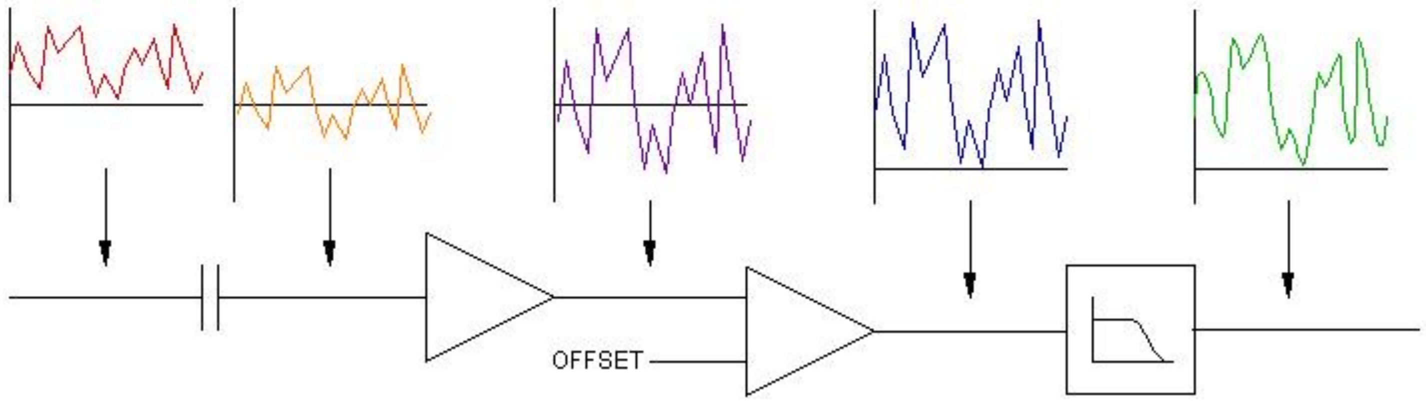
Introduction to Signal Conditioning



Presented by

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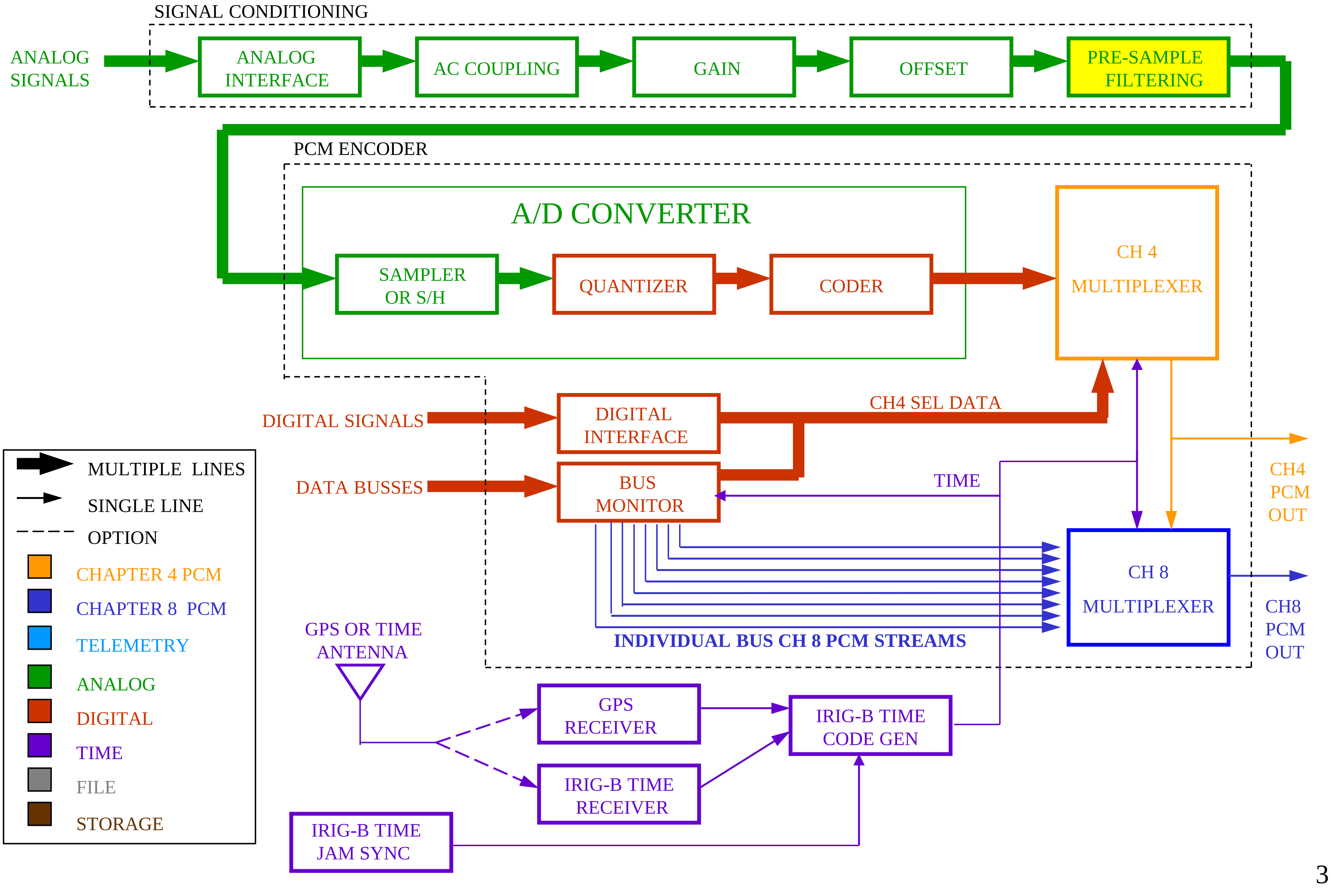
Data Acquisition System



Introduction

Filtering is a very important element of signal conditioning. Filters have many different characteristics which can greatly effect a measurement, and it is important to understand these characteristics in order to select the correct filter parameters for each specific measurement.

Mathematical Disclaimer

Filter designs are based on mathematical formulas. It would be impossible to explain how filtering works without referencing mathematical concepts.

However, where possible, some of these mathematical concepts will be shown graphically to get a non-mathematical understanding of filter properties.

The important equations will be highlighted in yellow.

Purpose of Pre-Sample Filtering

- To avoid aliasing effects during sampling.
- To remove unwanted data within a certain frequency range.
- To minimize noise.

Note:
Aliasing will be covered in Sampling Theory training.

Pre Sample Filters

The object of pre sample filtering is to attenuate unwanted frequency components such that they no longer contribute to the signal.

The amount of attenuation depends on the frequency of the incoming signal.

One-third of this presentation is preliminary information that must be covered in order to understand the characteristics of a filter.

Logarithms and dB

Numbers Raised to a Power

- A number N (called the base) raised to the power of *p* is represented as N^p.
- $N^0 = 1$ (any number raised to the power of 0 is 1)
- $N^1 = N$, ex: $4^1 = 4$
- $N^2 = N \times N$, ex: $3^2 = 3 \times 3 = 9$
- $N^3 = N \times N \times N$, ex: $5^3 = 5 \times 5 \times 5 = 125$
- $N^4 = N \times N \times N \times N$, ex: $2^4 = 2 \times 2 \times 2 \times 2 = 16$

and so on for $p = 5, 6, 7, \ldots$

Some Concepts to Understand First: What are logarithms?

- Logarithms express numbers in terms of the power of a specified base.

  $x = \log_b y$

- x is the power of the base, *b* that will obtain the quantity y, or in other words, what power, x of the base, *b* will yield y?
- In mathematical terms, $b^x = y$
- For our purposes, the base of the logarithm will be **10**. So we would use *log base ten* or in mathematical terms, $\log_{10}$.

Some Concepts to Understand First: Logarithms Example

- What is x, where $x = \log_{10}100$?
- In words, this is saying what power of 10 gives you the quantity 100?

or…

$$10^x = 100$$

- since $10^2 = 100$, then $x = 2$
- so

$$\log_{10} 100 = 2$$

Some Concepts to Understand First: Logarithms Example

- What is x, where $x = \log_{10}215$?
- In words, this is saying what power of 10 gives you the quantity 215?

or…

$$10^x = 215$$

- This is not obvious just by looking at it, because 215 is not a whole number power of 10. For this calculation, you must use a calculator.

$$\log_{10}215 = 2.33243846\ldots \text{ or about } 2.33$$

Some Concepts to Understand First: Undefined Logarithms

- What is x, where $x = \log_{10} 0$?
- In words, this is saying what power of 10 gives you a quantity of 0 ?

or…

$$10^x = 0$$

- There is no such number x that will give us 0. You can get us close by having x be a very large negative number ($-\infty$), but it will never equal 0.
- So taking $\log_{10} 0$ on your calculator will give you an error.

Some Concepts to Understand First: Undefined Logarithms

- What is x, where $x = \log_{10} -412$?
- In words, this is saying what power of 10 gives you a quantity of -412 ?

or…

- $10^x = -412$
- There is no such number x that will give us -412 or any other negative value for that matter.
- So taking $\log_{10} -412$ on your calculator will also give you an error.

Some Concepts to Understand First: What is a dB?

- A dB stands for decibel and it is a unit of power gain or attenuation. It simply describes the power gain logarithmically.

- It is similar to saying that a 30-inch long board is 2.5 feet long.

Converting Power Gain into dB

- The decibel equation is the logarithmic relationship of output **power** to input **power**.
- A factor of 10 results from the prefix *deci* in decibel.

$$G_{db} = 10 \log\left(\frac{P_{out}}{P_{in}}\right)$$

Relating Power and Voltage

- Many measurements are of voltage, we want to have voltage quantities in our equation.
- Voltage is related to power by,

$$P = \frac{V^2}{R}$$

Relating Power and Voltage

$$\frac{Pout}{Pin} = \frac{\left(\frac{Vout^2}{R}\right)}{\left(\frac{Vin^2}{R}\right)} = \frac{\left(\frac{\mathrm{Vout}^2}{\mathrm{Vin}^2}\right)}{\left(\frac{\mathrm{R}}{\mathrm{R}}\right)}$$

The R's will cancel, and we are left with

$$\frac{Pout}{Pin} = \frac{Vout^2}{Vin^2} = \left(\frac{Vout}{Vin}\right)^2$$

Power is proportional to voltage squared

Converting Voltage Gain into dB

$$\frac{Pout}{Pin} = \left(\frac{Vout}{Vin}\right)^2$$

Substitute into the decibel equation

$$G_{dB} = 10\, log\left(\frac{P_{out}}{P_{in}}\right)$$

$$G_{dB} = 10\, log\left(\frac{V_{out}}{V_{in}}\right)^2$$

The logarithmic property of exponents states that

$$\log(\boldsymbol{y^n}) = \boldsymbol{n} \cdot \boldsymbol{log}(\boldsymbol{y})$$

$$G_{db} = 20\log\left(\frac{V_{out}}{V_{in}}\right) = 10\log\left(\frac{P_{out}}{P_{in}}\right)$$

Converting Voltage Gain Into dB

- Now that we have done all the math, decibels can now be calculated in the following way. If you have a system where the magnitude of the gain or attenuation is:
- $G_v = V_{out}/V_{in}$, then

$$\boxed{G_{db} = 20\log\left(\frac{V_{out}}{V_{in}}\right) = 10\log\left(\frac{P_{out}}{P_{in}}\right)}$$

If G > 1, then G_{dB} is positive (Gain)
If G = 1, then G_{dB} is 0 (no Gain / no Attenuation)
If 0 < G < 1, then G_{dB} is negative (Attenuation)
G is always positive since we are only dealing with the magnitude of the gain or attenuation.

Some Decibel Calculation Examples

If you have a power gain of 2 (power doubled), what is the gain in dB?

$$G_{dB} = 10\log_{10}(2)$$
$$G_{dB} = 10 \times 0.301$$
$$G_{dB} = 3dB\ of\ gain$$

If you have a power gain of 0.5 (power halved), what is the gain in dB?

$$G_{dB} = 10\log_{10}(0.5)$$
$$G_{dB} = 10 \times (-0.301)$$
$$G_{dB} = -3\ dB\ gain\ or\ +3\ dB\ of\ attenuation$$

If you have a power gain of 10, what is the gain in dB?

$$G_{dB} = 10\log_{10}(10)$$
$$G_{dB} = 10 \times (1)$$
$$G_{dB} = 10\ dB\ gain$$

Some Decibel Calculation Examples

If you have a ***voltage*** gain of 2 (voltage doubled), what is the gain in dB?

$$G_{dB} = 20\log_{10}(2)$$
$$G_{dB} = 20 \times 0.602$$
$$G_{dB} = 6$$
$$G_{dB} = 6\ dB\ of\ gain$$

If you have a ***voltage*** gain of 0.5 (voltage halved), what is the gain in dB?

$$G_{dB} = 20\log_{10}(0.5)$$
$$G_{dB} = 20 \times (-0.602)$$
$$G_{dB} = -6$$
$$G_{dB} = -6\ dB\ gain\ or\ +6\ dB\ of\ attenuation$$

Some Decibel Calculation Examples

- If you have a **voltage** gain of 126, what is the gain in dB?

$$G_{dB} = 20\log_{10} G_v$$
$$G_{dB} = 20\log_{10}(126)$$
$$G_{dB} = 20 \times 2.1$$
$$G_{dB} = 42$$
$$G_{dB} = 42dB\ of\ gain$$

Converting dB's To Voltage Gain

- Taking $G_{dB} = 20 \log_{10} G$, and solving for G, we get:

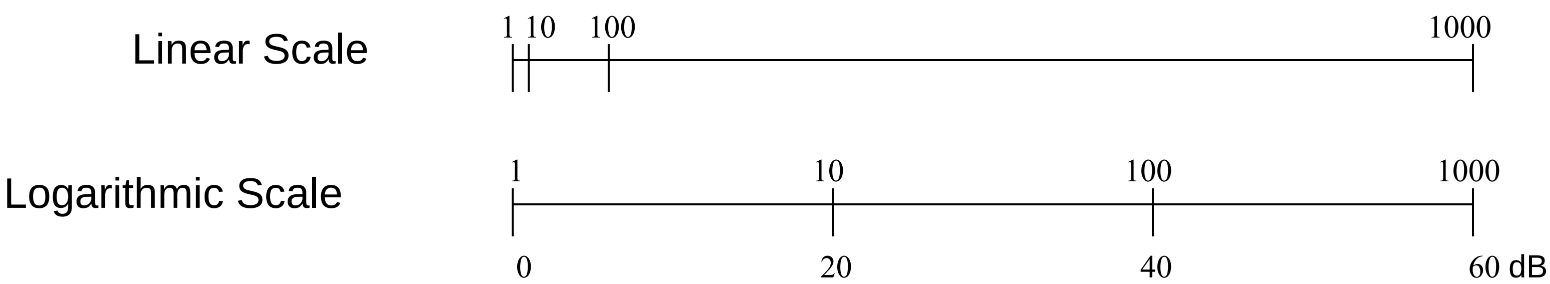
$$\boxed{G_v = 10^{\frac{G_{db}}{20}}}$$

45 dB would be:
$G_v = 10^{45/20}$
$G_v = 10^{2.25}$
$G_v = 177.83$

-36 dB would be:
$G_v = 10^{-36/20}$
$G_v = 10^{-1.8}$
$G_v = .01585$

Why Use Decibels?

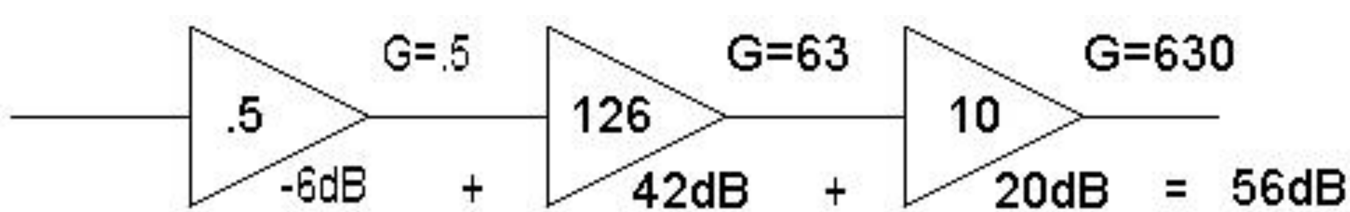
Decibels are used because large ranges in gain can be represented on a smaller, compressed scale. For example, to represent gains ranging from 1 to 1000 may be difficult to put on a linear scale, but in dB the range is from 0 to 60. Also, notice that the logarithmic scale has equal distances between 1 to 10, 10 to 100, and 100 to 1000.



Why Use Decibels?

Another advantage is that because decibels are a logarithmic quantity, they follow all the rules of logarithms, such that if you have several gains in series (normally multiplied), they are added in dB instead of being multiplied.

$$\log(x \cdot y) = \log(x) + \log(y)$$



dB vs dB_x

- dB is a unitless ratio
  - It is only gain or attenuation
- dB_x is a ratio with a known physical quantity

dBm	Power in decibels above 1 milliwatt
dBW	Power decibels above 1 watt
dBi	Antenna gain in decibels above isotropic

Signal Conditioning

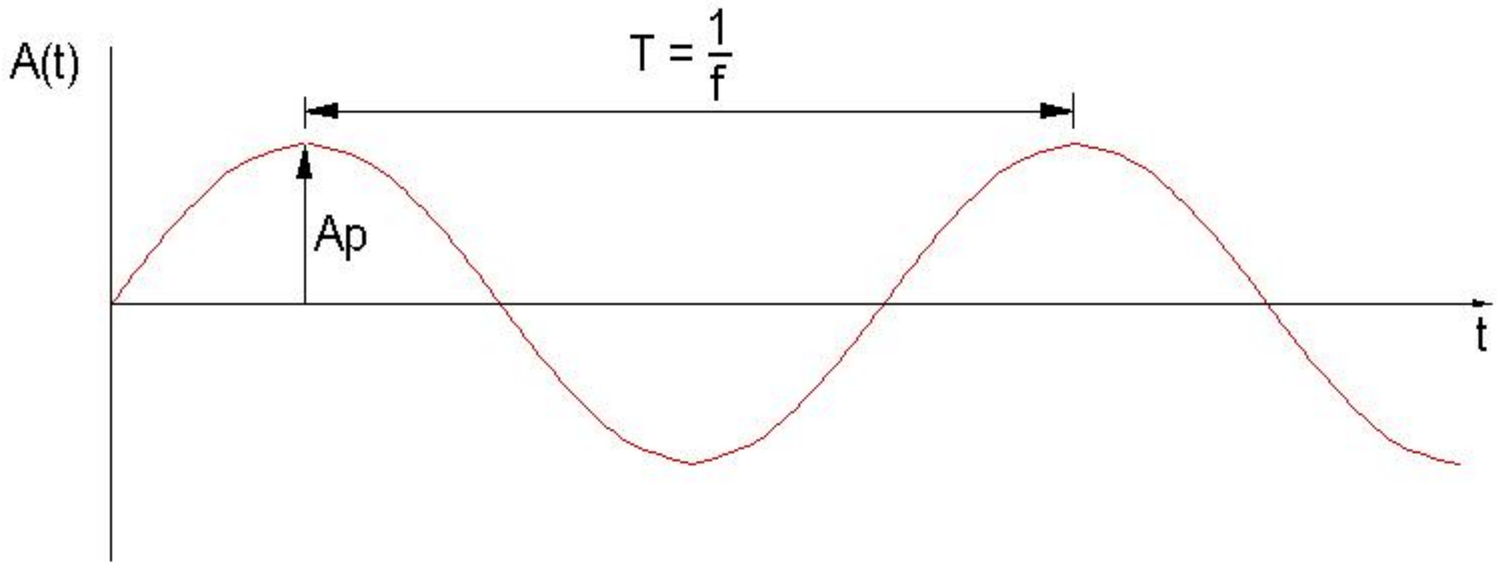
Frequencies and Signals

Some Concepts to Understand First: The Frequency Domain

- We are comfortable with measurements in the time domain. This is what we see on an oscilloscope or on a strip chart.

- Since filters are described in terms of frequency, they are represented in the frequency domain (as displayed on a spectrum analyzer). This makes a filter's characteristics easier to see.

A Sine Wave in the Time Domain



- In the time-domain, the horizontal (x) axis is time, and the vertical (y) axis is the amplitude of the signal. The sine wave has a period T, that is measured between two like points on the sine wave. The frequency ($f = 1/T$) of the sine wave is the inverse of the period.

Mathematical Conversion to the Frequency Domain: The Fourier Transform

- To mathematically convert functions from the time domain to the frequency domain, the Fourier Transform is used.
- If f(*t*) is a function with respect to time *t*, then the Fourier Transform of f(*t*) is

$$F(f) = \int_{-\infty}^{+\infty} f(t) e^{-j2\pi ft} dt$$

f(*t*) is a function of time

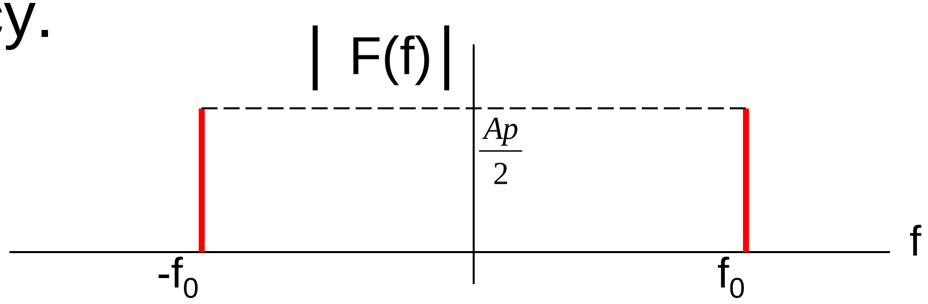
F(*f*) is a function of frequency

Sine Wave in the Frequency Domain

- A sine wave of amplitude A_p and frequency f_o is represented mathematically as $f(t) = A_p \sin(2\pi f_o t)$,
- doing all the math, its Fourier Transform would be:

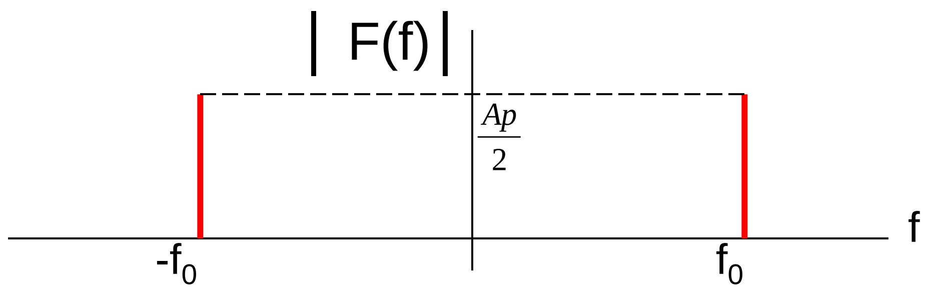
  $F(f) = (A_p/2)\ j\, [d(f - f_o) - d(f + f_o)]$
- where d is the Dirac (or Impulse) Function, defined as

  d(0) =1 and zero everywhere else.
- Observing the magnitude only, and ignoring the phase, the sine wave in the frequency domain would look like this with the x axis now representing the frequency.



Sine Wave in the Frequency Domain

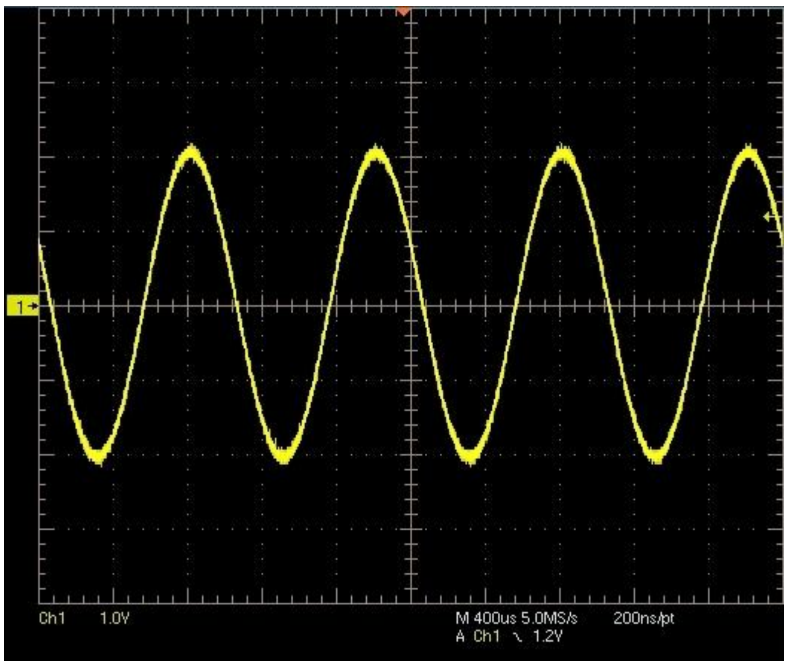
This is a mathematical representation of the sine wave in the frequency domain, which is why you will see that a negative frequency of $-f_o$ is present.

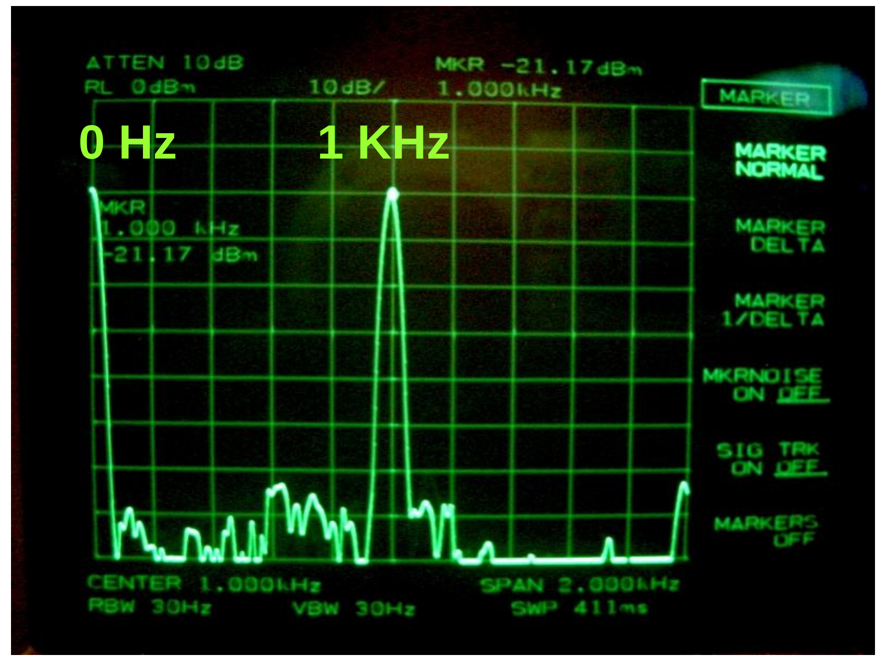


This will also be present on a spectrum analyzer, where negative frequency spikes are seen to the left of 0 Hz. Normally the negative frequencies are ignored.

