# A Programming Language for Quantum Oracle Construction

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August 2021

## 1 Abstract

Many quantum programs require circuits for addition, subtraction and logical operations. These circuits may be packaged within routines known as oracles. However, oracles can be tedious to code with current quantum frameworks. To solve this problem the author has developed a programming language named $Q$ for use on quantum computers to ease their creation through a C-style syntax for describing oracles. The compiler translates high-level code written in $Q$ and converts it into *OpenQASM*[1], a gate-based assembly language that runs on IBM Quantum Systems and compatible simulators.

## 2 Introduction

Quantum Computers were first conceptualized by the late physicist Richard Feynman[2] in a landmark paper in which he observed that classical techniques are inefficient when simulating quantum mechanics. Since the memory required to store a quantum state increases exponentially with the size of the system, Feynman proposed a new form of computer which would be capable of processing quantum information, hence the term *Quantum Computer*, in which the fundamental unit of information is a two-level system known as a qubit. Quantum gates are applied to qubits in order to change their state[3].

Unfortunately current quantum frameworks make it tedious to implement classical functions in the form of quantum circuits. This was the primary motivation behind Q where tasks such as addition, subtraction and multiplication on quantum states as well as relational operations can be performed in a user-friendly manner. This is aided by the Q compiler's resource estimator which heuristically computes the total number of qubits required to run a program before run-time and therefore allows dynamic resizing of registers at compile-time. Q's focus is on simplicity and expressiveness − the compiler generates OpenQASM, thus its output can be used with IBM's QISKit framework [1].

## 3 The Q Programming Language

A standard Q program consists of two main parts:

- Declaration of data
- Program statements to manipulate data

Statements and declarations are terminated with semicolons.

There are two types of data in Q:

- *Integer (int)*
- *Quantum-Integer (super)*

These two types distinguish between data allocated on two different devices, namely classical and quantum data. Data is assigned through a *variable declaration*. Variables are represented by identifiers. The values of both data-types are denoted by integers. However, for quantum data, the integer $n$ represents the upper-bound of a uniform superposition, which ranges from $\{0 : n - 1\}$. This provides a useful method for generating superposition states.

Expressions in Q can be of a quantum nature or classical, and are constructed using a selection of operators (infix notation) similar to C.

- Arithmetic operators are for addition, subtraction and multiplication (+, - , *, +=, -=, *=)
- Relational operators are for testing equality (==, !=) and order (>, <, >=, <=)
- Boolean operators are for testing Boolean functions (&, |)

## 3.1 Functions and Oracles

Subroutines are *functions* that have the option to return a value or *oracles* which, unlike functions, are first-class objects − they return a memory address pointing to the location of the oracle in classical memory. Since all quantum operations are unitary and therefore reversible, the bodies of all subroutines in Q are expanded inline.

Functions are declared using the *function* keyword. If a function returns a value, it is specified by preceding the keyword *function* with the type. This is followed by a function identifier followed by a list of parameters enclosed in parentheses. The standard form for declaring parameters is $(< type, identifier >, ...)$. Finally, the function's body is enclosed in opening and closing braces $\{< body > \}$. All parameters are passed-by-reference due to the no-cloning theorem which states that a quantum state cannot be cloned in its entirety [7].

The return statement saves whichever variable is returned from being uncomputed and the corresponding qubit register is returned and can be assigned a new identifier. The syntax for the return statement is the *return* keyword, followed by the variable name to be returned.

Although they have no return value, oracles are declared in a similar manner using the keyword *oracle* instead of the keyword *function.* The primary reason for oracles is the need to pass functions as parameters to other functions, this is because low-level memory manipulation is not supported and function-pointers cannot be used. Instead, oracles were introduced into the language for this purpose. When oracles are sent as input, a structure is passed which contains their address in memory amongst other metadata.

Figure 1: Functions and Oracles in Q

```
# This is a comment

super function some_function() {
   # do stuff
    # return stuff
}

oracle some_oracle() {
   # do stuff
}

function main() {
   # do stuff
}
```

### 3.2 Type System

Variable declarations in Q begin with a type specifier, followed by a variable identifier. The type system is designed to indicate where a variable should be allocated. A variable of type "int" declares an integer on a classical computer, whereas a declaration introduced with the keyword "super" allocates qubits on a quantum computer.

