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Corporate Financing: An Artificial Agent-based Analysis

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Abstract

We examine corporate security choice by simulating an economy populated by adaptive agents who learn about the structure of security returns and prices through experience. Through a process of evolutionary selection, each agent gravitates toward strategies that generate the highest payoffs. Despite the fact that markets are perfect and agents maximize value, a financing hierarchy emerges in which straight debt dominates other financing choices. Equity and convertible debt display significant underpricing. In general, the smaller the probability of loss to outside investors, the more likely the firm is to issue the security and the smaller the security's underpricing.

A MAJOR GOAL OF CORPORATE FINANCE RESEARCH is to explain corporate financing patterns. To explain security issuance preferences, researchers have extended the Modigliani and Miller (1958) analysis by allowing for the presence of market frictions such as transactions costs, taxes, and informational differences among agents (DeAngelo and Masulis (1980), Castanias (1983), and Myers and Majluf (1984)). These extensions, however, maintain the assumption that the security payoff structure is common knowledge.

The assumption of common knowledge places extraordinary demands on the abilities and knowledge of agents (Simon (1972), Russell and Thaler (1985), and Thaler (1992)). Agents must know not only the set of feasible states, but also the relationship among these states and the payoffs on assets. In addition, they must be able to use this knowledge to compute the payoffs on any potential portfolio of assets in any state of the world. Not surprisingly, actual economic agents do not appear to have these capabilities (see, e.g., Kahneman and Tversky (1979) and Kahneman, Slovic, and Tversky (1982)).

A realistic model of decision making should incorporate the fact that agents cannot precisely partition states of the world, that they can only construct fuzzy maps from states to outcomes, and that they lack the ability to fully exploit the knowledge they have by computing all of its consequences (see Mukerji (1997) for a detailed discussion). Despite these observed limitations in agent rationality,

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Miller (1977) argues that the forces of competition, by selecting agents whose strategies produce the highest payoffs, will drive aggregate outcomes to those predicted by rational-actor models. Because competitive selection appears to be an important component in any attempt to realistically model individual agents' decision making, a number of researchers have proposed theoretical models of economic evolution (e.g., Samuelson and Zhang (1992), Young (1996), and Routledge (1999)).

In this paper, using a genetic algorithm to model retrospective learning, we simulate corporate security issuance. In our simulations, a firm needs external financing to undertake a profitable investment project that features random future payoffs. The firm chooses from a menu of securities that contains debt, equity, and convertible debt. These securities are issued in a perfect market where investors compete solely on the basis of price for the right to purchase the securities. Proceeds from the issue, if sufficient, are used to fund the investment project. If proceeds are insufficient, the project is not undertaken. After project cash flows are realized, the cash flows are split between the firm and outside investors based on the terms specified by the security issued. Security issuance and pricing strategies are updated using a genetic algorithm. The genetic algorithm uses ordinal, rank-based selection criteria that rely on the relative success of strategies in the past.[1]

Our results indicate that the learning effects captured by our genetic algorithm can induce strong preferences over capital structure choices even in perfect and competitive markets. The preferences induced are consistent with observed empirical regularities—firms tend to rely primarily but not exclusively on debt financing. Although debt financing dominates, equity financing is also frequently employed. Other contracts that have been identified in the theoretical literature as optimal incentive contracts but which are infrequently observed in practice, such as do-or-die securities—securities that promise all cash flows to outsiders whenever firm cash flows fall below a threshold level and promise all cash flows (not just marginal cash flows) to insiders when firm cash flows beat the threshold (Innes (1990))—are also infrequently observed in our simulations.

Three regularities typify the outcomes of our simulations. First, the frequency with which securities are issued by the firm and the prices the securities command are highly correlated. Second, securities tend to be underpriced as documented in the empirical literature on initial public offerings (see, e.g., Beatty and Ritter (1986)). Third, firms tend to issue securities that promise outside investors "safety first" (see Roy (1952)); that is, they issue the security that minimizes the probability that outside investors realize a negative net payoff. Because debt provides safety first to outside investors, it dominates other securities, especially in the very long run.

Our results are driven by both two-sided learning and the rank-based process for selecting agent strategies. Because selection is based on rank alone, outperforming a strategy by a large amount garners no evolutionary advantage over outperforming it by a small amount. Thus, a few spectacular successes for a high-risk strategy are more than offset by the many cases where the high-risk strategy

[1] Arifovic (1996), among others, uses a similar rank-based selection criterion.

produces losses. It follows that, in general, low-risk strategies that result in relatively infrequent losses will be adopted more frequently than high-risk strategies.

Since the total risk of the project is fixed, the firm can avoid downside risk only by transferring such risk to outside investors. First, consider the situation where traders price securities correctly and the firm's security choice evolves. Because of the ordinal ranking procedure used by the genetic algorithm, despite the fact that all securities are correctly priced, securities that have the highest probability of producing the lowest payoffs to the firm are selected out. This process leads to the dominance of subordinated debt and equity.

In contrast, if the firm fixes on a security to issue while investors learn to price the security, the dynamic is completely different. Transferring risk to outsiders can generate large and frequent outsider losses. Adaptive responses to such losses produce low price responses and hence issue failure, which in turn causes the firm to forgo the profitable project. In this case, debt emerges as the security least likely to result in issue failure. To see this, suppose that debt is risk free. As long as investor bids are less than the face value of the debt, an investor's payoff is bounded below by zero. Thus, investors learn to bid aggressively for debt. In contrast, other securities can generate negative investor payoffs when the amount of the bid is greater than the security's payoff, prompting investors to adopt more conservative bidding strategies.

When both the firm and investors learn, the genetic algorithm trades off the firm's direct benefit from transferring downside risk against the greater likelihood of mispricing and issue failure caused by such a transfer. Our simulations show that the latter effect dominates, and debt is the most commonly issued security. Subordinated debt and equity are still popular. However, this popularity is not sustainable in the long run. In the long run, debt is the most popular security by far. For other securities, the failure rate is substantially higher.

These results show that imperfect learning provides an alternative explanation for some very basic empirical regularities in corporate financing patterns, an explanation that generates some empirical predictions that overlap with asymmetric information and moral hazard theories but also some unique predictions. Similar to the adverse selection theory of security design (e.g., Innes (1990) and Nachman and Noe (1994)), our results predict reliance on debt finance. However, our learning model's predictions contrast with the predictions of adverse selection and moral hazard models in a number of significant ways. For example, both adverse selection and moral hazard models also predict that, absent monotonicity constraints, firms will issue "do-or-die" securities which expose outsiders to significant risk of a zero payment. However, consistent with the safety-first criterion and in contrast to the predictions of both asymmetric information and moral hazard models, the do-or-die security is rarely issued in our simulations.[2]

[2] Another contrast between our results and asymmetric information and moral hazard analysis is that the predictions of these paradigms are very dependent on specific assumptions about how conditional cash flow distributions are ordered either by private information or by agent actions (see Innes (1990) and Nachman and Noe (1994)). The nature of parametric restrictions required for the safety-first ranking to govern the financing hierarchy in learning models remains to be determined.

The importance of safety first in our context is closely related to the importance of risk dominance, used in Young (1996). Risk dominance selects those equilibria that maximize the range of opponent strategies under which the equilibrium strategies are best responses. Thus, risk-dominant strategies can be viewed as maximizing safety with respect to the endogenous uncertainty generated by the strategic behavior of the opponent. In our simulations, debt maximizes safety to investors over the largest range of realizations for exogenous uncertainty. Thus, there is a fairly strong analogy between the results of our simulations of the rather complex security issuance game and theoretical results concerning risk dominance in much simpler 2×2 matrix games.[3] The key difference between risk dominance and safety first is that safety first minimizes the risk from exogenous cash flow uncertainty while risk dominance minimizes uncertainty concerning the endogenous moves of the other player.

Our results are also consistent with views on institutional learning emphasizing limited rationality and the role of information in organizations (e.g., Bernard (1938) and Nelson and Winter (1982)). In these theories, and also in our simulation, organizational learning produces routines based on past experience and, as such, are not readily adapted to deal with novel situations. Like our artificial agents, real-world financial institutions develop experience dealing with specific contract forms. Because of this experience, they develop pricing expertise for specific contracts, and these contracts become attractive sources of capital. In this way, a specific knowledge base grows up around each financial contract—for example, debt or equity. This expertise is not easily transferable to other contracts, leading intermediaries to specialize in a limited range of contracts.

Complementarities between investor and firm decisions are also central to the theory of security design, even in models not based on adaptive learning. For example, in Gale (1992), coordination failures between investors and firms engendered by complementarities can lead to the failure of Pareto optimality in the security design market. Firms, conjecturing that other firms will not issue market-completing securities, realize that the idiosyncratic risk associated with being the unique issuer of market-completing securities will make such securities unattractive to investors. Investors, recognizing that the security menu is very limited, will not collect information regarding their nonsecurity income stream and will have less incentive to purchase novel securities that hedge the risks associated with this income stream. In Pagano (1993), when few firms choose to use equity finance in a given nation because of lack of diversification, the nation's market portfolio will be very risky and thus command high risk pre-

[3] Note that, as well as being well beyond the scope of this paper, the application of Young's stochastic stability criteria to our problem is problematic. In our Bertrand competition game with a finite price set, corresponding to each security, with the exception of the junk contract, there exists a unique strict Nash subgame equilibrium—all investors offer the firm one tick less than the breakeven price. Thus, the issuance of each of the securities, with the exception of the junk contract, is supported by a Nash equilibrium. All of these equilibria produce the same payoffs to all of the agents. Thus, our problem, in contrast to problems where stochastic stability is typically employed, is not a problem of refining equilibria based on payoff dominance.

mia. The high risk premia will in turn produce a high cost of equity capital, and this cost of capital will discourage equity investment.