Time Domain/Frequency Domain

The main thing to understand is that a sine wave will appear as a spike at the frequency f_o in the frequency domain.

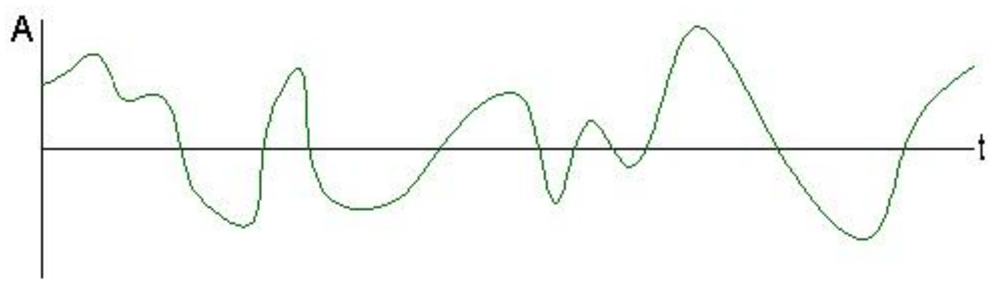




Screen shots of a 1KHz sine wave on an oscilloscope (time domain) and a spectrum analyzer (frequency domain). There is also a spike at –1 KHz (off to the left and not shown)

Multiple Sine Waves

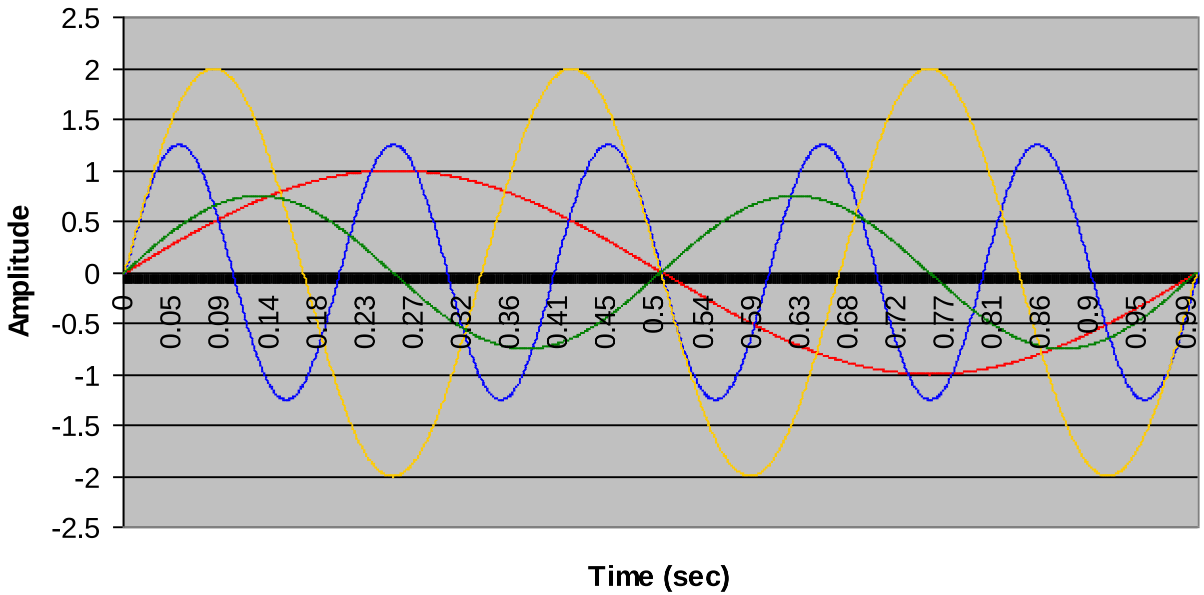
Signals that are measured on an aircraft are not pure sine waves. They contain numerous sine waves of different frequencies, amplitudes and phases. The sensor within the transducer detects the sum of all the frequency components present in the physical measurements made on the aircraft. The transducer then outputs an electrical signal which is proportional to the physical measurement.



Multiple Sine Waves

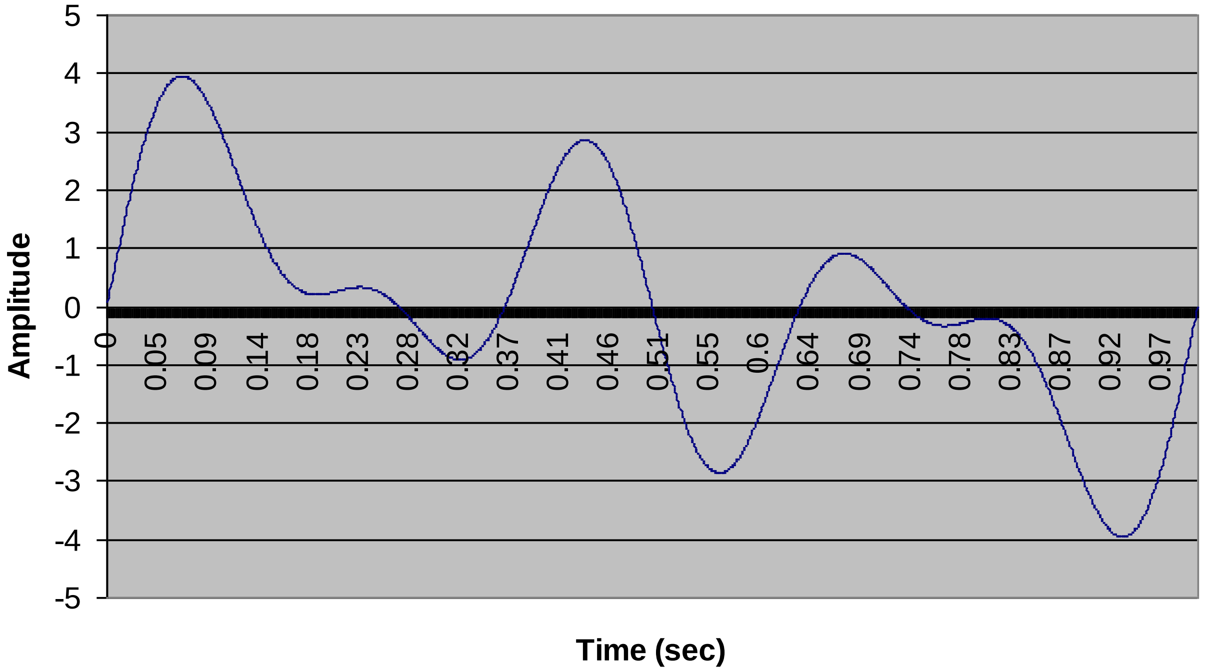
A composite signal is made up of the following sine waves:

color	amplitude	frequency	signal function
*	1.00	1 Hz	$\sin(2\pi t)$
*	0.75	2 Hz	$.75\sin(4\pi t)$
*	2.00	4 Hz	$2\sin(8\pi t)$
*	1.25	5 Hz	$1.25\sin(10\pi t)$



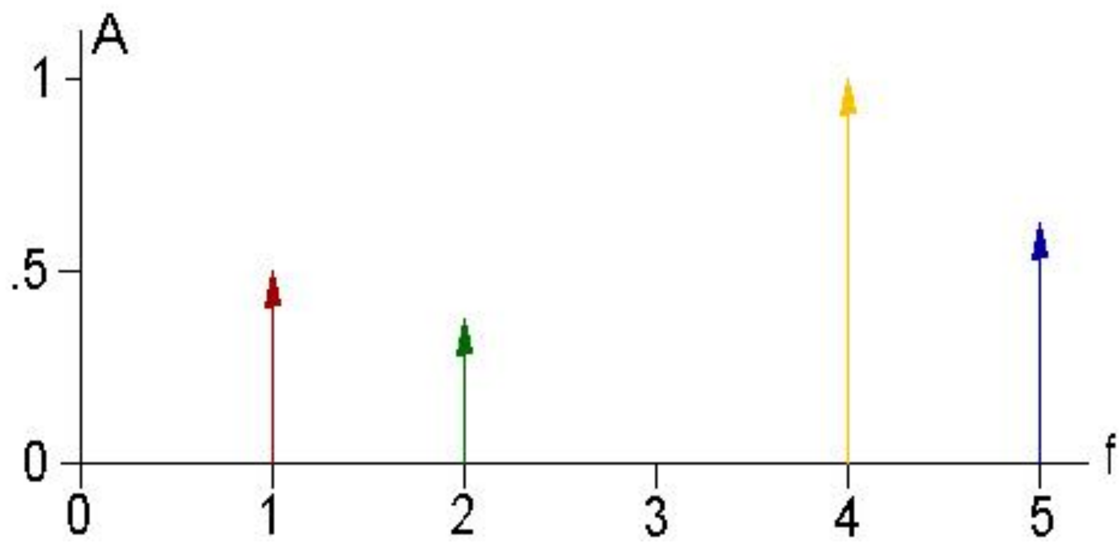
Multiple Sine Waves

Sum the four sine waves together and this is the resultant signal

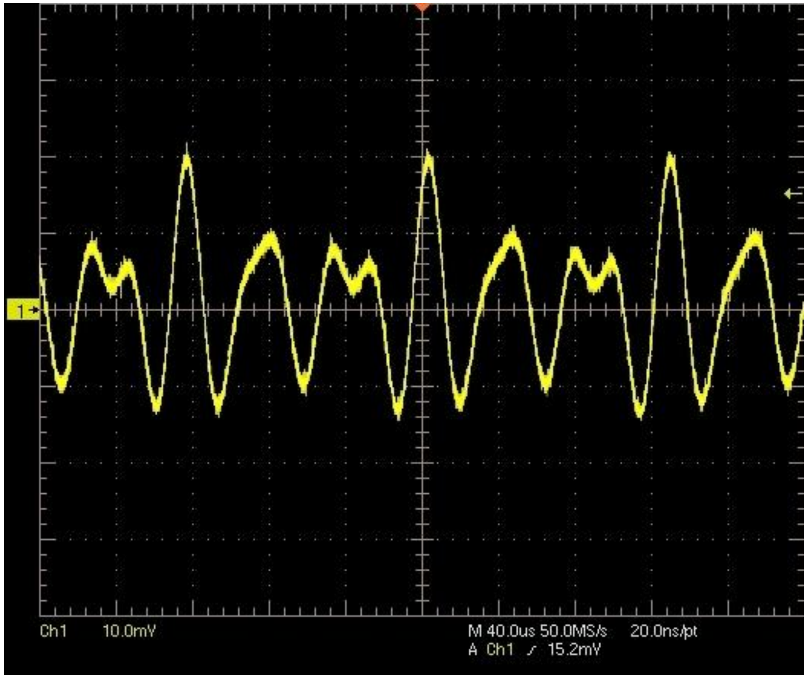


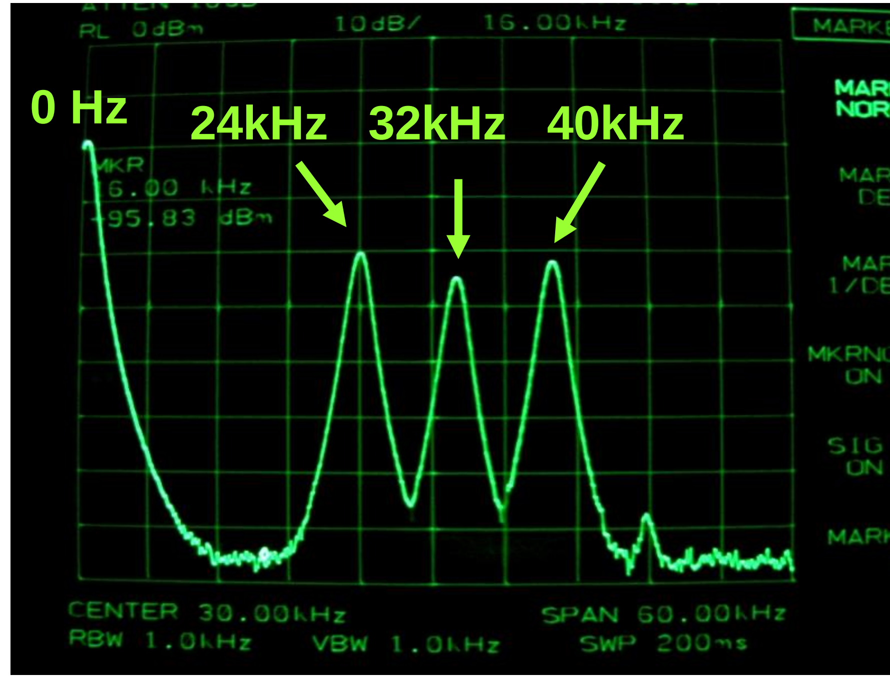
Multiple Sine Waves

- In the frequency domain, each sine wave will have a spike at its respective frequency and amplitude as shown below.



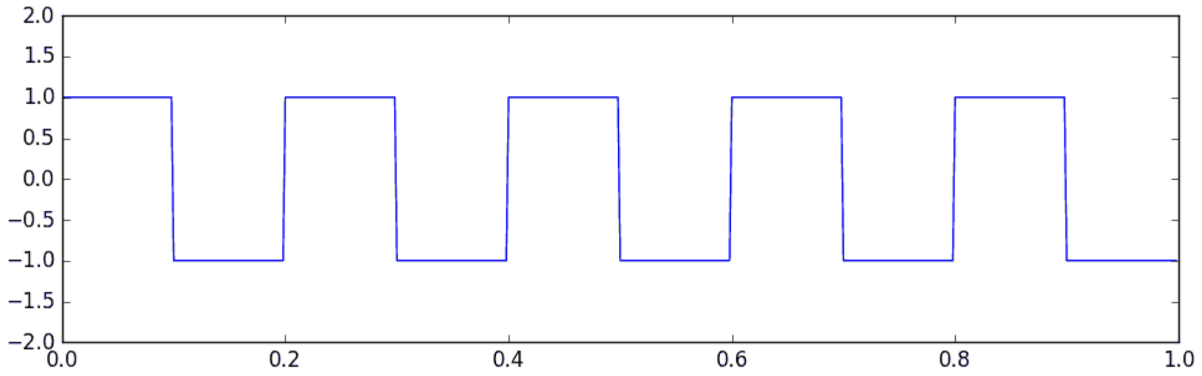
Time Domain/Frequency Domain





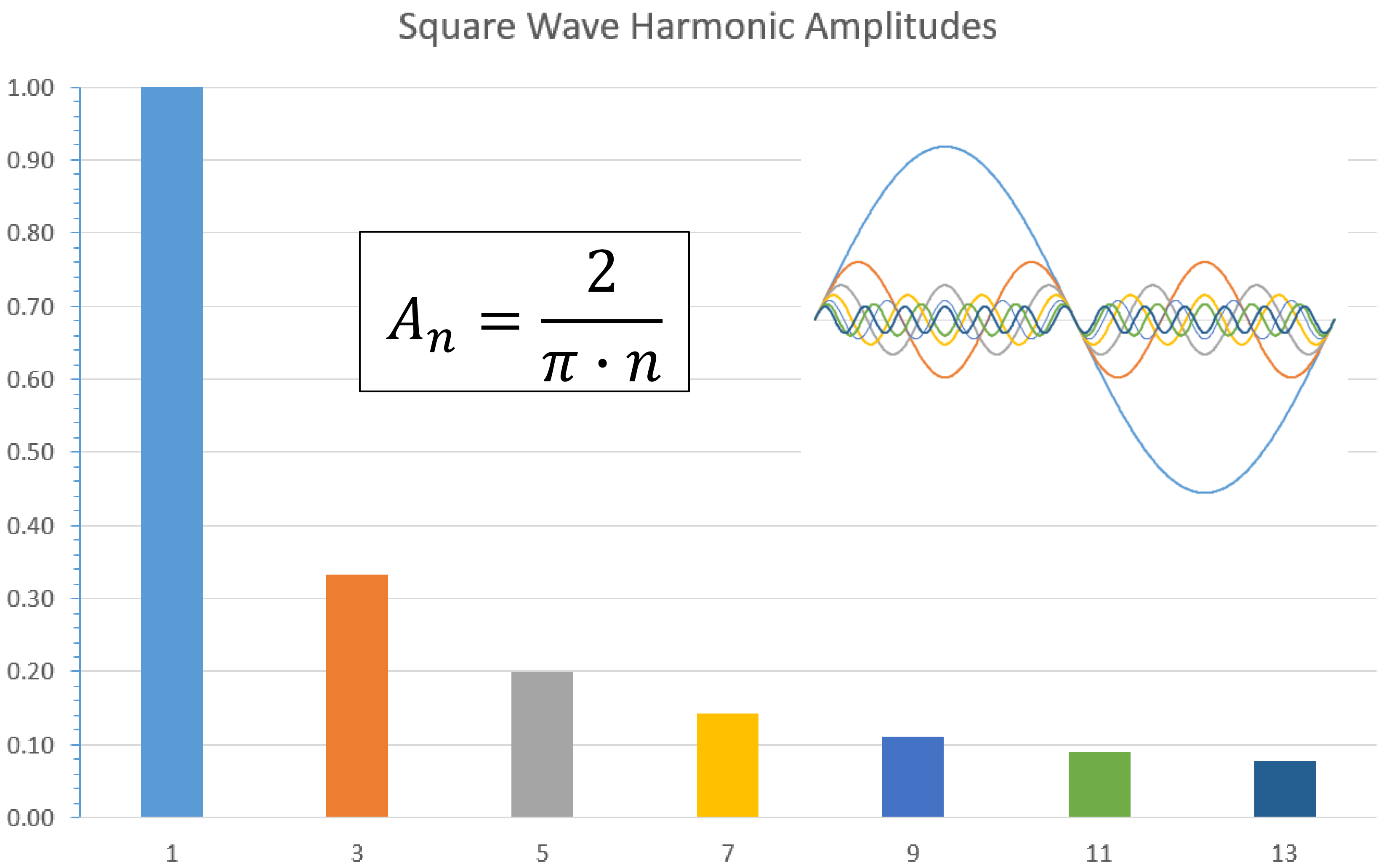
A composite of three sine waves with frequencies of 24 kHz, 32 kHz, and 40 kHz as seen on an oscilloscope (time domain) and a spectrum analyzer (frequency domain).

Square Waves



- **Square waves have frequency components at odd harmonics (i.e. multiples) of the square wave frequency**
  - **f_c, f_cx3, f_cx5, etc.**
  - **No energy at even harmonics**
- **Amplitude of harmonic levels decreases with frequency**
  - **But the number of harmonic components is infinite**