The "super" keyword for quantum variables is purely to provide a shorthand way to declare uniform superpositions. Values are assigned to either type of variable using the operator "=" as in C.

There is a limitation on the values that can be assigned to a quantum variable $-$ it must be a power of two, ie any quantum variable initialized with a value $x$ must satisfy $log_2(x) \in N$. This limitation enables the creation of a uniform superposition: $\sum_{i=0}^{x-1} \frac{1}{\sqrt{x}} |i_{10}\rangle$. When measured this state will collapse into a basis state $|i\rangle$ with probability $(\frac{1}{\sqrt{x}})^2 = \frac{1}{x}$. Reassignment of quantum variables is not allowed.

Figure 2: Type System in Q

```
function main() {
   super a = 4;
    int b = 2;
}
```

## 3.3 Operations

Operations can be performed on both quantum and classical data. The compiler maintains a resource estimator at compile-time during which it tracks each operation and dynamically resizes quantum registers as required. This is particularly useful when performing arithmetic operations since they can alter the size of a register from $n$ qubits to $m$ qubits − it saves the programmer from having to perform such tasks manually.

## 3.4 Conditional Expressions

Q supports the if-else model that permeates throughout most modern programming languages.

A conditional expression can be based on classical or quantum test conditions, but not both. Quantum conditional expressions introduce entanglement.

The syntax for declaring an if-statement is to use the "if" keyword followed by a test condition enclosed in parentheses, succeeded by the conditional-body enclosed within braces.

The syntax is similar for an elsif statement, which must succeed an if-statement or an elsif-statement. The only difference is instead of using the "if" keyword, one must use the "elsif" keyword.

An else-statement signals the default case. It must succeed an if-statement or an elsif-statement. To declare an else-statement, one must use the "else" keyword, followed by the body of the else-statement enclosed within braces.

Figure 3: Conditionals in Q

```
function main() {
   if(some_condition) {
      # do stuff
    }

    elsif(some_other_condition) {
      # do other stuff
    }

    else {
      # do something else
    }
}
```

## 3.5 Loops

Loops are treated as classical constructs within Q by expanding them inline during compile time. There are two forms of loops in the language:

- For loop
- While loop

### 3.5.1 For Loop

The syntax for declaring a for loop is to use the keyword “for” followed by a series of three expressions enclosed in parentheses and separated by commas.

The first expression should declare and/or initialize the classical data to be used in the loop. The second should specify the halt condition, that is, the circumstances required for the loop to terminate. As in C, the third condition specifies the modifications to be made to data on each iteration. After these three conditions the loop body is specified in braces.

### 3.5.2 While Loop

The syntax for declaring a while loop is to use the keyword “while” followed by a single condition enclosed in parentheses. The condition specifies the circumstances required for the loop to run. After this condition the loop body is specified in braces.

Figure 4: Loops in Q

```
function main() {
      # FOR
   for(int i = 0; i < 5; i+=1) {
      # Do stuff
    }

    # WHILE
    while(true) {
      # Do stuff
    }
}
```

## 3.6 Assembly Instructions

Q supports the basic quantum assembly instructions shown in Figure 5.

