time_running=0.0164, x=array([ 0.00347562,
-0.00695185]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765278,
gradient_value=array([-7.87901132e-07, 3.33644209e-07]))
```

```
(204/320) [04:54] step_size=0.08 max_time(5) min_gradient(1e-06)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentResult(n_iter=2874, time_running=0.5755, x=array([ 0.98450948,
-1.96902095]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231798,
gradient_value=array([-6.24073622e-07, -7.75478185e-07]))
```

```
(205/320) [04:50] step_size=0.08 max_time(5) min_gradient(1e-06)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=91, time_running=0.0186, x=array([ 0.00347596,
-0.00695153]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367658885, gradient_value=array([-9.22325613e-08,
9.76500772e-07]))
```

```
(206/320) [04:46] step_size=0.08 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentResult(n_iter=532, time_running=0.1079, x=array([ 0.00347644,
-0.00695196]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765295,
gradient_value=array([8.56243616e-07, 1.35256992e-07]))
```

```
(207/320) [04:43] step_size=0.08 max_time(5) min_gradient(1e-06)
starting_point=[ 2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=253, time_running=0.0511, x=array([ 0.98451068,
-1.96902107]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232233,
gradient_value=array([ 5.46402188e-07, -8.03088654e-07]))
```

```
(208/320) [04:39] step_size=0.08 max_time(5) min_gradient(1e-06)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=83, time_running=0.0169, x=array([ 0.003476 ,
-0.00695251]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367657086, gradient_value=array([-3.66240469e-08,
-9.45523825e-07]))
```

```
(209/320) [04:35] step_size=0.08 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=374, time_running=0.0751, x=array([ 0.98451013,
-1.9690212 ]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231893,
gradient_value=array([-1.35205611e-10, -9.54065089e-07]))
```

```
(210/320) [04:31] step_size=0.08 max_time(5) min_gradient(1e-06)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentResult(n_iter=3025, time_running=0.6101, x=array([ 0.00347629,
-0.00695167]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367654144, gradient_value=array([5.52754763e-07,
```



7.00923140e-07]))

(211/320) [04:28] step\_size=0.08 min\_gradient(0.0001) starting\_point=[  
0.63157895 -0.21052632]  
GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([ 0.63157895,  
-0.21052632]), f\_value=0.8402749009220171, gradient\_value=array([ 0.79782729,  
-0.20675964]))  
GradientDescentResult(n\_iter=58, time\_running=0.012, x=array([ 0.00351554,  
-0.00697501]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.49657248572312457, gradient\_value=array([ 7.81335346e-05,  
-4.37432428e-05]))

(212/320) [04:24] step\_size=0.08 min\_gradient(0.0001) starting\_point=[2.47368421  
0.10526316]  
GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([2.47368421,  
0.10526316]), f\_value=1.4971462897557586, gradient\_value=array([0.01276325,  
0.00331099]))  
GradientDescentResult(n\_iter=322, time\_running=0.0655, x=array([ 0.00351811,  
-0.00696427]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.4965724855740172, gradient\_value=array([ 8.35518536e-05,  
-2.27238611e-05]))

(213/320) [04:20] step\_size=0.08 min\_gradient(0.0001)  
starting\_point=[-0.68421053 0.10526316]  
GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([-0.68421053,  
0.10526316]), f\_value=0.8803879697816689, gradient\_value=array([-0.84858717,  
0.13183868]))  
GradientDescentResult(n\_iter=58, time\_running=0.0119, x=array([ 0.00343393,  
-0.00693764]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.4965724856265263, gradient\_value=array([-8.34777096e-05,  
2.69018948e-05]))

(214/320) [04:17] step\_size=0.08 min\_gradient(0.0001)  
starting\_point=[-1.47368421 -3.36842105]  
GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([-1.47368421,  
-3.36842105]), f\_value=1.4998295224940215, gradient\_value=array([-0.00084072,  
-0.00047196]))  
GradientDescentResult(n\_iter=2819, time\_running=0.5596, x=array([ 0.98443702,  
-1.9690838 ]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.9927420967397966, gradient\_value=array([-7.57806027e-05,  
-6.12374558e-05]))

(215/320) [04:13] step\_size=0.08 min\_gradient(0.0001) starting\_point=[0.10526316

```

1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=64, time_running=0.0135, x=array([ 0.00347389,
-0.00690329]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248599533305, gradient_value=array([-2.86839825e-06,
9.50389458e-05]))

(216/320) [04:10] step_size=0.08 min_gradient(0.0001) starting_point=[2.47368421
0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentResult(n_iter=505, time_running=0.1046, x=array([ 0.00352324,
-0.00694267]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724859966746, gradient_value=array([9.43847606e-05,
1.95639429e-05]))

(217/320) [04:06] step_size=0.08 min_gradient(0.0001) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=189, time_running=0.0391, x=array([ 0.9845837 ,
-1.96910307]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420973756516, gradient_value=array([ 6.72957711e-05,
-6.93300245e-05]))

(218/320) [04:03] step_size=0.08 min_gradient(0.0001)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=56, time_running=0.0115, x=array([ 0.00347118,
-0.0069996 ]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248591355574, gradient_value=array([-1.09581259e-05,
-9.29504491e-05]))

(219/320) [03:59] step_size=0.08 min_gradient(0.0001)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))

```

```
GradientDescentResult(n_iter=313, time_running=0.0647, x=array([ 0.9844786 ,  
-1.96912158]), stop_condition='min_gradient(0.0001)', f_value=0.992742097291186,  
gradient_value=array([-3.72234946e-05, -9.23176895e-05]))
```

```
(220/320) [03:56] step_size=0.08 min_gradient(0.0001) starting_point=[2.21052632  
2.          ]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.  
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))  
GradientDescentResult(n_iter=2998, time_running=0.5931, x=array([ 0.00350834,  
-0.00691519]), stop_condition='min_gradient(0.0001)', f_value=0.49657248607464,  
gradient_value=array([6.54497357e-05, 7.27761077e-05]))
```

```
(221/320) [03:53] step_size=0.08 max_time(5) starting_point=[ 0.63157895  
-0.21052632]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 0.63157895,  
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,  
-0.20675964]))  
GradientDescentIteration(n_iter=5020, time_running=1.0002, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=9807, time_running=2.0001, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=14814, time_running=3.0002, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=19820, time_running=4.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentResult(n_iter=24793, time_running=5.0, x=array([ 0.00347601,  
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([ 2.60208521e-18, -5.20417043e-18]))
```

```
(222/320) [03:52] step_size=0.08 max_time(5) starting_point=[2.47368421  
0.10526316]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,  
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,  
0.00331099]))  
GradientDescentIteration(n_iter=4953, time_running=1.0001, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=9972, time_running=2.0001, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([  
2.60208521e-18, -5.20417043e-18]))  
GradientDescentIteration(n_iter=14776, time_running=3.0002, x=array([
```

```

0.00347601, -0.00695203]], f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=19860, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=24940, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentResult(n_iter=24940, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 2.60208521e-18, -5.20417043e-18]))

```

```

(223/320) [03:50] step_size=0.08 max_time(5) starting_point=[-0.68421053
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=5024, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=10016, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=15086, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=20047, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentResult(n_iter=24907, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))

```