Square Waves

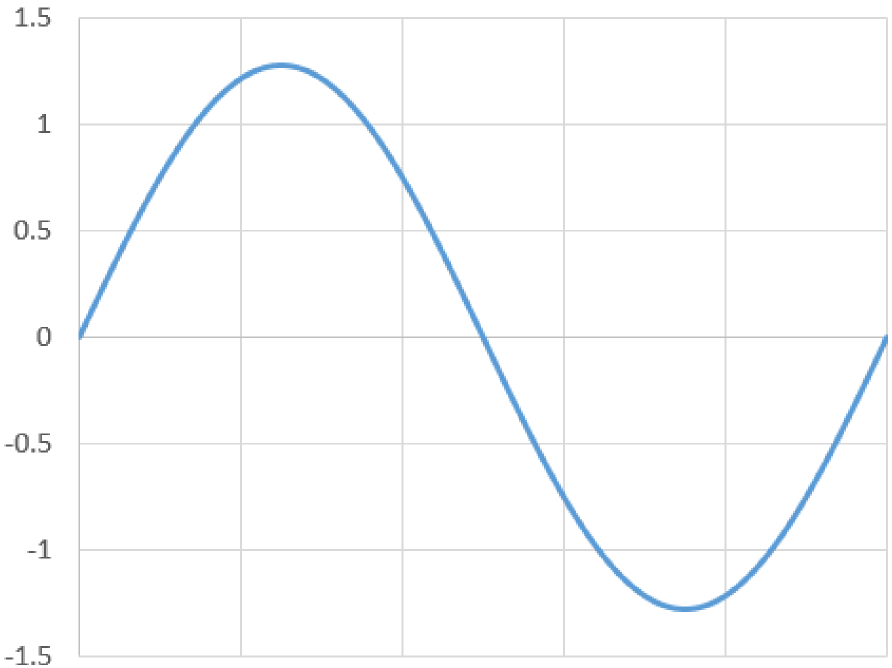


$$A_n = \frac{2}{\pi \cdot n}$$

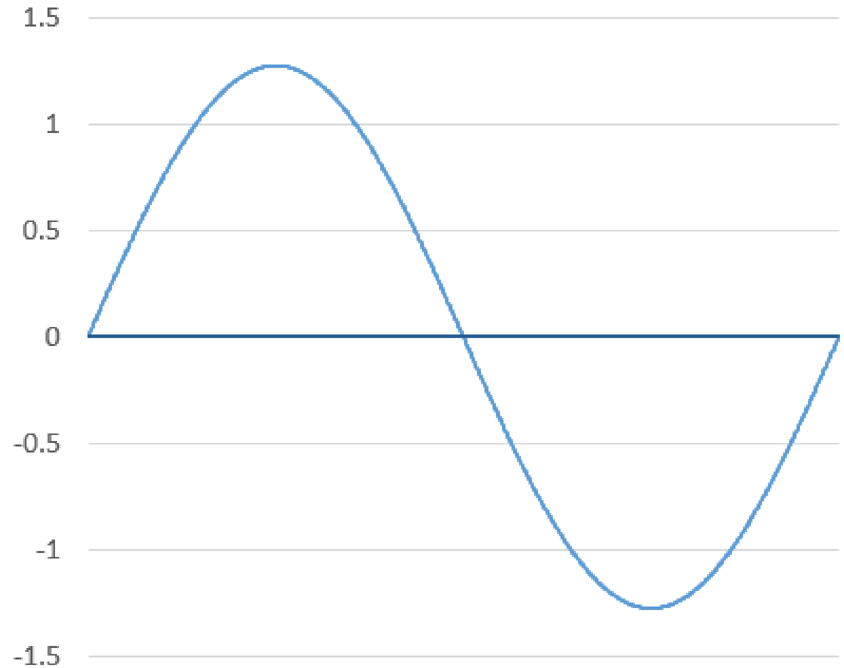
Building a Square Wave

Fundamental

<u>Sum</u>



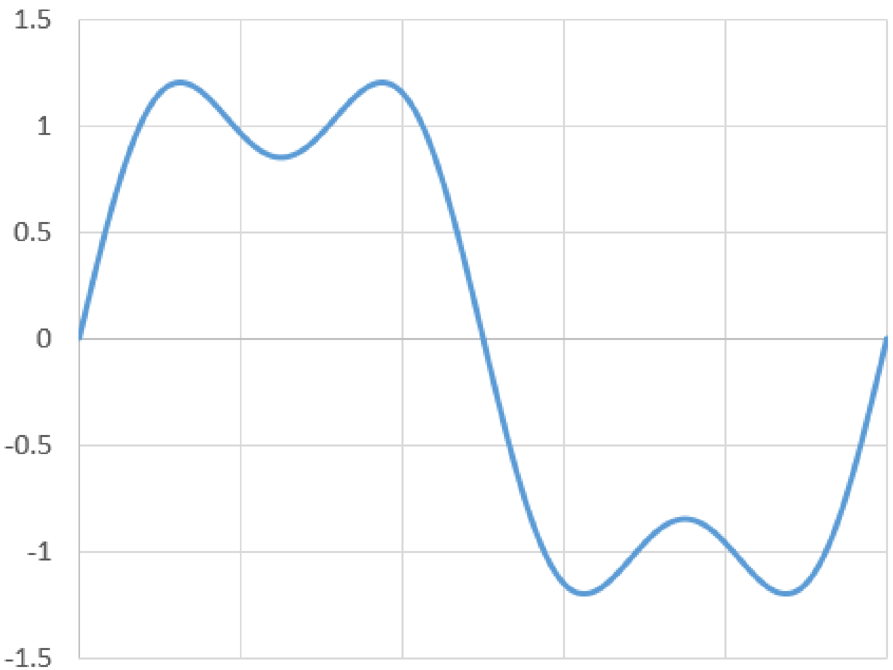
<u>Components</u>



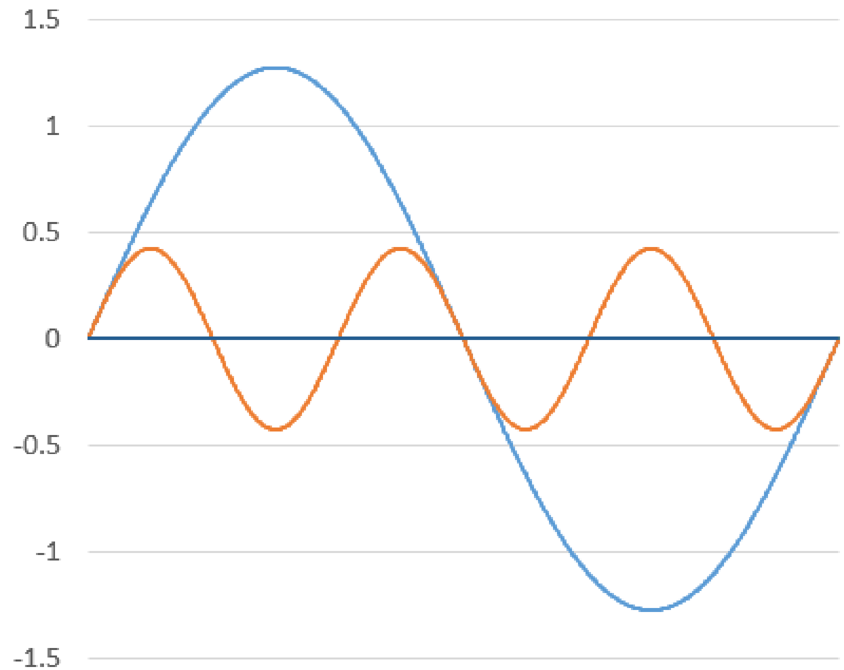
Building a Square Wave

Fundamental + 1 Harmonic

Sum



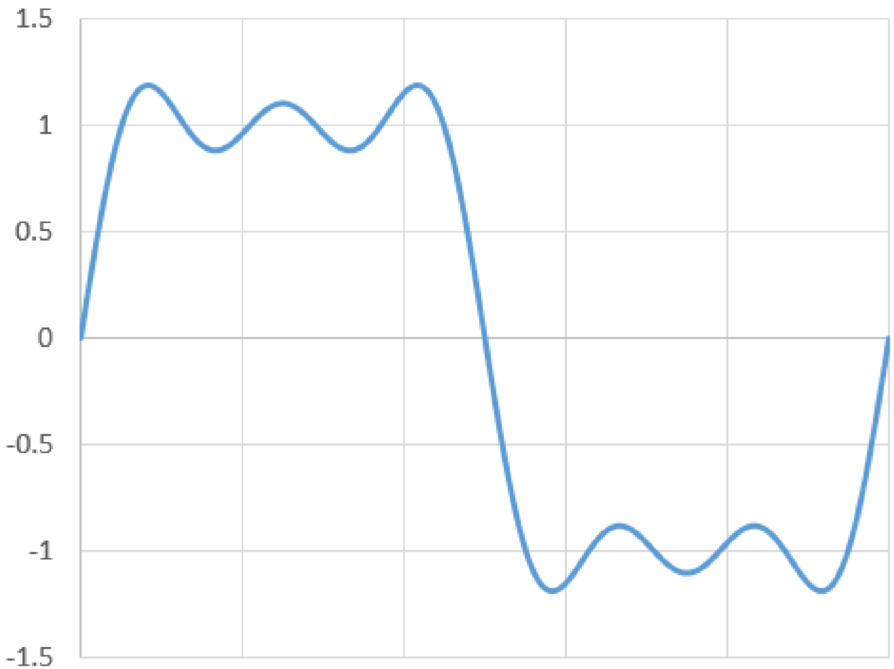
Components



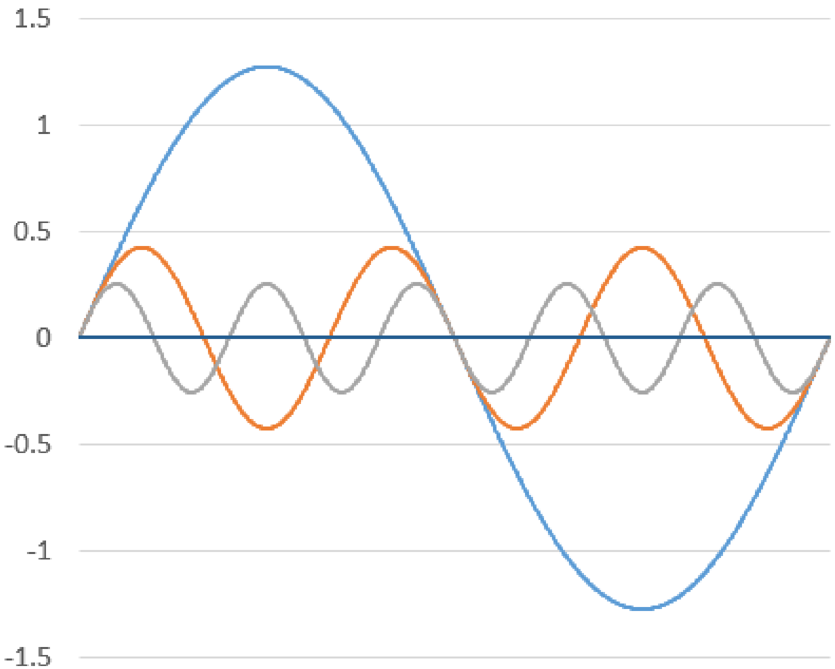
Building a Square Wave

Fundamental + 2 Harmonics

<u>Sum</u>



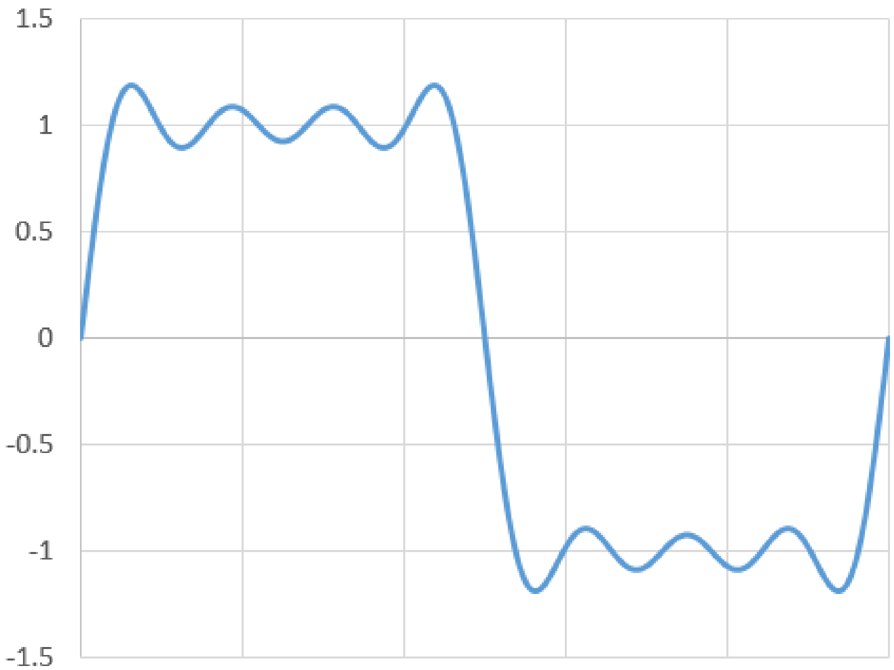
<u>Components</u>



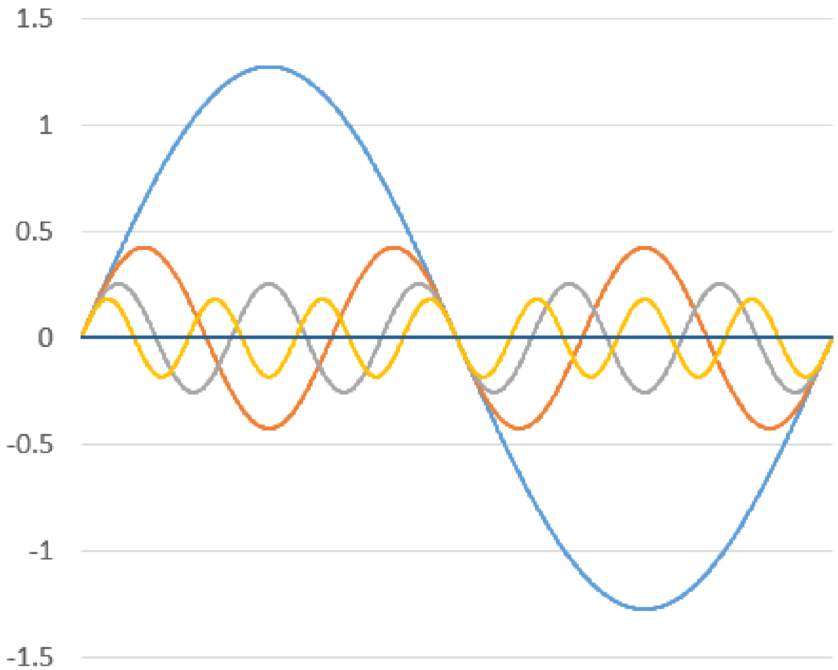
Building a Square Wave

Fundamental + 3 Harmonics

Sum



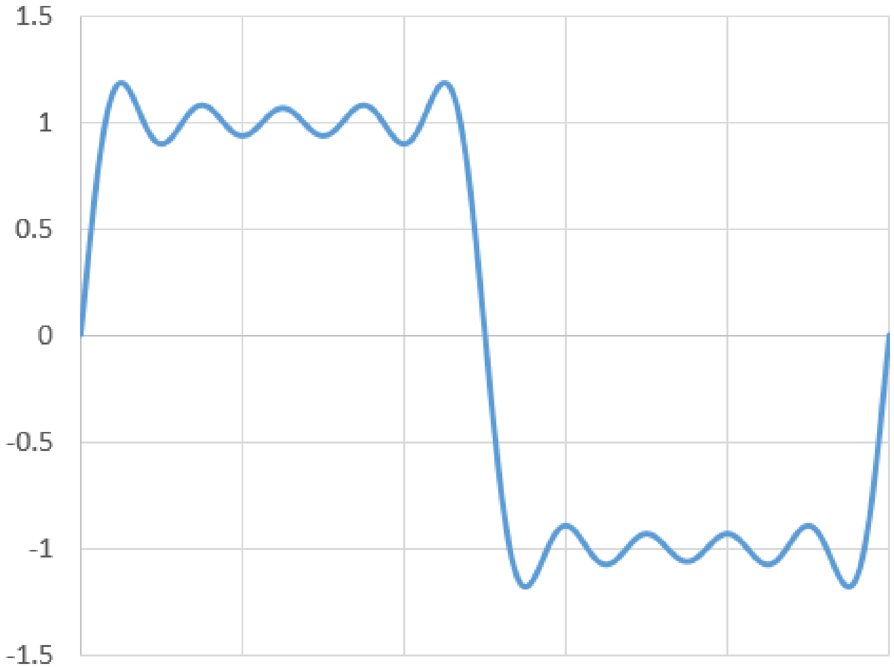
Components



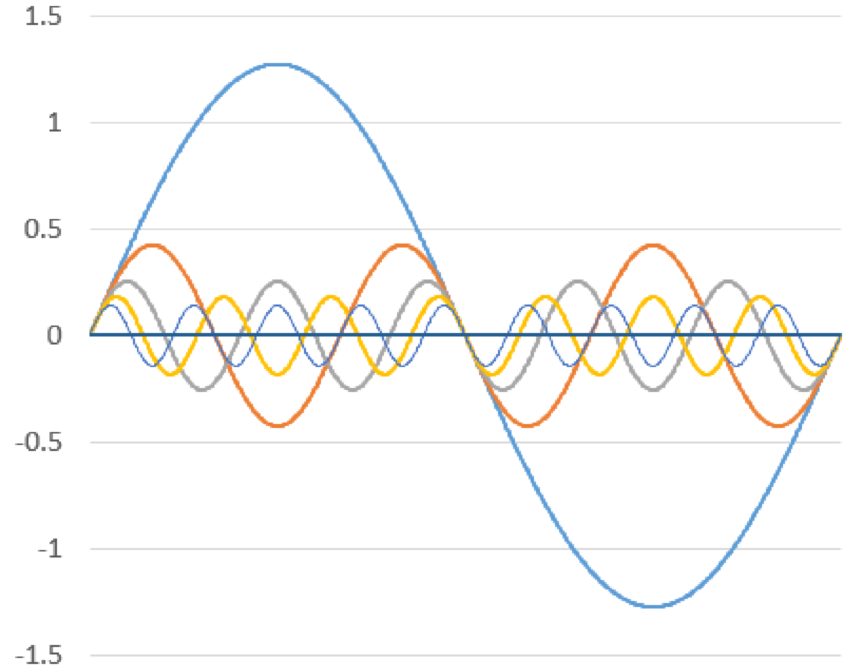
Building a Square Wave

Fundamental + 4 Harmonics

Sum



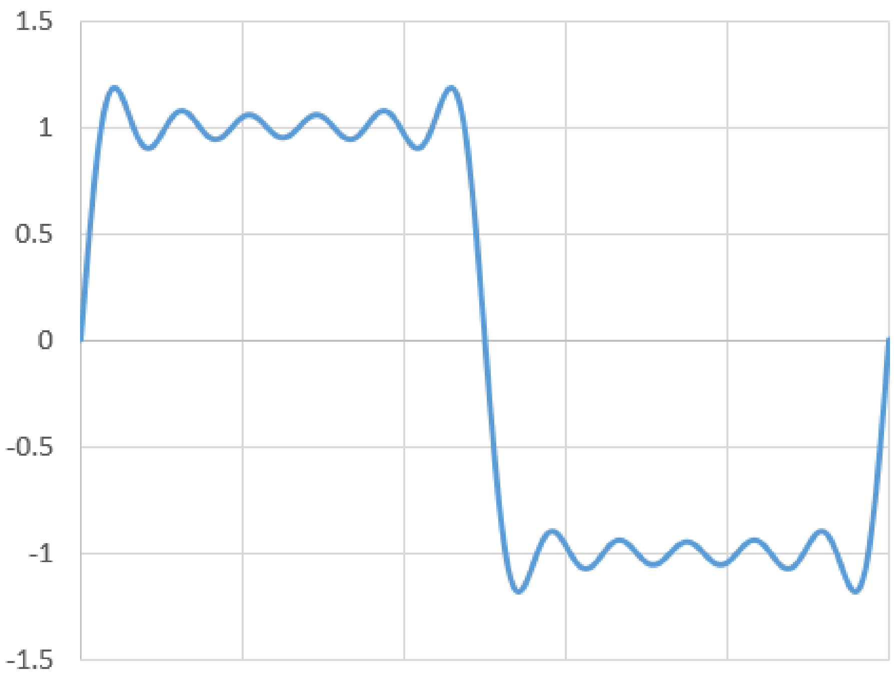
Components



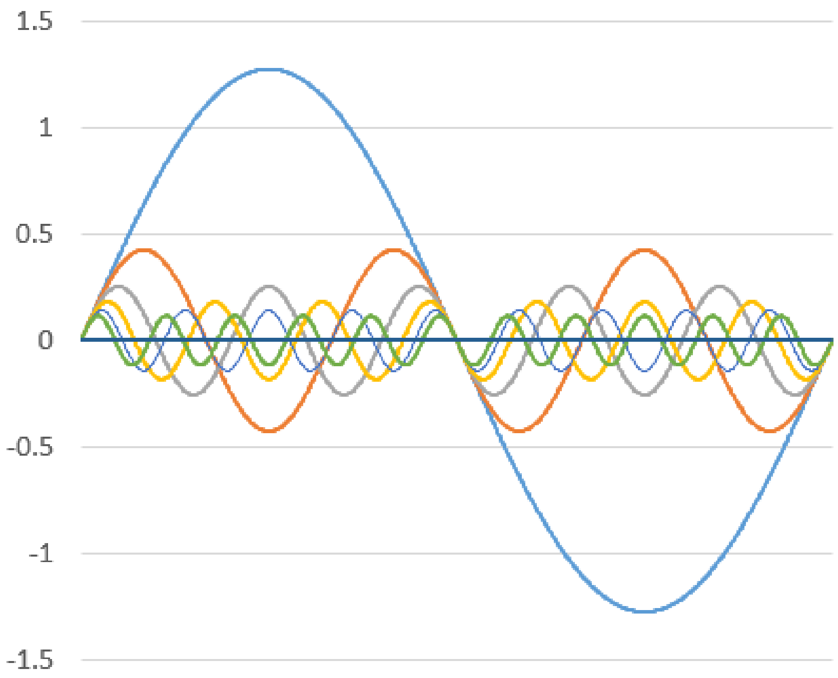
Building a Square Wave

Fundamental + 5 Harmonics

<u>Sum</u>



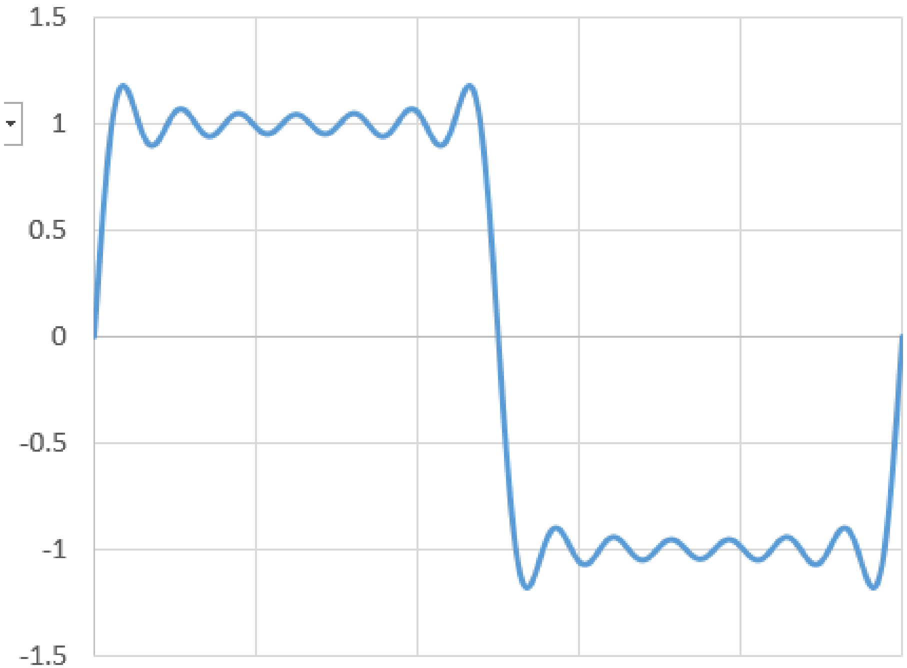
<u>Components</u>



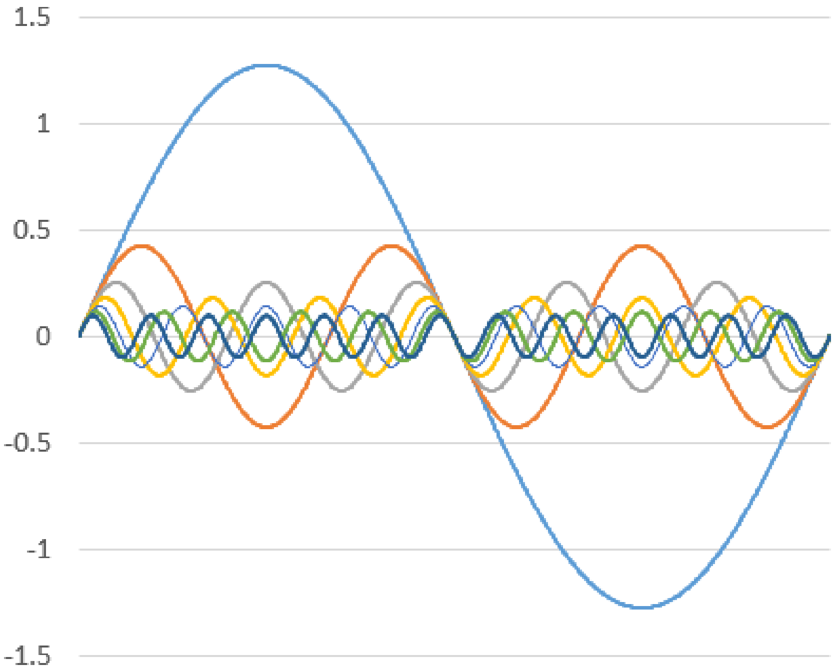
Building a Square Wave

Fundamental + 6 Harmonics

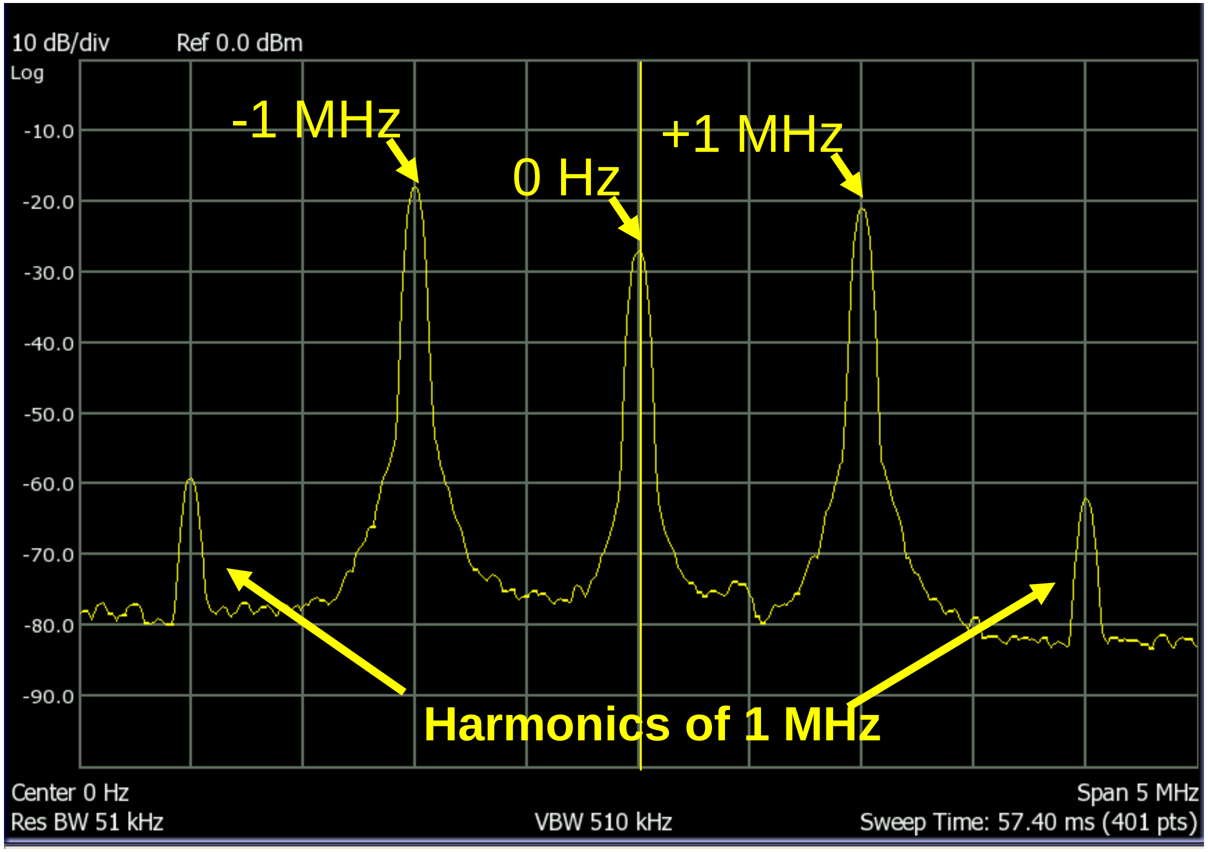
Sum



Components

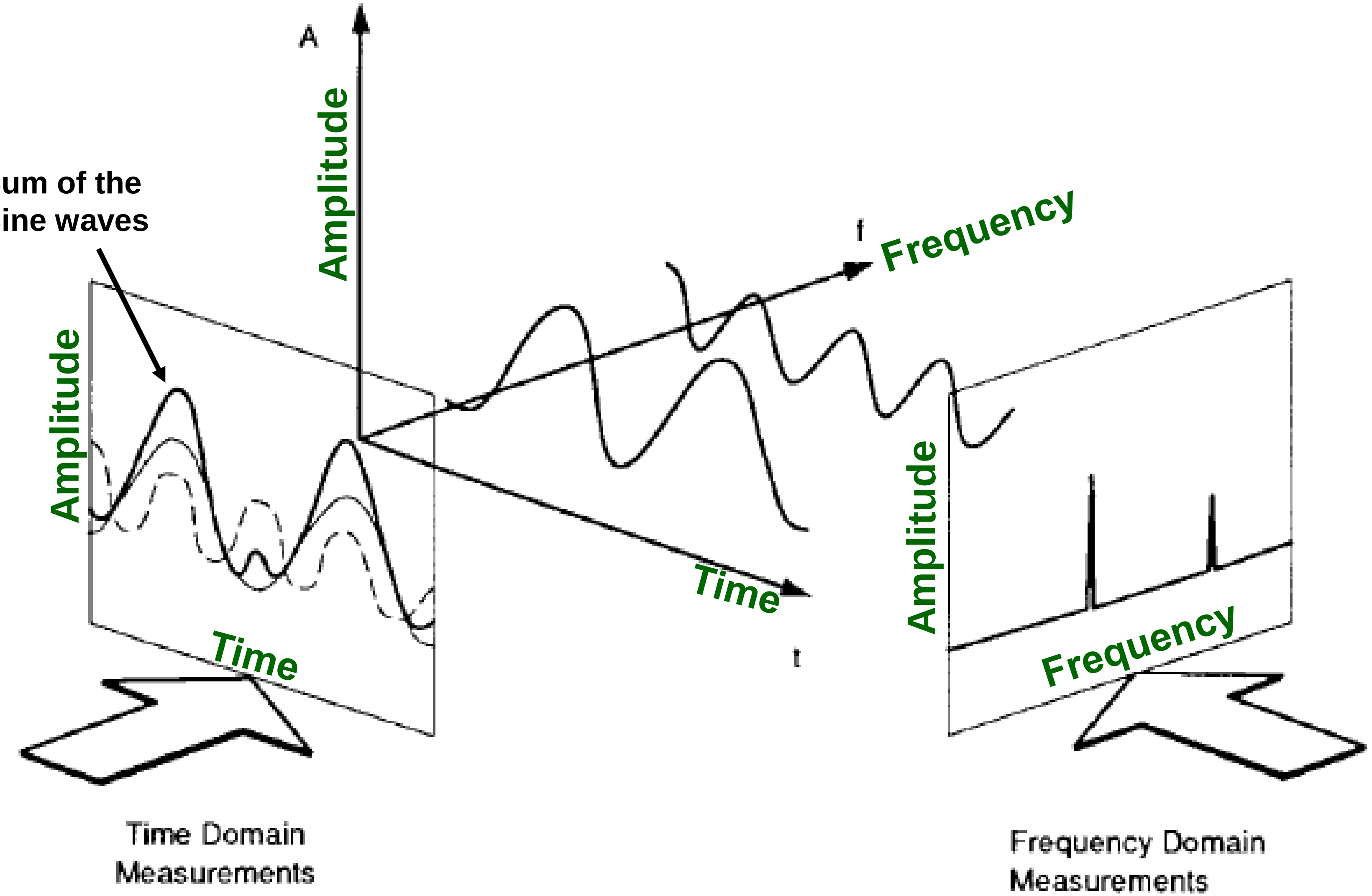


Side Band Levels



- 2nd harmonic is about ~45 dB below the peak
- -46 dB = -20 dB - 20 dB - 6 dB = 1/10 * 1/10 * 1/2 = 1/200 = 0.005
- For a 1 volt signal the 3rd harmonic signal would be about 5 mVolt

