1 - Type derivation for $(x + \ 2 \ > \ 3) \ \&\& \ y$ **with** $\Gamma \ = \ \{x : \ int, \ y : \ int\}$

$$\dfrac{\dfrac{\dfrac{VA}{\{x : \ int, \ y : \ int\} \vdash \ x \ : \ int} \quad \dfrac{CA}{\{x : \ int, \ y : \ int\} \vdash \ 2 \ : \ int}}{\{x : \ int, \ y : \ bool\} \vdash x \ + \ 2 \ : \ int}(AO) \quad \dfrac{CA}{\{x : \ int, \ y : \ int\} \vdash \ 3 \ : \ int}}{\{x : \ int, \ y : \ bool\} \vdash \ x \ + \ 2 \ > \ 3 \ \ : \ bool}(RO) \quad \dfrac{VA}{\{x : \ int, \ y : \ bool\} \vdash \ y \ : \ bool}}{\{x : \ int, \ y : \ bool\} \vdash \ (x \ + \ 2 \ > \ 3) \ \&\& \ y \ : \ bool}(LCR)$$

$\Longrightarrow$ Type derivation for code is correct and evaluates to *bool*

2 - Type Derivation for $fn\ f\ \Longrightarrow\ fn\ x\ \Longrightarrow\ if\ x\ then\ f\ true\ else\ 19$ **with** $\Gamma\ =\ \{\ \}$

$$\dfrac{\dfrac{\dfrac{VA}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ x\ :\ bool} \qquad \dfrac{\dfrac{VA}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ f:\ \varphi\ \rightarrow \varepsilon} \qquad \dfrac{CA}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ true\ :\ \varphi}\ (FA) \qquad \dfrac{CA}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ 19\ :\ \varepsilon}}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ f\ true\ :\ \varepsilon}\ (ITE)}{\{f:\ \beta,\ x:\ \delta\}\ \vdash\ if\ x\ then\ f\ true\ else\ 19\ :\ \varepsilon}\ (AF)}{\{f:\ \beta\}\ \vdash\ fn\ x\ \Longrightarrow\ if\ x\ then\ f\ true\ else\ 19\ :\ \gamma}\ (AF)$$

$$\{\ \}\ \vdash fn\ f\ \Longrightarrow\ fn\ x\ \Longrightarrow\ if\ x\ then\ f\ true\ else\ 19\ :\ \alpha$$

$$\begin{array}{l} \varepsilon\ =\ int \\ \varphi =\ bool \\ \beta\ =\ \varphi\ \rightarrow\ \lambda \\ \delta\ =\ bool \\ \gamma\ =\ \delta\ \rightarrow\ \varepsilon \\ \alpha\ =\ \beta\ \rightarrow\ \gamma \end{array} \quad \Longrightarrow \quad \begin{array}{l} \varepsilon\ =\ int \\ \varphi =\ bool \\ \beta\ =\ bool\ \rightarrow\ int \\ \delta\ =\ bool \\ \gamma\ =\ bool\ \rightarrow\ int \\ \alpha\ =\ (bool\ \rightarrow\ int)\ \rightarrow bool\ \rightarrow\ int \end{array}$$

$\Longrightarrow$ Type derivation for code is correct and evaluates to $(bool\ \rightarrow\ int)\ \rightarrow bool\ \rightarrow\ int$

3 - Type derivation for $let\ fun\ f\ x\ =\ 5\ <\ x\ in\ fn\ y\ =>\ (f\ 3)\ ||\ y\ end$ **with** $\Gamma\ =\ \{\ \}$

$$\dfrac{\dfrac{\dfrac{CA}{\{f:\beta\ \to\gamma, x:\beta\}\ \vdash\ 5\ :\ int}\quad\dfrac{VA}{\{f:\ \beta\ \to\gamma,\ x:\ \beta\}\ \vdash\ x\ :\ int}\ (RO)}{\{f:\ \beta\ \to\ \gamma,\ x:\ \beta\}\ \vdash\ 5\ <\ x\ :\ bool}\quad\dfrac{\dfrac{\dfrac{VA}{\{f:\ \beta\ \to\gamma,\ y:\ \varphi\}\ \vdash\ f:\ \psi\ \to\ bool}\quad\dfrac{CA}{\{f:\ \beta\ \to\gamma,\ y:\ \varphi\}\ \vdash\ 3:\ \psi}\ (FA)}{\{f:\ \beta\ \to\ \gamma,\ y:\ \varphi\}\ \vdash\ f\ 3:\ bool}\quad\dfrac{VA}{\{f:\ \beta\ \to\gamma,\ y:\ \varphi\}\ \vdash\ y:\ bool}\ (LCR)}{\{f:\ \beta\ \to\ \gamma,\ y:\ \varphi\}\ \vdash\ (f\ 3)\ ||\ y\ :\ bool}\ (AF)}{\{f:\ \beta\ \to\gamma\}\ \vdash\ fn\ y\ \Longrightarrow\ (f\ 3)\ ||\ y\ end\ :\ \alpha}\ (LF)$$
$$\{\ \}\ \vdash\ let\ fun\ f\ x\ =\ 5\ <\ x\ in\ fn\ y\ \Longrightarrow\ (f\ 3)\ ||\ y\ end\ :\ \alpha$$