```

(224/320) [03:49] step_size=0.08 max_time(5) starting_point=[-1.47368421
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5002, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))
GradientDescentIteration(n_iter=9947, time_running=2.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))
GradientDescentIteration(n_iter=14744, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([

```

```

6.81746326e-16, -1.36349265e-15]))
GradientDescentIteration(n_iter=19672, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
6.81746326e-16, -1.36349265e-15]))
GradientDescentResult(n_iter=24631, time_running=5.0, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 6.81746326e-16, -1.36349265e-15]))

```

```

(225/320) [03:48] step_size=0.08 max_time(5) starting_point=[0.10526316
1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=4800, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=9673, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=14515, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=19378, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentResult(n_iter=24284, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))

```

```

(226/320) [03:46] step_size=0.08 max_time(5) starting_point=[2.47368421
0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=4815, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=9627, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=14461, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=19273, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))

```

```
GradientDescentResult(n_iter=24063, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 2.60208521e-18, -5.20417043e-18]))
```

```
(227/320) [03:45] step_size=0.08 max_time(5) starting_point=[ 2.73684211
-2.42105263]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
```

```
GradientDescentIteration(n_iter=4917, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))
```

```
GradientDescentIteration(n_iter=9848, time_running=2.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))
```

```
GradientDescentIteration(n_iter=14843, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
6.81746326e-16, -1.36349265e-15]))
```

```
GradientDescentIteration(n_iter=19884, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
6.81746326e-16, -1.36349265e-15]))
```

```
GradientDescentIteration(n_iter=24876, time_running=5.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
6.81746326e-16, -1.36349265e-15]))
```

```
GradientDescentResult(n_iter=24876, time_running=5.0001, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 6.81746326e-16, -1.36349265e-15]))
```

```
(228/320) [03:43] step_size=0.08 max_time(5) starting_point=[-0.15789474
-0.52631579]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
```

```
GradientDescentIteration(n_iter=4885, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
```

```
GradientDescentIteration(n_iter=9600, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
```

```
GradientDescentIteration(n_iter=14362, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
```

```
GradientDescentIteration(n_iter=19314, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
```

```
GradientDescentResult(n_iter=24263, time_running=5.0, x=array([ 0.00347601,
```

```
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([ 2.60208521e-18, -5.20417043e-18]))
```

```
(229/320) [03:42] step_size=0.08 max_time(5) starting_point=[-0.68421053  
-3.36842105]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,  
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,  
-0.01238275]))  
GradientDescentIteration(n_iter=4974, time_running=1.0002, x=array([ 0.98451006,  
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,  
-1.36349265e-15]))  
GradientDescentIteration(n_iter=9749, time_running=2.0001, x=array([ 0.98451006,  
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,  
-1.36349265e-15]))  
GradientDescentIteration(n_iter=14461, time_running=3.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([  
6.81746326e-16, -1.36349265e-15]))  
GradientDescentIteration(n_iter=19477, time_running=4.0002, x=array([  
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([  
6.81746326e-16, -1.36349265e-15]))  
GradientDescentResult(n_iter=24281, time_running=5.0, x=array([ 0.98451006,  
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,  
gradient_value=array([ 6.81746326e-16, -1.36349265e-15]))
```

```
(230/320) [03:40] step_size=0.08 max_time(5) starting_point=[2.21052632 2.  
]  
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.  
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))  
GradientDescentIteration(n_iter=4824, time_running=1.0003, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([2.60208521e-18,  
5.20417043e-18]))  
GradientDescentIteration(n_iter=9679, time_running=2.0002, x=array([ 0.00347601,  
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([2.60208521e-18,  
5.20417043e-18]))  
GradientDescentIteration(n_iter=14615, time_running=3.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416,  
gradient_value=array([2.60208521e-18, 5.20417043e-18]))  
GradientDescentIteration(n_iter=19730, time_running=4.0001, x=array([  
0.00347601, -0.00695203]), f_value=0.4965724836763416,  
gradient_value=array([2.60208521e-18, 5.20417043e-18]))  
GradientDescentResult(n_iter=24758, time_running=5.0, x=array([ 0.00347601,  
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,  
gradient_value=array([2.60208521e-18, 5.20417043e-18]))
```

```
(231/320) [03:39] step_size=0.08 max_iterations(20000) starting_point=[
```

```

0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=5018, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=9995, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=14950, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=19883, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0238, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 2.60208521e-18,
-5.20417043e-18]))

```

```

(232/320) [03:37] step_size=0.08 max_iterations(20000)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5010, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=9962, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=15000, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9908, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 2.60208521e-18,
-5.20417043e-18]))

```

```

(233/320) [03:35] step_size=0.08 max_iterations(20000)
starting_point=[-0.68421053 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=4873, time_running=1.0001, x=array([ 0.00347601,

```



```

-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=9916, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=14907, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=19819, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0372, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-2.60208521e-18,
5.20417043e-18]))

```

```

(234/320) [03:33] step_size=0.08 max_iterations(20000)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5065, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))
GradientDescentIteration(n_iter=10079, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
6.81746326e-16, -1.36349265e-15]))
GradientDescentIteration(n_iter=15099, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
6.81746326e-16, -1.36349265e-15]))
GradientDescentResult(n_iter=20000, time_running=3.9492, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))

```

```

(235/320) [03:31] step_size=0.08 max_iterations(20000)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5056, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))
GradientDescentIteration(n_iter=10120, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18,  5.20417043e-18]))

```

```

GradientDescentIteration(n_iter=15146, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-2.60208521e-18, 5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9545, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-2.60208521e-18,
5.20417043e-18]))

```

```

(236/320) [03:29] step_size=0.08 max_iterations(20000)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=4999, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=10035, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=14888, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=19942, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0119, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 2.60208521e-18,
-5.20417043e-18]))

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(237/320) [03:27] step_size=0.08 max_iterations(20000) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=4977, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))
GradientDescentIteration(n_iter=10056, time_running=2.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
6.81746326e-16, -1.36349265e-15]))
GradientDescentIteration(n_iter=15051, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
6.81746326e-16, -1.36349265e-15]))
GradientDescentIteration(n_iter=19928, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([

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6.81746326e-16, -1.36349265e-15]))
GradientDescentResult(n_iter=20000, time_running=4.0153, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))

(238/320) [03:25] step_size=0.08 max_iterations(20000)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentIteration(n_iter=4858, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=9714, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=14608, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentIteration(n_iter=19571, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
2.60208521e-18, -5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0865, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 2.60208521e-18,
-5.20417043e-18]))