Time Domain/Frequency Domain



Oscilloscope View

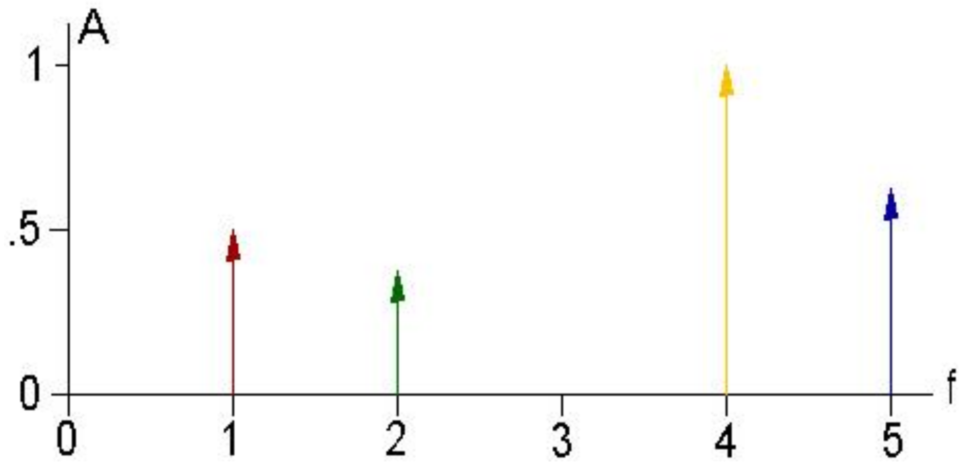
Spectrum Analyzer View

Signal Conditioning

Filters

The Purpose of Filters

- *The object of pre sample filtering is to attenuate unwanted frequency components* [4 Hz and 5 Hz] *such that they no longer contribute to the signal.*



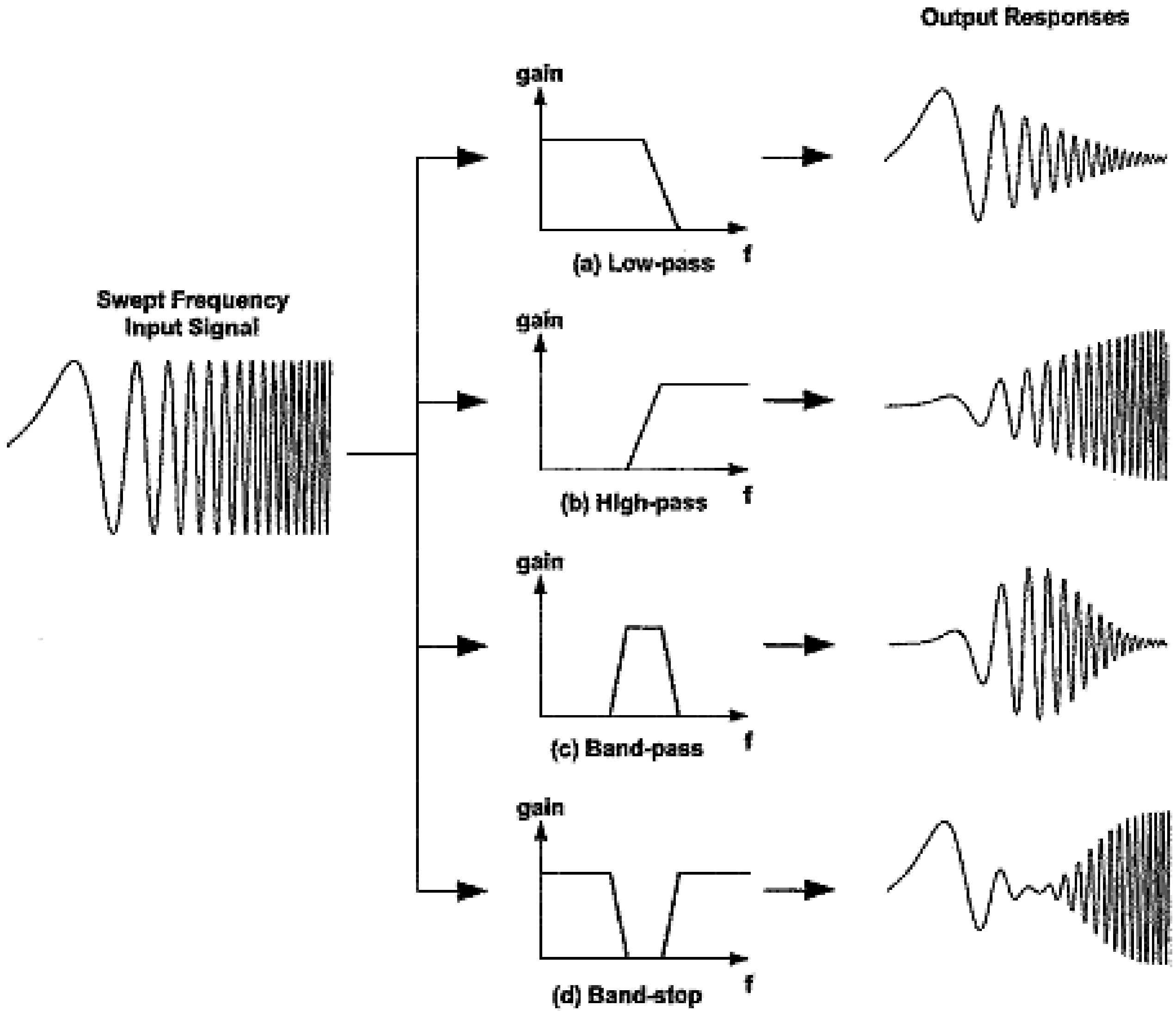
- The amount of attenuation depends on the frequency of the incoming signal component.
- 1 Hz and 2 Hz no attenuation (Gain = 1)
- 4 Hz and 5 Hz fully attenuated (Gain = 0)

Types of Filters

- Low Pass Filter - passes frequencies lower than the cutoff frequency.
- High Pass Filter - passes frequencies higher than the cutoff frequency (as in AC Coupling).
- Band Pass Filter - passes frequencies within a frequency band.
- Band Stop Filter - rejects frequencies within a frequency band (as in 400 Hz notch filter).

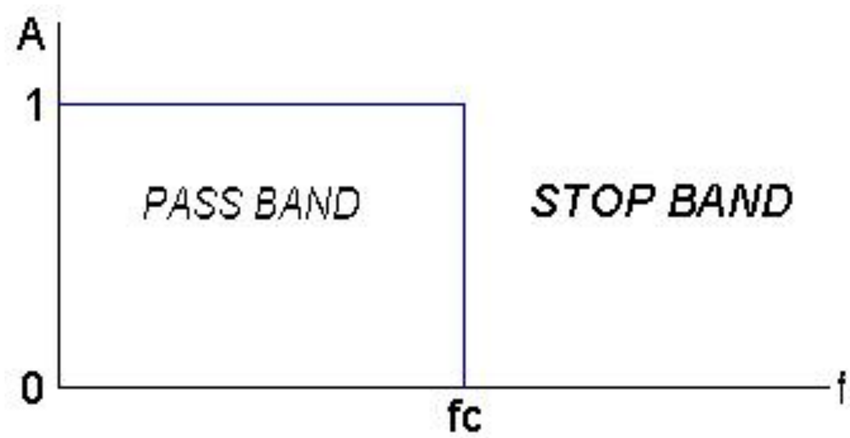
For pre-sample filtering, we will mainly be concerned with low pass filters.

Types of Filters



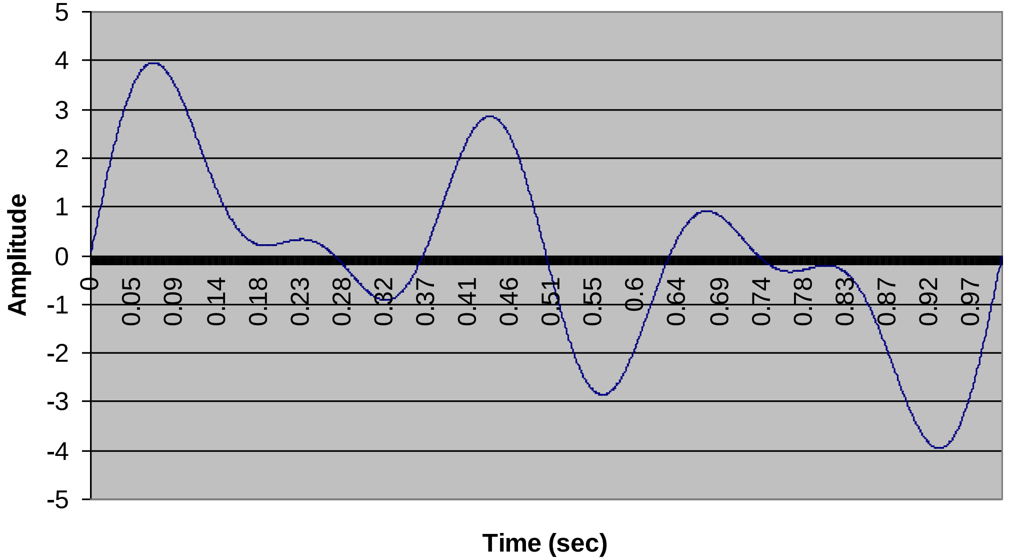
The Ideal Low-Pass Filter

The ideal low-pass filter has a gain of 1 in the pass band and a gain of 0 in the stop band. The cutoff frequency is $f_{c.}$ All frequencies below f_c are passed through the filter, but no frequencies above f_c are passed through. The frequency response of a low pass filter looks like the diagram below.



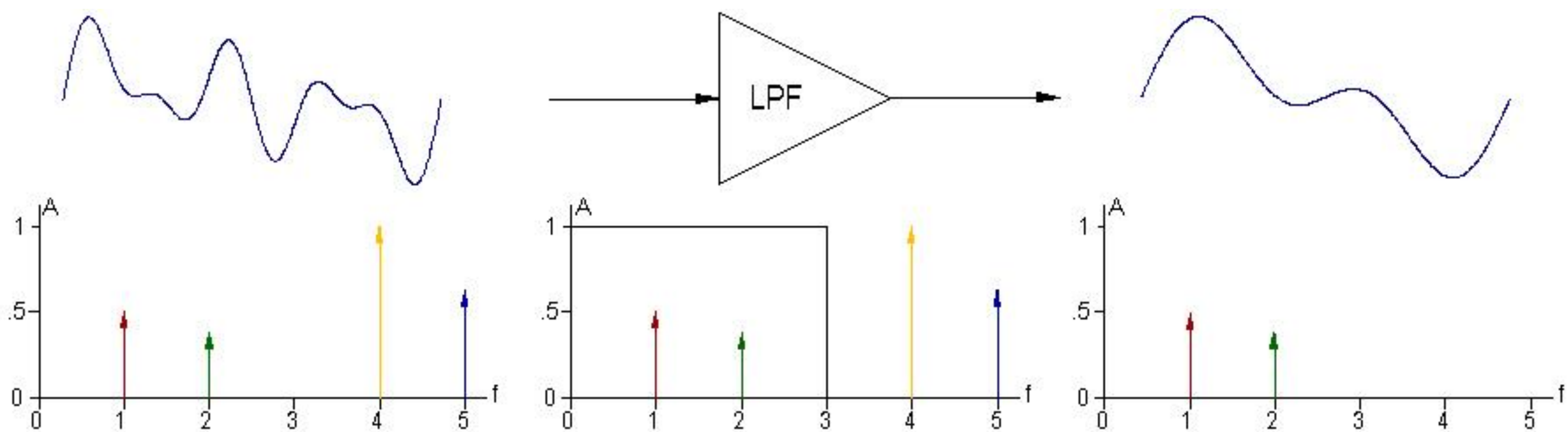
The Ideal Low-Pass Filter Example

In our earlier example, we had a signal comprised of four sine waves with frequencies 1, 2, 4, and 5 Hz. What would we get if this signal is passed through an ideal low pass filter with a cutoff frequency f_c=3 Hz?



The Ideal Low-Pass Filter Example

First, the signal must be observed in the frequency domain in order to see the effects of the filter. Only frequency components within the pass band of the filter pass through with a gain of 1, all other frequencies are removed (gain of 0).



Real Low Pass Filters

- Ideal filters don't exist
- Filters are designed by selecting a *transfer function* that is non-ideal but has some "optimum" property
- Some transfer function optimum properties include:
  - Flat gain within the pass band
  - Steep attenuation outside the pass band
  - "Well behaved" phase characteristics
- All filter designs begin with an optimum transfer function equation
- When you select a filter type be mindful of what kind of performance you are trying to optimize

Some Concepts to Understand First: What are Imaginary Numbers?

Imaginary numbers arise in mathematics where an even numbered root is taken of a negative number.

What is the square root of -16?

$$\sqrt{-16}$$

$$\sqrt{(-1)(4)(4)}$$

$$4\sqrt{-1}$$

$\sqrt{-1}$ does not exist
You cannot take the square root of a negative number.

Some Concepts to Understand First: What are Imaginary Numbers?

- Since $\sqrt{-1}$ appears so often in mathematics, it was defined to be an imaginary number and assigned the symbol *j* in electronics (assigned the symbol *i* in other disciplines such as mathematics).
- So for the example

$$\sqrt{-16}$$

$$\sqrt{-16} = 4\sqrt{-1} = 4j$$

Some Concepts to Understand First: What are Complex Numbers?

- Complex numbers contain both real and imaginary numbers.
  - For example:

$$\underbrace{3}_{\text{real}} + \underbrace{j4}_{\text{imaginary}}$$

  - Complex conjugates

$$3 \pm j4$$

shorthand for

$3 + j4$ and $3 - j4$

Some Concepts to Understand First: Why Use Complex Numbers?

- When solving transfer functions it is common to have complex roots.
  - For the normalized Butterworth filter transfer function

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{s^2 + \sqrt{2}s + 1} = \frac{1}{\left(s - \frac{1}{\sqrt{2}}(1+j)\right)\left(s - \frac{1}{\sqrt{2}}(1-j)\right)}$$

- Real signals have Frequency <u>and</u> Phase. These are easily expressed by the complex number form.

$$\cos(\omega t) + j\sin(\omega t)$$

Some Concepts to Understand First: Transfer Function

- Transfer functions show the relationship between an input and an output
- Filters have one input and one output
- The function could be expressed as a function of time or as a function of frequency but in filter design it is much more natural to express as a function of frequency

$$H(s) = \frac{V_{out}}{V_{in}}$$

- Complex frequency "*s*" is normally used

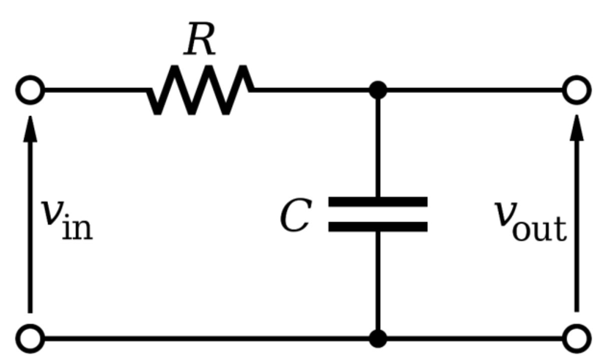
$$s = \sigma + j\omega = \sigma + j2\pi f$$

Some Concepts to Understand First: RC Filter Transfer Function Example

- In this example we start with a circuit and derive the transfer function
- In filter design we start with a transfer function and derive (or synthesize) the circuit

- An RC filter transfer function
  - Output (magnitude and phase) is defined for any and all input frequencies

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{1}{sRC + 1}$$

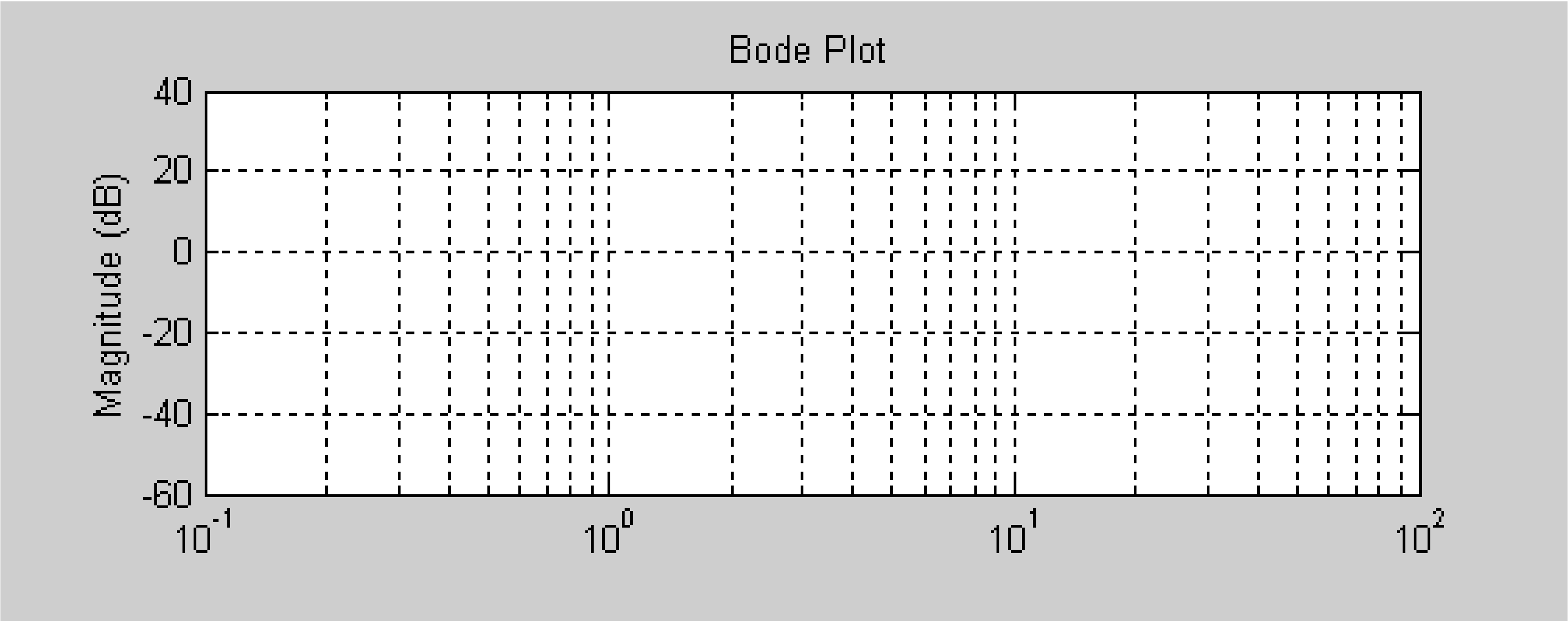


Some Concepts to Understand First: What is a Bode Plot?

- Bode Plots are semi-log charts of the magnitude of the gain expressed in dB versus frequency on a logarithmic scale. The Bode Plot shows the characteristic features of a filter which will be shown in detail later.

- An accompanying graph displays the phase delay as a function of frequency on a logarithmic scale.

Bode Plot Graph



Notice that the x-axis does not go down to 0 or has negative values, why would this be?

Because the x-axis is on a logarithmic scale, and remember that you cannot take the log of 0 or a negative value.

Some Concepts to Understand First: RC Filter Transfer Function

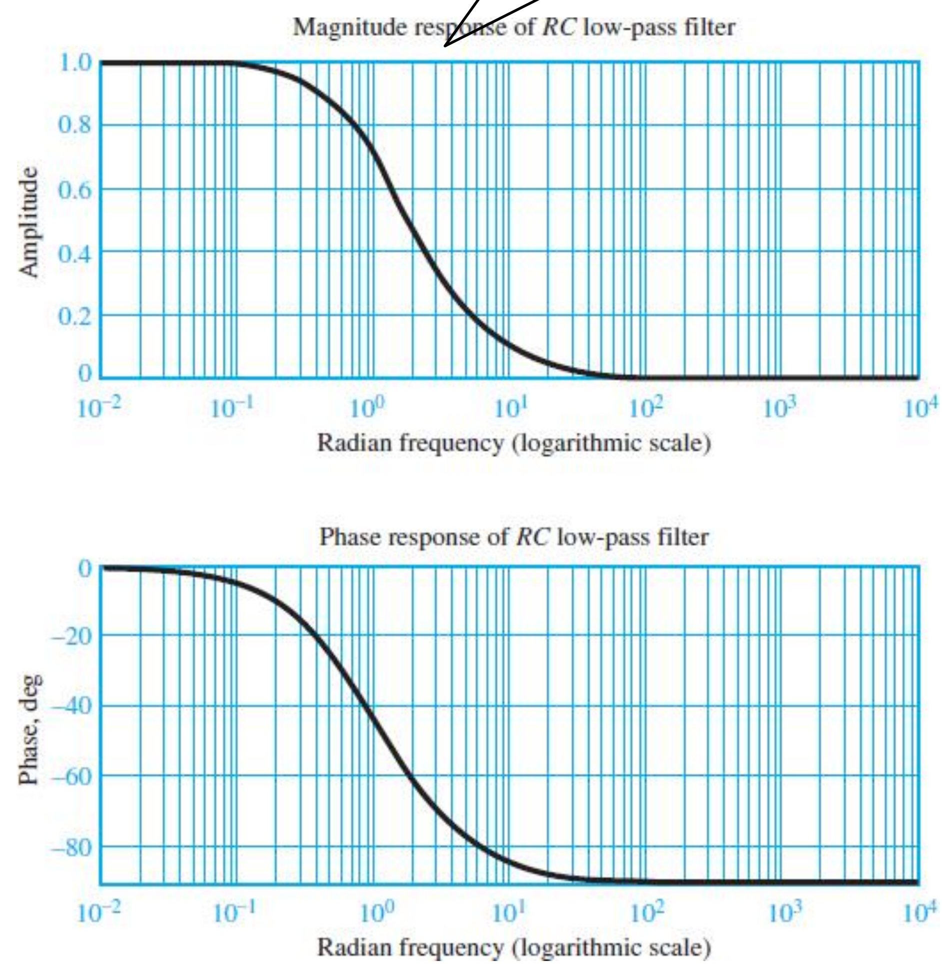
Note: Not a Bode Plot

Normally we are only interested in using the real frequency response component so *"s"* is simplified to *"f"*

Remember $s = \sigma + j\omega = \sigma + j2\pi f$

So for $\sigma = 0$ and $\omega = 2\pi f$

$$H(f) = \frac{1}{j2\pi fRC + 1}$$

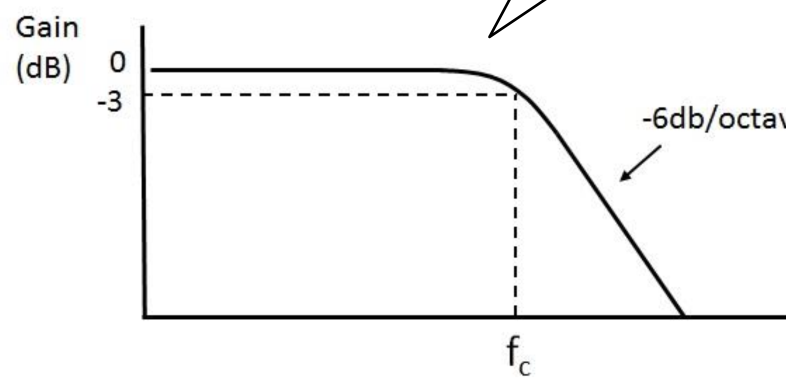


Some Concepts to Understand First: RC Filter Time Constant

To find the half power frequency

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{(2\pi f RC)^2 + 1}} = \frac{1}{\sqrt{2}}$$

Note: Bode Plot



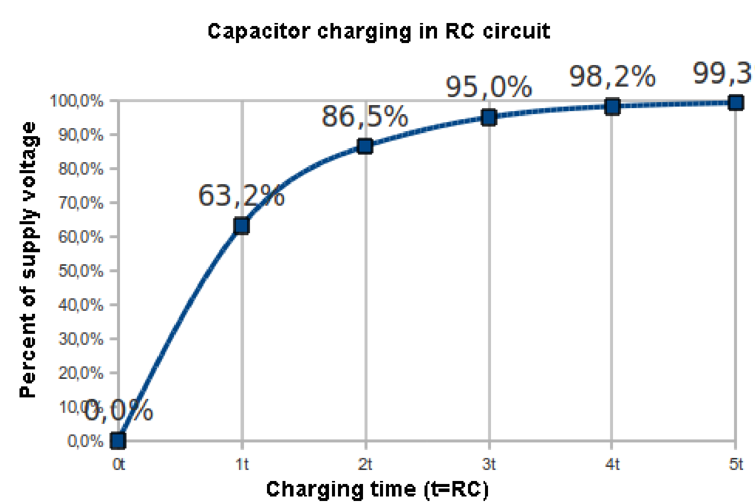
Solve for half power (i.e. 3 dB) corner frequency

$$T_c = RC \text{ seconds}$$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi T_c} \text{ Hz}$$

DC Charge rate is exponential

$$V_{\%} = 1 - e^{-\frac{t}{T_c}}$$



Butterworth Filter

- The Butterworth filter is the most useful filter for most applications
  - Flat gain response across the passband
- First described by the British engineer and physicist Stephen Butterworth in 1930
  - At the time filter design was largely trial and error
- Butterworth developed a family of polynomials
  - Maximally flat in the pass band
  - Rolls off towards zero with increasing frequency
  - Increased “order” or complexity allows increased performance

Butterworth Filter Polynomial

n	Factors of Polynomial $B_n(s)$
1	$(s+1)$
2	$(s^2+1.4142s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.7654s+1)(s^2+1.8478s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$
6	$(s^2+0.5176s+1)(s^2+1.4142s+1)(s^2+1.9319s+1)$
7	$(s+1)(s^2+0.4450s+1)(s^2+1.2470s+1)(s^2+1.8019s+1)$
8	$(s^2+0.3902s+1)(s^2+1.1111s+1)(s^2+1.6629s+1)(s^2+1.9616s+1)$
9	$(s+1)(s^2+0.3473s+1)(s^2+s+1)(s^2+1.5321s+1)(s^2+1.879s+1)$
10	$(s^2+0.3129s+1)(s^2+0.9080s+1)(s^2+1.4142s+1)(s^2+1.7820s+1)(s^2+1.9754s+1)$

$$\mathrm{H}(s) = \frac{1}{B_n(s)} \qquad \mathrm{G}(\omega) = |\mathrm{H}(\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}$$