Our work is part of an emerging stream of research on dynamic learning. For example, in Young (1996), agents play strategies that are best responses to the sampled history of play. In our simulations, the strategy played in each round is determined by a random draw. Ex post, after a strategy has failed to produce good results for an agent, its representation in the agent's pool of strategies shrinks. Thus, our approach to learning features ex post selection but no anticipatory learning. For this reason, it resembles the case-based learning paradigm of Gilboa and Schmeidler (1995).

However, in contrast to Gilboa and Schmeidler (1995), we implement retrospective learning using the genetic algorithm paradigm first articulated in Holland (1975). This technique has already been applied by a number of researchers to simulate the evolution of economic and strategic competition. For example, Arifovic (1996) simulates exchange-rate determination; Arifovic (1994) and Chen and Yeh (1996) simulate the evolution of market-clearing demand and supply functions; Noe and Pi (2000) simulate the evolution of free-riding behavior in takeovers; and Routledge (2001) simulates information acquisition in financial markets. In closely related work, Arifovic and Eaton (1998) examine genetic algorithm learning in the context of a pure coordination game. In both our work and Arifovic and Eaton, agents have difficulties learning how to respond to strategies rarely played by other agents. This effect ensures that the more frequently a strategy is played, the more attractive the strategy becomes, and this positive feedback effect leads agents to converge to playing a few strategies even when many equivalent strategies are available.[4] However, in contrast to Arifovic and Eaton, ours is not a coordination game, and the alternative strategies, although they produce the same Nash equilibrium payoffs to the parties, differ systematically in the degree of learning risk exposure they generate. The results of our analysis show that this exposure has a significant impact on security issuance behavior.

The remainder of the paper is organized as follows. In Section I, we detail the security market structure, describe the agents and their objectives, choice sets, and the sequence of decisions. In Section II, we describe the genetic algorithm employed in our analysis. Section III discusses the underlying forces driving our results and Section IV discusses the outcomes of the simulations. In Section V, we study the robustness of these results, and the effects of security characteristics are assessed in Section VI. The paper concludes with a brief discussion and summary of our analysis in Section VII.

I. The Capital Market

Consider a firm attempting to raise capital to finance a project that generates a random end-of-period payoff of X that is uniformly distributed over the interval $[0, 100]$. To undertake the project the firm needs to raise I units of capital, which it

[4] The importance of positive feedback in such systems is also noted in Arthur (1989).

attempts to raise by selling *securities*. Securities are mappings from the firm's cash flow to a payment to outside investors. Outside investors respond to a security issue by offering a price for the security. The security is sold to the investor who offers the highest price. Ties are broken by splitting the security into equal shares and assigning one share to each of the high bidders; that is, all high bidders share equally in the contribution and the payoff.

If the firm is able to raise at least I by selling securities, the project is undertaken and its cash flows are shared between the firm and outside investors in accordance with the terms of the security now held by outside investors. In addition to receiving all the project's end-of-period cash flows in excess of payments to outside investors, cash in excess of the required investment I is paid out as a dividend to insiders. If investor bids are not high enough, the project is not undertaken. In the simulations, the capital needed to undertake the investment was set at 32.5.

The firm chooses among six securities: junk (s_0), debt (s_D), equity (s_E), do-or-die (s_{DD}), convertible debt (s_C), and subordinated-convertible debt (s_S).[5] The basic set of simulations is performed using only these six securities. Our choice of these six securities was guided by the existing capital structure literature. The debt, convertible debt, and equity securities are included in the sample because of the dominant role they play in corporate financing. The subordinated-convertible debt and do-or-die securities are included in the simulations because it is argued that they are optimal security designs (see Brennan and Kraus (1987) and Innes (1990), respectively). The junk security is included as a rough check for the plausibility of the results. With the exception of s_0, the parameters chosen for each security ensure that its intrinsic value (i.e., its expected payoffs) equals 37.5.

Table I, describes payoffs and characteristics of these securities. With the exception of the junk security and the do-or-die security, the securities are standard. The junk security is worthless in that it provides the investor with no payoff. The do-or-die security provides investors with all the project's cash flow if the cash flow is less than 86.60, and no cash flow otherwise. The payoff on the subordinated convertible is standard. However, in our simulations we can only allow for one external claimant on the firm's cash flow. Thus, when subordinated convertible debt is considered, the firm is the senior creditor and receives all the cash flows if the realized cash flow is between zero and 10.[6]

Because of the perfect-market and rational-agent environment, the equilibrium outcome of this capital-raising scenario is transparent: The firm issues a security whose parameters ensure that its value is at least I. Investors set a price for the security equal to its value. Thus, the firm is indifferent among all

[5] Restricting the number of securities in the simulations is necessary for technical reasons provided in Footnote 10.

[6] In the absence of this assumption, we would not have been able to include any form of subordinated claim into the analysis, as this would require three distinct classes of agents (the firm, investors who hold the subordinated claim, and those that hold the senior claim), while the other securities require only two classes of agents—the firm and investors who hold that claim.

Table I

Securities Employed in Our Central Simulations

This table presents the payoffs and characteristics of the securities employed in the simulations. Panel A presents investor payoffs from these securities. Panel B presents other security characteristics including expected payoff to the investor (*Mean*), variance of security payoff to the investor (*Variance Investor*), variance of security payoff to the firm (*Variance Firm*), probability of a positive net payoff to outside investors if they purchase securities at a price equal to the investment level of 32.5 (*PositiveInv*), probability of generating the highest payoff to the firm (*HighFirm*), and probability of generating the lowest payoff to the firm (*LowFirm*).

Panel A: Functional Form of the Securities

Security	Investor Payoff as a Function of Cash Flow Realization x
Junk	$s(x) = 0$
Debt	$s(x) = \min(x, 50)$
Equity	$s(x) = 0.75x$
Do-or-die	$s(x) = \begin{cases} x \text{ if } x \leq 86.60 \\ 0 \text{ if } x > 86.60 \end{cases}$
Convertible	$s(x) = \min(x, 30) + 0.96 \max(x - 50, 0)$
Subordinated	$s(x) = \min[\max(x - 10, 0), 60] + 0.75 \max(x - 80, 0)$

Panel B: Characteristics of the Securities

Security	Mean	Variance		Probability		
		Investor	Firm	PositiveInv	HighFirm	LowFirm
Junk	0.00	0.00	833.33	0.00	—	—
Debt	37.50	260.42	260.42	0.68	0.16	0.50
Equity	37.50	468.75	52.08	0.57	0.00	0.00
Do-or-die	37.50	758.81	1012.20	0.54	0.13	0.87
Convertible	37.50	417.75	91.08	0.47	0.31	0.13
Subordinated	37.50	588.75	32.08	0.58	0.40	0.00

securities with the exception of s_0. In the following sections, we investigate whether this indifference extends to an economy populated by adaptive agents.

II. Genetic Algorithm Application

We simulate the behavior of agents in the capital-raising situation described in the previous section by playing a "virtual" game between a virtual firm and two virtual investors. Agent strategies are determined by a genetic algorithm. Each *run* of the genetic-algorithm simulation consists of a number of *rounds*.

In each round, the virtual players select strategies and receive payoffs. The firm first picks a security-issuance *strategy* from among six pure strategies. Each of these strategies corresponds to issuing one of the six securities described above. A strategy is thus a number, which takes a value from the set $\{0, 1, 2, 3, 4, 5\}$, with choosing s_0, s_D, s_E, s_{DD}, s_C, and s_S are represented by the numbers 0, 1, 2, 3, 4, and

5, respectively. The firm has a pool of 80 *chromosomes*. Each of these chromosomes encodes one of the six possible strategies. In each round, one chromosome is drawn at random from the pool of 80 chromosomes. The firm plays the strategy encoded by the drawn chromosome.

The initial composition of the chromosome pool is uniform, with each security having an equal probability of being included in the pool. After each round, the fitness of each of the strategies is evaluated based on the profitability of the strategy in the last round in which it was used. Profitability is measured by the realized cash flows received by the firm. The chromosome pool is then updated via a process of *selection* and *mutation*. Selection involves replacement of all chromosomes that encode the least profitable strategy with chromosomes that encode the most profitable strategy. After selection, each chromosome is subject to possible mutation. The probability of mutation for each chromosome in each round is one percent. Mutation results in the chromosome being replaced by a new randomly encoded chromosome. The mutant is equally likely to be encoded with each one of the six possible strategies.

