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## 6. Mixture models for matrix completion

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We can now extend our Gaussian mixture model to predict actual movie ratings. Let  $\mathbf{X}$  be a data matrix. The rows of this matrix correspond to users and columns specify movies.  $x_{u,i}$  is the rating value of user  $u$  for movie  $i$  (if available). Both  $n$  and  $d$  are typically quite large. The ratings are from one to five stars and are mapped to integers  $\{1, 2, 3, 4, 5\}$ . We will set  $x_{u,i} = 0$  when the rating is missing.

In a realistic setting, most of the entries of  $\mathbf{X}$  are missing. For this reason, we define  $C_u$  as the set of column indexes (column indexes) that user  $u$  has rated and  $H_u$  as its complement (the set of remaining movies we wish to predict ratings for). We use  $|C_u|$  to denote the number of observed ratings for user  $u$ . From the point of view of our mixture model, each user  $u$  is an example  $\mathbf{x}^{(u)} = \mathbf{x}[u, :]$ . Since the coordinates of  $\mathbf{x}^{(u)}$  are missing, we need to focus the model during training on just the observed ratings. To this end, we use  $\mathbf{x}_{C_u}^{(u)} = \{x_i^{(u)} : i \in C_u\}$  as the vector of only observed ratings. If  $C_u = \{0, \dots, d-1\}$ , then a user  $u$  with a rating vector  $\mathbf{x}^{(u)} = (5, 4, 0, 0, 2)$ , where zeros indicate missing ratings, has  $C_u = \{0, 1, 4\}$ ,  $H_u = \{2, 3\}$ , and  $\mathbf{x}_{C_u}^{(u)} = (5, 4, 2)$ .

In this part, we will extend our mixture model in two key ways.

- First, we are going to estimate a mixture model based on partially observed ratings. Since the ratings are missing, we need to handle missing values.
- Second, since we will be dealing with a large, high-dimensional data set, we will need to handle numerical underflow issues. To this end, you should perform most of your computations using logarithms. Remember,  $\log(a \cdot b) = \log(a) + \log(b)$ . This can be useful to remember when  $a$  and  $b$  are small. In these cases, addition should result in fewer numerical underflow issues than multiplication.

An additional numerical optimization trick that you will find useful is the LogSumExp trick. If you wish to evaluate  $y = \log(\exp(x_1) + \dots + \exp(x_n))$ . We define  $x^* = \max\{x_1, \dots, x_n\}$ . Then  $y = x^* + \log(\exp(x_1 - x^*) + \dots + \exp(x_n - x^*))$ . This is just another trick to handle numerical stability.

## Marginalizing over unobserved coordinates

If  $\mathbf{x}^{(u)}$  were a complete rating vector, the mixture model from Part 1 would simply say that the probability  $P(\mathbf{x}^{(u)} | \theta) = \sum_{j=1}^K \pi_j N(\mathbf{x}^{(u)}; \mu^{(j)}, \sigma_j^2 I)$ . In the presence of missing values, we must consider the probability  $P(\mathbf{x}_{C_u}^{(u)} | \theta)$  that is over only the observed values. This marginal corresponds to the mixture density  $P(\mathbf{x}^{(u)} | \theta)$  over all the unobserved coordinate values. In our case, this is computed as follows.

The mixture model for a complete rating vector is written as:

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