1 - Type derivation for $(x + \ 2 \ > \ 3) \ \&\& \ y$ with $\Gamma \ = \ \{x: \ int, \ y: \ int\}$

$$\dfrac{\dfrac{\dfrac{\dfrac{VA}{\{x: \ int, \ y: \ int\} \ \vdash \ x \ : \ int} \quad \dfrac{CA}{\{x: \ int, \ y: \ int\} \ \vdash \ 2 \ : \ int}}{\{x: \ int, \ y: \ bool\} \ \vdash x \ + \ 2 \ : \ int}(AO) \quad \dfrac{CA}{\{x: \ int, \ y: \ int\} \ \vdash \ 3 \ : \ int}}{\{x: \ int, \ y: \ bool\} \ \vdash \ x \ + \ 2 \ > \ 3 \ \ : \ bool}(RO) \quad \dfrac{VA}{\{x: \ int, \ y: \ bool\} \ \vdash \ y \ : \ bool}}{\{x: \ int, \ y: \ bool\} \ \vdash \ (x \ + \ 2 \ > \ 3) \ \&\& \ y \ : \ bool}(LCR)$$

$\Longrightarrow$ Type derivation for code is correct and evaluates to *bool*

2 - Type Derivation for $fn\ f \implies fn\ x \implies if\ x\ then\ f\ true\ else\ 19$ with $\Gamma = \{\ \}$

$$\dfrac{\dfrac{\dfrac{\dfrac{VA}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ x\ :\ bool} \quad \dfrac{\dfrac{VA}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ f:\ \varphi\ \to \varepsilon} \quad \dfrac{CA}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ true\ :\ \varphi}}{\{f:\ \beta,\ x:\delta\}\ \vdash\ f\ true\ :\ \varepsilon}\,(FA) \quad \dfrac{CA}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ 19\ :\ \varepsilon}}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ if\ x\ then\ f\ true\ else\ 19\ :\ \varepsilon}\,(ITE)}{\{f:\ \beta\}\ \vdash\ fn\ x \implies if\ x\ then\ f\ true\ else\ 19\ :\ \gamma}\,(AF)}{\{\ \}\vdash fn\ f \implies fn\ x \implies if\ x\ then\ f\ true\ else\ 19\ :\ \alpha}\,(AF)$$

$$\begin{array}{l} \varepsilon = int \\ \varphi = bool \\ \beta = \varphi \to \lambda \\ \delta = bool \\ \gamma = \delta \to \varepsilon \\ \alpha = \beta \to \gamma \end{array} \quad \implies \quad \begin{array}{l} \varepsilon = int \\ \varphi = bool \\ \beta = bool \to int \\ \delta = bool \\ \gamma = bool \to int \\ \alpha = (bool \to int) \to bool \to int \end{array}$$

$\implies$ Type derivation for code is correct and evaluates to $(bool \to int) \to bool \to int$

3 - Type derivation for $let\ fun\ f\ x\ =\ 5\ <\ x\ in\ fn\ y\ =>\ (f\ 3)\ ||\ y\ end$ with $\Gamma\ =\ \{\ \}$

$$\dfrac{\dfrac{\dfrac{CA}{\{f : \beta \to \gamma, x : \beta\} \vdash 5 : int} \quad \dfrac{VA}{\{f : \beta \to \gamma,\ x : \beta\} \vdash x : int}}{\{f : \beta \to \gamma,\ x : \beta\} \vdash 5 < x : \gamma}\,(RO) \qquad \dfrac{\dfrac{\dfrac{\dfrac{VA}{\{f : \beta \to \gamma,\ y : \varphi\} \vdash f : \psi \to \sigma} \quad \dfrac{CA}{\{f : \beta \to \gamma,\ y : \varphi\} \vdash 3 : \psi}}{\{f : \beta \to \gamma,\ y : \varphi\} \vdash f\ 3 : bool}\,(FA) \quad \dfrac{VA}{\{f : \beta \to \gamma,\ y : \varphi\} \vdash y : bool}}{\{f : \beta \to \gamma,\ y : \varphi\} \vdash (f\ 3)\ ||\ y : bool}\,(LCR)}{\{f : \beta \to \gamma\} \vdash fn\ y \implies (f\ 3)\ ||\ y\ end : \alpha}\,(AF)}{\{\ \} \vdash let\ fun\ f\ x = 5 < x\ in\ fn\ y \implies (f\ 3)\ ||\ y\ end : \alpha}\,(LF)$$

$$\begin{aligned}
\beta &= int \\
\gamma &= int \\
\alpha &= \varphi \to bool \\
\psi &= \beta \\
\sigma &= \gamma \\
\psi &= int \\
\varphi &= bool
\end{aligned}
\quad \implies \quad
\begin{aligned}
\beta &= int \\
\gamma &= int \\
\alpha &= bool \to bool \\
\sigma &= int \\
\psi &= int \\
\varphi &= bool
\end{aligned}$$

$\implies$ Type derivation for code is correct and evaluates to $bool \to bool$