Investors respond to a security issued by the firm by making a bid for that security. Each investor makes a single bid that is determined by the investor's pricing strategy. This pricing strategy is drawn at random from a strategy pool associated with that security. For each security, each investor has a strategy pool consisting of 80 chromosomes. Each chromosome is encoded with a strategy. The strategies are binary strings of 0s and 1s that decode to a six-place, base-two representation of a number, n, between 0 and 63. For example, the string 000010 decodes to the number 2. If the strategy encoded by the drawn chromosome decodes to the number n, the price bid by the investor is $100\frac{n}{63}$. The security is awarded to the investor submitting the highest bid, with ties being broken by splitting both the payment for the security and the security's payoffs among the highest bidding investors.

Initially, the 80 chromosomes in each pool are randomly encoded, using a uniform distribution over strategies. After each round, chromosome pools are updated. This involves selection, crossover, mutation, and election.[7] Selection is performed first and involves estimation of the profitability of each strategy. A strategy's estimated profitability is the investor's profit from using that strategy, assuming that the other investor continues to choose the strategy selected in the previous round. Strategies are then ranked in terms of expected profitability. The selection process is completed by replacing the 10 chromosomes encoding the least profitable strategies with 10 chromosomes encoding the most profitable strategies.

A new chromosome pool is produced from the postselection pool as follows. First, two chromosomes are randomly chosen from the pool, and, with a probability of 0.6, they are *crossed*.[8] A crossing of two chromosomes involves splitting each

[7] The process for *selection*, *mutation*, *crossover*, and *election* employed here is similar to the process used in Arifovic (1996).

[8] Note that only chromosomes representing investor strategies are "crossed." Because we permit only six different firm strategies, using crossover to generate novel strategies for firms is not necessary.

chromosome into two parts at a randomly selected position and then swapping the parts between the pair of chromosomes. For example, suppose that two chosen chromosomes are encoded with 010101 and 111000. If the randomly selected position for splitting the chromosomes is three, two new chromosomes are generated with the chromosome encoded with 010101 being recoded with 011000, and the chromosome encoded with 111000 being recoded with 110101. The two new chromosomes are then subject to mutation with probability 0.003. Mutation involves replacing the chromosome with a new chromosome that is randomly encoded using a uniform probability measure over the 64 permitted strategies. The new chromosomes generated through crossover and mutation, together with the two original chromosomes, are then subjected to *election*; that is, only the two chromosomes encoding the most profitable strategies are added to the new chromosome pool. Each time this process of crossover, mutation, and election is repeated, two new chromosomes are added to the new pool. The process continues until the new pool contains 80 chromosomes. This completion of the new chromosome pool marks the end of a round.

A run of the genetic algorithm starts with the initial encoding of chromosome pools for the firm and the two investors. For the first five rounds, the firm's chromosome pool is not updated. The investors can, however, learn from the inception of the run. This procedure is followed to give investors some additional learning time.[9] A run ends either when chromosomes encoding a single security comprise 95 percent of the firm's chromosome pool or when 2,000 rounds have elapsed, whichever comes first.[10] Each experiment consists of 1,000 runs.

III. Underlying Forces Driving the Results

Because of their controlled nature and the low cost of producing new data, artificial-agent simulations produce a great deal of data. Copious data, combined with the lack of any closed-form solution characterizing the data, make framing an overarching interpretation of results difficult. Nevertheless, based on our inspection of the data, we have developed an overall rationale for our results.

A central factor driving our results is two-sided learning: The firm learns which security is best to issue at the same time that investors learn how to price the securities issued by the firm. On the one hand, because investors can learn to price securities only if the firm issues them, investor sophistication in pricing a given security depends on the frequency with which the firm issues that security.

[9] The firm has one pool for learning, whereas the investors have six pools that must evolve simultaneously. In each round, only one of the six pools is updated. The initial five rounds of no updating by the firm are designed to avoid a situation in which the firm has converged to a security before the investors' pools even start to evolve. Allowing for more rounds where only investor chromosome pools are updated did not significantly affect our results.

[10] Because we aim to examine the optimal investor reaction to each possible security issuance by the firm, a chromosome pool must be constructed corresponding to each possible security choice. For learning to occur in a given investor, the security corresponding to the pool must be issued with reasonable frequency. Thus, given a reasonable limitation on the number of rounds, only a small number of securities can be included in the analysis.

On the other hand, the sophistication of investor pricing of a given security affects the willingness of the firm to issue that security. Thus, two-sided learning engenders a mutual dependence between firm and investor learning and actions, making it difficult to isolate all the factors driving our results. To better isolate the forces that drive the results, we now step back and examine two one-sided learning problems—(1) the learning problem of the firm and (2) the learning problem of the investors. Later we combine the insights from these one-sided problems to explain the results of our more complex two-sided learning experiment.

First, consider the case of a firm issuing securities to a market that prices all securities at their *ex ante* value and never revises its pricing schedule based on experience. In this case, investors will always finance the firm, and on average, will break even regardless of the security issued by the firm. Moreover, because the project has positive net present value and the project is always funded by investors, the firm will always avoid the worst possible outcome—not receiving funding for the project. However, *ex post*, for some realizations of the end-of-period cash flow, some securities will do better than others.

How will firm learning evolve in this setting? One might expect the evolution of firm learning to be random. However, this is not the case. In the genetic algorithm used in our analysis, securities are selected out (in) of the firm's chromosome pool if they generate the lowest (highest) payoff to the firm. Thus, if investor learning were not important, the relative ranking of securities to the firm would depend simply on the likelihood of their producing the highest and lowest payoffs to the firm. These probabilities are easily computed. Whenever investors fail to fund the project, all securities provide the firm with the same payoffs. When the project is funded by investors, the contract terms and the random payoff on the project determine a security's relative payoff. Thus, the likelihood that one security will be ranked higher than another by the firm depends on the likelihood that the security produces a higher contractual payoff. Using this fact, in Table I, we present for each security the probabilities of generating the highest and lowest firm payoffs.[11]

Subordinated debt and equity have a zero probability of producing the lowest payoff. Thus, they are highly unlikely to be selected out of the firm's chromosome pool. Further, subordinated debt has the highest probability of producing the highest payoff to the firm, making it the most likely candidate to replace chromosomes that are selected out. Although the identity of the worst security from the firm's perspective is unclear because no security is dominated by all others in both dimensions, a case can be made for the do-or-die security or debt to attain the lowest ranking.

This conjecture is supported by our simulation results in Table II. To isolate the effect of firm learning, we populated investor chromosome pools entirely with pricing strategies corresponding to the intrinsic value of the securities and did not allow for them to be updated. In these simulations, subordinated debt

[11] As it is obvious that the junk security will always produce the highest payoff to firms (assuming they can find some investor to buy it), it is excluded from the comparisons in Table I.

Table II

Outcomes of Simulations with Fixed Investor Responses

This table presents outcomes when investors' responses are fixed at the intrinsic value of the securities. It presents the frequency with which securities were issued in the final round (*Freq.*), and the frequency with which the security was issued in the final round of a nonconvergent simulation (*Nonconverge*). A simulation is nonconvergent when no one security ever accounted for at least 95 percent of the firm's chromosome pool over 2,000 rounds. The average fraction of the security in the final chromosome pool (*Average Fraction*) across all 1,000 runs are reported in the table as well. Standard deviations appear in parentheses. The results were generated by simulations employing a mutation rate for the firm of 0.01, and investment level of 32.5 (Net Present Value (NPV) = 17.5).

Security	Freq.	Nonconverge	Average Fraction
Junk	0	0	0.001
			(0.004)
Debt	18	18	0.019
			(0.059)
Equity	356	335	0.367
			(0.207)
Do-or-die	4	4	0.005
			(0.017)
Convertible	71	70	0.076
			(0.141)
Subordinated	551	388	0.533
			(0.235)

emerges as the dominant equilibrium security choice and debt performs poorly. Equity is also a frequent choice, accounting for 36 percent of security. This is not surprising given that equity also has a zero probability of producing the lowest firm payoff, making it unlikely to be selected out of the firm's chromosome pool.

Now consider a scenario in which the firm issues a fixed security, but investors learn. In this case, each time the security's realized payoff is less than the funds provided by investors, strategies that do not fund the project at all dominate strategies that involve funding the project, allowing strategies that mandate underfunding to multiply at the expense of strategies that mandate funding. This multiplication causes securities featuring high investor risk to be more underpriced and more susceptible to issue failure. In contrast to the fixed-security price case, securities that minimize the probability of investor loss—that is, securities that provide "safety-first" for outside investors—tend to dominate.

Devising a ranking for securities from the investor's perspective is complex. Even if the firm's issuance choice is fixed, an investor's optimal strategy depends on that of the competing investor. This precludes a simple univariate ranking of investor bidding strategies and thus security rankings. However, we know that the genetic algorithm will never select in and will frequently select out strategies that produce investor losses. Thus, to roughly capture investor preferences across securities, we compute the likelihood that each security produces an inflow to investors that is greater than the capital required to finance the project. Under this criterion, debt ranks the highest, followed by subordinated debt. Note that

Table III

Outcomes of Simulations Fixing Firm Issue Strategies

This table presents outcomes of simulations in which investors learn to bid for one security for 2,000 rounds. It presents the number of instances in which security issues failed in the final round (*Failures*), the mean values of bid amount paid by the winning investor (*Investor Payment*) in the final round, averages values (*Average Bid*), and standard deviations (*Std. Dev. Bid*) of investor chromosome pools. These averages are conditioned on the security issued in the final round. Standard deviations appear in parentheses. The results were generated by simulations employing crossover and mutation rates for investors at 0.6 and 0.003, respectively, and investment level of 32.5 (NPV = 17.5).