Where

ω is the angular frequency in radians per second

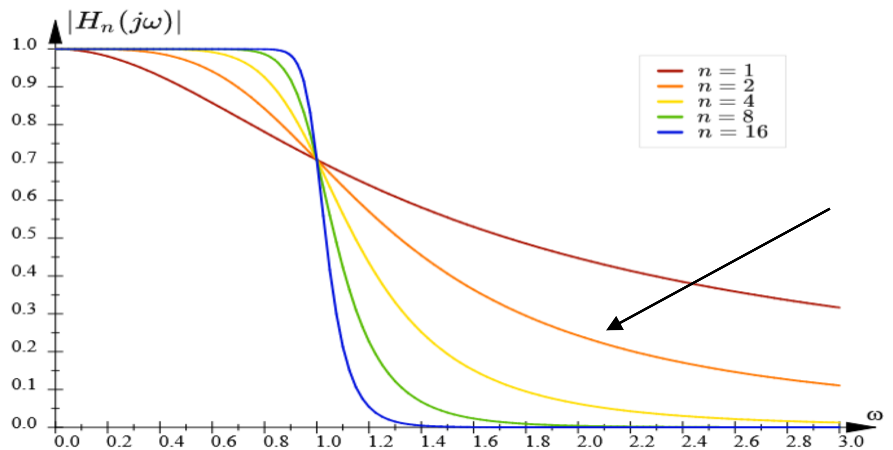
n is the number of poles in the filter

Butterworth Filter Polynomial

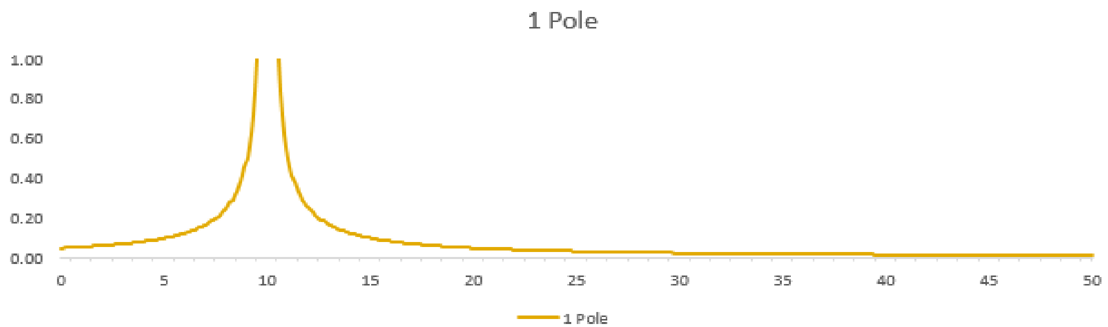
For example for a Butterworth filter with degree 2 the gain versus frequency is

$$\mathrm{H}(s) = \frac{1}{B_2(s)} \qquad \mathrm{G}(f) = \frac{1}{\sqrt{1 + (2\pi f)^4}}$$

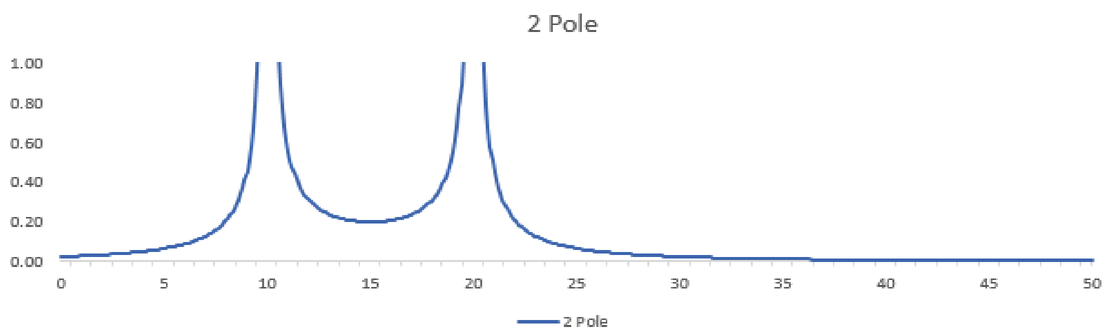
- At DC (*f* = 0) the gain is 1
- As *f* increases the gain decreases monotonically



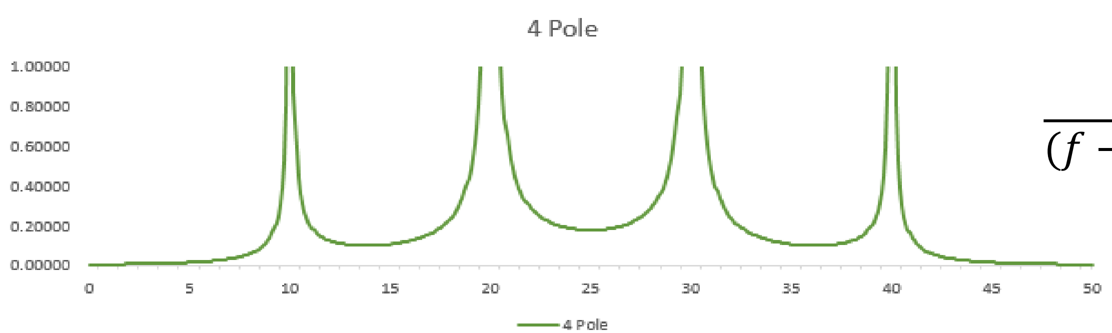
Some Concepts to Understand First: What is a Function Pole



$$\frac{1}{(f-10)}$$



$$\frac{1}{(f-10)(f-20)}$$



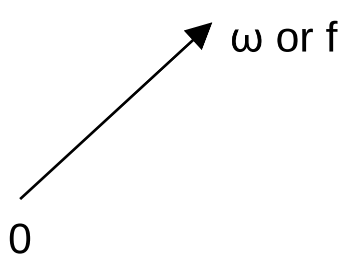
$$\frac{1}{(f-10)(f-20)(f-30)(f-40)}$$

Some Concepts to Understand First: Complex Frequency Plane

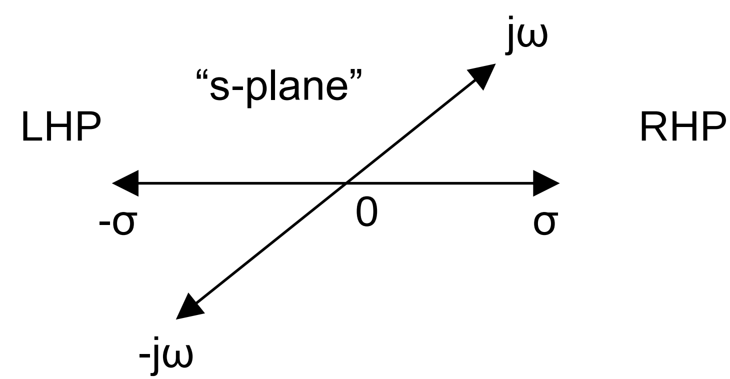
- In the real world frequency is just a real number
  - Cycles per second or Hz ("f")
  - Radians per second ("ω")
- In the mathematics world it is convenient to allow frequency to be a complex number
- Complex frequency "*s*" has two independent components

$$s = \sigma + j\omega \Longrightarrow H(\sigma + j\omega)$$

Real Only Frequency



Complex Valued Frequency

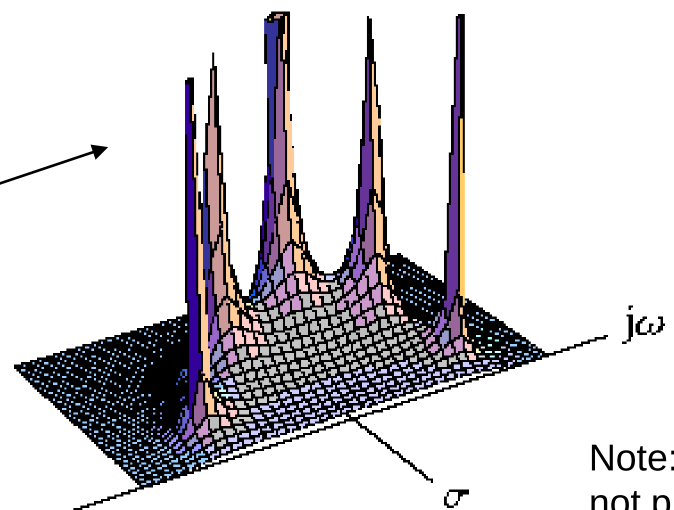


Some Concepts to Understand First: What is a Function Pole

- Transfer Function Amplitude is a function of complex frequency "*s*"

$$H(s)$$

Function Poles



Note: Positive values for σ are not plotted here

Plot of $H(s)$ for values of σ and $j\omega$

Some Concepts to Understand First: What are Poles in a Filter?

The amplitude response (transfer function) of a Butterworth Filter is described mathematically in the following equation:

$$H(s) = \frac{1}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$$

where

$\boldsymbol{n}$ is the order of the equation

$\boldsymbol{a_n}$'s are constants

$\boldsymbol{s}$ is complex frequency

Example

$$H(s) = \frac{1}{B_3(s)} = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

Some Concepts to Understand First: What are Poles in a Filter?

Factoring the polynomial in the denominator:

$$H(s) = \frac{1}{(s+s_n)(s+s_{n-1})(s+s_{n-2})\ldots(s+s_1)}$$

It is easy to see that $H(s) = \infty$ (i.e. a pole) when

$$s = -s_n$$
$$s = -s_{n-1}$$
$$s = -s_{n-2}$$
$$\ldots$$
$$s = -s_1$$

Some Concepts to Understand First: What are Poles in a Filter?

Usually it is more convenient to have quadratic factors:

$$H(s) = \frac{1}{(s^2 + a_n s + b_n)(s^2 + a_{n-1} s + b_{n-1}) \ldots (s^2 + a_1 s + b_1)}$$

The s_n's take on the form of complex conjugate pairs

$$s_n = \sigma_n \pm j\omega_n$$

When solving for s, you get the roots of the denominator or the ***poles*** of the transfer function in complex conjugate pairs of the form

$$\sigma \pm j\omega_n$$

Butterworth Filter Polynomial

For a Butterworth filter with degree 3

$$\mathrm{H}(s) = \frac{1}{B_3(s)} = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

Factor the denominator

$$s^3 + 2s^2 + 2s + 1 = (s+1)\left(s + \frac{1}{2} + \frac{\sqrt{3}}{2}j\right)\left(s + \frac{1}{2} - \frac{\sqrt{3}}{2}\,j\right)$$

When the value of the complex frequency "*s*" is

$s = -1$ or $s = -\left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)$ or $s = -\left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)$

then one of the factors of $B_3(s)$ goes to zero and

$$\mathrm{H}(s) = \infty$$

Butterworth Filter Polynomial

The 2nd and 3rd poles are a complex conjugate pair

$$s = -1 \qquad s = -\left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right) \qquad s = -\left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)$$

A shorthand way of writing this is:

$$s = -1 \qquad s = -\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}j\right)$$

Complex conjugate pairs arise from quadratic factors

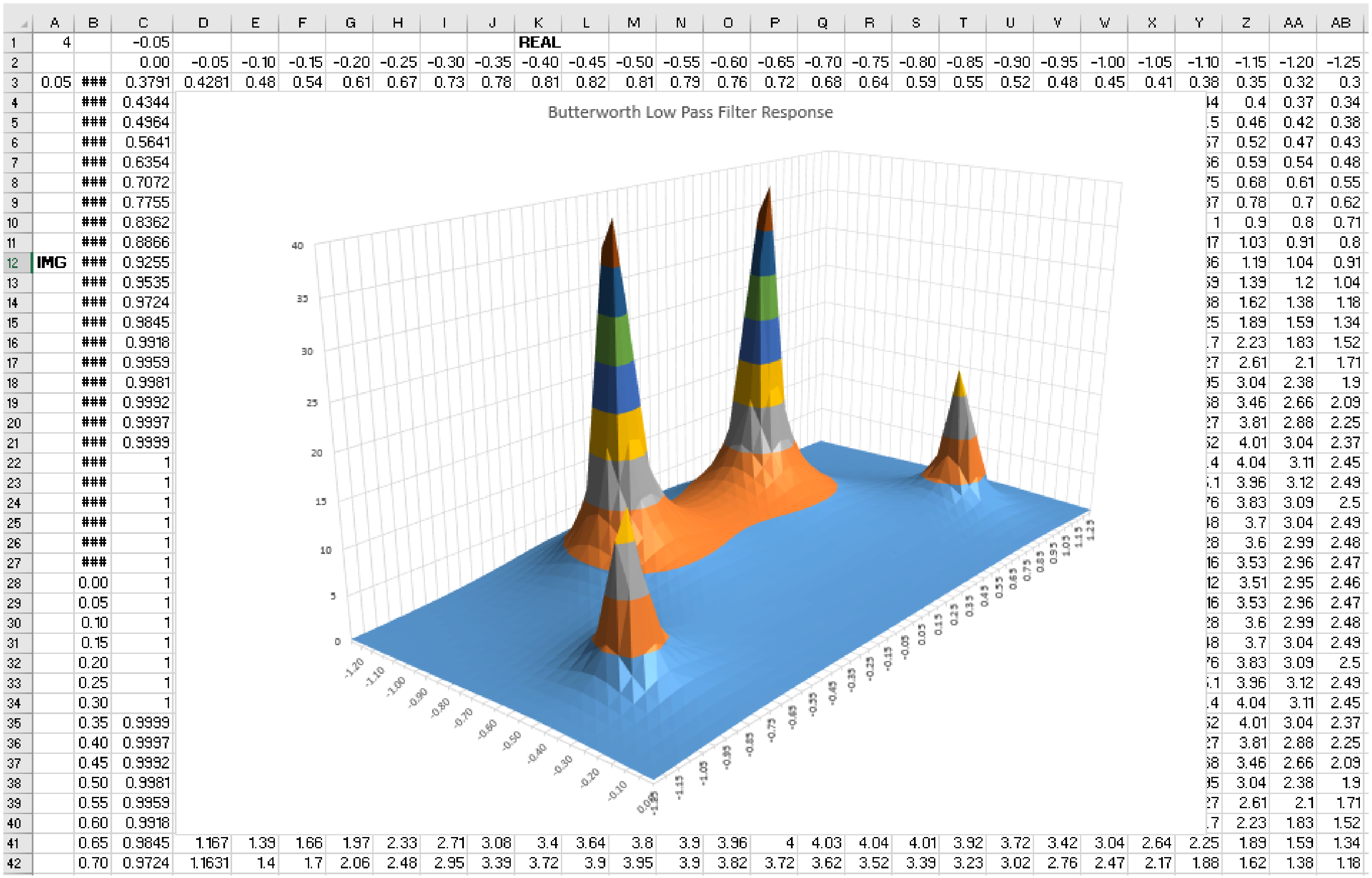
Butterworth Polynomial Factors

n	Factors of Polynomial $B_n(s)$
1	$(s+1)$
2	$(s^2+1.4142s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.7654s+1)(s^2+1.8478s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$
6	$(s^2+0.5176s+1)(s^2+1.4142s+1)(s^2+1.9319s+1)$
7	$(s+1)(s^2+0.4450s+1)(s^2+1.2470s+1)(s^2+1.8019s+1)$
8	$(s^2+0.3902s+1)(s^2+1.1111s+1)(s^2+1.6629s+1)(s^2+1.9616s+1)$
9	$(s+1)(s^2+0.3473s+1)(s^2+s+1)(s^2+1.5321s+1)(s^2+1.879s+1)$
10	$(s^2+0.3129s+1)(s^2+0.9080s+1)(s^2+1.4142s+1)(s^2+1.7820s+1)(s^2+1.9754s+1)$

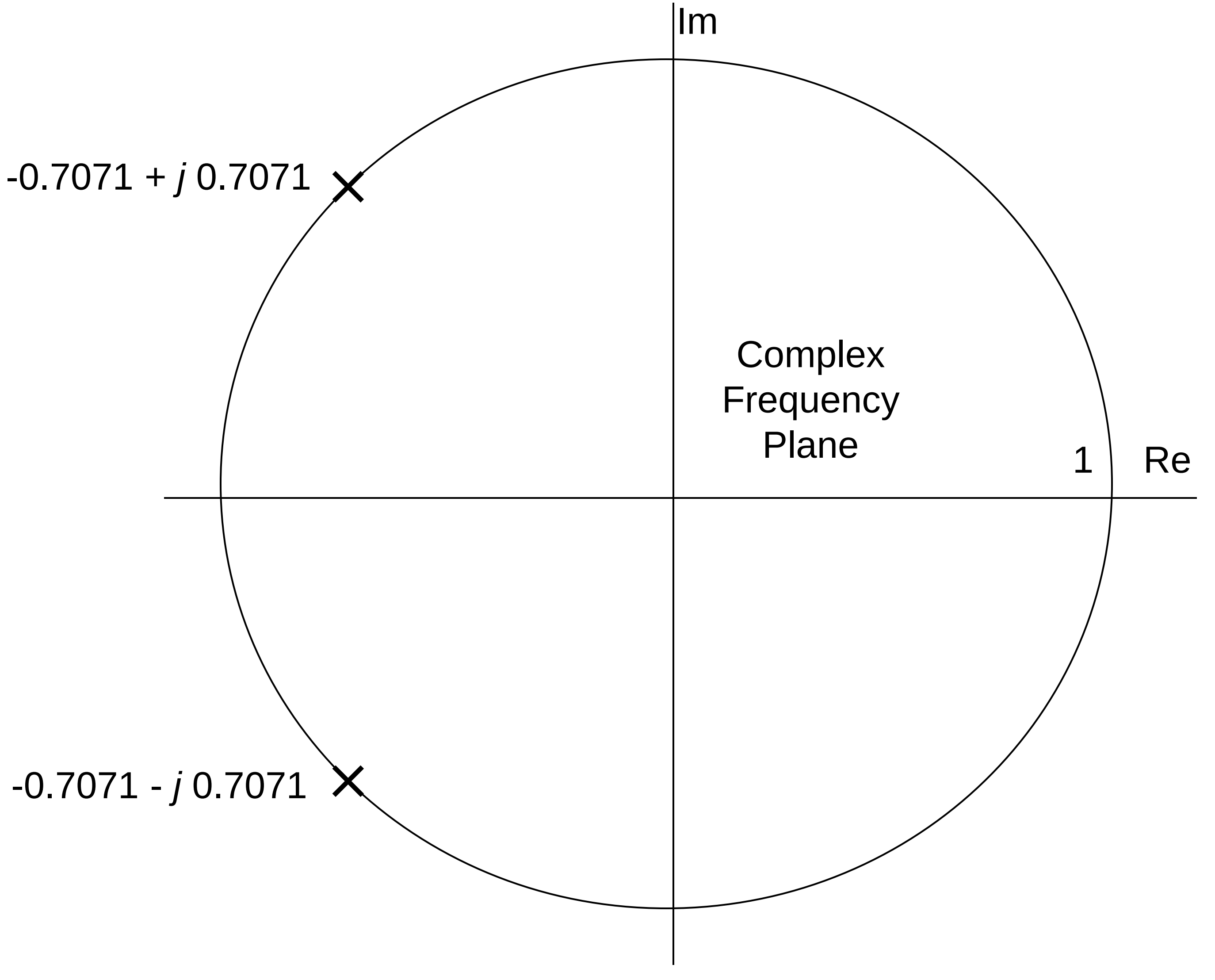
Note quadratic factors
Each will have a
complex conjugate pair

Order	Pole Location
1	$-1 \pm j\,0$
2	$-0.707 \pm j\,0.707$
3	$-1 \pm j\,0$, $-0.5 \pm j\,0.866$
4	$-0.924 \pm j\,0.383$, $-0.383 \pm j\,0.924$
5	$-1 \pm j\,0$, $-0.809 \pm j\,0.588$, $-0.309 \pm j\,0.951$
6	$-0.966 \pm j\,0.259$, $-0.707 \pm j\,0.707$, $-0.259 \pm j\,0.966$
7	$-1 \pm j\,0$, $-0.901 \pm j\,0.434$, $-0.624 \pm j\,0.782$, $-0.222 \pm j\,0.975$
8	$-0.981 \pm j\,0.195$, $-0.832 \pm j\,0.556$, $-0.556 \pm j\,0.832$, $-0.195 \pm j\,0.981$

