12. Linear Independence, Subspaces and Dimension (Optional)

Vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are said to be **linearly dependent** if there exist scalars $c_1, \ldots, c_n$
are zero and (2) $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n = 0$.

Otherwise, they are said to be **linearly independent** : the only scalars $c_1, \ldots, c_n$ that s
$c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n = 0$ are $c_1 = \cdots = c_n = 0$.

The collection of non-zero vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n \in \mathbb{R}^m$ determines a **subspace** of $\mathbb{R}^m$,
linear combinations $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n$ over different choices of $c_1, \ldots, c_n \in \mathbb{R}$. The
subspace is the size of the **largest possible, linearly independent** sub-collection of the
$\mathbf{v}_1, \ldots, \mathbf{v}_n$.

Row and Column Rank (Optional)

0 points possible (ungraded)

Suppose $\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$. The rows of the matrix, $(1, 3)$ and $(2, 6)$, span a subspace c

1

✔ . This is the **row rank** of $\mathbf{A}$.

The columns of the matrix, $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ span a subspace of dimension

1

✔ . This is the **column rank** of $\mathbf{A}$.

We will be using these ideas when studying **Linear Regression**, where we will work wit
rectangular matrices.

Submit

You have used 2 of 3 attempts

Rank of a matrix (Optional)

0 points possible (ungraded)

In general, row rank is always equal to the column rank, so we simply refer to this comr
a matrix.

What is the largest possible rank of a 2×2 matrix?

None of the above

✖

Submit You have used 3 of 3 attempts

Examples of Rank (Optional)

0 points possible (ungraded)

What is the rank of ?

1 ✔

What is the rank of ?

2 ✔

What is the rank of ?

0 ✔

What is the rank of ?

2 ✔

What is the rank of ?

3 ✔

Submit

You have used 1 of 3 attempts

Invertibility of a matrix (Optional)

0 points possible (ungraded)

An matrix is invertible if and only if has full rank, i.e. .

Which of the following matrices are invertible? Choose all that apply.

< Previous

Next >