(239/320) [03:23] step_size=0.08 max_iterations(20000)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=4876, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))
GradientDescentIteration(n_iter=9790, time_running=2.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))
GradientDescentIteration(n_iter=14680, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
6.81746326e-16, -1.36349265e-15]))
GradientDescentIteration(n_iter=19324, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
6.81746326e-16, -1.36349265e-15]))
GradientDescentResult(n_iter=20000, time_running=4.1431, x=array([ 0.98451006,

```

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-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 6.81746326e-16,
-1.36349265e-15]))
```

```
(240/320) [03:21] step_size=0.08 max_iterations(20000)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=4773, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([2.60208521e-18,
5.20417043e-18]))
GradientDescentIteration(n_iter=9757, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([2.60208521e-18,
5.20417043e-18]))
GradientDescentIteration(n_iter=14768, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([2.60208521e-18, 5.20417043e-18]))
GradientDescentIteration(n_iter=19773, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([2.60208521e-18, 5.20417043e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0462, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([2.60208521e-18,
5.20417043e-18]))
```

```
(241/320) [03:19] step_size=0.09 max_time(5) min_gradient(1e-06)
starting_point=[ 0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=75, time_running=0.0156, x=array([ 0.00347638,
-0.00695228]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367653145, gradient_value=array([ 7.13524038e-07,
-4.82485178e-07]))
```

```
(242/320) [03:16] step_size=0.09 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentResult(n_iter=309, time_running=0.0636, x=array([ 0.00347646,
-0.0069522 ]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765711,
gradient_value=array([ 8.90851568e-07, -3.31691268e-07]))
```

```
(243/320) [03:12] step_size=0.09 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053  0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=74, time_running=0.0155, x=array([ 0.00347555,
-0.00695182]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765907,
gradient_value=array([-9.11196274e-07,  3.86071559e-07]))
```

```
(244/320) [03:09] step_size=0.09 max_time(5) min_gradient(1e-06)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentResult(n_iter=2555, time_running=0.5187, x=array([ 0.98450952,
-1.9690209 ]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231167,
gradient_value=array([-5.77572537e-07, -7.29709307e-07]))
```

```
(245/320) [03:06] step_size=0.09 max_time(5) min_gradient(1e-06)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=80, time_running=0.0157, x=array([ 0.00347596,
-0.00695152]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765985,
gradient_value=array([-9.57950306e-08,  9.95391008e-07]))
```

```
(246/320) [03:03] step_size=0.09 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentResult(n_iter=472, time_running=0.0955, x=array([ 0.00347647,
-0.00695196]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367655704, gradient_value=array([9.17046922e-07,
1.43711229e-07]))
```

```
(247/320) [03:00] step_size=0.09 max_time(5) min_gradient(1e-06)
starting_point=[ 2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=224, time_running=0.0459, x=array([ 0.98451069,
-1.96902109]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232476,
```

```
gradient_value=array([ 5.57533457e-07, -8.21305640e-07]))
```

```
(248/320) [02:57] step_size=0.09 max_time(5) min_gradient(1e-06)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=73, time_running=0.0157, x=array([ 0.003476 ,
-0.00695251]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765693,
gradient_value=array([-3.33086372e-08, -9.42305382e-07]))
```

```
(249/320) [02:53] step_size=0.09 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=332, time_running=0.0688, x=array([ 0.98451013,
-1.96902119]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231831,
gradient_value=array([ 4.66178520e-09, -9.47998639e-07]))
```

```
(250/320) [02:50] step_size=0.09 max_time(5) min_gradient(1e-06)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentResult(n_iter=2688, time_running=0.5485, x=array([ 0.00347631,
-0.00695164]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765817,
gradient_value=array([6.05082062e-07, 7.68873734e-07]))
```

```
(251/320) [02:47] step_size=0.09 min_gradient(0.0001) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=51, time_running=0.0124, x=array([ 0.00351567,
-0.00697523]), stop_condition='min_gradient(0.0001)', f_value=0.496572485742588,
gradient_value=array([ 7.83813256e-05, -4.41613264e-05]))
```

```
(252/320) [02:44] step_size=0.09 min_gradient(0.0001) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentResult(n_iter=286, time_running=0.0562, x=array([ 0.0035174 ,
```

```
-0.00696423]), stop_condition='min_gradient(0.0001)',  
f_value=0.49657248551436456, gradient_value=array([ 8.21437102e-05,  
-2.26522372e-05]))
```

```
(253/320) [02:41] step_size=0.09 min_gradient(0.0001)  
starting_point=[-0.68421053  0.10526316]  
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,  
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,  
0.13183868]))  
GradientDescentResult(n_iter=51, time_running=0.0102, x=array([ 0.0034338 ,  
-0.00693751]), stop_condition='min_gradient(0.0001)',  
f_value=0.4965724856406673, gradient_value=array([-8.37264252e-05,  
2.71549736e-05]))
```

```
(254/320) [02:38] step_size=0.09 min_gradient(0.0001)  
starting_point=[-1.47368421 -3.36842105]  
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,  
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,  
-0.00047196]))  
GradientDescentResult(n_iter=2506, time_running=0.5031, x=array([ 0.98443965,  
-1.96908187]), stop_condition='min_gradient(0.0001)',  
f_value=0.9927420964277265, gradient_value=array([-7.30742951e-05,  
-5.93563900e-05]))
```

```
(255/320) [02:35] step_size=0.09 min_gradient(0.0001) starting_point=[0.10526316  
1.05263158]  
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,  
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,  
0.68763702]))  
GradientDescentResult(n_iter=57, time_running=0.012, x=array([ 0.003474 ,  
-0.00690888]), stop_condition='min_gradient(0.0001)',  
f_value=0.4965724854946581, gradient_value=array([-2.81486687e-06,  
8.41439149e-05]))
```

```
(256/320) [02:32] step_size=0.09 min_gradient(0.0001) starting_point=[2.47368421  
0.73684211]  
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,  
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,  
0.00205835]))  
GradientDescentResult(n_iter=449, time_running=0.092, x=array([ 0.00351985,  
-0.00694343]), stop_condition='min_gradient(0.0001)',  
f_value=0.49657248567366535, gradient_value=array([8.76016744e-05,  
1.79859554e-05]))
```

```
(257/320) [02:29] step_size=0.09 min_gradient(0.0001) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=168, time_running=0.0345, x=array([ 0.98458129,
-1.96910087]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420970660386, gradient_value=array([ 6.50641003e-05,
-6.75220670e-05]))
```

```
(258/320) [02:26] step_size=0.09 min_gradient(0.0001)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=49, time_running=0.0101, x=array([ 0.00347102,
-0.00700198]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248614218696, gradient_value=array([-1.13361212e-05,
-9.76014079e-05]))
```

```
(259/320) [02:23] step_size=0.09 min_gradient(0.0001)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=278, time_running=0.0564, x=array([ 0.98447916,
-1.96912114]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420972304548, gradient_value=array([-3.66448586e-05,
-9.18960172e-05]))
```

```
(260/320) [02:21] step_size=0.09 min_gradient(0.0001) starting_point=[2.21052632
2.
]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentResult(n_iter=2665, time_running=0.5279, x=array([ 0.00350665,
-0.00691697]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724858403084, gradient_value=array([6.20279318e-05,
6.92538573e-05]))
```

```
(261/320) [02:18] step_size=0.09 max_time(5) starting_point=[ 0.63157895
-0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
```



```

-0.20675964]))
GradientDescentIteration(n_iter=5009, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=10058, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=15172, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=20313, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=25402, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 1.73472348e-18, -3.46944695e-18]))

```

```

(262/320) [02:16] step_size=0.09 max_time(5) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5019, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=10110, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=15158, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=20042, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=25108, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 1.73472348e-18, -3.46944695e-18]))

