1 - Type derivation for $(x + \ 2 \ > \ 3) \ \&\& \ y$ with $\Gamma \ = \ \{x : \ int, \ y : \ int\}$

$$\dfrac{\dfrac{\dfrac{\dfrac{VA}{\{x: \ int, \ y: \ int\} \ \vdash \ x \ : \ int} \quad \dfrac{CA}{\{x: \ int, \ y: \ int\} \ \vdash \ 2 \ : \ int}}{\{x: \ int, \ y: \ bool\} \ \vdash x \ + \ 2 \ : \ int} (AO) \quad \dfrac{CA}{\{x: \ int, \ y: \ int\} \ \vdash \ 3 \ : \ int}}{\{x: \ int, \ y: \ bool\} \ \vdash \ x \ + \ 2 \ > \ 3 \ \ : \ bool} (RO) \quad \dfrac{VA}{\{x: \ int, \ y: \ bool\} \ \vdash \ y \ : \ bool}}{\{x: \ int, \ y: \ bool\} \ \vdash \ (x \ + \ 2 \ > \ 3) \ \&\& \ y \ : \ bool} (LCR)$$

$\Longrightarrow$ Type derivation for code is correct and evaluates to *bool*

2 - Type Derivation for $fn\ f \implies fn\ x \implies if\ x\ then\ f\ true\ else\ 19$ with $\Gamma = \{\ \}$

$$
\dfrac{\dfrac{\dfrac{\dfrac{VA}{\{f:\ \beta,\ x:\ \delta\} \vdash x\ :\ bool} \qquad \dfrac{\dfrac{VA}{\{f:\ \beta,\ x:\ \delta\} \vdash f:\ \varphi \to \varepsilon} \qquad \dfrac{CA}{\{f:\ \beta,\ x:\ \delta\} \vdash true\ :\ \varphi}}{\{f:\ \beta,\ x:\delta\} \vdash f\ true\ :\ \varepsilon}\,(FA) \qquad \dfrac{CA}{\{f:\ \beta,\ x:\ \delta\} \vdash 19\ :\ \varepsilon}}{\{f:\ \beta,\ x:\ \delta\} \vdash if\ x\ then\ f\ true\ else\ 19\ :\ \varepsilon}\,(ITE)}{\{f:\ \beta\} \vdash fn\ x \implies if\ x\ then\ f\ true\ else\ 19\ :\ \gamma}\,(AF)}{\{\ \} \vdash fn\ f \implies fn\ x \implies if\ x\ then\ f\ true\ else\ 19\ :\ \alpha}\,(AF)
$$

$$
\begin{array}{l}
\varepsilon = int \\
\varphi = bool \\
\beta = \varphi \to \lambda \\
\delta = bool \\
\gamma = \delta \to \varepsilon \\
\alpha = \beta \to \gamma
\end{array}
\implies
\begin{array}{l}
\varepsilon = int \\
\varphi = bool \\
\beta = bool \to int \\
\delta = bool \\
\gamma = bool \to int \\
\alpha = (bool \to int) \to bool \to int
\end{array}
$$

$\implies$ Type derivation for code is correct and evaluates to $(bool \to int) \to bool \to int$

3 - Type derivation for $let\ fun\ f\ x\ =\ 5\ <\ x\ in\ fn\ y\ =>\ (f\ 3)\ ||\ y\ end$ with $\Gamma\ =\ \{\ \}$

$$\dfrac{\dfrac{\dfrac{CA}{\{f:\beta \to \gamma, x:\beta\} \vdash 5 : int} \quad \dfrac{VA}{\{f:\beta \to \gamma,\ x:\beta\} \vdash x : int}}{\{f:\beta \to \gamma,\ x:\beta\} \vdash 5 < x : \gamma}(RO) \quad \dfrac{\dfrac{\dfrac{\dfrac{VA}{\{f:\beta \to \gamma,\ y:\varphi\} \vdash f : \psi \to \sigma} \quad \dfrac{CA}{\{f:\beta \to \gamma,\ y:\varphi\} \vdash 3 : \psi}}{\{f:\beta \to \gamma,\ y:\varphi\} \vdash f\ 3 : bool}(FA) \quad \dfrac{VA}{\{f:\beta \to \gamma,\ y:\varphi\} \vdash y : bool}}{\{f:\beta \to \gamma,\ y:\varphi\} \vdash (f\ 3)\ ||\ y : bool}(LCR)}{\{f:\beta \to \gamma\} \vdash fn\ y \Longrightarrow (f\ 3)\ ||\ y\ end : \alpha}(AF)}{\{\ \} \vdash let\ fun\ f\ x = 5 < x\ in\ fn\ y \Longrightarrow (f\ 3)\ ||\ y\ end : \alpha}(LF)$$

$$\begin{aligned} \beta &= int \\ \gamma &= int \\ \alpha &= \varphi \to bool \\ \psi &= \beta \\ \sigma &= \gamma \\ \psi &= int \\ \varphi &= bool \end{aligned} \quad \Longrightarrow \quad \begin{aligned} \beta &= int \\ \gamma &= int \\ \alpha &= bool \to bool \\ \sigma &= int \\ \psi &= int \\ \varphi &= bool \end{aligned}$$

$\Longrightarrow$ Type derivation for code is correct and evaluates to $bool \to bool$