$$\begin{array}{l}\beta\ =\ int\\ \gamma\ =\ bool\\ \alpha\ =\ \varphi\ \to\ bool\\ \psi\ =\ \beta\\ \gamma\ =\ bool\\ \psi\ =\ int\\ \varphi\ =\ bool\end{array}\quad\Longrightarrow\quad\begin{array}{l}\beta\ =\ int\\ \gamma\ =\ bool\\ \alpha\ =\ bool\ \to\ bool\\ \sigma\ =\ int\\ \psi\ =\ int\\ \varphi\ =\ bool\end{array}$$

$\Longrightarrow$ Type derivation for code is correct and evaluates to $bool\ \to\ bool$

4 - Type inference for $fn\ x\ \Longrightarrow\ if\ x\ then\ x\ +\ 1\ else\ x(1)$

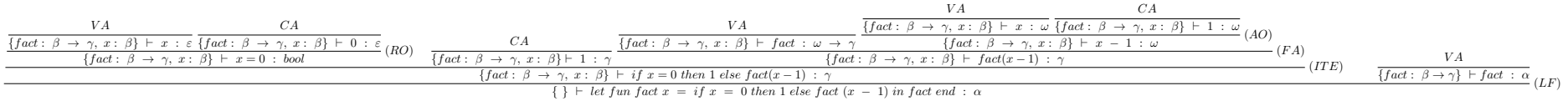
$$\dfrac{\dfrac{\dfrac{VA}{\{x:\ \beta\}\ \vdash\ x\ :bool} \quad \dfrac{\dfrac{VA}{\{x:\ \beta\}\ \vdash\ x\ :\ int} \quad \dfrac{CA}{\{x:\ \beta\}\ \vdash\ x:\ int}\ (AO)}{\{x:\ \beta\}\ \vdash\ x\ +\ 1\ :\ int} \quad \dfrac{\dfrac{VA}{\{x:\ \beta\}\vdash\ x\ :\ \varphi\ \rightarrow\ \gamma} \quad \dfrac{CA}{\{x:\ \beta\}\ \vdash\ 1\ :\ \varphi}\ (FA)}{\{x:\ \beta\}\ \vdash\ x(1):\ \gamma}\ (ITE)}{\{\ x:\ \beta\ \}\ \vdash\ if\ x\ then\ x\ +\ 1\ else\ x(1)\ :\ \gamma}\ (AF)}{\{\ \}\ \vdash\ fn\ x\ \Longrightarrow\ if\ x\ then\ x\ +\ 1\ else\ x(1)\ :\ \alpha}$$

Constraints :

$\varphi = int$
$\beta\ =\ \varphi\ \rightarrow\ \gamma$
$\beta\ =\ bool$
$\gamma\ =\ int$
$\beta\ =\ bool$
$\alpha\ =\ \beta\ \rightarrow\ \gamma$

$$\begin{array}{lll} & Unify(\ \{\varphi = int,\ \beta\ =\ \varphi\ \rightarrow\ \gamma,\ \beta\ =\ bool,\ \gamma\ =\ int,\ \beta\ =\ bool,\ \alpha\ =\ \beta\ \rightarrow\ \gamma\}\ ) & :\ Eliminate \\ \Longrightarrow & Unify(\ \{\beta\ =\ int\ \rightarrow\ \gamma,\ \beta\ =\ bool,\ \gamma\ =\ int,\ \beta\ =\ bool,\ \alpha\ =\ \beta\ \rightarrow\ \gamma\}\ )\ \circ\ \{\varphi\ =\ int\} & :\ Eliminate \\ & Unify(\ \{int\ \rightarrow\ \gamma\ =\ bool,\ \gamma\ =\ int,\ int\ \rightarrow\ \gamma\ =\ bool,\ \alpha\ =\ (int\ \rightarrow\ \gamma)\ \rightarrow\ \gamma\}\ )\ \circ\ \{\beta\ =\ int\ \rightarrow\gamma\}\ \circ\ \{\varphi\ =\ int\} & :\ Unification\ Failure \end{array}$$