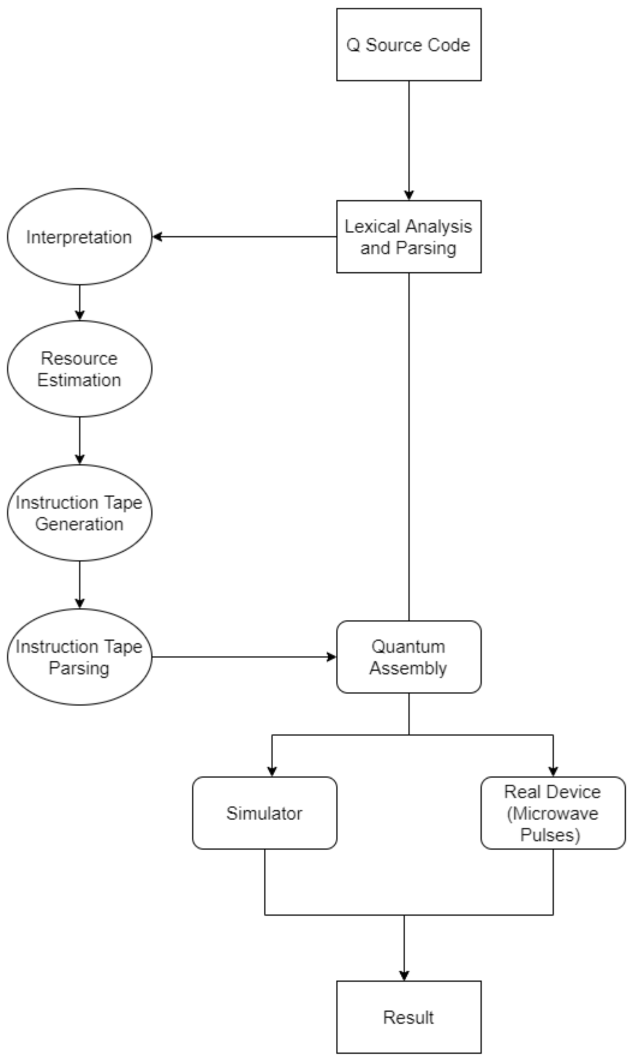
Figure 5: Q: Quantum Gates

```
function main() {
   # Hadamard gate
   H(foo);
    # X/NOT gate
    X(foo);
    # Y gate
    Y(foo);
    # Z gate
    Z(foo);
    # Rotate X gate
    RX(foo, angle);
    # Rotate Z gate
    RZ(foo, angle);
    # Rotate Y gate
    RY(foo, angle);
    # Apply phase
    P(foo, angle);
    # S gate
    S(foo);
    # T gate
    T(foo);
    # Controlled-Not gate
    CX(foo, bar);
    # Controlled-Z gate
    CZ(foo, bar);
    # Controlled phase
    CP(foo, bar, angle);
}
```

### 3.7 Compiler Details

The Q compiler makes use of mechanisms such as register-size tracking in order to perform mathematical operations. Furthermore, ancillary registers used in such operations are dealt with internally and are uncomputed when not required and reset to zero automatically by maintaining an internal instruction tape intermediate program representation. They are referenced by the compiler as ancillaX where X is the number of ancillary registers at the time of creation decremented by one. Likewise, cmpX registers are used for storing results of relational operations. Purely classical expressions are evaluated at compile-time, leaving only quantum code to be compiled for later execution.

Figure 6: Q Compiler Structure Flowchart



### 3.7.1 Addition and Subtraction

The addition operator in Q is based on the Quantum Fourier Transform (QFT) [3] and is implemented as an optimized version of the method proposed by Draper[4]. The algorithm to perform $r = x + y$, is executed as followed:

1. Initialize two registers, $x$ and $y$
2. Initialize a register $r$ to $|0\rangle^{\otimes n}$, where $n = floor(log_2(x+y)) + 1$
3. Apply $H^{\otimes n}$ to $r$ to obtain the state:

$$(\frac{|0\rangle + |1\rangle}{\sqrt{2}})^{\otimes n} = \frac{1}{\sqrt{2^n}}(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \otimes ...(|0\rangle + |1\rangle)$$

4. Apply a series of controlled phase operations to store the Fourier Transform of $x$ in $r$:

$$\frac{1}{\sqrt{2^n}}(|0\rangle + e^{2\pi i 0.x_n} |1\rangle) \otimes (|0\rangle + e^{2\pi i 0.x_{n-1}...x_n} |1\rangle) \otimes ...(|0\rangle + e^{2\pi i 0.x_1 x_2 ... x_n} |1\rangle)$$

5. Apply a series of controlled phase operations to add the Fourier Transform of $y$ into $r$. $r$ now holds the sum of $x$ and $y$ in the Fourier Basis.

$$\frac{1}{\sqrt{2^n}}(|0\rangle + e^{2\pi i(0.x_n + 0.y_n)} |1\rangle) \otimes (|0\rangle + e^{2\pi i(0.x_{n-1}...x_n + 0.y_{n-1}...y_n)} |1\rangle) \otimes ...(|0\rangle + e^{2\pi i(0.x_1 x_2 ... x_n + 0.y_1 y_2 ... y_n)} |1\rangle)$$

= QFT|x + y⟩

6. Apply $QFT^\dagger$ to $result$ to retrieve $|x+y\rangle$ in the computational basis:

$$QFT^\dagger QFT \, |x+y\rangle = |x+y\rangle$$

Note: To perform subtraction, the controlled operations in step 5 are inverted

### 3.7.2 Multiplication

The multiplication operator in Q is similarly based on the QFT.