Security	Failures	Investor Payment	Average Bid	Std. Dev. Bid
Debt	40	38.39[a]	35.55[b]	2.43
		(4.40)	(1.47)	(1.45)
Equity	304	31.55[c]	25.36[b]	4.20
		(7.88)	(3.29)	(3.31)
Do-or-die	378	30.38[c]	23.78[b]	4.70
		(8.82)	(1.50)	(1.45)
Convertible	759	23.80[c]	17.35[b]	5.69
		(10.16)	(3.24)	(3.32)
Subordinated	251	32.90[c]	26.66[b]	3.93
		(7.56)	(1.55)	(4.07)

[a]The magnitude of overpricing is significant at the five percent confidence level.
[b]The average bid is significantly lower than the intrinsic value of the security at the five percent confidence level.
[c]The magnitude of underpricing is significant at the five percent confidence level.

the do-or-die security, which ranks low in the firm's ranking, also ranks among the lowest in the investor's ranking.

The predictive power of the safety-first ranking is confirmed by the simulation results in Table III. In these simulations, we isolated the effect of investor learning by fixing the security issued by the firm. Investors, in contrast, could update their pricing strategies as described in the previous section. Rankings of securities based on the difference between their intrinsic value and mean values of pricing strategies in investor chromosome pools exactly correspond to the pecking order based on frequency of issue failure. This ranking is also identical to the security ranking based on the mean dispersion of pricing strategies as captured by investors' chromosome pools.

IV. Learning and Security Choices

Having examined the effects of firm and investor learning in isolation, we are in a position to better understand the dynamics of two-sided learning. Issue failure causes the insiders to lose the net present value of the project, 17.5. This loss of net present value is a much larger penalty than the largest mispricing loss suffered if the project is financed, that is, the intrinsic value of the security less the minimum proceeds required to fund the investment, $37.5 - 32.5 = 5$. Thus, strategies

Table IV

Outcomes with Two-Sided Learning

This table presents outcomes of the final rounds of 1,000 simulation runs. It presents the frequency with which securities were issued in the final round of the runs (*Freq.*), the number of instances in which security issues failed in the final round (*Failures*), and the frequency with which a run lasted 2,000 rounds because no one security accounted for at least 95 percent of the firm's chromosome pool at an earlier round (*Nonconverge*). The table also contains mean values of firm payoffs (*Firm Payoff*), investor payoffs (*Investor Payoff*), winning bids (*Winning Bid*), and losing bids (*Losing Bid*). These averages are conditioned on the security issued in the final round. Standard deviations appear in parentheses. The results were generated by simulations employing crossover and mutation rates for investors of 0.6 and 0.003, respectively, a mutation rate for the firm of 0.01, and investment level of 32.5 (NPV = 17.5).

Security	Freq.	Failures	Non-converge	Firm Payoff	Investor Payoff	Winning Bid	Losing Bid
Junk	4	4	0	0.00	0.00	16.67	11.11
				(0.00)	(0.00)	(6.15)	(8.09)
Debt	519	4	2	19.88	0.25	38.50[a]	33.75
				(17.11)	(13.67)	(3.60)	(9.26)
Equity	151	8	0	14.60	0.08	35.43[b]	26.09
				(8.29)	(18.21)	(3.47)	(11.69)
Do-or-die	48	3	0	16.86	0.01	34.85[b]	25.99
				(32.94)	(25.60)	(6.09)	(11.43)
Convertible	21	3	0	13.85	1.90	32.28[b]	17.08
				(10.11)	(16.80)	(8.05)	(12.79)
Subordinated	257	5	0	16.20	− 0.05	36.40[b]	28.44
				(6.99)	(22.76)	(4.15)	(10.79)

[a] The magnitude of overpricing is significant at the five percent confidence level.
[b] The magnitude of underpricing is significant at the five percent confidence level.

that engender issue failure become prime targets for elimination by the genetic selection process. As we argued earlier, securities that produce highly risky investor payoffs are more likely to induce investor strategies that produce failure. When the investor learning effect is salient, the firm will eschew low firm risk securities in favor of low investor risk securities; that is, the firm will tend to choose the security that minimizes the probability of investor loss, the safety-first security. In our simulations, this security is debt. Our argument for the importance of safety first for the selection of the debt security is confirmed by the general dominance of debt in the simulations.

Outcomes of the central simulations are summarized in Table IV, Table V, and Table VI. These tables present two types of data: (1) the actual pricing and issuing patterns of the agents (Table IV) and (2) the characteristics of the agents' chromosome pools (Table V and Table VI). The statistical properties of agents' chromosome pools allow us to go beyond a mere examination of strategies actually played by agents to explore the equilibrium probability distributions used to generate these strategies.

The results provided in these tables are fairly straightforward. Debt is the most commonly employed security, followed by subordinated debt and equity. Conver-

Table V

Investor Chromosome Pool Averages with Two-Sided Learning

This table presents the mean (*Mean*), standard deviation (*Std. Dev.*), minimum value (*Minimum*), and maximum value (*Maximum*) of investor chromosome pool averages in the final round across all 1,000 runs employing an investment level of 32.5 (NPV = 17.5). These results were generated by simulations employing crossover and mutation rates for investors of 0.6 and 0.003, respectively, and a mutation rate for the firm of 0.01.

Security	Mean	Std. Dev.	Minimum	Maximum
Junk	13.19	3.89	2.10	53.89
Debt	30.70[a]	10.15	1.53	49.17
Equity	20.30[a]	9.46	1.59	47.62
Do-or-die	22.20[a]	10.01	0.62	44.17
Convertible	16.90[a]	7.68	0.12	50.32
Subordinated	21.95[a]	10.26	0.52	50.79

[a] The average bid is significantly lower than the intrinsic value of the security at the five percent confidence level.

Table VI

Standard Deviations of Investor Chromosome Pools with Two-Sided Learning

This table presents the mean (*Mean*), standard deviation (*Std. Dev.*), minimum value (*Minimum*), and maximum value (*Maximum*) of the standard deviation of the investor chromosome pools in the final round across all 1,000 runs employing an investment level of 32.5 (NPV = 17.5). These results were generated by simulations employing crossover and mutation rates for investors of 0.6 and 0.003, respectively, and a mutation rate for the firm of 0.01.

Security	Mean	Std. Dev.	Minimum	Maximum
Junk	8.54	2.12	1.06	31.30
Debt	3.93	4.32	0.00	17.19
Equity	6.78	4.05	0.00	23.53
Do-or-die	6.27	4.29	0.00	24.46
Convertible	7.89	3.59	0.00	29.84
Subordinated	6.36	4.28	0.00	29.46

tible debt and the do-or-die security are a distant fourth and fifth. The junk security is seldom issued. With regard to security pricing, with the exception of the junk security and debt, all securities are underpriced; convertible debt displays the greatest underpricing. Failure rates for security issues are lower than in the one-sided learning simulations where investors learn to bid for securities. These results are quite robust. They persist across a number of different experimental treatments. The following subsections study the issue frequency, investor valuation, and issue failure in detail.

A. *Frequency of Security Issuance*

From Table IV, an examination of the frequency with which each of the securities is issued in the final round of the simulations indicates that debt dominates the other securities. Debt is issued 51.9 percent of the time in the final rounds of the simulations. Subordinated debt is also issued quite frequently, accounting for 25.7 percent of the outcomes. Equity follows, accounting for 15.1 percent of the outcomes. The junk security is issued least frequently, accounting for less than 0.4 percent of observations.

An examination of security characteristics presented in Table I indicates that the ordering of securities in terms of their issuance frequency is not explained by the dispersion of their payoffs. The ordering is, however, consistent with the safety-first criterion developed by Roy (1952). The ranking of average bid prices in Table IV is also consistent with our safety-first interpretation. Note that the ranking of securities is insensitive to whether rankings are based on the average winning bids or on the average losing bids. Further, the contracts with a higher probability of generating an investor loss display higher price variation—variation in the winning bid.

Table V and Table VI also offer insights into the desirability of the various securities. The ordering of securities in terms of the mean bids implied by investor chromosome pools corresponds closely with the ranking of the securities in terms of their relative frequency of issuance and their ranking in terms of the safety-first criterion. This consistency is not surprising given that realized prices and security choices represent draws from the chromosome pool formed in the previous rounds.

The only aberration appears to be the ranking of the do-or-die security, which is consistently ranked ahead of equity in terms of the mean bids implied by the chromosome pools. We believe that this aberration may be a direct result of the nonmonotonicity of the payoff of the do-or-die security. The do-or-die security is the only nonmonotone security considered in our analysis, and this property can produce "cobweb" dynamics. High realizations of random cash flows produce high firm payoffs, leading the contract to multiply in the firm's chromosome pool. High cash flows simultaneously generate low payoffs for investors, and these low payoffs cause chromosomes representing favorable (i.e., high) bids to be selected out. Low realizations lead to a reverse pattern of evolution—the contract is selected out of the firm's pool while favorable price responses multiply in each investor's chromosome pool. Thus, for do-or-die contracts, low representation in the firm's chromosome pool will occur when investors are favorably disposed to the contract. This may explain why the do-or-die security has a higher mean price implied by the chromosome pools but a lower issue frequency ranking than equity.