5 - Type inference for $let\ fun\ fact\ x\ =\ if\ x\ =\ 0\ then\ 1\ else\ fact\ (x\ -\ 1)\ in\ fact\ end$



Constrains :

$\alpha\ =\ \beta\ \rightarrow\ \gamma$
$\omega =\ int$
$\beta\ =\ \omega$
$\beta\ \rightarrow\ \gamma\ =\ \omega\ \rightarrow\ \gamma$
$\gamma\ =\ int$
$\varepsilon\ =\ int$
$\beta\ =\ \varepsilon$

$Unify(\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma,\ \omega =\ int,\ \beta\ =\ \omega,\ \beta\ \rightarrow\ \gamma\ =\ \omega\ \rightarrow\ \gamma,\ \gamma\ =\ int,\ \varepsilon\ =\ int,\ \beta\ =\ \varepsilon\ \})$: *Eliminate*

$Unify(\ \{\omega =\ int,\ \beta\ =\ \omega,\ \beta\ \rightarrow\ \gamma\ =\ \omega\ \rightarrow\ \gamma,\ \gamma\ =\ int,\ \varepsilon\ =\ int,\ \beta\ =\ \varepsilon\ \})\ \circ\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma\}$: *Eliminate*

$Unify(\ \{\beta\ =\ int,\ \beta\ \rightarrow\ \gamma\ =\ int\ \rightarrow\ \gamma,\ \gamma\ =\ int,\ \varepsilon\ =\ int,\ \beta\ =\ \varepsilon\ \})\ \circ\ \{\omega\ =\ int\}\ \circ\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma\}$: *Eliminate*

$Unify(\ \{int\ \rightarrow\ \gamma\ =\ int\ \rightarrow\ \gamma,\ \gamma\ =\ int,\ \varepsilon\ =\ int,\ int\ =\ \varepsilon\ \})\ \circ\ \{\beta\ =\ int\}\ \circ\ \{\omega\ =\ int\}\ \circ\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma\}$: *Decompose*

$Unify(\ \{int\ =\ int,\ \gamma\ =\ \gamma,\ \gamma\ =\ int,\ \varepsilon\ =\ int,\ int\ =\ \varepsilon\ \})\ \circ\ \{\beta\ =\ int\}\ \circ\ \{\omega\ =\ int\}\ \circ\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma\}$: *Delete*

$Unify(\ \{\gamma\ =\ \gamma,\ \gamma\ =\ int,\ \varepsilon\ =\ int,\ int\ =\ \varepsilon\ \})\ \circ\ \{\beta\ =\ int\}\ \circ\ \{\omega\ =\ int\}\ \circ\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma\}$: *Delete*

$\Longrightarrow\ Unify(\ \{\gamma\ =\ int,\ \varepsilon\ =\ int,\ int\ =\ \varepsilon\ \})\ \circ\ \{\beta\ =\ int\}\ \circ\ \{\omega\ =\ int\}\ \circ\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma\}$: *Eliminate*

$Unify(\ \{\varepsilon\ =\ int,\ int\ =\ \varepsilon\ \})\ \circ\ \{\gamma\ =\ int\}\ \circ\ \{\beta\ =\ int\}\ \circ\ \{\omega\ =\ int\}\ \circ\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma\}$: *Eliminate*

$Unify(\ \{int\ =\ int\ \})\ \circ\ \{\varepsilon\ =\ int\}\ \circ\ \{\gamma\ =\ int\}\ \circ\ \{\beta\ =\ int\}\ \circ\ \{\omega\ =\ int\}\ \circ\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma\}$: *Delete*

$Unify(\{\ \})\ \circ\ \{\varepsilon\ =\ int\}\ \circ\ \{\gamma\ =\ int\}\ \circ\ \{\beta\ =\ int\}\ \circ\ \{\omega\ =\ int\}\ \circ\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma\}$: *Identity*

$\{\ \}\ \circ\ \{\varepsilon\ =\ int\}\ \circ\ \{\gamma\ =\ int\}\ \circ\ \{\beta\ =\ int\}\ \circ\ \{\omega\ =\ int\}\ \circ\ \{\alpha\ =\ \beta\ \rightarrow\ \gamma\}$

$\Longrightarrow \{\varepsilon\ =\ int\ ,\ \gamma\ =\ int,\ \beta\ =\ int,\ \omega\ =\ int,\ \alpha\ =\ int\ \rightarrow\ int\}$