```

```

(263/320) [02:14] step_size=0.09 max_time(5) starting_point=[-0.68421053
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=5057, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))

```

```

GradientDescentIteration(n_iter=10092, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=15100, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=20064, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentResult(n_iter=25008, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))

```

```

(264/320) [02:12] step_size=0.09 max_time(5) starting_point=[-1.47368421
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5042, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))
GradientDescentIteration(n_iter=9987, time_running=2.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))
GradientDescentIteration(n_iter=14999, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentIteration(n_iter=20035, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentResult(n_iter=25127, time_running=5.0, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 5.82867088e-16, -1.16573418e-15]))

```

```

(265/320) [02:10] step_size=0.09 max_time(5) starting_point=[0.10526316
1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=4992, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=10040, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=15024, time_running=3.0002, x=array([

```

```

0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=20084, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=25162, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentResult(n_iter=25162, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))

```

```

(266/320) [02:09] step_size=0.09 max_time(5) starting_point=[2.47368421
0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0005, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=5029, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=10046, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=15139, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=20190, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=25243, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 1.73472348e-18, -3.46944695e-18]))

```

```

(267/320) [02:07] step_size=0.09 max_time(5) starting_point=[ 2.73684211
-2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=5089, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))
GradientDescentIteration(n_iter=10177, time_running=2.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentIteration(n_iter=15256, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([

```

```

5.82867088e-16, -1.16573418e-15]))
GradientDescentIteration(n_iter=20465, time_running=4.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentIteration(n_iter=25571, time_running=5.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentResult(n_iter=25571, time_running=5.0001, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 5.82867088e-16, -1.16573418e-15]))

```

```

(268/320) [02:05] step_size=0.09 max_time(5) starting_point=[-0.15789474
-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentIteration(n_iter=4995, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=9825, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=14886, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=19944, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=25039, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=25039, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 1.73472348e-18, -3.46944695e-18]))

```

```

(269/320) [02:03] step_size=0.09 max_time(5) starting_point=[-0.68421053
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=5059, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))
GradientDescentIteration(n_iter=10097, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))

```

```

GradientDescentIteration(n_iter=15134, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentIteration(n_iter=20170, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentResult(n_iter=25182, time_running=5.0, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 5.82867088e-16, -1.16573418e-15]))

```

```

(270/320) [02:01] step_size=0.09 max_time(5) starting_point=[2.21052632 2.
]

```

```

GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5062, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([1.73472348e-18,
3.46944695e-18]))
GradientDescentIteration(n_iter=10061, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=15115, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=20124, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))
GradientDescentResult(n_iter=25205, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))

```

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(271/320) [01:59] step_size=0.09 max_iterations(20000) starting_point=[
0.63157895 -0.21052632]

```

```

GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=4796, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=9846, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=14745, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=19751, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([

```

```

1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0468, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 1.73472348e-18,
-3.46944695e-18]))

(272/320) [01:57] step_size=0.09 max_iterations(20000)
starting_point=[2.47368421 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5035, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=10147, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=15167, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9451, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 1.73472348e-18,
-3.46944695e-18]))

(273/320) [01:55] step_size=0.09 max_iterations(20000)
starting_point=[-0.68421053 0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=4998, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=10065, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=15108, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18, 3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9725, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-1.73472348e-18,
3.46944695e-18]))

(274/320) [01:52] step_size=0.09 max_iterations(20000)

```

```

starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5051, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))
GradientDescentIteration(n_iter=10110, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentIteration(n_iter=14995, time_running=3.0003, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentIteration(n_iter=19973, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentResult(n_iter=20000, time_running=4.0061, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))

```

```

(275/320) [01:50] step_size=0.09 max_iterations(20000)
starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5024, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=10070, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=15091, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9768, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-1.73472348e-18,
3.46944695e-18]))

```

```

(276/320) [01:48] step_size=0.09 max_iterations(20000)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=5009, time_running=1.0002, x=array([ 0.00347601,

```

```

-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=10062, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=15161, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=3.9634, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 1.73472348e-18,
-3.46944695e-18]))

```

```

(277/320) [01:46] step_size=0.09 max_iterations(20000) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=4971, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))
GradientDescentIteration(n_iter=9918, time_running=2.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))
GradientDescentIteration(n_iter=15012, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentIteration(n_iter=19672, time_running=4.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentResult(n_iter=20000, time_running=4.0669, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))

```

```

(278/320) [01:43] step_size=0.09 max_iterations(20000)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentIteration(n_iter=4885, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=9888, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))

```



```

GradientDescentIteration(n_iter=14905, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=19910, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0182, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 1.73472348e-18,
-3.46944695e-18]))

```

```

(279/320) [01:41] step_size=0.09 max_iterations(20000)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=4985, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))
GradientDescentIteration(n_iter=10007, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentIteration(n_iter=14828, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentIteration(n_iter=19569, time_running=4.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
5.82867088e-16, -1.16573418e-15]))
GradientDescentResult(n_iter=20000, time_running=4.0913, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 5.82867088e-16,
-1.16573418e-15]))

```

```

(280/320) [01:39] step_size=0.09 max_iterations(20000)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=4951, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([1.73472348e-18,
3.46944695e-18]))
GradientDescentIteration(n_iter=10045, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=15036, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))

```

```
GradientDescentResult(n_iter=20000, time_running=3.9843, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([1.73472348e-18,
3.46944695e-18]))
```

```
(281/320) [01:37] step_size=0.1 max_time(5) min_gradient(1e-06) starting_point=[
0.63157895 -0.21052632]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
```

```
GradientDescentResult(n_iter=67, time_running=0.0142, x=array([ 0.00347636,
-0.00695227]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765136,
gradient_value=array([ 6.76926011e-07, -4.62196378e-07]))
```

```
(282/320) [01:34] step_size=0.1 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.10526316]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
```

```
GradientDescentResult(n_iter=278, time_running=0.0572, x=array([ 0.00347642,
-0.00695219]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765328,
gradient_value=array([ 8.11209805e-07, -3.07822711e-07]))
```

```
(283/320) [01:31] step_size=0.1 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 0.10526316]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
```

```
GradientDescentResult(n_iter=66, time_running=0.0137, x=array([ 0.00347557,
-0.00695183]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367657747, gradient_value=array([-8.85124515e-07,
3.78817108e-07]))
```

```
(284/320) [01:28] step_size=0.1 max_time(5) min_gradient(1e-06)
starting_point=[-1.47368421 -3.36842105]
```

```
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
```

```
GradientDescentResult(n_iter=2299, time_running=0.4565, x=array([ 0.98450952,
-1.96902092]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920231302,
gradient_value=array([-5.83982210e-07, -7.42495184e-07]))
```

```
(285/320) [01:25] step_size=0.1 max_time(5) min_gradient(1e-06)
```

```

starting_point=[0.10526316 1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=72, time_running=0.0153, x=array([ 0.00347596,
-0.00695159]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765303,
gradient_value=array([-8.56250106e-08, 8.52542804e-07]))

```