Given two integers, $k$ and $l$, the algorithm computes their product $j$.

The algorithm is as follows:

(a) Initialize registers $k$ as multiplicand, $l$ as multiplier and $j$ as $|0\rangle^n$ where n is the size of the output in bits and defaults to $n = sizeof(k) + sizeof(l)$

(b) Apply $H^{\otimes n}$ to $j$, resulting in the state:

$$(\frac{|0\rangle + |1\rangle}{\sqrt{2}})^{\otimes n}) = \frac{1}{\sqrt{2^n}}(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \otimes ...(|0\rangle + |1\rangle)$$

The goal is to transform the state described above to the Fourier Transform of $k \cdot l = j$

(c) $\text{QFT}|j\rangle = \frac{1}{\sqrt{2^n}}(|0\rangle + e^{2\pi i 0.j_1 j_2 \ldots j_n}|1\rangle) \otimes (|0\rangle + e^{2\pi i 0.j_2 \ldots j_n}|1\rangle) \otimes ...(|0\rangle + e^{2\pi i 0.j_n}|1\rangle)$, therefore it is desirable to obtain the relative phase factor $e^{2\pi i 0.j_1 j_2 \ldots j_n}$.

This can be achieved through multiplying the binary fractional forms of the multiplier and multiplicand: $(0.k)(0.l) = 0.kl = (\frac{k_1}{2^1} + \frac{k_2}{2^2} ... \frac{k_n}{2^n}) \cdot (\frac{l_1}{2^1} + \frac{l_2}{2^2} ... \frac{l_n}{2^n}) =$

$$\frac{k_1 l_1}{2^2} + \frac{k_1 l_2}{2^3} \dots \frac{k_1 l_n}{2^{n+1}} +$$
$$\frac{k_2 l_1}{2^3} + \frac{k_2 l_2}{2^4} \dots \frac{k_2 l_n}{2^{n+2}} +$$
$$\frac{k_n l_1}{2^{n+1}} + \frac{k_n l_2}{2^{n+2}} \dots \frac{k_n l_n}{2^{2n}}$$
$$= C$$

This is implemented as multi-controlled phase rotations ($P$) applied to a single qubit of $j$, to produce the phase $e^{2\pi i 0.j_1 j_2 \ldots j_n} = e^{2\pi i C}$, where $P$ corresponds to the following matrix:

$\begin{bmatrix} 1 & 0 \\ 0 & e^{2\pi i \theta} \end{bmatrix}$ and $\theta$ is equal to the $\frac{1}{2^x}$ phase angles.

(d) Apply relative phases $e^{2\pi i 2^m C}$ to each qubit $j_i$, where $m$ is the size of the output

(e) Apply $QFT^\dagger$ (Inverse QFT) on $j$ to retrieve the product in the computational basis

# 4 Examples

## 4.1 Quantum Search

The quantum search algorithm, first published in 1996 by Lov K. Grover [5], offers a polynomial-time advantage over the best known classical algorithm for searching for an item in a set of unordered data.

The algorithm is as follows:

(a) Initialize a superposition of the search space

(b) Apply an oracle, $O$, to the superposition. The oracle is designed in such a way that it performs an operation that applies a phase of $\pi$ if a term in the superposition fits a specified search constraint.

(c) Apply diffusion operator, ie $|s\rangle\langle s| - I$ where $|s\rangle$ is the initial search space.

(d) Repeat steps 1-3 $\frac{\pi}{4}\sqrt{N}$ times where N is the size of the search space.

(e) Measure result.

The following is an application of the quantum search algorithm to search for all $x$ where $4x < 4$ and $0 \leq x \leq 7$. The only solution is the state 0, and the algorithm should discover it with high probability.

The first figure depicts the algorithm written in Q, and the second figure shows the same algorithm written in IBM's QISKit.