B. Bid Shading and Underpricing

In addition to providing insights into security rankings, the chromosome pool statistics also provide data on investor bidding strategies. Bertrand competition implies that the winning bid must equal the highest tick below a security's intrinsic value. In our algorithm, this is 100(23/63), which is approximately 36.5. The mean of the bids encoded by an investor's chromosome pool is an unbiased estimate of the bid he or she is willing to make for the security. In general, the mean bids are lower than the intrinsic value of the securities.[12] The chromosome pools indicate bid shading of 18 percent for the debt security. This rises to 42 percent for the subordinated debt security, and 46 percent for equity.

The difference between the intrinsic value of a security and the prices paid by investors captures underpricing. A comparison of the winning bids from Table IV with the mean chromosome pool values indicates that the underpricing of contracts is lower than the extent of bid shading implied by the chromosome pools. With the exception of the junk security and convertible debt, underpricing remains in the single digits for securities issued in the final round of the simulation. This phenomenon may result from the selection bias—the firm does not randomly choose a security but tends to select the security that garners the highest prices from investors. The discrepancy between observed underpricing and bid shading implied by investors' chromosome pools may also be partially explained by the fact that, in the simulations, the highest bid is chosen as the transaction price. Thus, even if both investors, on average, submit bids less than their true value, the winning bid can on average exceed true value. In fact, in some instances, this "winner's curse" effect is strong enough to more than offset the bid shading implied by investor chromosome pools. For example, the average winning bid for debt exceeds its intrinsic values.

C. Variance of Bids

The data in Table VI highlight another apparent consistency across the chromosome pools resulting from the simulations: The three most frequently issued securities display lower average bid variance than the three least frequently issued securities.[13] This result is consistent with the observation that the more frequently issued securities have lower probabilities of inducing losses, as well as higher cash flows conditioned on losses occurring. Consequently, the algorithm leads to fewer radical adjustments in optimal pricing strategies and, thus, to more homogenous chromosome pools of price responses.

D. Failure Rates

A comparison of the failure rates for security issues in Tables III and IV indicates that failure rates are generally lower in simulations employing two-sided learning. In the one-sided learning simulations where the firm's security choice was fixed, failure rates ranged from 4 percent for debt to 76 percent for convertible debt. In the two-sided learning simulations, failure rates ranged from 7 percent for debt to 14 percent for convertible debt. The junk security had a 100 percent failure rate in the two-sided learning simulations. The relatively low failure rates for the nondebt securities in the two-sided learning simulations probably

[12] The junk security is the exception. It has an intrinsic value of zero; because all prices are restricted to being no less than zero, by construction, investors always overbid for the junk security.

[13] Once again, the do-or-die security is the exception. The average price variance implied by the chromosome pool for this security is lower than that implied for equity.

resulted from the ability of the firm to switch from securities receiving poor investor responses, an option that was not available to the firm in the one-sided learning simulations described in Table III. The slightly higher failure rates for debt in the two-sided learning simulations probably resulted from investors being offered fewer opportunities to price debt relative to the one-sided learning simulations.

E. Learning in the Very Long Run

The simulations reported in Tables IV, V, and VI were stopped when a single security accounted for at least 95 percent of the firm's gene pool or when a run had lasted 2,000 rounds. As Young (1996) demonstrates, in a system with background noise provided by mutation, such convergence may not represent the limiting behavior of the algorithm over the very long run. Outcomes of simulations run to capture the very long run limiting behavior of our genetic algorithm are presented in Table VII. Each of these simulations lasted one million rounds.

A comparison of the security issuance frequencies in Tables IV and VII indicates that debt is much more dominant in the very long run as its frequency of issuance climbs from 51.9 percent to 83.5 percent. Issuance of equity and subordinated securities drops by over 70 percent over the very long run. The dominance of debt over the very long run is also reflected in other aspects of the outcomes. The average winning bid for debt increases slightly. In contrast, the winning bids for the other securities fall precipitously, with winning bids for equity and subordinated falling almost 10 percent. This decline in the average winning bids for the other securities is driven by increased bid shading as captured by the drop in mean bids implied by investor chromosome pools. In contrast, the bid shading for debt financing is reduced by almost 35 percent. Convergence to debt in the very long run indicates that debt possesses "the largest basin of attraction," that is, the highest likelihood of the system converging following a destabilizing mutation.

Note that these very long run simulations were terminated after a fixed number of rounds rather than when a threshold level of convergence was reached. Because of the constant background level of mutation, the gene pools may not be stable despite the length of the simulations.[14] Because the properties of the system were frequently measured when the gene pools were not stable, particularly in the case of securities other than straight debt, it is not surprising that issue failure and bid shading is higher than in the results reported in Tables IV and V.

F. Global versus Local Attractors

The above evidence supports the view that ease of learning how to price securities can result in a pecking order, with debt emerging as the preferred security. However, these results are derived from a system in which, *a priori*, all securities

[14] An examination of the firm's chromosome pool at the end of the one million rounds indicates that in 21.3 percent of the simulations, no security accounted for over 95 percent of the firm's chromosome pool. When debt was the security issued in the millionth round, in 14.4 percent of the simulations no single security accounted for over 95 percent of the firm's chromosome pool. This statistic varied between 51 percent and 80 percent for the other securities.

have an equal chance of being issued. Following Routledge (2001), we run five sets of simulations to ascertain if a similar result would obtain if the system started at a different point.

For each simulation, we picked one security, which we call the candidate security, from the firm's choice set. Only the junk security was not selected. The system was then started at a fixed point, with the firm's chromosome pool populated solely with chromosomes encoding the candidate security. Investors' chromosome pools corresponding to the candidate strategy were populated only with bids encoding the intrinsic value of the security. Chromosome pools corresponding to noncandidate strategies were encoded with bids of zero. The remainder of the simulation procedure remained unchanged, with simulations being terminated after either 2000 rounds or when a security accounted for 95 percent of the firm's chromosome pool.

Table VIII presents the results from these simulations. In all simulations, with the exception of the convertible debt simulation, the system converged to the candidate security more than 50 percent of the time. This indicates that these securities are local attractors. Once again safety first appears to influence

Table VII

Outcomes in the Very Long Run

This table presents outcomes of 1,000 simulation runs, each of which lasts one million rounds. It presents the frequency with which securities were issued in the final round (*Freq.*), the number of instances in which security issues failed in the final round (*Failures*), and the number of runs ending with no one security accounting for over 95 percent of firm gene pool representation (*Nonconverge*). The table also contains the average amount paid by the winning investor (*Investor Payment*) in the final round. This average is conditioned on the security issued in the final round. The mean values of bid averages (*Average Bid*) and standard deviations (*Std. Dev. Bid*) of the chromosome pools of the last 1,000 rounds across all 1,000 runs are reported in the table as well. Standard deviations appear in parentheses. The results were generated by simulations employing crossover and mutation rates for investors of 0.6 and 0.003, respectively, a mutation rate for the firm of 0.01, and investment level of 32.5 (NPV = 17.5).

Security	Freq.	Failures	Nonconverge	Investor Payment	Average bid	Std. Dev. bid
Junk	10	10	6	15.56	13.12	5.78
				(8.33)	(6.48)	(9.02)
Debt	835	29	120	38.93[a]	33.71[b]	2.86
				(4.33)	(2.87)	(3.66)
Equity	45	13	23	31.68[c]	16.23[b]	6.11
				(9.76)	(9.75)	(7.84)
Do-or-die	21	5	11	32.12[c]	16.09[b]	6.18
				(7.17)	(3.58)	(3.36)
Convertible	15	14	12	22.43[c]	14.58[b]	6.34
				(8.81)	(8.65)	(8.98)
Subordinated	74	19	41	32.39[c]	16.40[b]	5.99
				(9.42)	(3.57)	(3.64)

[a] The magnitude of overpricing is significant at the five percent confidence level.
[b] The average bid is significantly lower than the intrinsic value of the security at the five percent confidence level.
[c] The magnitude of underpricing is significant at the five percent confidence level.

Table VIII
Local versus Global Attractors

This table presents the outcomes of the final rounds of five sets of 1,000 simulation runs with different initial conditions. Each panel presents the outcomes of 1,000 simulations for a candidate security. In each simulation, the firm's initial gene pool contains only the candidate security. The investors' initial gene pool for the candidate security contains only the correct pricing bid, $\frac{23}{63}(100)$. The investors' initial gene pools for other securities contain only the bid zero. This table presents the frequency with which securities were issued in the final round (*Freq.*), the number of instances in which security issues failed in the final round (*Failures*), and the frequency with which a run lasted 2,000 rounds because no one security accounted for at least 95 percent of the firm's chromosome pool at an earlier round (*Nonconverge*). The table also contains mean values of bid amount paid by the winning investor (*Investor Payment*). This average is conditioned on the security issued in the final round. The mean values of bid averages (*Average Bid*) and standard deviations (*Std. Dev. Bid*) of the chromosome pools across all 1,000 runs are reported in the table as well. Standard deviations appear in parentheses. The results were generated by simulations employing crossover and mutation rates for investors of 0.6 and 0.003, respectively, a mutation rate for the firm of 0.01, and investment level of 32.5 (NPV = 17.5).

