Stevens Institute of Technology
School of Business

Special Topics in Dynamic Optimization
Homework Assignment 1
Due: 18 Oct 2015

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1 Resource Extraction

A monopolist resource owner is endowed with a given initial quantity of resources (R_0, $R_t \geqslant 0$. The goal of the monopolist is to maximize the infinite sum of discounted profit streams. The demand for the exhaustible resource is specified by a general function $P = D(X, Q)$, X is the demand-shifter parameter. The time horizon is infinite. Extraction is costless.

1.1 Analytical Solution using Optimal Control Approach

Setup the Hamiltonian and analyze the problem under the following conditions.

1. No tax
2. Environmental tax on a unit of resource extraction (paid by the producer)
3. Environmental tax on a unit of consumption (paid by consumers)

1.2 Numerical Solution of the Dynamic Programming Model

Now go back to the original problem, set up the Bellman Equation (Dynamic Programming), and solve it numerically using the Value Function Iteration approach.

The annual interest rate $r = 5\%$. Assume a step size of $\Delta R = 0.01$. Use $R_0 = 100$ as the upper bound of the state variable.

1. Plot the value function as a function of remaining reserves.
2. Plot the path of extraction and prices over time (up to 100 year) for $R_0 \in \{20, 40, 60, 80, 100\}$
3. Plot the profit of resource owner over time for 100 years, starting at $R_0 = 100$.

Since you are solving the problem numerically you need to enforce a specific function form for demand. Solve the problem using two alternative forms for the demand function:

- $P = X - bQ$, $X = 100$, $b = 5$

- $P = XQ^{\gamma}$. $X = 100$, pick an arbitrary value of $\gamma \in (-1, 0)$

1.3 Costly Extraction

Add extraction costs (per unit) to the problem. Solve the previous problems under two alternative specifications for the marginal extraction cost:

- Constant extraction cost per unit $C = 0.5$

- Level dependent convex marginal extraction cost: $C_t = 0.5 + (1 - \frac{R_t}{100})^2$

Solve the problem for both specifications of the demand functions.

1.4 Limited Extraction Rate

Solve the problem assuming the extraction rate is bounded by $q \leqslant \overline{Q}$ per year. Fix the initial endowment at $R_0 = 100$. Assume zero extraction costs.

Change the extraction rate limit and plot the following graphs:

1. Value function for each value of extraction constraint in the range of $\overline{Q} \in [0, 3]$. (take a step size of $\Delta\overline{Q} = 0.1$ for the constraint vector). Assume $R_0 = 100$. (Remember: you are not plotting the value function as a function of initial reserves)

2. Time dynamics of extraction rate (production) and equilibrium prices for $\overline{q} \in \{0.5, 1, 1.5, 2, 2.5\}$. Plot all 5 curves in a single graph. Limit the time to 100 periods. Assume $R_0 = 100$.

Solve the problem for both specifications of the demand functions.