Figure 7: Example Q program for Quantum Search Algorithm

```
# oracle takes a superposition as input
oracle some_oracle(super var) {
   # if condition satisfied apply phase of pi to var
   if(var * 4 < 4) {
      mark(var,pi);
   }
}


function main() {
   #declare a uniform superposition of 3 qubits
   super variable = 8;
   # "filter" function is diffusion operator
   filter(some_oracle(variable));
   measure variable;
}
```

Figure 8: QISKit Code for Quantum Search Algorithm

```
# import libraries
from qiskit.circuit.library.arithmetic import IntegerComparator
from qiskit.circuit.library import QFT, GroverOperator
from qiskit.visualization import plot_histogram
from qiskit import *
from math import pi

# create and initialize registers
input_reg = QuantumRegister(3,name="input")
output_reg = QuantumRegister(5, name="output")
qc1 = QuantumCircuit(input_reg, output_reg)
qc1.h(input_reg)
qc = QuantumCircuit(input_reg, output_reg

# set up multiplication circuit based on QFT
phase = pi*1/2**2*4
phase_copy = phase*2
for i in range(5):
    qc.h(output_reg[i])
    for j in range(3):
        if i >= 3 and j == 0 or (j==1 and i==4):
            phase /= (2**1)
            continue
        qc.cp(phase,input_reg[j], output_reg[i])
        phase /= (2 ** 1)
    phase = phase_copy
    phase_copy *= 2
 # apply inverse QFT
qft_circ = QFT(num_qubits=5).inverse()
qc.compose(qft_circ, qubits=[3,4,5,6,7], inplace=True)
qc1.compose(qc, qubits=range(8), inplace=True)
# compare with 4 (less than)
comparison_circ = IntegerComparator(5, 4, geq=False,name="comparison_circ")
circ_final = QuantumCircuit(15,3)
circ_final.compose(qc1, inplace=True)
circ_final.compose(comparison_circ, qubits=range(3,13), inplace=True)
# apply phase
circ_final.z(8)
# uncomputation
circ_final.compose(comparison_circ.inverse().to_gate(), qubits=range(3,13),
inplace=True)
circ_final.compose(qc.inverse().to_gate(), qubits=range(8), inplace=True)
# diffusion operator applied once
circ_final.h(range(3))
circ_final.x(range(3))
circ_final.mct([0,1,2], 13, 14, mode="basic")
circ_final.x(range(3))
circ_final.h(range(3))
# measurement
circ_final.measure(range(3), range(3))
```

## 4.2 Deutsch-Josza Algorithm

The algorithm was first proposed by Deutsch [6] in 1985 and later revised in 1992 by Deutsch and Josza.

Given a function $f$, this algorithm checks if it is constant or balanced , where $f$ is guaranteed to be on or the other.
If $f$ is constant, $f$ returns either 0 or 1 for all inputs, otherwise it is balanced and returns 0 for half of all inputs and 1 for the other half.

The algorithm proceeds as follows:

(a) Initialize state $|s\rangle$ as a superposition: $\frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle$.

(b) Apply $f$ to $|s\rangle$ and XOR the result with a qubit in the $|-\rangle$ state.

(c) Apply Hadamard Gate ($H$), on $|s\rangle$.

(d) Measure $|s\rangle$.

(e) If $|s\rangle$ is measured to be zero, then $f$ is constant else it is balanced.

The following is an application of the Deutsch-Josza algorithm for a function $f$ whereby $f(x) = 1$ if $x + 7 > 14$ and $f(x) = 0$ if $x + 7 \leq 14$ where $0 \leq x \leq 15$. This function is balanced and must return a non-zero integer $-$ in this case the bit-string $1000_2$.

Figure 9: Example Q program for the Deutsch-Josza Algorithm

```
function deutsch_josza(super inputs) {
   # if condition satisfied store result in
   # 1-qubit register (called cmp0 internally)
   if(inputs + 7 > 14) {
      # XOR the contents of cmp0 with a qubit in the state |->
      mark(inputs,pi);
   }
}

function main() {
   # initialize superposition of 4 qubits
   super test = 16;
   deutsch_josza(test);
   # apply interference with Hadamard Gate
    H(test);
   # store result in a classical register creg_test
   measure test;
}
```