Security	Freq.[a]	Failures[b]	Non-converge	Investor Payment	Average Bid	Std. Dev. Bid
			Panel A: Debt as a Candidate Security			
Junk	5	5	0	0.00[c]	0.19[c]	0.80[c]
				(0.00)	(0.77)	(1.13)
Debt	982	8	0	40.45[c]	37.06[c]	2.57[c]
				(3.08)	(1.45)	(2.00)
Equity	3	3	0	0.00[c]	0.27[c]	1.10[c]
				(0.00)	(7.71)	(0.82)
Do-or-die	3	3	0	8.47[c]	0.25[c]	1.06[c]
				(14.66)	(3.44)	(1.96)
Convertible	4	4	0	0.00[c]	0.24[c]	1.02[c]
				(0.00)	(1.02)	(1.37)
Subordinated	3	2	0	11.64[c]	0.28[c]	1.05[c]
				(20.16)	(2.11)	(1.99)
			Panel B: Equity as a Candidate Security			
Junk	7	7	0	2.49[c]	2.28[c]	3.58[c]
				(3.41)	(3.68)	(6.71)
Debt	125	7	0	36.24[c]	5.64[c]	3.02[c]
				(7.73)	(3.48)	(3.98)
Equity	745	38	0	36.27[c]	25.77[c]	4.15[c]
				(4.21)	(11.32)	(5.02)
Do-or-die	32	7	0	32.19	3.32[c]	3.80[c]
				(15.71)	(3.36)	(3.61)
Convertible	14	7	0	21.09[c]	2.69[c]	3.74[c]
				(18.85)	(11.75)	(9.62)
Subordinated	77	6	0	38.69[c]	4.67[c]	3.79[c]
				(10.14)	(3.92)	(4.10)
			Panel C: Do-or-die as a Candidate Security			
Junk	9	9	0	2.82[c]	2.52[c]	3.91[c]
				(4.26)	(3.69)	(11.71)
Debt	173	7	0	36.56[c]	6.97[f]	3.39[c]
				(6.72)	(3.45)	(4.17)

Table VIII—*continued*

Equity	60	7	0	34.15	4.27[c]	4.05[c]
				(10.70)	(12.41)	(5.18)
Do-or-die	664	40	0	36.25	25.14[c]	4.54[c]
				(4.87)	(3.59)	(3.89)
Convertible	14	7	0	21.77[c]	3.23[c]	4.35[c]
				(18.06)	(8.11)	(9.27)
Subordinated	80	7	0	37.76	4.89[c]	4.16[c]
				(11.49)	(3.78)	(4.03)
Panel D: Convertible as a Candidate Security						
Junk	7	7	0	6.35[c]	4.73[c]	5.81[c]
				(7.56)	(4.71)	(10.09)
Debt	346	10	0	37.10[c]	14.24[c]	4.70[c]
				(6.08)	(3.34)	(4.05)
Equity	115	11	0	33.87	7.94[c]	5.76[c]
				(9.53)	(15.33)	(9.91)
Do-or-die	95	10	0	35.77	7.81[c]	5.96[c]
				(10.85)	(3.98)	(3.72)
Convertible	252	38	1	33.79	17.84[c]	5.67[c]
				(6.18)	(10.23)	(12.50)
Subordinated	185	8	0	39.10[c]	9.96[c]	5.93[c]
				(8.30)	(3.99)	(4.31)
Panel E: Subordinated as a Candidate Security						
Junk	6	6	0	1.85[c]	1.52[c]	2.63[c]
				(2.91)	(3.13)	(5.94)
Debt	62	7	1	34.49[c]	3.28[c]	2.41[c]
				(11.89)	(3.11)	(3.74)
Equity	27	5	1	30.63[c]	2.24[c]	2.87[c]
				(12.43)	(8.64)	(3.73)
Do-or-die	20	4	0	32.54	2.38[c]	2.96[c]
				(13.64)	(3.09)	(3.56)
Convertible	11	7	0	15.73[c]	1.72[c]	2.95[c]
				(17.68)	(5.53)	(11.04)
Subordinated	874	49	0	37.30[c]	29.61[c]	3.68[c]
				(4.73)	(3.64)	(3.95)

[a]The distribution is significantly different from the distribution of issue frequency in the base case (Table IV) at the five percent level based on the Pearson chi-square test.
[b]The failure rate is significantly different from the failure rate in the base case (Table IV) at the five percent level based on the Cochran–Mantel–Haenszel test.
[c]The value is significantly different from the corresponding value in the base case (Table IV) at the five percent level based on a t-test.

the outcome. Table VIII indicates that the rate of convergence to a security is monotonically increasing in the security's safety-first rating. The security with the second lowest safety first ranking, do-or-die, was the convergent limit only approximately two thirds of the time. The large probability of investor losses embodied by convertible debt combined with the emergence of more favorable prices for other securities resulting from mutation, ensures that the system frequently moves away from convertible debt. Thus, convertible debt, the worst security by the safety first criterion, was the limiting value only 25 percent of the time.[15]

V. Comparative Statics

We now describe outcomes from simulations designed to examine the sensitivity of our results to changes in the underlying economic structure. First, we examine the impact of changes in market microstructure. Next, we examine the effects of changes in profitability of the firm's project. We find that the results of the simulation are, for the most part, robust to these perturbations.

A. Market Microstructure

In the base-case analysis, the highest bidder is awarded the financing contract at the price bid. To determine whether our outcomes are driven by our choice of market mechanism, we ran five simulations employing alternative mechanisms. In each of these simulations, the financing contract continued to be awarded to the highest bidder. However, the price paid by the winning investor was a weighted average of the high bid and the low bid. The weights placed on the winning investor's bid were 0.8, 0.6, 0.4, 0.2, and 0.

Table IX, Panel A, reports the outcomes of the simulation employing a second-price auction; that is, simulations where the weight placed on the winning investor's bid was zero, resulting in the winning investor paying the price quoted by the losing investor.[16] As the results show, in terms of issue frequency, debt financing continues to dominate, and subordinated debt and equity retain their positions in the hierarchy. However, the change in auction mechanism significantly altered the composition of investor chromosome pools by raising the mean bids reflected in their chromosome pools. Under the altered market mechanism, this result is not surprising given that higher investor bids, while increasing the likelihood of winning the bidding, need not translate into higher transaction prices. This is likely to have favored chromosomes encoding higher prices in the evolution of investors' pricing strategies.

The change in the auction mechanism also significantly increased the frequency with which the firm failed to raise sufficient capital in the final rounds of the 1,000 runs. In a second-price auction, the sensitivity of an investor's payoff to his bid is reduced. This lowered sensitivity lowers the strength of the feedback effect for learning. Consequently, as can be seen from Table IX, the investors' chromosome pools remained more heterogeneous. This heterogeneity in investor chromosome pools translated into greater dispersion in responses to the firm's security issuance decisions which, in turn, discouraged the convergence of the

[15] When the mutation rate of investor chromosome pools falls from 0.003 to 0.001, even convertible debt becomes the predominant limiting security. Results from simulations employing a mutation rate of 0.001 for investor chromosome pools are available from the authors upon request.

[16] We obtained similar results in the other simulations. The results are available from the authors.

Table IX

Microstructure and Security Choices

This table presents outcomes of the final rounds of two sets of 1,000 simulation runs employing different market structures. It presents the frequency with which securities were issued in the final round (*Freq.*), the number of instances in which security issues failed in the final round (*Failures*), and the frequency with which a run lasted 2,000 rounds because no one security accounted for at least 95 percent of the firm's chromosome pool at an earlier round (*Nonconverge*). The table also contains mean values of bid amount paid by the winning investor (*Investor Payment*). This average is conditioned on the security issued in the final round. The mean values of bid averages (*Average Bid*) and standard deviations (*Std. Dev. Bid*) of the chromosome pools across all 1,000 runs are reported in the table as well. Standard deviations appear in parentheses. The results were generated by simulations employing crossover and mutation rates for investors of 0.6 and 0.003, respectively, a mutation rate for the firm of 0.01, and investment level of 32.5 (NPV = 17.5).

Security	Freq.[a]	Failures[b]	Nonconverge	Investor Payment	Average Bid	Std. Dev. Bid
			Panel A: Second-price Auction			
Junk	19	19	9	7.69[c]	20.84[c]	12.48[c]
				(7.64)	(12.76)	(19.74)
Debt	527	82	26	40.12[c]	46.12[c]	10.18c
				(10.78)	(6.41)	(6.71)
Equity	130	40	24	34.53	31.33[c]	12.56[c]
				(12.67)	(23.32)	(16.78)
Do-or-die	97	31	24	34.28	32.04[c]	12.77[c]
				(14.16)	(6.24)	(6.54)
Convertible	39	25	16	20.80[c]	27.21[c]	12.84[c]
				(16.16)	(19.24)	(21.01)
Subordinated	188	48	26	37.07	34.36[c]	12.24[c]
				(13.28)	(6.66)	(6.73)
			Panel B: Smaller Tick Size			
Junk	4	4	0	12.70	13.17	8.70[c]
				(8.50)	(3.23)	(10.01)
Debt	503	5	3	37.93[c]	30.26	3.85
				(3.30)	(1.76)	(4.33)
Equity	137	7	1	35.06	19.76	6.87
				(3.65)	(9.97)	(7.23)
Do-or-die	49	1	0	35.96	22.16	6.47
				(3.33)	(4.30)	(3.32)
Convertible	18	5	0	32.19	16.69	8.09
				(7.96)	(9.25)	(10.41)
Subordinated	289	13	0	36.21	21.75	6.44
				(4.62)	(4.05)	(4.28)

[a] The distribution is significantly different from the distribution of issue frequency in the base case (Table IV) at the five percent level based on the Pearson chi-square test.
[b] The failure rate is significantly different from the failure rate in the base case (Table IV) at the five percent level based on the Cochran–Mantel–Haenszel test.
[c] The value is significantly different from the corresponding value in the base case (Table IV) at the five percent level based on a t-test.

firm's chromosome pool to a single issuance strategy. In fact, in the simulations employing the second-price auction, the firm's chromosome pool failed to converge to a single security over 10 percent of the time compared with 0.2 percent in the base case simulations. Thus, the higher failure rates for security issuance may be a product of the reduced feedback effects of the second-price auction.