```

(286/320) [01:23] step_size=0.1 max_time(5) min_gradient(1e-06)
starting_point=[2.47368421 0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentResult(n_iter=424, time_running=0.0856, x=array([ 0.00347651,
-0.00695196]), stop_condition='min_gradient(1e-06)', f_value=0.4965724836765888,
gradient_value=array([9.82508561e-07, 1.52680044e-07]))

```

```

(287/320) [01:20] step_size=0.1 max_time(5) min_gradient(1e-06) starting_point=[
2.73684211 -2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=201, time_running=0.0415, x=array([ 0.98451069,
-1.96902109]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232519,
gradient_value=array([ 5.58359029e-07, -8.25198408e-07]))

```

```

(288/320) [01:17] step_size=0.1 max_time(5) min_gradient(1e-06)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=65, time_running=0.0134, x=array([ 0.00347601,
-0.00695251]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367656714, gradient_value=array([-2.98469295e-08,
-9.37935281e-07]))

```

```

(289/320) [01:15] step_size=0.1 max_time(5) min_gradient(1e-06)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentResult(n_iter=298, time_running=0.061, x=array([ 0.98451014,
-1.96902123]), stop_condition='min_gradient(1e-06)', f_value=0.9927420920232153,
gradient_value=array([ 7.17885873e-09, -9.77554637e-07]))

```

```
(290/320) [01:12] step_size=0.1 max_time(5) min_gradient(1e-06)
starting_point=[2.21052632 2.          ]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentResult(n_iter=2419, time_running=0.4889, x=array([ 0.0034763 ,
-0.00695165]), stop_condition='min_gradient(1e-06)',
f_value=0.49657248367656287, gradient_value=array([5.78614745e-07,
7.39902828e-07]))
```

```
(291/320) [01:09] step_size=0.1 min_gradient(0.0001) starting_point=[ 0.63157895
-0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentResult(n_iter=45, time_running=0.0095, x=array([ 0.00351943,
-0.0069775 ]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248615658484, gradient_value=array([ 8.58147726e-05,
-4.84870384e-05]))
```

```
(292/320) [01:07] step_size=0.1 min_gradient(0.0001) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentResult(n_iter=257, time_running=0.0525, x=array([ 0.00351849,
-0.00696468]), stop_condition='min_gradient(0.0001)',
f_value=0.49657248561584527, gradient_value=array([ 8.43111940e-05,
-2.35044427e-05]))
```

```
(293/320) [01:04] step_size=0.1 min_gradient(0.0001) starting_point=[-0.68421053
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentResult(n_iter=45, time_running=0.0096, x=array([ 0.00342979,
-0.00693612]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724860318768, gradient_value=array([-9.16783793e-05,
2.97570190e-05]))
```

```
(294/320) [01:01] step_size=0.1 min_gradient(0.0001) starting_point=[-1.47368421
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-1.47368421,
```

```
-3.36842105]], f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentResult(n_iter=2255, time_running=0.4551, x=array([ 0.98443762,
-1.96908371]), stop_condition='min_gradient(0.0001)', f_value=0.992742096689561,
gradient_value=array([-7.51875784e-05, -6.11259055e-05]))
```

```
(295/320) [00:59] step_size=0.1 min_gradient(0.0001) starting_point=[0.10526316
1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentResult(n_iter=51, time_running=0.0108, x=array([ 0.00347399,
-0.0069105 ]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724853602125, gradient_value=array([-2.87879243e-06,
8.09652845e-05]))
```

```
(296/320) [00:56] step_size=0.1 min_gradient(0.0001) starting_point=[2.47368421
0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentResult(n_iter=404, time_running=0.0818, x=array([ 0.00351844,
-0.00694378]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724855464849, gradient_value=array([8.47924305e-05,
1.72724357e-05]))
```

```
(297/320) [00:54] step_size=0.1 min_gradient(0.0001) starting_point=[ 2.73684211
-2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentResult(n_iter=151, time_running=0.031, x=array([ 0.98458027,
-1.96910009]), stop_condition='min_gradient(0.0001)',
f_value=0.9927420969472238, gradient_value=array([ 6.41037060e-05,
-6.68886155e-05]))
```

```
(298/320) [00:51] step_size=0.1 min_gradient(0.0001) starting_point=[-0.15789474
-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentResult(n_iter=44, time_running=0.0095, x=array([ 0.0034716 ,
-0.00699808]), stop_condition='min_gradient(0.0001)',
f_value=0.4965724857706291, gradient_value=array([-1.00716254e-05,
```

-8.99847214e-05]))

(299/320) [00:49] step\_size=0.1 min\_gradient(0.0001) starting\_point=[-0.68421053  
-3.36842105]

GradientDescentIteration(n\_iter=0, time\_running=0.0003, x=array([-0.68421053,  
-3.36842105]), f\_value=1.495486345997162, gradient\_value=array([-0.01518909,  
-0.01238275]))

GradientDescentResult(n\_iter=250, time\_running=0.0606, x=array([ 0.98447973,  
-1.96912069]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.9927420971687226, gradient\_value=array([-3.60578692e-05,  
-9.14618652e-05]))

(300/320) [00:46] step\_size=0.1 min\_gradient(0.0001) starting\_point=[2.21052632  
2. ]

GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([2.21052632, 2.  
]), f\_value=1.4998617341927332, gradient\_value=array([0.00061125, 0.00055312]))

GradientDescentResult(n\_iter=2399, time\_running=0.4781, x=array([ 0.00350247,  
-0.00692154]), stop\_condition='min\_gradient(0.0001)',  
f\_value=0.4965724853034495, gradient\_value=array([5.35836345e-05,  
6.02282503e-05]))

(301/320) [00:44] step\_size=0.1 max\_time(5) starting\_point=[ 0.63157895  
-0.21052632]

GradientDescentIteration(n\_iter=0, time\_running=0.0002, x=array([ 0.63157895,  
-0.21052632]), f\_value=0.8402749009220171, gradient\_value=array([ 0.79782729,  
-0.20675964]))

GradientDescentIteration(n\_iter=4784, time\_running=1.0002, x=array([ 0.00347601,  
-0.00695203]), f\_value=0.4965724836763416, gradient\_value=array([  
1.73472348e-18, -3.46944695e-18]))

GradientDescentIteration(n\_iter=9820, time\_running=2.0002, x=array([ 0.00347601,  
-0.00695203]), f\_value=0.4965724836763416, gradient\_value=array([  
1.73472348e-18, -3.46944695e-18]))

GradientDescentIteration(n\_iter=14863, time\_running=3.0001, x=array([  
0.00347601, -0.00695203]), f\_value=0.4965724836763416, gradient\_value=array([  
1.73472348e-18, -3.46944695e-18]))

GradientDescentIteration(n\_iter=19930, time\_running=4.0001, x=array([  
0.00347601, -0.00695203]), f\_value=0.4965724836763416, gradient\_value=array([  
1.73472348e-18, -3.46944695e-18]))

GradientDescentResult(n\_iter=24966, time\_running=5.0, x=array([ 0.00347601,  
-0.00695203]), stop\_condition='max\_time(5)', f\_value=0.4965724836763416,  
gradient\_value=array([ 1.73472348e-18, -3.46944695e-18]))

(302/320) [00:42] step\_size=0.1 max\_time(5) starting\_point=[2.47368421  
0.10526316]