Figure 10: QISKit Code for Deutsch Josza Algorithm

```
# import libraries for integer comparisons and addition
from qiskit.circuit.library.arithmetic import WeightedAdder,
IntegerComparator
from qiskit import QuantumRegister, ClassicalRegister, QuantumCircuit

# create circuits for addition and integer comparison
addition_circuit = WeightedAdder(7, [8,4,2,1, 4,2,1], name="adder_circ")
comparison_circuit = IntegerComparator(5, 15, name="comparison_circ")

# create registers and circuits for input and the integer "seven"
input_register = QuantumRegister(4, name="input_register")
seven = QuantumRegister(3, name="seven")
input_reg_to_circ = QuantumCircuit(input_register)
integer_seven_circ = QuantumCircuit(seven)

# apply HADAMARD on input to generate uniform superposition
input_reg_to_circ.h(input_register)
# flip all 3 bits in register to represent "7" in binary
integer_seven_circ.x(seven)
# add summands as input to addition circuit
addition_circuit.compose(qc.to_gate(), qubits=[0,1,2,3], front=True,
inplace=True)
addition_circuit.compose(qc1.to_gate(), qubits=[4,5,6], front=True,
inplace=True)
# create final circuit to hold all circuits
circuit_final = QuantumCircuit(22,4)
# append addition circuit in front of empty circuit
circuit_final.compose(addition_circuit.to_gate(), qubits=range(17),
front=True, inplace=True)
# add comparison circuit after addition circuit with the input
# being the sum from the previous circuit
circuit_final.compose(comparison_circuit.to_gate(),
qubits=[7,8,9,10,11,17,18,19,20,21], inplace=True)
# apply phase
circuit_final.z(17)
# uncomputation of circuits
circuit_final.compose(comparison_circuit.inverse().to_gate(),
qubits=[7,8,9,10,11,17,18,19,20,21], inplace=True)
circuit_final.compose(addition_circuit.inverse().to_gate(),
qubits=range(17), inplace=True)
# measure input register
circuit_final.measure(range(4), range(4))
```

# 5 Conclusion

The author developed a new programming language for quantum computation that allows a higher-level description of oracle functions than available in existing frameworks. Although the language compiler can be used as a standalone tool, it was designed to be used alongside other OpenQASM-based frameworks such as QISKit.

# 6 Acknowledgements

The author thanks Prof David Abrahamson of TCD Compiler Design Optimization, Programming Language Design, for his guidance and supervision in the creation of this paper, and Prof Brendan Tangney of TCD Computer Science and Statistics, Dr Keith Quille, Lecturer of Computing in TUDublin, Tallaght Campus, and CSInc, for their insights and assistance. Thanks are also due to Dr Lee O'Riordan from the Irish Center For High-End Computing and Dr Peter Rohde from UTS Australia, for their suggestions and feedback.

# 7 Appendices

## 7.1 Lexical Specification in Regex

digit = [0..9] number = digit+ letter = [a..z] | [A..Z] identifier = letter (letter | digit | “$_{''})*type=“int''}$—“$super''$

semicolon = “;”

lparen = “(”

rparen = “)”

lbrace = “{”

rbrace = “}”

assign = “=”

operator = “+” |“-” | “*” | “+=” | “-=” | “*=” | “¡” | “¿” | “¡=” | “¿=” | “==” | “!=”’

keyword = “return” | “measure” | “function”

conditional = “if” | “elsif” | “else”

loop = “for” | “while”

### 7.2 EBNF Specification of Syntax

program = function , {function};
function = “function”, whitespace, identifier, lparen, {type, identifier}, rparen, lbrace, body, rbrace;
body = {assignment | operation — loop — cond};
assignment = type, whitespace, identifier, “=”, integer;
operation = identifier | integer — operation , operator, identifier | integer — operation;
loop = for | while;
for = ”for” lparen, assignment, semicolon, operation, semicolon, operation, rparen, lbrace, body, rbrace;
while = “while”, lparen, operation, rparen, lbrace, body, rbrace;
cond = if | elsif | else;
if = “if”, lparen, operation, rparen, lbrace, body, rbrace;
elsif = “elsif”, lparen, operation, rparen, lbrace, body, rbrace;
else = “else”, lbrace, body, rbrace;
type = (“super” | “int”);
identifier = char, {char | “” | digit};
integer = digit, {digit};
char = (“a” | “b” | “c” | “d” | “e” | “f” | “g” | “h” | “i” | “j” | “k” | “l” | “m” | “n” | “o” | “p” | “q” | “r” | “s” | “t” | “u” | “v” | “w” | “x” | “x” | “y” | “z” | “A” | “B” | “C” | “D” | “E” | “F” | “G” | “H” | “I” | “J” | “K” | “L” | “M” | “N” | “O” | “P” | “Q” | “R” | “S” | “T” | “U” | “V” | “W” | “X” | “Y” | “Z”);
digit = (0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9);
lparen = “(”;
rparen = “)”;
lbrace = “{”;
rbrace = “}”;
whitespace = “ ”;

# 8 References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