To examine whether outcomes are affected by the range of permissible security prices, we simulated 1,000 runs employing different chromosome pools for the investors. In the base case runs, investor chromosome pools were restricted to encoding prices ranging from 0 to 100, and the initial chromosome pools were constructed using uniform random draws from this range. This implies that, a priori, investors overpriced securities because the expected bid in the initial gene pool was 50. While leaving the structure of investor chromosome pools unchanged, the new simulation restricted investor chromosome pools to encode prices between 0 and 50, with the initial chromosome pool being constructed using uniform random draws from this range. Thus, the initial expected bid from the investors is lower than the intrinsic value of the securities. As a byproduct of the change, the tick size is halved. As is apparent from Panel B of Table IX, which contains the results from these simulations, the change in initial investor population did not alter the results significantly.

B. Project Profitability

Just as there are good reasons to suspect that microstructure may play a role in financing choices, there are good reasons to believe that the profitability of a firm's investment may play a role. We ran two sets of simulations to examine this effect. One increased profitability by employing an investment level of 30, while the other decreased profitability by employing an investment level of 35. Results from these simulations are reported in Table X.

The ranking of securities in terms of the frequency of issuance is invariant to the level of investment. However, the degree to which debt dominates varies significantly. At higher investment levels, debt becomes more dominant as other contracts recede in importance. The frequency of subordinated debt issuance declines by approximately 42 percent as the investment level increases from 30 to 35. Equity displays an even greater decline in the frequency of issuance for the same change in investment level, approximately 50 percent.

The safety-first criterion, which is able to explain the frequency of security issuance in the base-case simulations, also explains the relationship between security choice and investment level. As the investment level increases from 30 to 35, all securities are more likely to generate losses for investors. At the same time, the advantage of the debt security relative to the other securities with respect to the likelihood of avoiding losses increases. Thus, safety first predicts, and the simulation results confirm, that debt becomes more dominant as the investment level increases. The steep decline in the relative frequency with which equity is issued is consistent with the fact that equity experiences the largest increase in the probability of loss as investment increases.

Table X

Project Profitability and Security Choices

This table presents outcomes of the final rounds of two sets of 1,000 simulation runs employing different investment levels. This table presents the frequency with which securities were issued in the final round (*Freq.*), the number of instances in which security issues failed in the final round (*Failures*), and the frequency with which a run lasted 2,000 rounds because no one security accounted for at least 95 percent of the firm's chromosome pool at an earlier round (*Nonconverge*). The table also contains mean values of bid amount paid by the winning investor (*Investor Payment*). This average is conditioned on the security issued in the final round. The mean values of bid averages (*Average Bid*) and standard deviations (*Std. Dev. Bid*) of the chromosome pools across all 1,000 runs are reported in the table as well. Standard deviations appear in parentheses. The results were generated by simulations employing crossover and mutation rates for investors of 0.6 and 0.003, respectively, and a mutation rate for the firm of 0.01.

Security	Freq.[a]	Failures[b]	Nonconverge	Investor Payment	Average Bid	Std. Dev. Bid
			Panel A: Investment Level = 30 (NPV = 20)			
Junk	6	6	1	17.46	12.31[c]	7.80
				(9.04)	(4.04)	(2.22)
Debt	387	2	6	37.33[c]	29.43[c]	3.54[c]
				(4.20)	(9.52)	(4.02)
Equity	198	6	2	34.04[c]	20.16	5.59[c]
				(4.39)	(9.91)	(4.12)
Do-or-die	32	3	1	32.89	21.97	5.38[c]
				(4.58)	(9.72)	(4.24)
Convertible	16	4	0	29.76	15.97[c]	7.12[c]
				(8.29)	(7.64)	(3.64)
Subordinated	361	9	5	34.71[c]	22.58	5.34[c]
				(4.48)	(10.64)	(4.36)
			Panel B: Investment Level = 35 (NPV = 15)			
Junk	3	3	0	16.93	14.84[c]	9.16[c]
				(1.83)	(4.47)	(2.25)
Debt	591	9	0	39.32[c]	31.49[c]	4.27[c]
				(3.23)	(10.47)	(4.53)
Equity	102	6	0	37.75[c]	19.60[c]	7.79[c]
				(3.59)	(8.92)	(3.92)
Do-or-die	77	10	0	37.00	21.64	7.46[c]
				(6.08)	(10.06)	(4.31)
Convertible	20	3	0	36.11	17.94[c]	8.55[c]
				(9.30)	(7.39)	(8.55)
Subordinated	207	9	0	38.38[c]	22.01	7.13[c]
				(3.62)	(10.22)	(4.29)

[a] The distribution is significantly different from the distribution of issue frequency in the base case (Table IV) at the five percent level based on the Pearson chi-square test.
[b] The failure rate is significantly different from the failure rate in the base case (Table IV) at the five percent level based on the Cochran–Mantel–Haenszel test.
[c] The value is significantly different from the corresponding value in the base case (Table IV) at the five percent level based on a t-test.

The level of bid shading varies with the firm's investment need. Bid shading on debt falls from 22 percent at an investment level of 30 to 16 percent at an investment level of 35. The extent of bid shading for subordinated debt is about 40 percent, whereas the bid shading for equity is between 46 percent and 48 percent. In the latter two cases, there does not appear to be any monotonic relationship between the extent of bid shading and the level of investment.

If price responses are insensitive to the firm's required investment, as one might expect, given that the expected payoff from investing in securities is unchanged, the frequency of issue failure ought to increase with the investment level. However, the data presented in Tables IV and X reveal no such trend is evident. Overall failure rates ranged from 2.7 percent when the investment was 32.5, to 4.0 percent for an investment level of 35.

The nonmonotone relationship between issue failure and required investment is surprising given that the average variance of price responses increases with the required investment level and no trend appears in the mean prices implied by investor gene pools. Taken in isolation, these facts suggest an increase in the failure rate with the investment level. Apparently, the natural increase in the probability of issue failure is thwarted by changes in the firm's issuance strategy at higher investment levels—the firm increases its reliance on securities that command the highest prices (i.e., debt). By issuing high-priced securities, the firm prevents the failure rate from increasing.

VI. Security Characteristics and Security Choice

In the previous section, we document contract choices that are, to a great extent, consistent with the safety-first ranking. We now describe the outcomes of two sets of simulations that are designed to examine whether the safety-first criterion may proxy for other statistical rankings of the security cash flow distributions, such as mean and variance rankings. The first examines the effects of changing the mean contract payoff while fixing the safety-first rating. The second examines the effect of changing the variance of contract payoff while fixing both the safety-first rating and the mean contract payoff.

A. Mean of Contract Payoff

To examine the effects of differences in expected payoff while holding safety-first ranking constant, we ran the genetic algorithm with six simple debt contracts. The promised payments on these contracts were set at 35, 40, 45, 50, 55, and 60. Other details regarding these contracts are presented in Table XI. We considered two levels of required investment: 32.50 and 35.00. Note that, under the safety-first criteria, all debt contracts with a face value in excess of the funds raised have the same safety-first ranking.

Details for these simulations are presented in Table XII. Our results show that, apart from contracts that do not provide outside investors break-even returns, the mean of the contract payoff has little effect on the frequency of issuance of a contract. When the investment level is 32.50, contracts with high mean values

Table XI
Debt Security Characteristics

This table presents characteristics of six debt securities with different face values employed to examine the impact of changes in expected payoff on financing decisions while holding safety-first rankings constant. These characteristics include expected security payoff (*Mean*), variance of expected security payoff (*Var.*), and probability of a positive net payoff to outside investors if they purchase securities at a price equal to the investment level of 32.5 (*Prob.*).

Face Value	Mean	Var.	Prob.
35	28.88	105.40	0.675
40	32.00	149.33	0.675
45	34.88	201.23	0.675
50	37.50	260.42	0.675
55	39.88	325.82	0.675
60	42.00	396.00	0.675

have a slightly higher frequency of being issued, while at the higher investment level of 35.00, contracts with a lower mean value have a somewhat higher issuance frequency. Thus, the first-order implication of our results for contract selection is that safety-first seems to be the dominant determinant of contract choices. Interestingly, failure rates are uniformly lower at the higher investment level. This result appears to be a product of the uniformly higher investor bid prices as captured by the average values of investor chromosome pools.