```

GradientDescentIteration(n_iter=0, time_running=0.0002, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=5069, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=10149, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=15183, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=20191, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=25199, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 1.73472348e-18, -3.46944695e-18]))

```

(303/320) [00:39] step\_size=0.1 max\_time(5) starting\_point=[-0.68421053  
0.10526316]

```

GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=5037, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=10103, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=14958, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=20018, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentResult(n_iter=25079, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))

```

(304/320) [00:37] step\_size=0.1 max\_time(5) starting\_point=[-1.47368421  
-3.36842105]

```

GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5048, time_running=1.0002, x=array([ 0.98451006,

```

```

-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 4.8398785e-16,
-9.6797570e-16]))
GradientDescentIteration(n_iter=10115, time_running=2.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentIteration(n_iter=15154, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentIteration(n_iter=20143, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentIteration(n_iter=25207, time_running=5.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentResult(n_iter=25207, time_running=5.0001, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 4.8398785e-16, -9.6797570e-16]))

```

```

(305/320) [00:35] step_size=0.1 max_time(5) starting_point=[0.10526316
1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=5056, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=10110, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=15160, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=20154, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=25262, time_running=5.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentResult(n_iter=25262, time_running=5.0001, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))

```

```

(306/320) [00:33] step_size=0.1 max_time(5) starting_point=[2.47368421
0.73684211]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,

```



```

0.00205835]))
GradientDescentIteration(n_iter=5008, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=9861, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=14908, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=19959, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=25046, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 1.73472348e-18, -3.46944695e-18]))

```

```

(307/320) [00:30] step_size=0.1 max_time(5) starting_point=[ 2.73684211
-2.42105263]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,
-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=5059, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 4.8398785e-16,
-9.6797570e-16]))
GradientDescentIteration(n_iter=10111, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentIteration(n_iter=15166, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentIteration(n_iter=20230, time_running=4.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentResult(n_iter=25302, time_running=5.0, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 4.8398785e-16, -9.6797570e-16]))

```

```

(308/320) [00:28] step_size=0.1 max_time(5) starting_point=[-0.15789474
-0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentIteration(n_iter=5054, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))

```

```

GradientDescentIteration(n_iter=10118, time_running=2.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=15174, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=20220, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=25061, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 1.73472348e-18, -3.46944695e-18]))

```

```

(309/320) [00:26] step_size=0.1 max_time(5) starting_point=[-0.68421053
-3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))
GradientDescentIteration(n_iter=5050, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 4.8398785e-16,
-9.6797570e-16]))
GradientDescentIteration(n_iter=10118, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentIteration(n_iter=15179, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentIteration(n_iter=20235, time_running=4.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentResult(n_iter=25283, time_running=5.0, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_time(5)', f_value=0.992742092022676,
gradient_value=array([ 4.8398785e-16, -9.6797570e-16]))

```

```

(310/320) [00:24] step_size=0.1 max_time(5) starting_point=[2.21052632 2.
]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=5016, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([1.73472348e-18,
3.46944695e-18]))
GradientDescentIteration(n_iter=10060, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=15057, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,

```

```

gradient_value=array([1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=20122, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))
GradientDescentResult(n_iter=25175, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))

(311/320) [00:21] step_size=0.1 max_iterations(20000) starting_point=[
0.63157895 -0.21052632]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 0.63157895,
-0.21052632]), f_value=0.8402749009220171, gradient_value=array([ 0.79782729,
-0.20675964]))
GradientDescentIteration(n_iter=5019, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=10075, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=15070, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=3.988, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 1.73472348e-18,
-3.46944695e-18]))

(312/320) [00:19] step_size=0.1 max_iterations(20000) starting_point=[2.47368421
0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.10526316]), f_value=1.4971462897557586, gradient_value=array([0.01276325,
0.00331099]))
GradientDescentIteration(n_iter=4794, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=9789, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=14716, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=19629, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0745, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',

```

```
f_value=0.4965724836763416, gradient_value=array([ 1.73472348e-18,
-3.46944695e-18]))
```

```
(313/320) [00:16] step_size=0.1 max_iterations(20000)
starting_point=[-0.68421053  0.10526316]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
0.10526316]), f_value=0.8803879697816689, gradient_value=array([-0.84858717,
0.13183868]))
GradientDescentIteration(n_iter=5047, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=9981, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=14908, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=19832, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0347, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-1.73472348e-18,
3.46944695e-18]))
```

```
(314/320) [00:14] step_size=0.1 max_iterations(20000)
starting_point=[-1.47368421 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-1.47368421,
-3.36842105]), f_value=1.4998295224940215, gradient_value=array([-0.00084072,
-0.00047196]))
GradientDescentIteration(n_iter=5022, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 4.8398785e-16,
-9.6797570e-16]))
GradientDescentIteration(n_iter=10059, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentIteration(n_iter=15083, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentResult(n_iter=20000, time_running=3.9964, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 4.8398785e-16,
-9.6797570e-16]))
```

```
(315/320) [00:12] step_size=0.1 max_iterations(20000) starting_point=[0.10526316
```

```

1.05263158]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([0.10526316,
1.05263158]), f_value=1.1734107031477512, gradient_value=array([0.06871534,
0.68763702]))
GradientDescentIteration(n_iter=4718, time_running=1.0187, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=9553, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=14524, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentIteration(n_iter=19390, time_running=4.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([-1.73472348e-18,  3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=4.1227, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([-1.73472348e-18,
3.46944695e-18]))

```

```

(316/320) [00:09] step_size=0.1 max_iterations(20000) starting_point=[2.47368421
0.73684211]

```

```

GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.47368421,
0.73684211]), f_value=1.4986896501592173, gradient_value=array([0.00641913,
0.00205835]))
GradientDescentIteration(n_iter=5035, time_running=1.0003, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=9787, time_running=2.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=14655, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=19545, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0958, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 1.73472348e-18,
-3.46944695e-18]))

```

```

(317/320) [00:07] step_size=0.1 max_iterations(20000) starting_point=[
2.73684211 -2.42105263]

```

```

GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([ 2.73684211,

```

```

-2.42105263]), f_value=1.4794927226497707, gradient_value=array([ 0.07123899,
-0.01727565]))
GradientDescentIteration(n_iter=4649, time_running=1.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 4.8398785e-16,
-9.6797570e-16]))
GradientDescentIteration(n_iter=9447, time_running=2.0001, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 4.8398785e-16,
-9.6797570e-16]))
GradientDescentIteration(n_iter=14517, time_running=3.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentIteration(n_iter=19552, time_running=4.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentResult(n_iter=20000, time_running=4.0892, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 4.8398785e-16,
-9.6797570e-16]))

```

```

(318/320) [00:04] step_size=0.1 max_iterations(20000)
starting_point=[-0.15789474 -0.52631579]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.15789474,
-0.52631579]), f_value=0.7457052280513097, gradient_value=array([-0.26802165,
-0.7343479 ]))
GradientDescentIteration(n_iter=4991, time_running=1.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=10073, time_running=2.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=14895, time_running=3.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentIteration(n_iter=19939, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416, gradient_value=array([
1.73472348e-18, -3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=4.0126, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([ 1.73472348e-18,
-3.46944695e-18]))

```

