Time series examination

This exam is to be returned exactly one week after the date printed above. Please attach any necessary supporting evidence as well as code to the report to be handed in at the due date.

For this exam you may use any programming language you may desire. Please hand in all the code in an appendix to the exam (or attached in a separate zip file).

Problem 1: Consider the following ARMA(3,1) model for the time series r_t :

$$r_t = 0.3 + 0.2r_{t-1} - 0.4r_{t-2} + 0.3r_{t-3} + a_t - 0.5a_{t-1}$$

Check whether or not the model is stationary. If the model is stationary calculate the theoretical covariances. Calculate the theoretical ACF.

Problem 2: Simulate 1000 observations for the process in problem 1. Use $r_0 = r_1 = r_2 = 0$. Then based on this path estimate the coefficients in the model for a ARMA(3,1) model. Give 95% confidence intervals for all the parameters. Are all the true parameters within the confidence intervals? How about 90% confidence intervals? 99%?

For the next four problems please refer to the attached data file "AAPL08.04.21-25.csv". The file contains minute data for five consecutive days for the Apple equity (ticker AAPL). The columns are not labeled but they are in order from left to right: Ticker, the time of the record, the next four columns are the opening price, high price, low price and close price all within the respective minute, the seventh column is the volume of shares traded, next is the number of trades within that minute, the average price within that minute, then there are 4 columns not important for our purposes.

Problem 3: Analyze the data and determine exactly (based on internal data consistencies) which of the four columns mentioned in the data description

are the opening price, high price, low price and close price. Explain. The time series to be fitted is the returns time series either simple or continuous compounding - your choice from the **closing minute data**. Create the time series data to be fitted and remove the return between days (i.e., calculated using the final minute of one day and the first minute of the next day.

Problem 4: Perform a unit root non-stationarity test. If unit roots are present remove them. Explain. If the data does not exhibit any unit roots fit using whichever criterion you desire an ARMA model. Please present ample details as to the reason you chose the respective model.

Problem 5: Obtain the residuals in the model in problem 3. Test for ARCH effects in the data.

Problem 6: If ARCH effects are present in the data please fit at least three Conditional heteroscedastic models to the data. For the main series use the model estimated in problem 3.

If ARCH models are not present in the data fit a neural network with 100 hidden layer nodes.

Obtain return predictions from all the models you considered. Then compare with the actual 30 minute returns obtained from the data in the attached file "AAPL.test.csv". What was the average squared prediction error? Please provide comments.