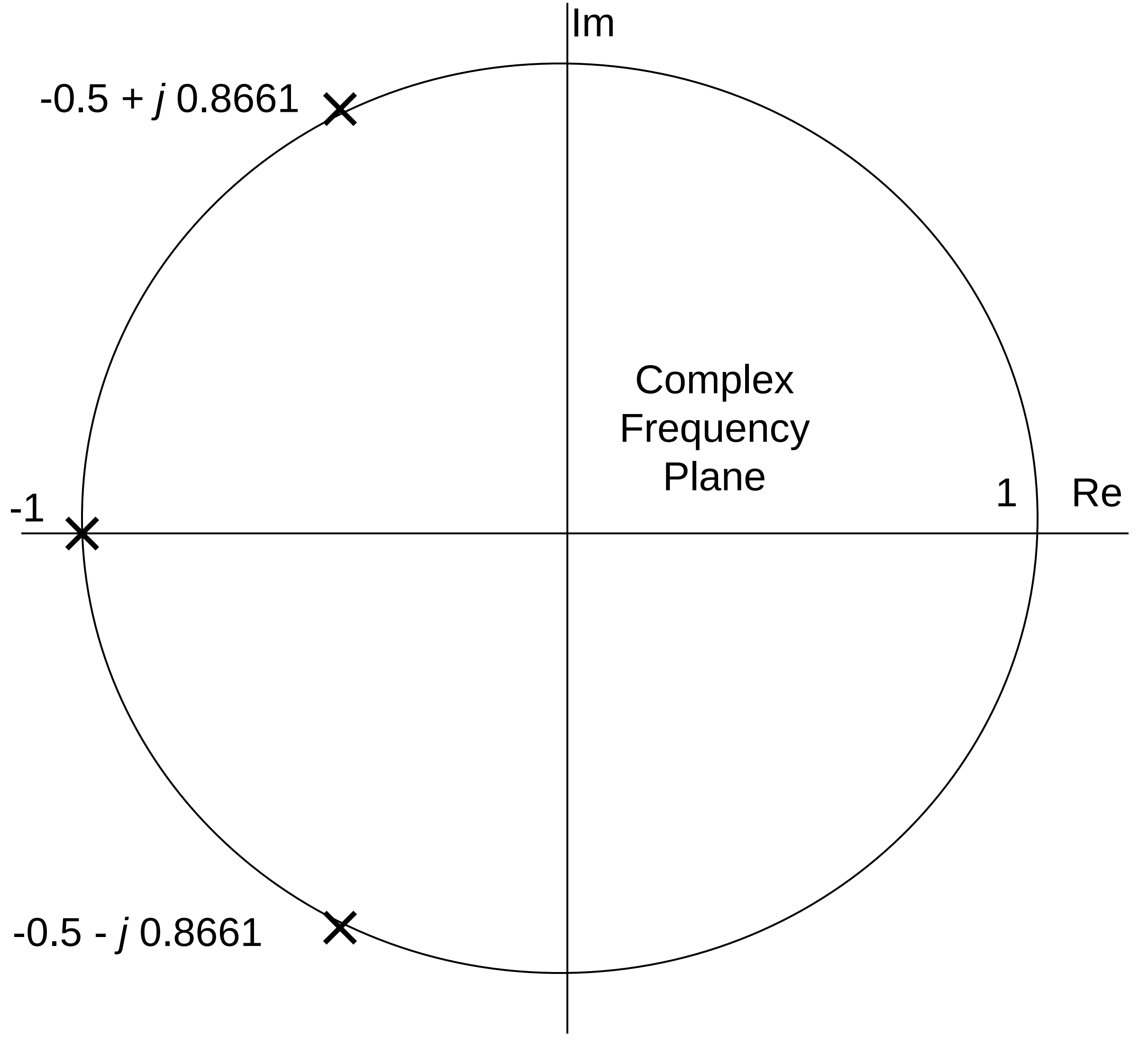
A Look At Butterworth Poles



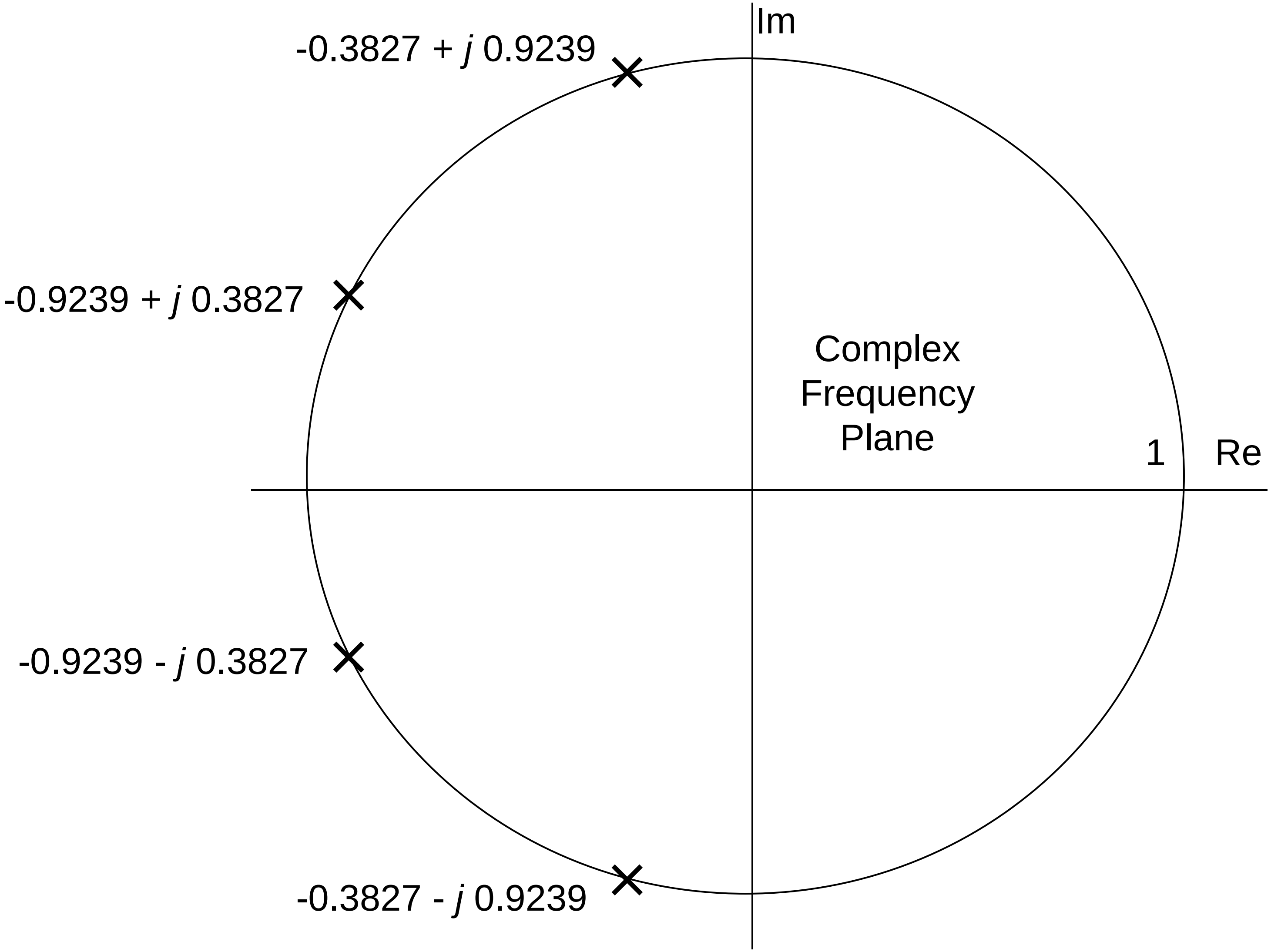
Poles of a 2-pole Butterworth Filter



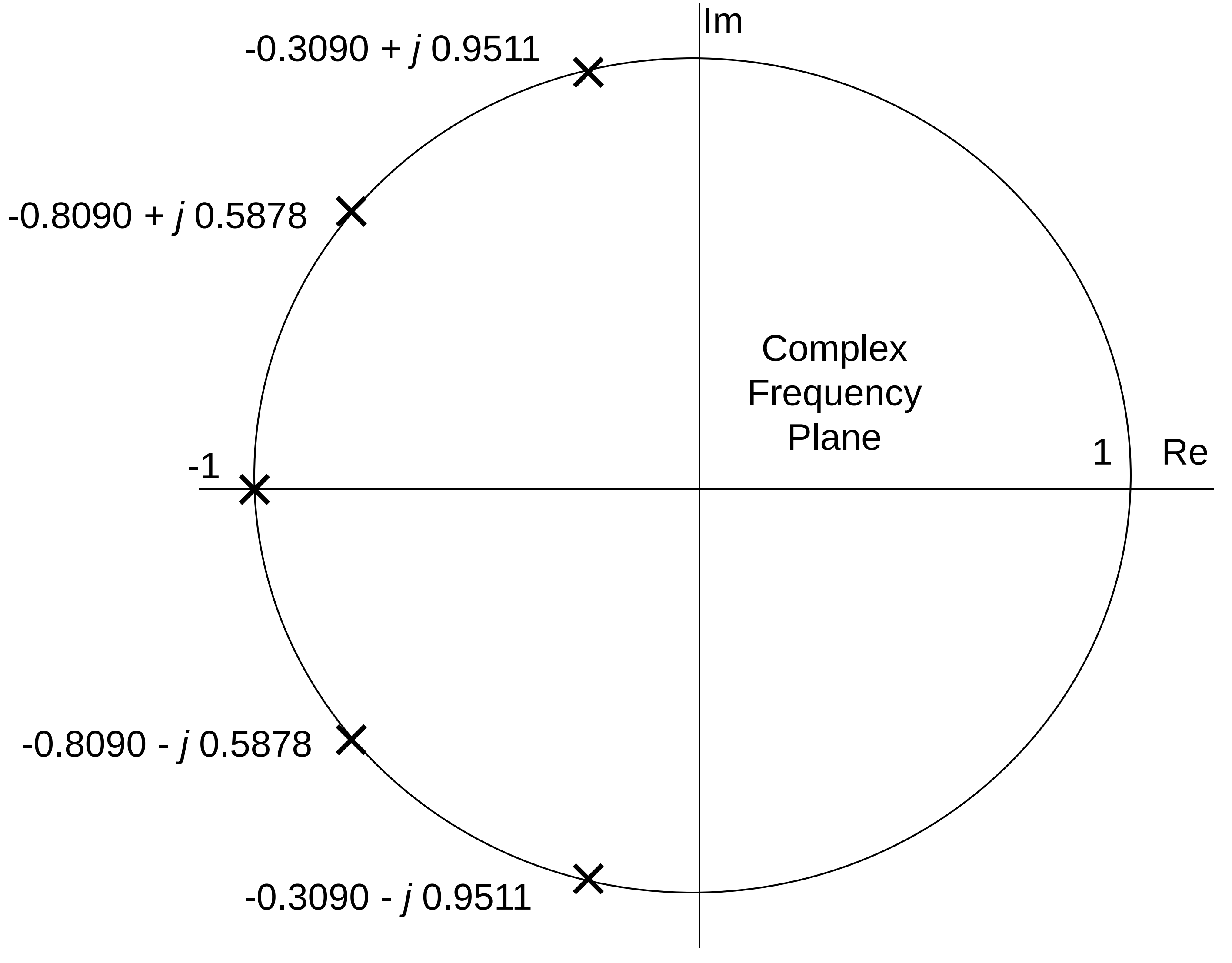
Poles of a 3-pole Butterworth Filter



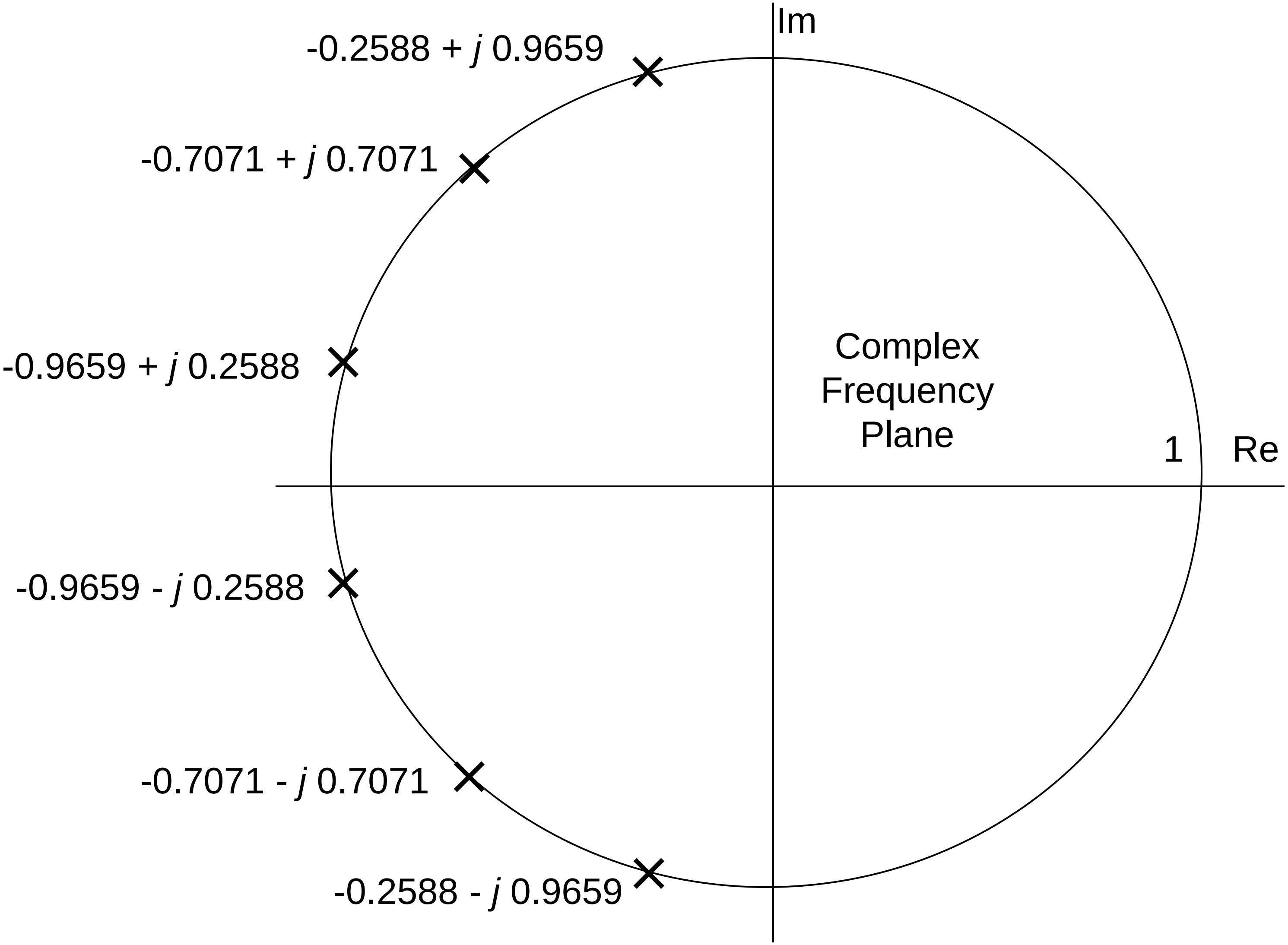
Poles of a 4-pole Butterworth Filter



Poles of a 5-pole Butterworth Filter

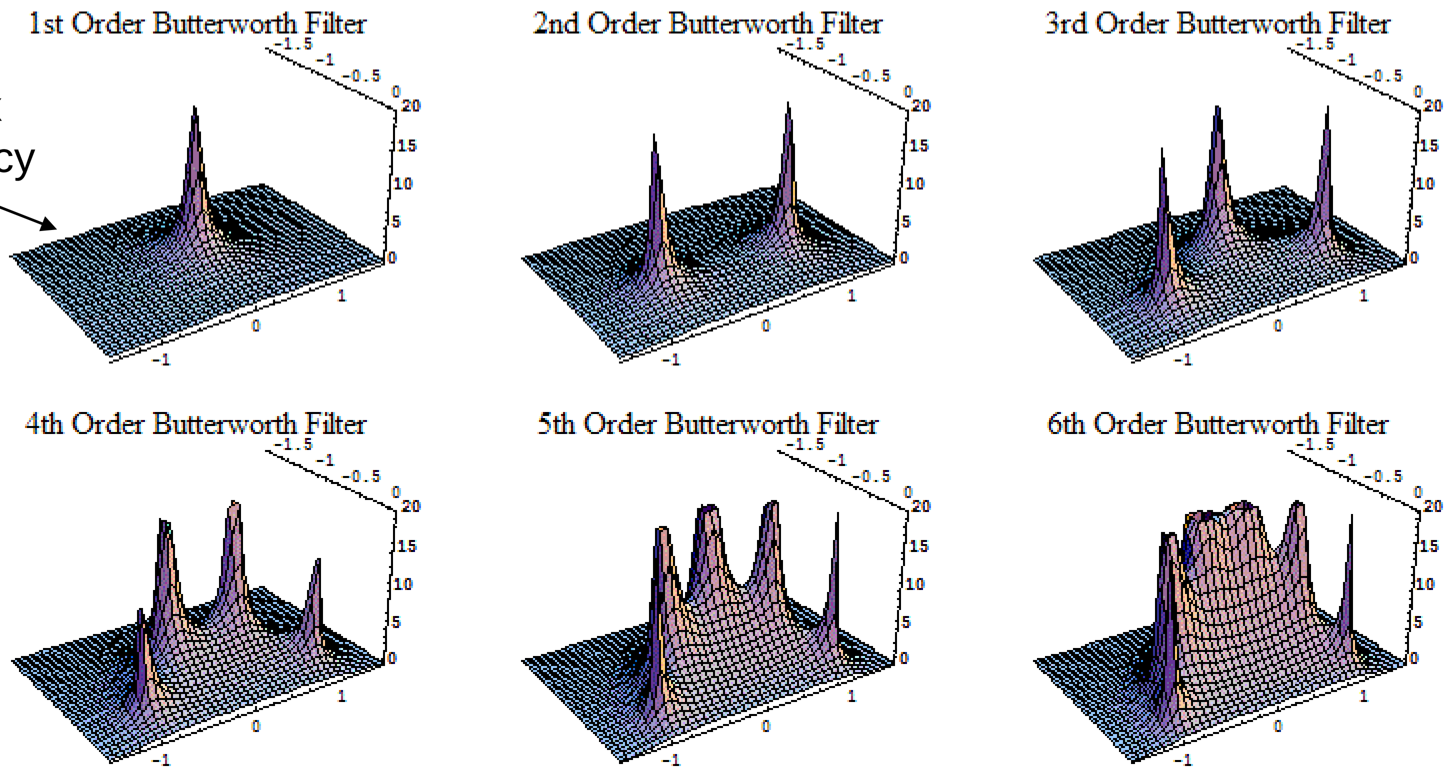


Poles of a 6-pole Butterworth Filter



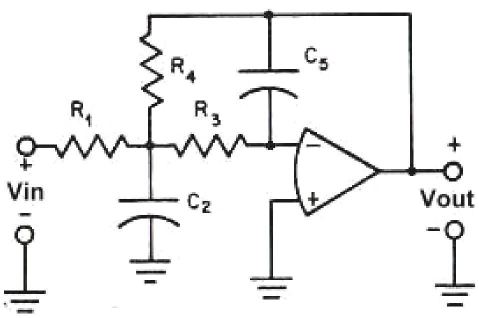
Butterworth Poles

Complex
Frequency
Plane

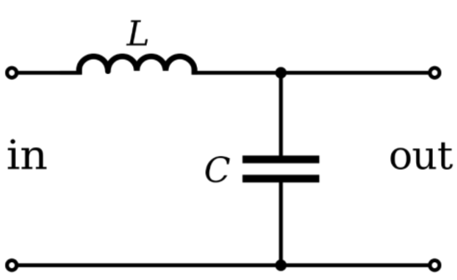


Poles and Order of a Filter

- You will hear these terms used interchangeably when describing filters.
- A 2-pole filter is the same as a 2^{nd} order filter.
- In general the number of poles (i.e. the filter order) is the same as the number of capacitors and/or inductors in the filter circuit.



2 C's → 2 Poles



1 L + 1 C → 2 Poles

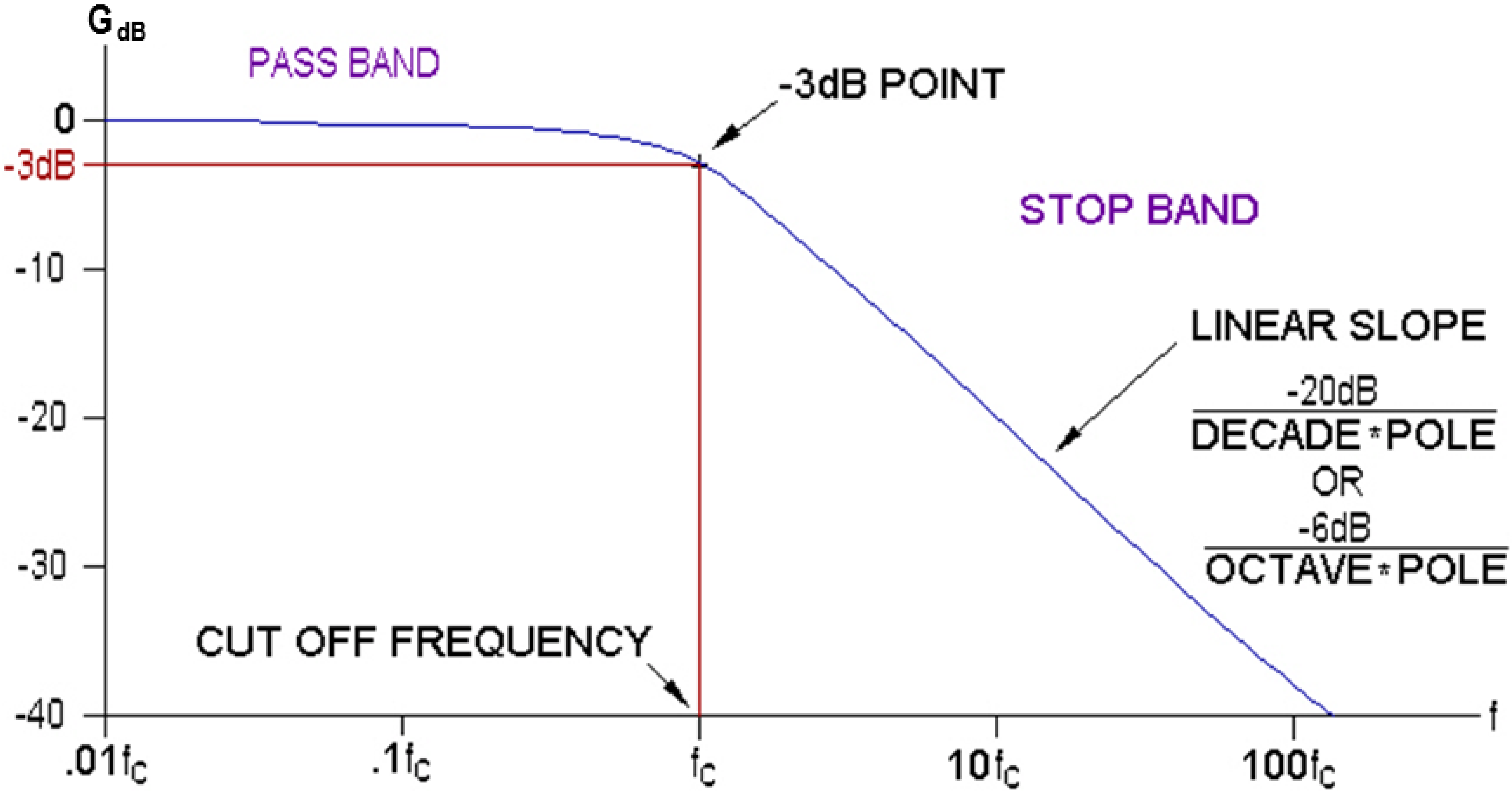
Describing Filter Characteristics

Some Concepts to Understand First: What is an Octave and a Decade?

- An *Octave* is a doubling in frequency.
- 220 Hz to 440 Hz is an Octave
- 1000 Hz to 2000 Hz is an Octave

- A *Decade* is a multiplication of 10 in frequency.
- 100 Hz to 1000 Hz is a Decade
- 200 Hz to 2000 Hz is a Decade

Characteristics of a Butterworth Filter Magnitude of the Frequency Response (Bode Plot)

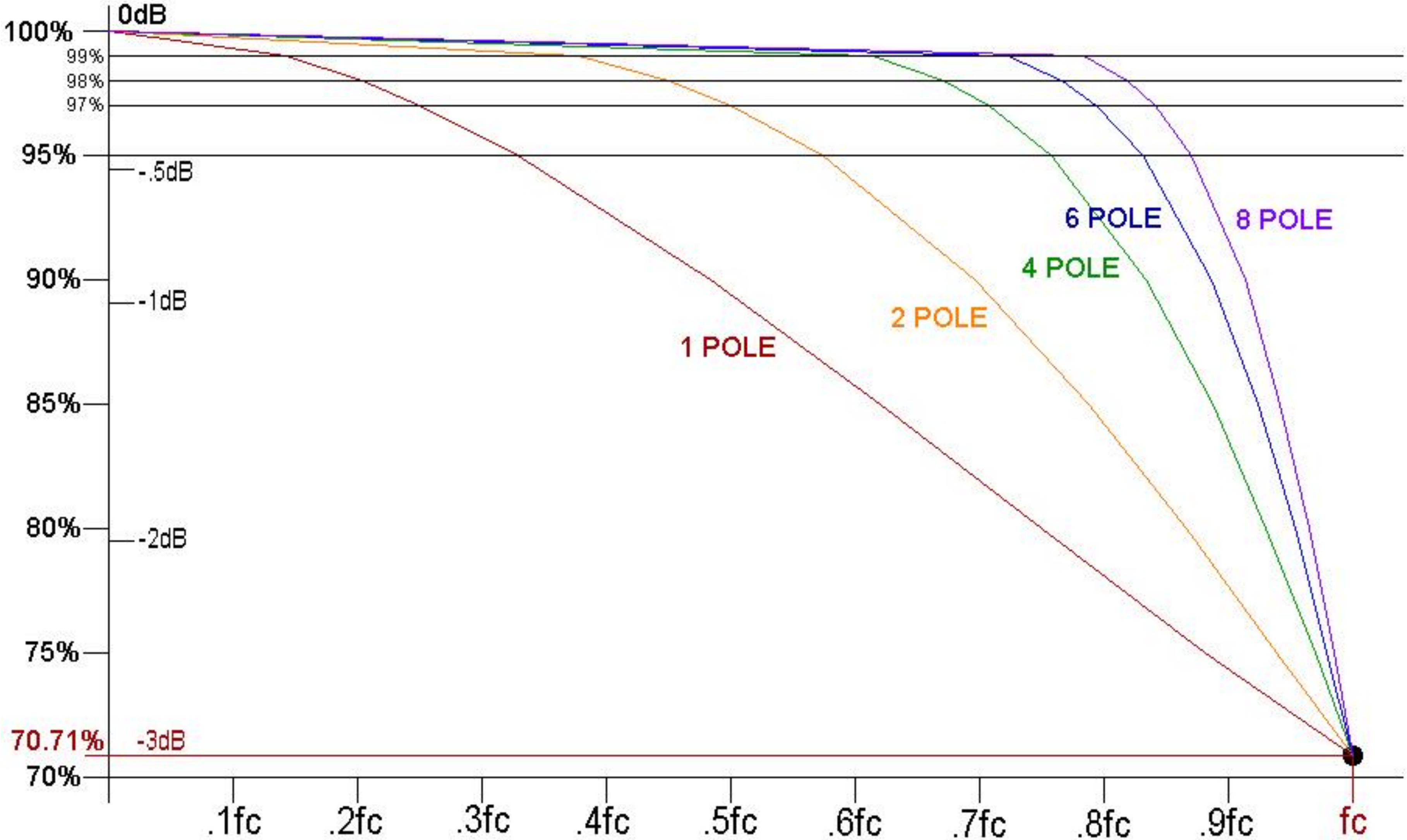


The Pass Band

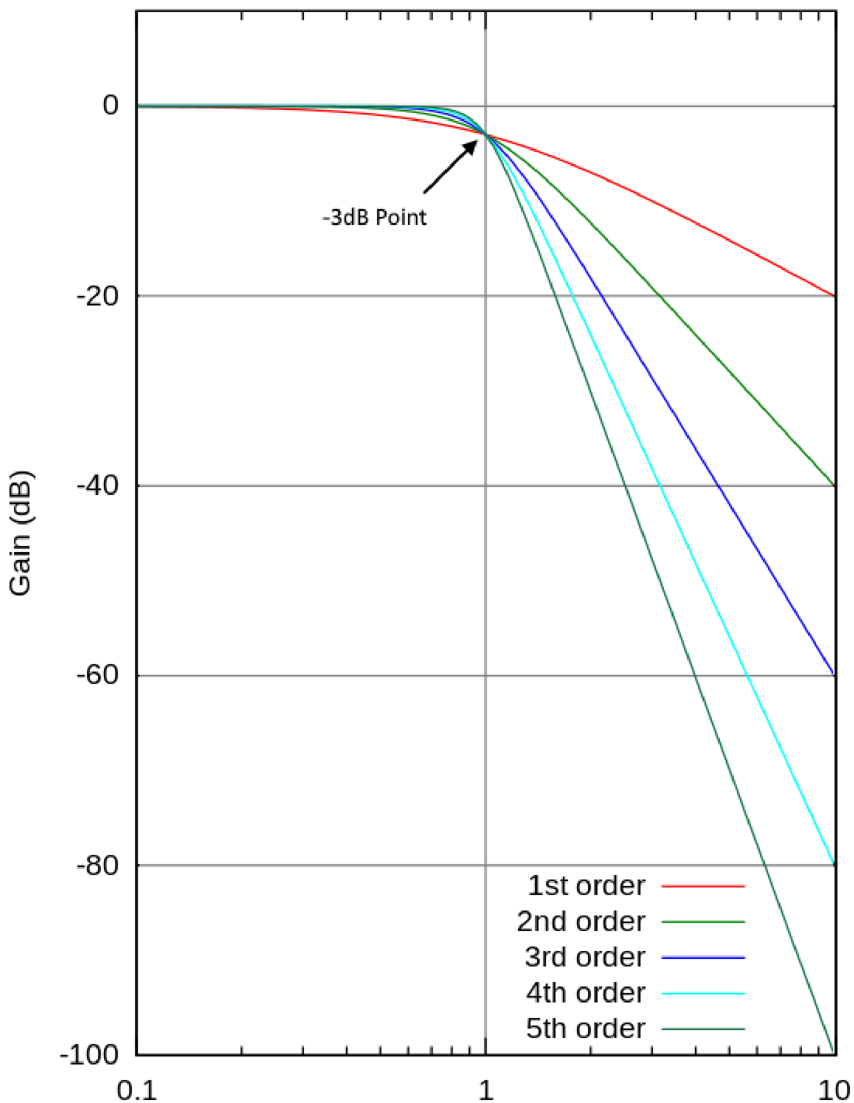
- Because the data of interest will be in the pass band, it is very important to understand what the attenuation is in that band.

- The Butterworth filter attenuates the signal even in the pass band. It is important to know by how much.

