MSBD5004 Mathematical Methods for Data Analysis
Homework 5

Due date: 22 May, Friday

1. Find the Fourier series for the following 1-periodic function
$$f(t) = t, \quad -\frac{1}{2} \leq t < \frac{1}{2}.$$

2. Find the sum
$$\frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots + \frac{1}{n^4} + \ldots$$
(*Hint: Consider the Fourier series for the function* $f(t) = t^2$ *on* $\left[-\frac{1}{2}, \frac{1}{2}\right)$ *and* $f(t+k) = f(t)$ *for all integer* k.)

3. Find a function $g(t)$ such that: for any $f(t)$, the convolution $f * g$ is the ideal low pass filter that retains only the frequencies in the interval $(-1, 1)$.

4. Find the Fourier transform of the function
$$f(t) = \begin{cases} 1 - |t|, & -1 \leq t \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

5. Compute the Discrete Fourier Transform of $[1\ 1\ 2\ 2]^T$.

6. Prove the discrete convolution theorem:
$$A_N(f * g) = (A_N f) \cdot (A_N g),$$
where $f, g \in \mathbb{C}^N$ are vectors, $A_N \in \mathbb{C}^{N \times N}$ is the discrete Fourier transform matrix, $\cdot$ is the entrywise multiplication, and $*$ is the discrete convolution defined by $(f * g)(m) = \sum_{n=0}^{N-1} f(n) g(m - n \mod N)$ for $m = 0, 1, \ldots, N-1$.

7. Let f be a vector and let $\tau(f)$ be the cyclic shift by 1 position to the right. What is $F(\tau(f))$ in relation to $F(f)$? Here $F(f)$ is the discrete Fourier transform of f.