B. Variance of Contract Payoff

To examine the effect of changes in the variance of contract payoff while maintaining both the safety-first rating and the mean payoff, we included the original debt contract and a set of five convertible debt contracts in the set of contracts employed. These contracts are differentiated by the fraction of value contributed by the debt and the option features of the contracts, with increases in the value of the option feature exactly offsetting decreases in the value of the debt feature of the contract. Details of the parameters are presented in Table XIII.

The results of this simulation, presented in Table XIV indicate that the variance of contractual payoffs appears to negatively affect issuance probabilities. However, this effect appears relatively weak in that higher variance does not preclude the issuance of the contract on a fairly frequent basis. The summary of chromosome pool characteristics indicates that bid shading is not significant. However, bid shading does appear to increase with the variance of security payoffs. Once again, the "winner's curse" effect appears to outweigh the bid shading implied by investor chromosome pools. Failure rates are once again uniformly lower at the higher investment level. Again this result appears to be a product of the uniformly higher investor bid prices as captured by the average values of investor chromosome pools.

Table XII

Contract Value and Security Choices

This table presents outcomes of the final rounds of 1,000 simulation runs employing debt securities with varying face values. It presents the frequency with which securities were issued in the final round (*Freq.*), the number of instances in which security issues failed in the final round (*Failures*), and the frequency with which a run lasted 2,000 rounds because no one security accounted for at least 95 percent of the firm's chromosome pool at an earlier round (*Nonconverge*). The table also contains mean values of bid amount paid by the winning investor (*Investor Payment*). This average is conditioned on the security issued in the final round. The mean values of bid averages (*Average Bid*) and standard deviations (*Std. Dev. Bid*) of the chromosome pools across all 1,000 runs are reported in the table as well. Standard deviations appear in parentheses. The results were generated by simulations employing crossover and mutation rates for investors of 0.6 and 0.003, respectively, and a mutation rate for the firm of 0.01.

Face Value	Freq.	Failures	Nonconverge	Investor Payment	Average Bid	Std. Dev. Bid
			Panel A. Investment Level = 32.5 (NPV = 17.5)			
35	47	8	4	29.82	5.54[a]	5.83
				(9.83)	(7.37)	(12.22)
40	125	3	1	36.74[b]	8.47[a]	5.52
				(3.81)	(3.33)	(3.61)
45	146	15	1	35.34	8.97[a]	5.56
				(9.60)	(11.35)	(17.13)
50	143	8	1	37.07	8.90[a]	5.42
				(6.96)	(3.66)	(4.69)
55	302	9	0	43.35[b]	15.10[a]	5.82
				(9.04)	(12.02)	(16.32)
60	237	6	1	44.41[b]	13.53[a]	6.09
				(8.81)	(3.71)	(4.59)
			Panel B: Investment Level = 35 (NPV = 15)			
35	1	1	0	19.05	15.47[a]	9.71
				(0.00)	(4.16)	(2.00)
40	88	1	10	37.66[b]	23.86[a]	7.01
				(0.92)	(9.96)	(4.66)
45	205	1	7	40.16[b]	26.73[a]	6.07
				(2.38)	(10.77)	(4.77)
50	271	6	13	41.02[b]	28.63[a]	5.48
				(4.03)	(11.06)	(4.68)
55	229	2	8	42.32[b]	28.21[a]	5.89
				(3.57)	(11.37)	(4.87)
60	206	4	7	42.19	28.60[a]	5.44
				(4.94)	(11.37)	(4.58)

[a] The average bid is significantly lower than the intrinsic value of the security at the five percent confidence level.

[b] The magnitude of overpricing is significant at the five percent confidence level.

Table XIII

Characteristics of Convertible Securities

This table presents characteristics of one debt and five convertible debt securities employed to examine the effect of changes in the variance of security payoffs on financing choices while holding constant safety-first ranking and expected payoff. All the convertible debt securities have the same conversion trigger point of 60, but differ in terms of their face values and conversion ratios. These characteristics include expected security payoff (*Mean*), variance of expected security payoff (*Var.*), and probability of a positive net payoff to outside investors if they purchase securities at a price equal to the investment level of 32.5 (*Prob.*).

Security	Face Value	Conversion Ratio	Mean	Var.	Prob.
Debt	50	—	37.50	260.42	0.675
Convertible	48	0.1275	37.50	261.86	0.675
Convertible	46	0.2600	37.50	266.63	0.675
Convertible	44	0.3975	37.50	275.41	0.675
Convertible	42	0.5400	37.50	288.92	0.675
Convertible	40	0.6875	37.50	307.92	0.675

VII. Conclusion

To examine corporate financing choices, we simulate adaptive learning by value-maximizing firms and investors in frictionless securities markets. Financing decisions are governed by the risks engendered by financial market learning. Our results are consistent with observed empirical regularities. For example, in our simulations, firms rely heavily on debt financing and suffer underpricing when they issue equity. Just as in the "real world," learning is not perfect, securities tend to be underpriced, and new issues may fail to raise sufficient capital.

This analysis represents an initial attempt to understand the role of learning in financing decisions. The conclusions are necessarily tentative because only a single specification of financial market structure is considered: One firm and two investors who compete on the basis of price. Future investigation should consider the effect of varying the number of investors and firms in the market. Different competitive structures should also be considered, such as allowing the firms to fix both the security design and the price, and having investors respond using accept–reject decisions. All the questions considered in this analysis can be reconsidered using modified market structures. Further, while our analysis highlights the central role of safety first in corporate financing decisions, it also demonstrates that other security characteristics may also play a role in these decisions. We believe that investigations of the role of other security characteristics on financing decisions could yield further insights into corporate financing decisions.

Our analysis simplifies the market environment by specifying investors with identical learning capacities. However, it will be of interest to examine the impact of investor heterogeneity on market outcomes. For example, investors can be endowed with varying learning capacities. This variation can occur along a number of dimensions. Some investors may be "smarter" than others because they possess larger chromosome pools. Differences in crossover rates among investors

Table XIV
Contract Variance and Security Choices

This table presents outcomes of the final rounds of 1,000 simulation runs employing one debt security and five convertible bonds with different face values and conversion ratios. This table presents the frequency with which securities were issued in the final round (*Freq.*), the number of instances in which security issues failed in the final round (*Failures*), and the frequency with which a run lasted 2,000 rounds because no one security accounted for at least 95 percent of the firm's chromosome pool at an earlier round (*Nonconverge*). The table also contains mean values of bid amount paid by the winning investor (*Investor Payment*). This average is conditioned on the security issued in the final round. The mean values of bid averages (*Average Bid*) and standard deviations (*Std. Dev. Bid*) of the chromosome pools across all 1,000 runs are reported in the table as well. Standard deviations appear in parentheses. The results were generated by simulations employing crossover and mutation rates for investors of 0.6 and 0.003, respectively, and a mutation rate for the firm of 0.01.

Security	Face Value	Conversion Ratio	Freq.	Failures	Nonconverge	Investor Payment	Average Bid	Std. Dev. Bid
Panel A: Investment Level = 32.5 (NPV = 17.5)								
Debt	50	—	164	6	0	37.82	10.20[a]	5.57
						(6.27)	(12.92)	(12.53)
Convertible	48	0.1275	189	7	0	37.61	10.82[a]	5.59
						(6.50)	(3.76)	(3.80)
Convertible	46	0.2600	146	13	1	35.93[b]	9.10[a]	5.82
						(9.14)	(13.37)	(12.29)
Convertible	44	0.3975	173	11	2	36.71	10.06[a]	5.75
						(6.74)	(3.74)	(3.83)
Convertible	42	0.5400	165	8	1	36.87	9.62[a]	5.73
						(5.85)	(11.82)	(12.68)
Convertible	40	0.6875	163	9	0	35.87[b]	10.49[a]	5.76
						(7.18)	(3.81)	(3.93)
Panel B: Investment Level = 35 (NPV = 15)								
Debt	50	—	225	7	18	41.27[c]	27.69[a]	5.73
						(3.50)	(11.17)	(4.68)
Convertible	48	0.1275	224	2	16	41.40[c]	27.77[a]	5.72
						(2.90)	(11.10)	(4.63)
Convertible	46	0.2600	179	4	12	40.60[c]	26.88[a]	6.05
						(3.38)	(10.80)	(4.64)
Convertible	44	0.3975	143	1	20	40.10[c]	26.04[a]	6.15
						(1.86)	(10.66)	(4.66)
Convertible	42	0.5400	131	4	12	38.65	25.73[a]	6.55
						(4.57)	(10.43)	(4.77)
Convertible	40	0.6875	98	4	13	38.06	25.40[a]	6.41
						(2.94)	(10.23)	(4.66)

[a] The average bid is significantly lower than the intrinsic value of the security at the five percent confidence level.
[b] The magnitude of underpricing is significant at the five percent confidence level.
[c] The magnitude of overpricing is significant at the five percent confidence level.

might produce investors with varying levels of "creativity." Investors might also differ in the speed with which they fix on strategies as a result of differences in the speed with which they rule out less profitable strategies. Another dimension in which investors might differ is experience; some investors might enter the market only after others have had an opportunity to learn.

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