```

(319/320) [00:02] step_size=0.1 max_iterations(20000)
starting_point=[-0.68421053 -3.36842105]
GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([-0.68421053,
-3.36842105]), f_value=1.495486345997162, gradient_value=array([-0.01518909,
-0.01238275]))

```

```

GradientDescentIteration(n_iter=5060, time_running=1.0002, x=array([ 0.98451006,
-1.96902012]), f_value=0.992742092022676, gradient_value=array([ 4.8398785e-16,
-9.6797570e-16]))
GradientDescentIteration(n_iter=10122, time_running=2.0001, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentIteration(n_iter=15165, time_running=3.0002, x=array([
0.98451006, -1.96902012]), f_value=0.992742092022676, gradient_value=array([
4.8398785e-16, -9.6797570e-16]))
GradientDescentResult(n_iter=20000, time_running=3.9569, x=array([ 0.98451006,
-1.96902012]), stop_condition='max_iterations(20000)',
f_value=0.992742092022676, gradient_value=array([ 4.8398785e-16,
-9.6797570e-16]))

```

```

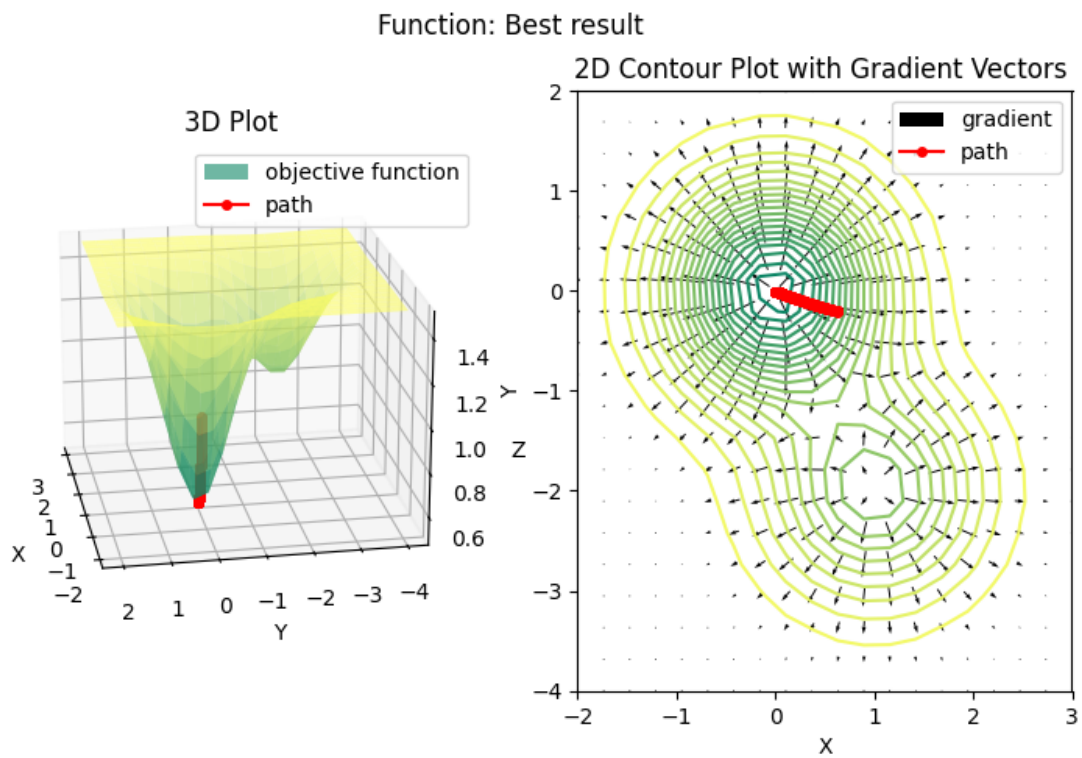
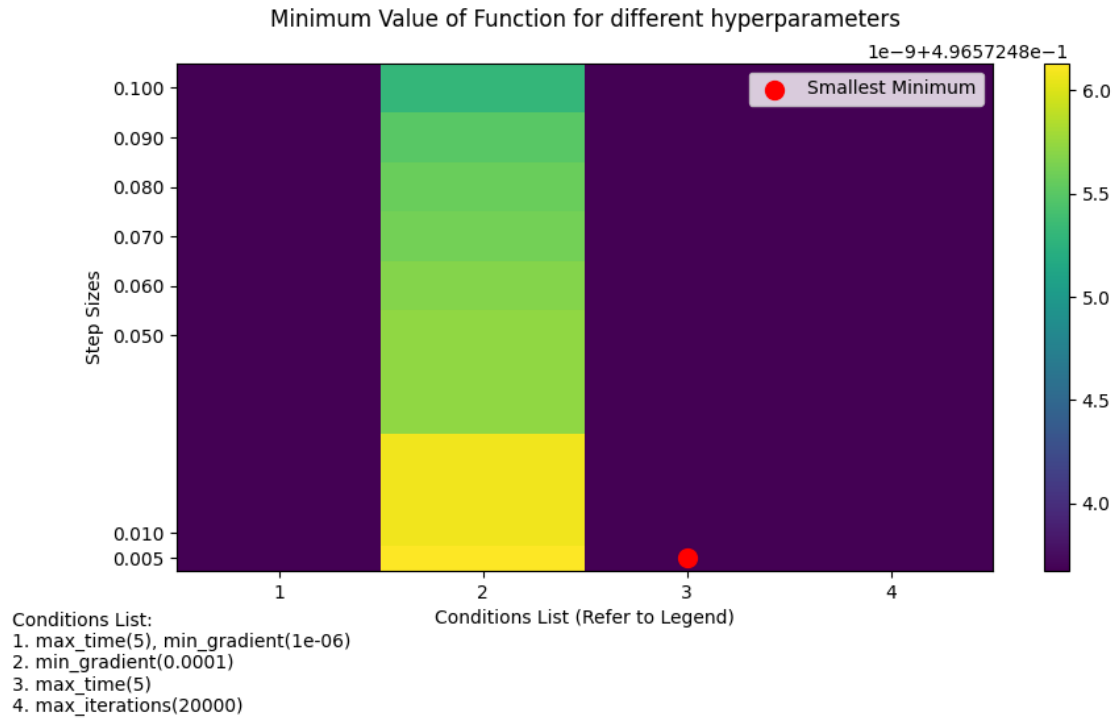
(320/320) [00:00] step_size=0.1 max_iterations(20000) starting_point=[2.21052632
2.          ]

```

```

GradientDescentIteration(n_iter=0, time_running=0.0003, x=array([2.21052632, 2.
]), f_value=1.4998617341927332, gradient_value=array([0.00061125, 0.00055312]))
GradientDescentIteration(n_iter=4999, time_running=1.0002, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([1.73472348e-18,
3.46944695e-18]))
GradientDescentIteration(n_iter=9894, time_running=2.0001, x=array([ 0.00347601,
-0.00695203]), f_value=0.4965724836763416, gradient_value=array([1.73472348e-18,
3.46944695e-18]))
GradientDescentIteration(n_iter=14920, time_running=3.0001, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))
GradientDescentIteration(n_iter=18730, time_running=4.0002, x=array([
0.00347601, -0.00695203]), f_value=0.4965724836763416,
gradient_value=array([1.73472348e-18, 3.46944695e-18]))
GradientDescentResult(n_iter=20000, time_running=4.34, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_iterations(20000)',
f_value=0.4965724836763416, gradient_value=array([1.73472348e-18,
3.46944695e-18]))