The Pass Band



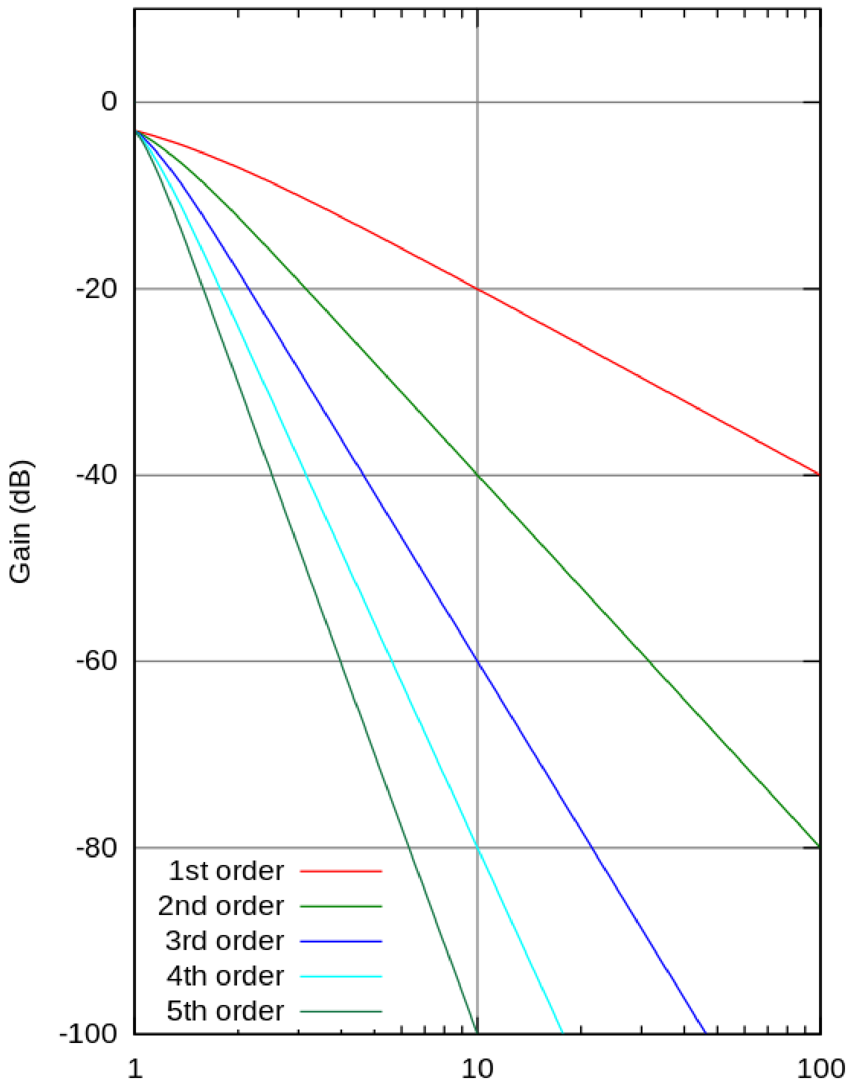
At the Cut Off Frequency



- No matter what the order of the filter is, the frequency response curve always goes through the -3dB point at the cutoff frequency

- Note that there are many definitions of bandwidth. The -3 dB point is common but not the only one

The Stop Band

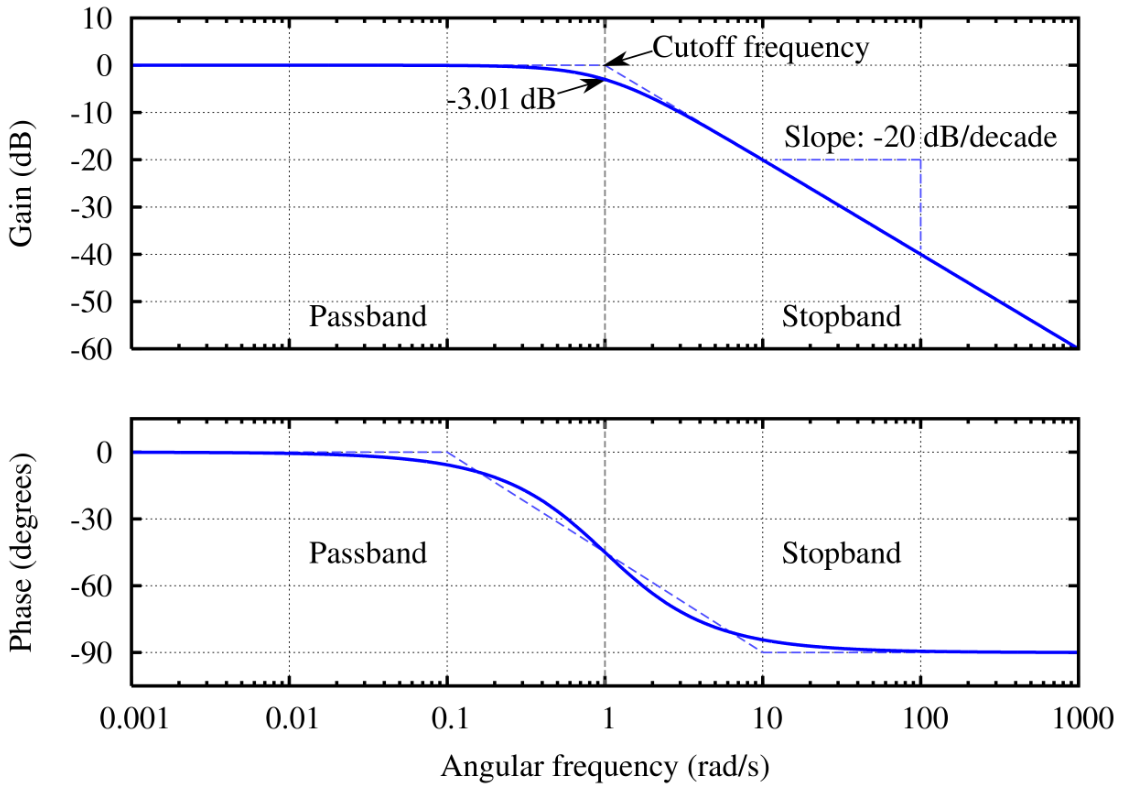


The sharpness of the drop-off is dependent upon the number of poles as seen in the diagram.

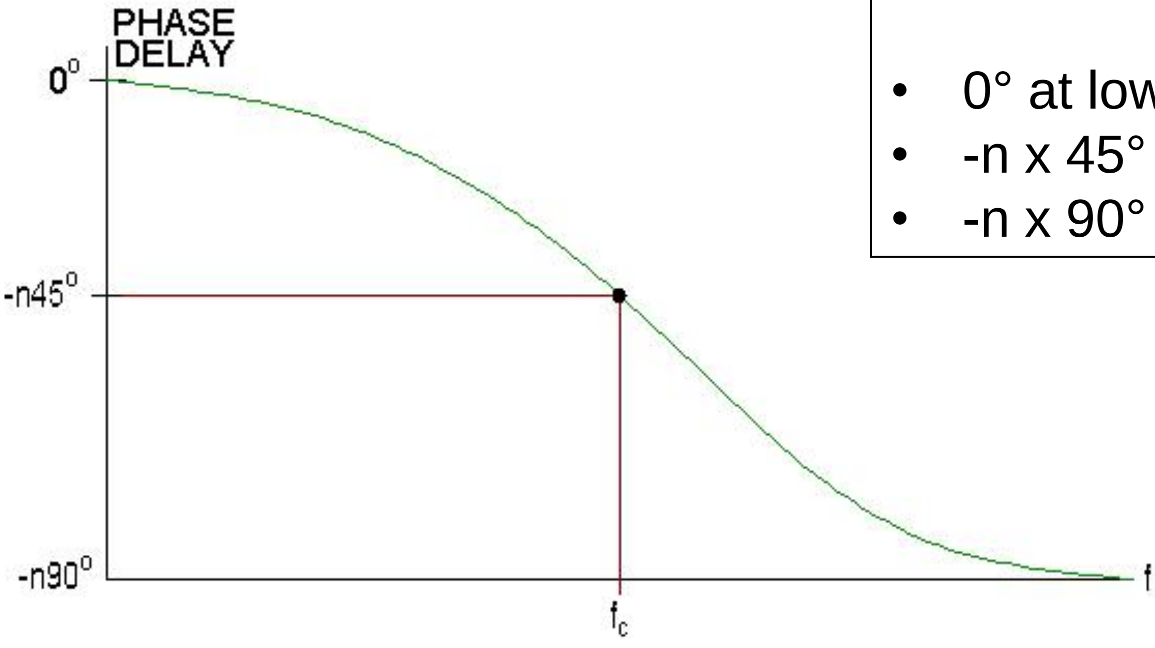
Each pole contributes
-20dB/decade
or
-6dB/octave

Butterworth Filter Phase Delays

Butterworth filters have phase delays plotted below. If a signal has two frequency components in phase as they enter the filter, they will be delayed slightly different as they exit the filter.



Butterworth Filter Phase Delays

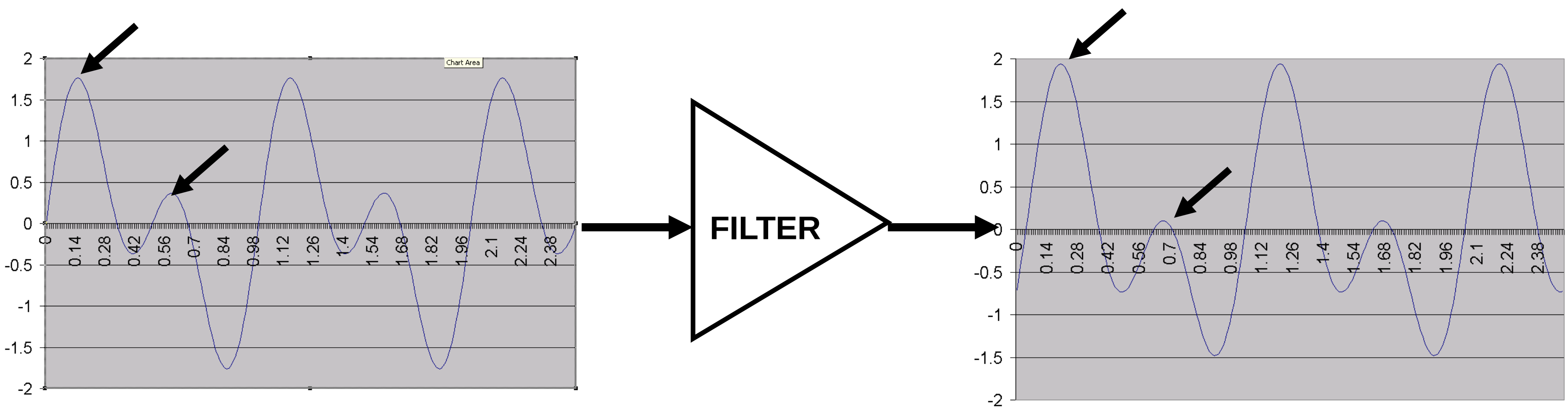


For an n-pole filter, the phase delay varies

- 0° at low frequencies
- -n x 45° at f_c
- -n x 90° at high frequencies

Graphical Illustration of Distortion Due to the Phase Delay

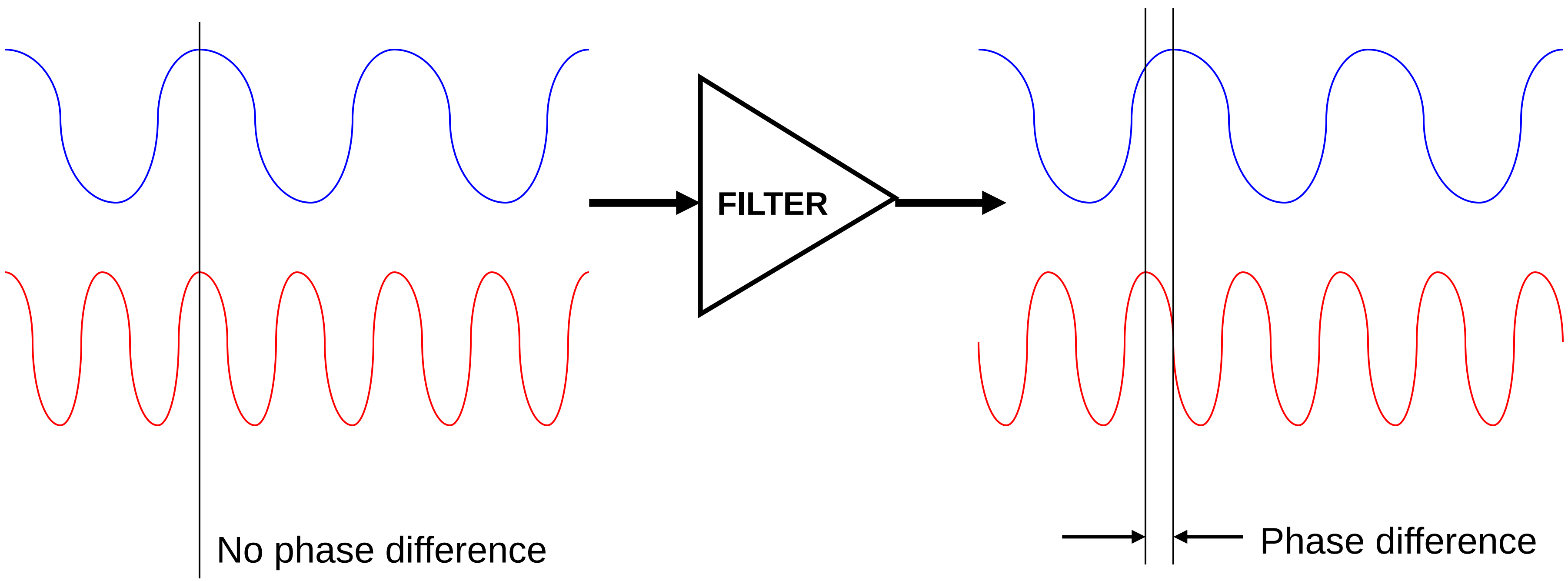
A signal containing two frequencies *f* and 2*f* enters the filter, and then is distorted due to the phase delay of the filter. The output shown is somewhat exaggerated to illustrate the effects of phase delay on the individual frequency components.



As you can see the peaks of the signal change due to phase shifting of the individual frequencies.

Graphical Illustration of Distortion Due to the Phase Delay

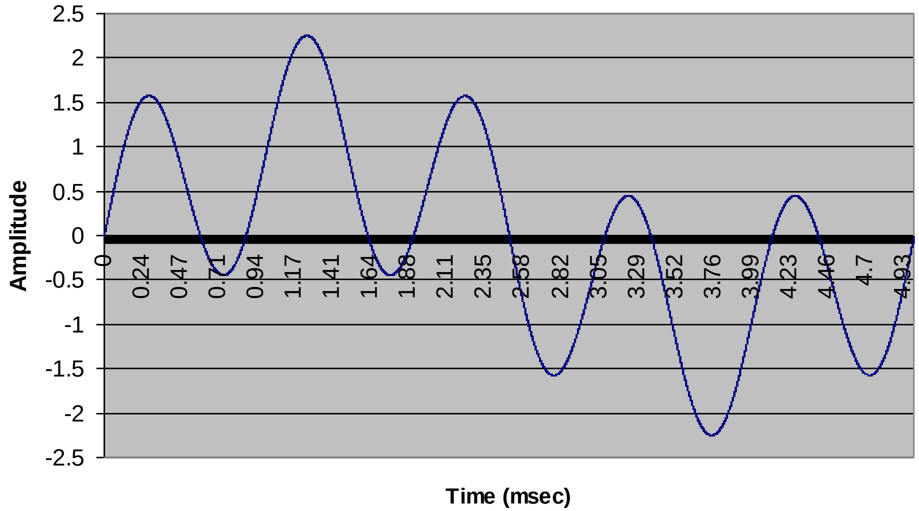
Separating out the two sine waves making up the signal, you can see what happens as they pass through the filter.

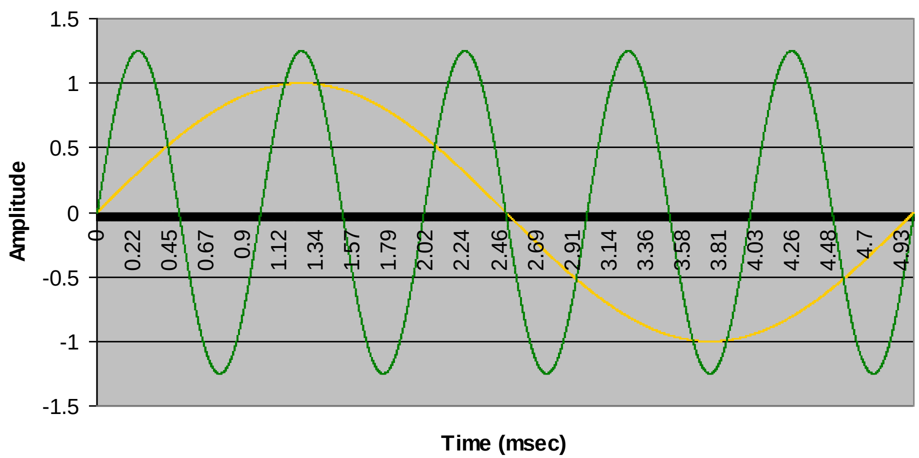


The difference in phase at the output of the filter causes distortion in the signal.

Phase Delay Calculation Example

Say we have a signal with two frequencies, one at the cutoff frequency of 1000 Hz, the other at 200 Hz.





Phase Delay Example

- For a 6-pole Butterworth filter, the delays will be as follows:

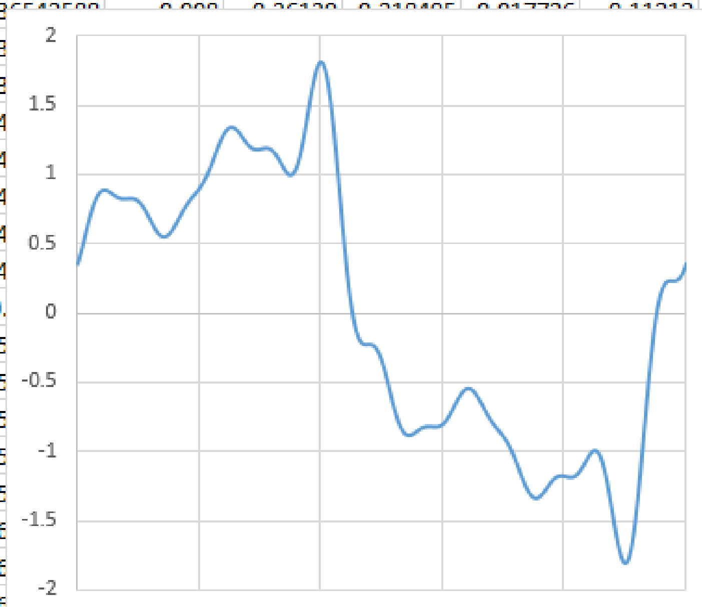
signal	phase delay	time delay
1000 Hz	-270°	-270/(360x1000) = -750 µs
200 Hz	approx -50°	-50/(360x200) = -694 µs

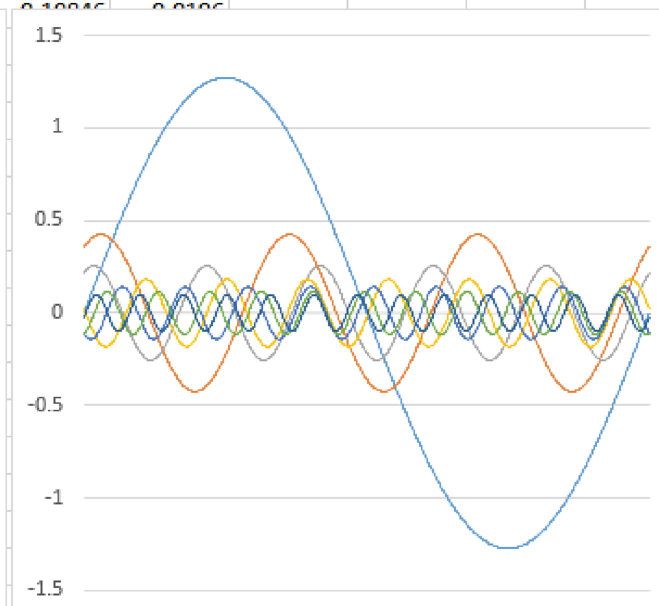
- The two frequency components will be delayed by

  -694 - (-750) = 56 µs of each other
- So the two frequency components of the signal will enter the filter at the same time, but the 200 Hz component will be output 56 µs before the 1000 Hz component.

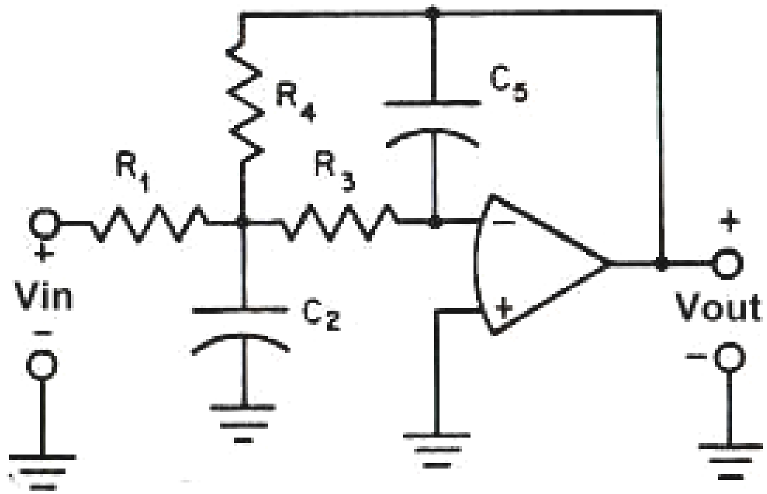
Phase Delay Example

			Enable Harmonic Plot						
			☑	☑	☑	☑	☑	☑	☑
		Harmonic	1	3	5	7	9	11	13
		Amplitude	0.63662	0.212207	0.127324	0.090946	0.070736	0.057875	0.048971
		Time Delay	0	1	1	3	4	5	6
	Time Step	Signal Sum							
0	0	0.35165212	0	0.357131	0.214279	0.025669	-0.10707	-0.11099	-0.02737
1	0.00628319	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	0.01256637	0.3							
3	0.01884956	0.3							
4	0.02513274	0.4							
5	0.03141593	0.4							
6	0.03769911	0.4							
7	0.0439823	0.4							
8	0.05026548	0.4							
9	0.05654867	0.							
.0	0.06283185	0.5							
.1	0.06911504	0.5							
.2	0.07539822	0.5							
.3	0.08168141	0.5							
.4	0.08796459	0.5							
.5	0.09424778	0.6							
.6	0.10053096	0.6							
.7	0.10681415	0.6							
.8	0.11309734	0.65270916	0.143693	0.413091	0.254644	-0.11009	-0.13492	-0.00453	0.090819
.9	0.11938052	0.66938189	0.151639	0.414853	0.254561	-0.11635	-0.1323	0.003472	0.093508





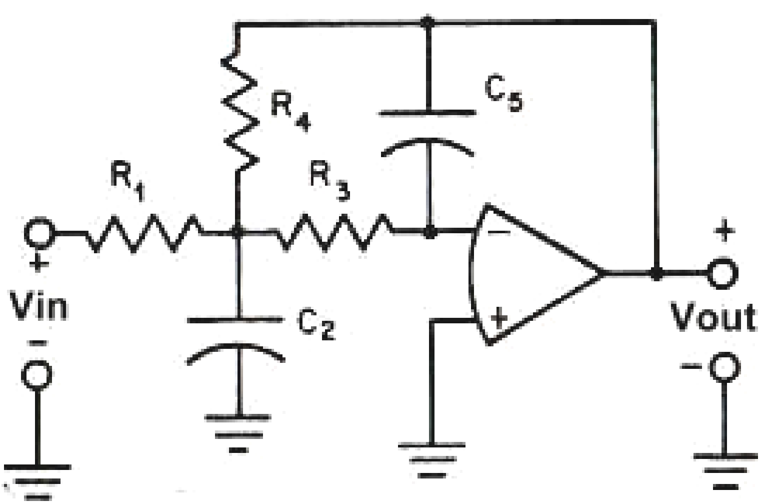
Butterworth Filters
A Practical Implementation



The classic Multi-Feedback (or Rauch) 2-pole low pass op amp filter

Butterworth Filters
A Practical Implementation

- The impedance of the capacitors used to implement a filter are dependent on frequency. Therefore, the filter's characteristics are frequency dependent.
- The diagram shows a 2-pole Butterworth filter.



The capacitors in the diagram have an impedance that is dependent on the frequency of V_{in}.

$$Z_c = \frac{1}{sC} = \frac{1}{j\omega C} = \frac{1}{j2\pi f C}$$

The varying impedance, varies the gain of the amplifier with respect to frequency.

Butterworth Filters
A Practical Implementation

The transfer function of the filter looks like the following equation containing the R and C's of the circuit. Note that the number of capacitors used in the circuit is also the number of poles in the equation.

$$H(s) = \frac{-1/_{R_1 R_3 C_2 C_5}}{s^2 + s\left(\frac{1}{C_2}\right)\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) + \frac{1}{R_3 R_4 C_2 C_5}}$$

What Determines the Characteristics of a 2-pole Butterworth Filter?