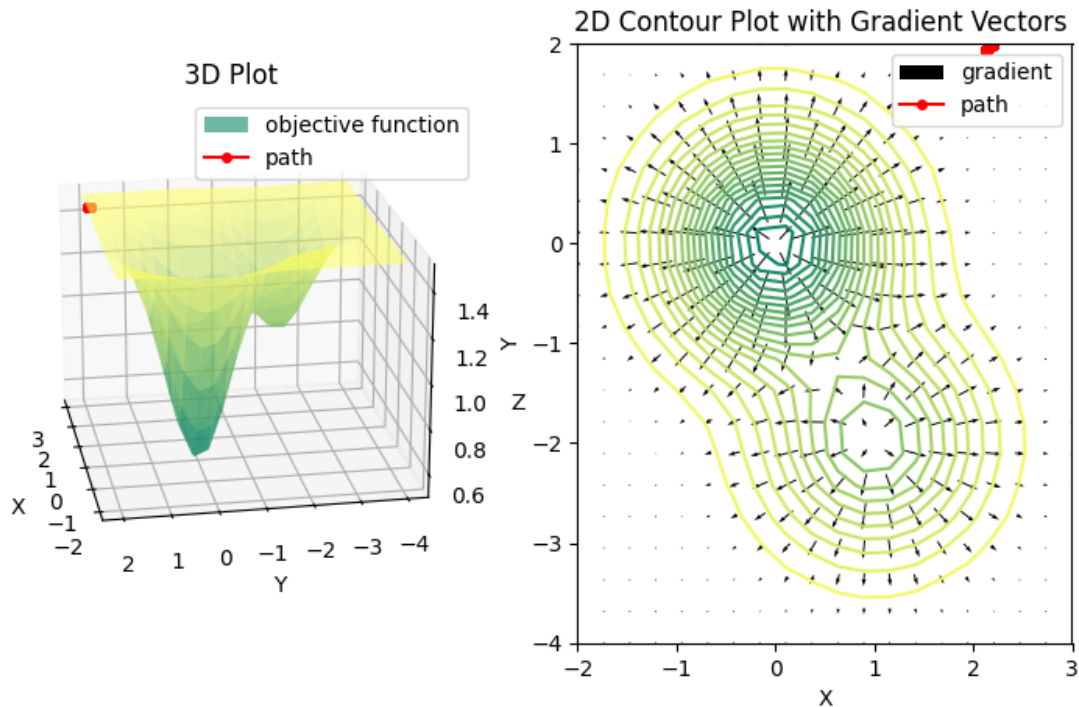
```





```
GradientDescentResult(n_iter=25393, time_running=5.0, x=array([ 0.00347601,
-0.00695203]), stop_condition='max_time(5)', f_value=0.4965724836763416,
gradient_value=array([ 4.33680869e-17, -8.67361738e-17]))
```

Function: Worst result



```
GradientDescentResult(n_iter=20000, time_running=3.9899, x=array([2.1287997 ,
1.92604607]), stop_condition='max_iterations(20000)',
f_value=1.4997364676168319, gradient_value=array([0.00112196, 0.00101526]))
```

## 1.5 Wyniki i wnioski

Powyższe eksperymenty pozwoliły mi wyznaczyć minimalne wartości obu funkcji i ich argumenty (wartość  $x$  i  $f\_value$  pod wykresem “Best result”). Są one bardzo bliskie faktycznych wartości wyznaczonych analitycznie.

### 1.5.1 Przebieg eksperymentu

Dla obu funkcji przetestowałem 32 konfiguracje parametrów (8 rozmiarów kroków \* 4 zestawy warunków stopu) dla 10 losowo wyznaczanych punktów z wybranej dziedziny (zawierającej minimum).

### 1.5.2 Funkcja 1

**Rozmiar kroku** Przetestowałem 8 rozmiarów kroku [0.005, 0.01, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1] które początkowo wyznaczyłem *empirycznie* jako potencjalnie dobre. Większe rozmiary kroku na

ogół nie doprowadzały do zbieżności, a mniejsze nie dawały lepszych wyników. Stąd wytypowałem właśnie te wartości

Okazało się, że dla funkcji 1 najlepsze wyniki uzyskałem dla rozmiaru kroku 0.1. Dla niektórych punktów startowych był on za duży i powodował *przestrzelenie* minimum i wypadnięcie z dziedziny. Jednak przy 10-krotnym powtórzeniu eksperymentu dla losowych punktów startowych ten rozmiar kroku dostarczył najlepsze wyniki. Warto zaznaczyć, że inne rozmiary skoku dawały bardzo podobne wyniki. Mniejsze rozmiary kroku dawały na ogół nieznacznie gorsze wyniki w sensownym czasie działania (prawdopodobnie byłyby one lepsze gdyby zwiększyć liczbę iteracji. Niestety już w obecnej konfiguracji eksperyment zajmuje 30 minut na moim komputerze).

**Warunki stopu** Postanowiłem również zbadać wpływ warunków stopu na wyniki. Przetestowałem 4: 1. `max_time(5)` lub `min_gradient(1e-6)` 2. `min_gradient(1e-4)` 3. `max_time(5)` 4. `max_iterations(20000)`

Wyniki dla warunków 1, 3 i 4 były bardzo zbliżone (najlepsze dla 3) dlatego nie pozwalają na wyciągnięcie wartościowych wniosków. Natomiast warunek 2 znacząco pogorszył działanie algorytmu, zatrzymując go zanim znajdzie minimum (wniosek - należy przyjmować mniejsze minimalne wartości gradientu niż  $1e-4$ ). Przy małych rozmiarach kroku podobne zjawisko zachodziło dla warunku 4.

### 1.5.3 Funckja 2

**Rozmiar kroku** Dla funkcji 2 zbadałem te same potencjalne rozmiary kroku. W tym przypadku znowu przy 10-krotnym wywołaniu prezentowały się one bardzo podobnie, jednak najlepsze wyniki uzyskałem dla najmniejszego rozmiaru kroku - 0.05. Dla większości warunków rozmiar kroku nie miał większego znaczenia, jednak dla warunku 2 (`min_gradient(1e-4)`) okazał się kluczowy. Dla mniejszych rozmiarów kroku algorytm utykał w minimum lokalnym i nie był w stanie go opuścić. Widoczne jest to na wykresie "Worst result".

**Warunki stopu** Przetestowałem te same warunki stopu co dla funkcji 1. Ponownie warunki 1, 3 i 4 nie różniły się znacząco (najlepszy ponownie 3). Warunek 2 znacząco pogorszył wyniki (nawet bardziej niż dla funkcji 1), ponieważ powodował on utknięcie w minimum lokalnym.