- The roots of the denominator will be the poles of the filter.
- The DC gain, $H_o = -R_4/R_1$ (usually H_o is set to -1 with $R_1 = R_4$).
- The cutoff frequency,

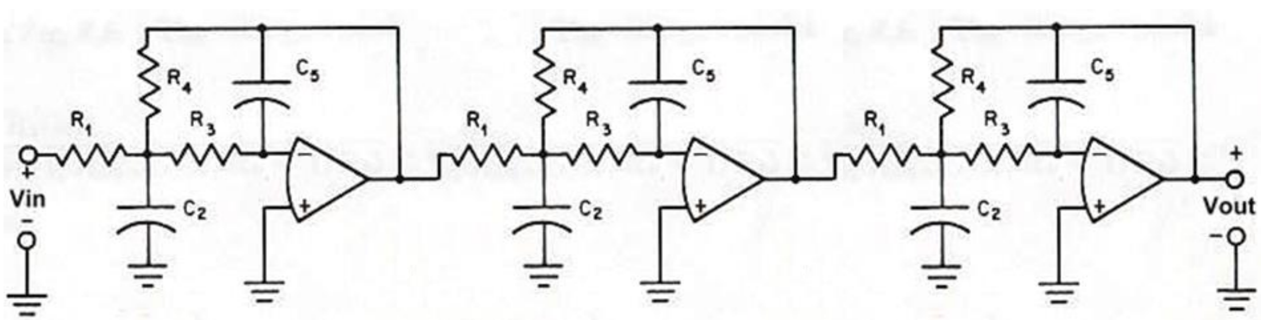
$$f_c = \frac{1}{\sqrt{4\pi^2 R_3 R_4 C_2 C_5}}$$

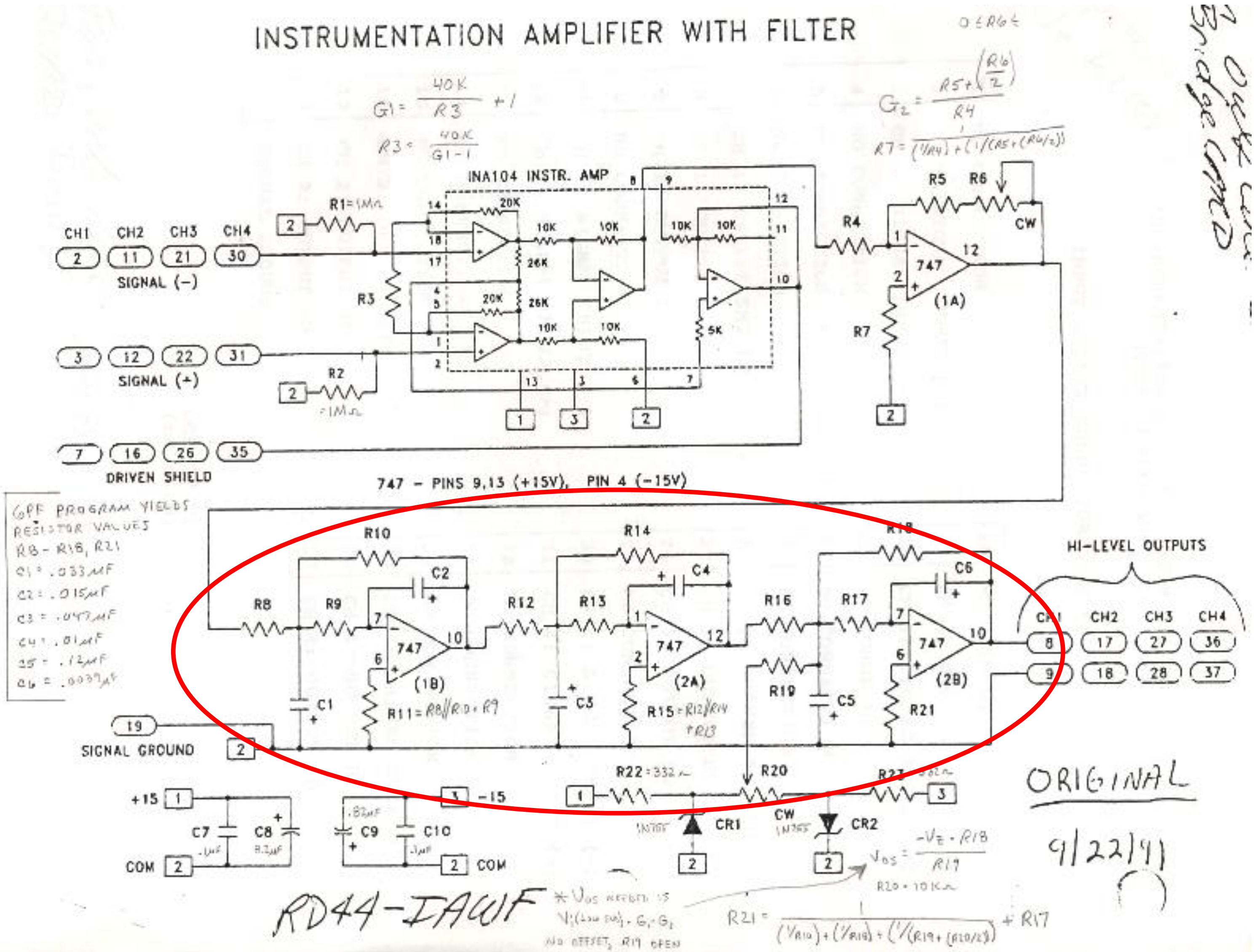
- The phase of the filter is

$$\varphi = \arctan\left(\frac{Re[H(j\omega)]}{Im[H(j\omega)]}\right)$$

Designing Butterworth Filters With More Than 2 Poles

- The two-pole filter shown earlier is used in series to get Butterworth filters with 3, 4, 5, 6, etc . . . poles. Usually you see filters with an even number of poles.
- A 6-pole filter will have three, two-pole stages in series as shown below:





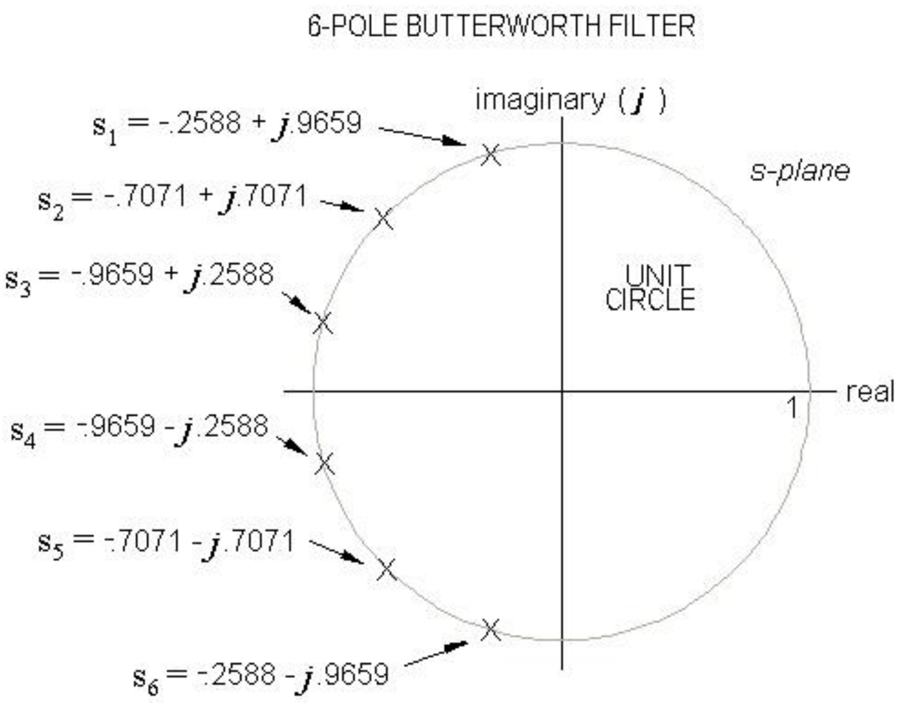
Note that our 2-pole filter stage contains an additional resistor (R11, R15, R21) which is used as an input offset bias current limitor to make the op amp look like the ideal op amp.

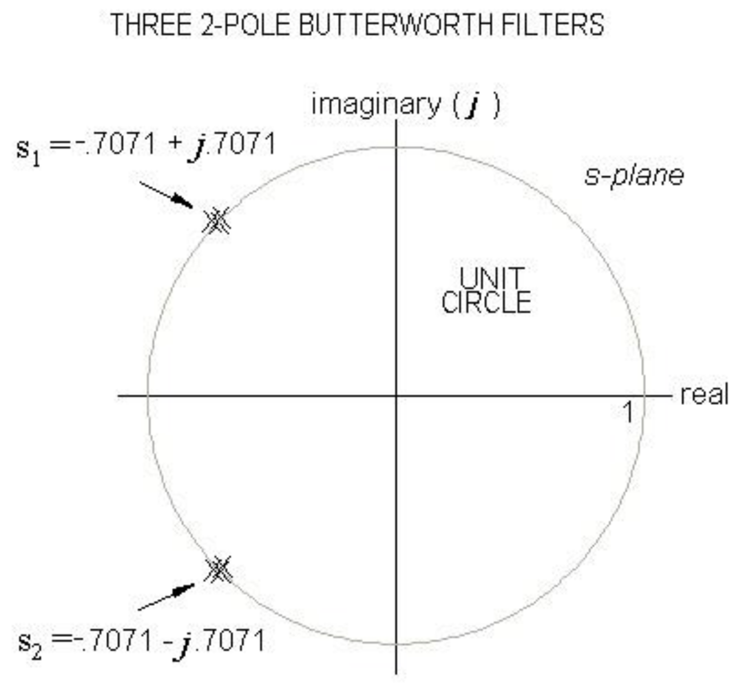
Butterworth Filters With 6 Poles

- In a six-pole Butterworth filter, each of the three two-pole filters in series are not the same as a two-pole Butterworth filter. They have different positioned poles.
- In a six-pole Butterworth filter, the three stages work together to get the classic Butterworth frequency response.
- You can't just string together 3 identical 2-pole Butterworth stages

Why Three 2-pole Butterworth Filters in Series are Not the Same as a 6-pole Butterworth Filter

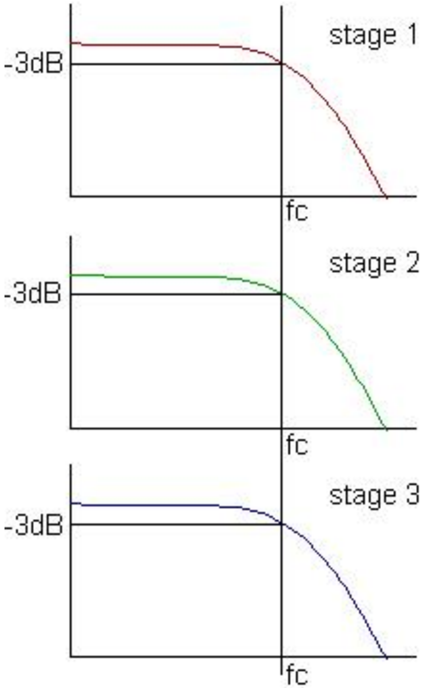
- **Pole locations for a 6-pole Butterworth and three 2-pole Butterworth filters in series.**

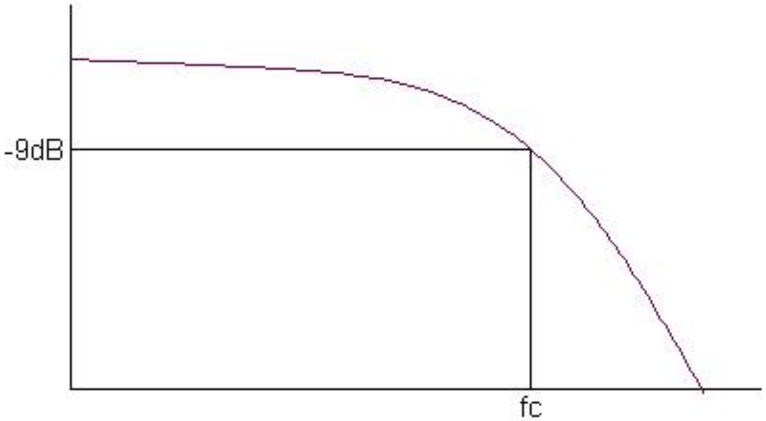




Why Three 2-pole Butterworth Filters in Series are Not the Same as a 6-pole Butterworth Filter

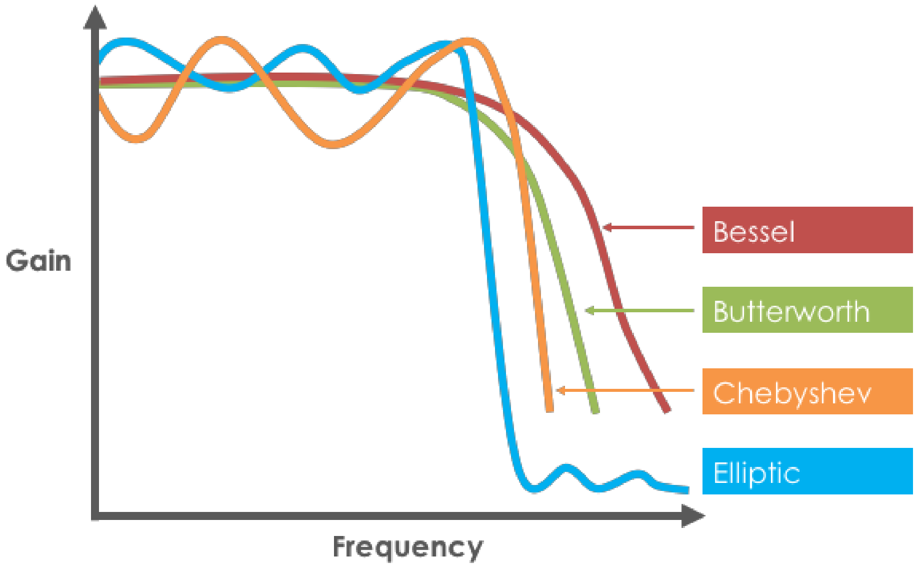
In dB, gains are added when each stage is in series. If you add all of the gains at f_c (-3dB), your output will have a gain of -9dB at the cutoff frequency.





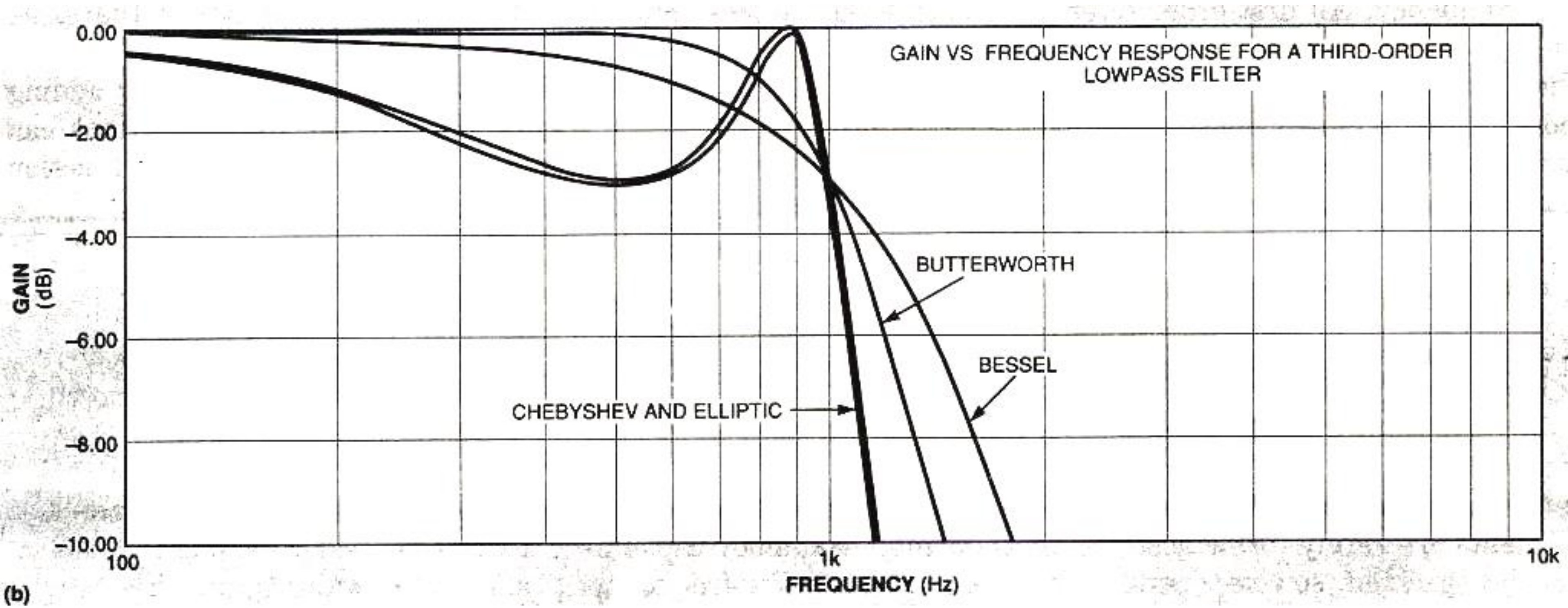
Optimum Filter Types

- **Butterworth**
- **Chebyshev**
- **Bessel**
- **Elliptic**



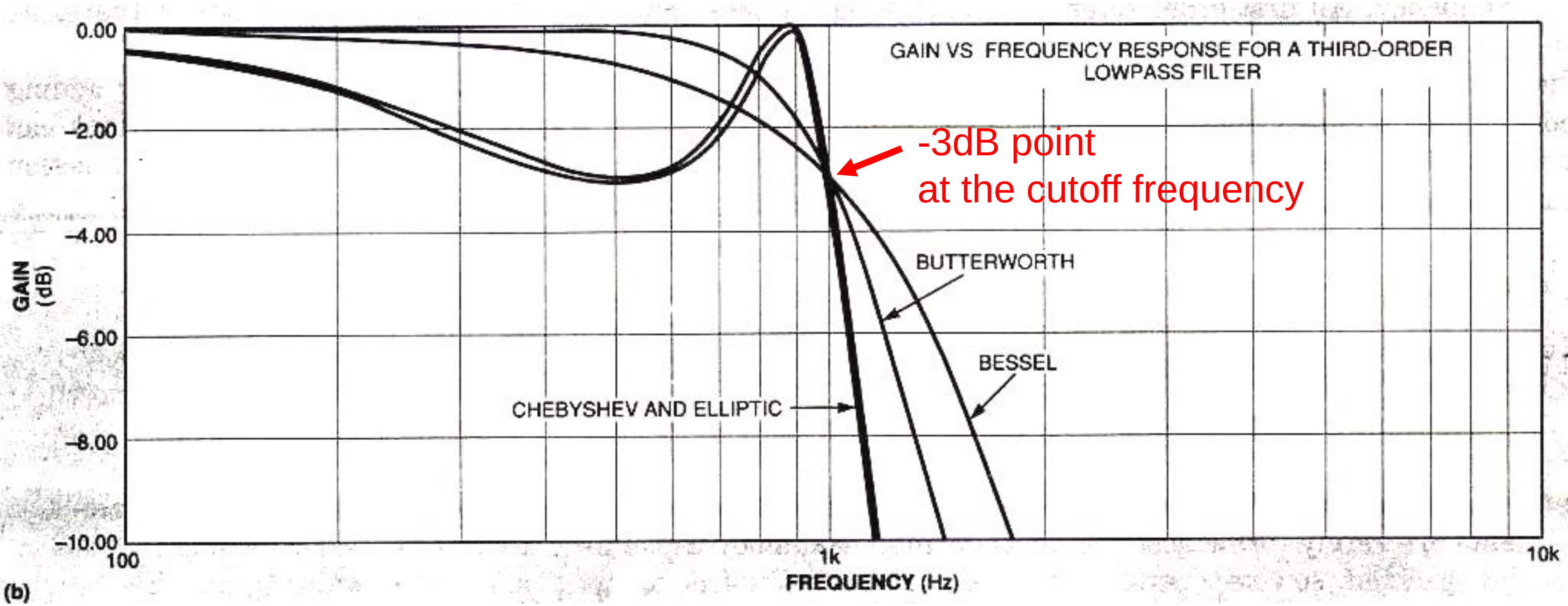
Comparison of Real LPF's

This is a Bode Plot that compares the magnitudes of a filter with a 1KHz cutoff frequency in each of the four filter representations.



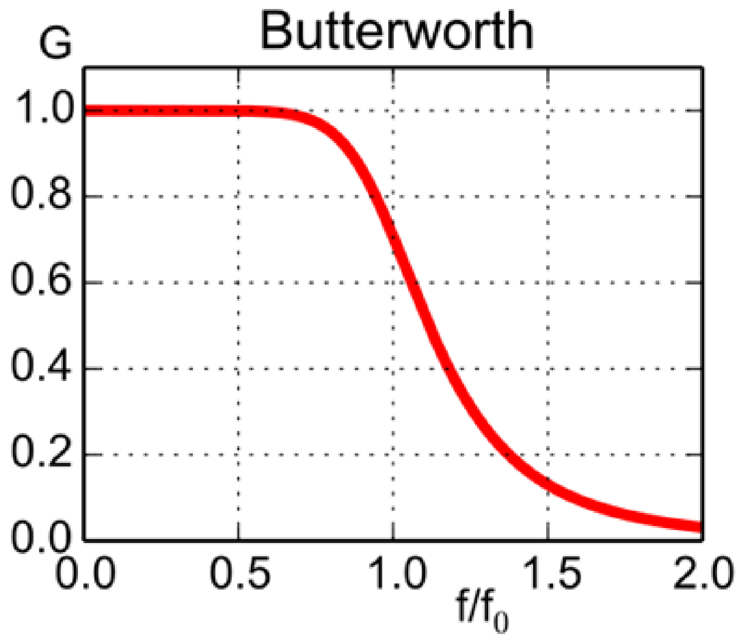
Comparison of Real LPF's

Another view of the Bode plots showing more detail around the cutoff frequency defined to be the -3dB point.



Optimum Filter Types - Butterworth

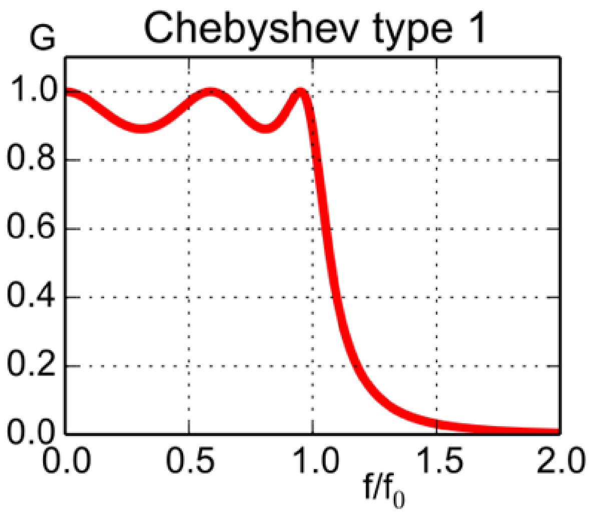
- Pros
  - Maximally flat passband
  - Moderate phase distortion
  - Easy to analyze
  - Easy to implement
- Cons
  - Slow attenuation drop off

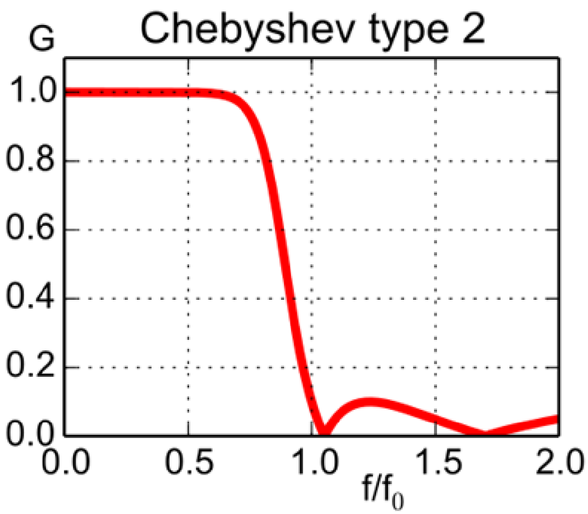


Optimum Filter Types - Chebyshev

- Pros
  - Steeper attenuation drop off
  - Easy to implement
- Cons
  - Ripples in the passband
  - Poor phase distortion

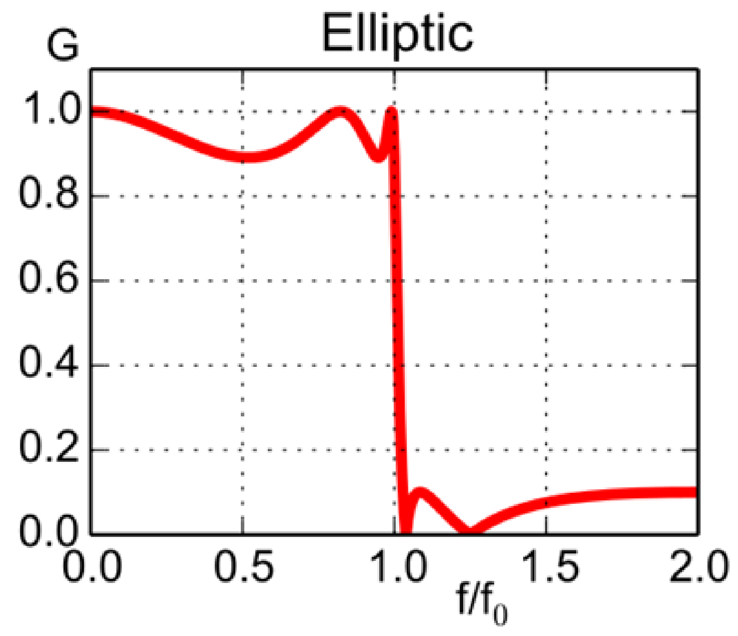
- Inverse Chebyshev has ripples in the stop band





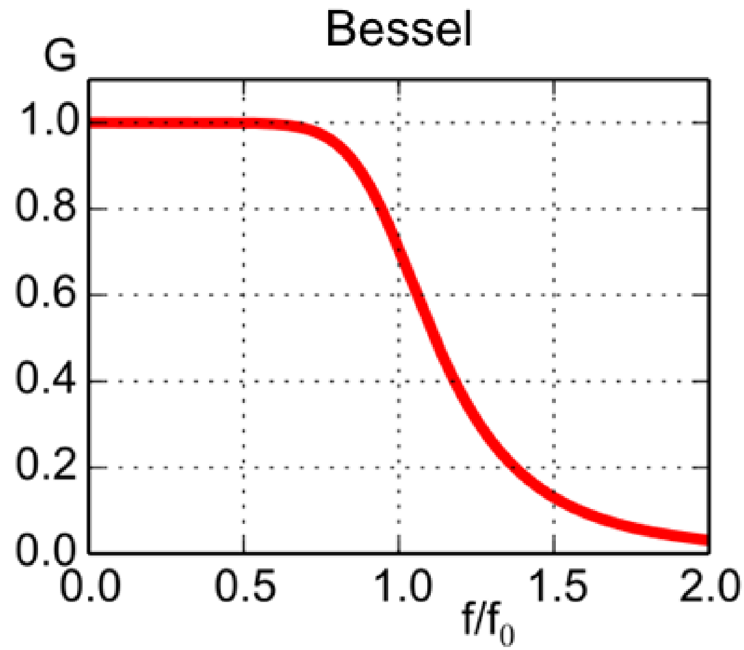
Optimum Filter Types - Elliptic

- Pros
  - Sharpest cutoff
- Cons
  - Poor phase distortion
  - More complicated implementation
  - Ripples in the pass band and stop band



Optimum Filter Types - Bessel

- Pros
  - Ideal linear phase response
  - Constant group (i.e. time) delay
  - Good for use with square waves
- Cons
  - Slow attenuation drop off



Because of their constant group delays, Bessel filters are commonly used for the pre-modulation filtering of digital signals.

Optimum Filter Transfer Functions

- **Butterworth**

$$G_n(\omega) = \frac{1}{\sqrt{1+\omega^{2n}}}$$

- **Chebyshev**

$$G_n(\omega) = \frac{1}{\sqrt{1+\varepsilon^2 T_n^2(\omega)}}$$

T_n is a Chebyshev polynomial of the n^{th} order

- **Bessel**

$$G_n(\omega) = \frac{\theta_n(0)}{\theta_n(\omega)}$$

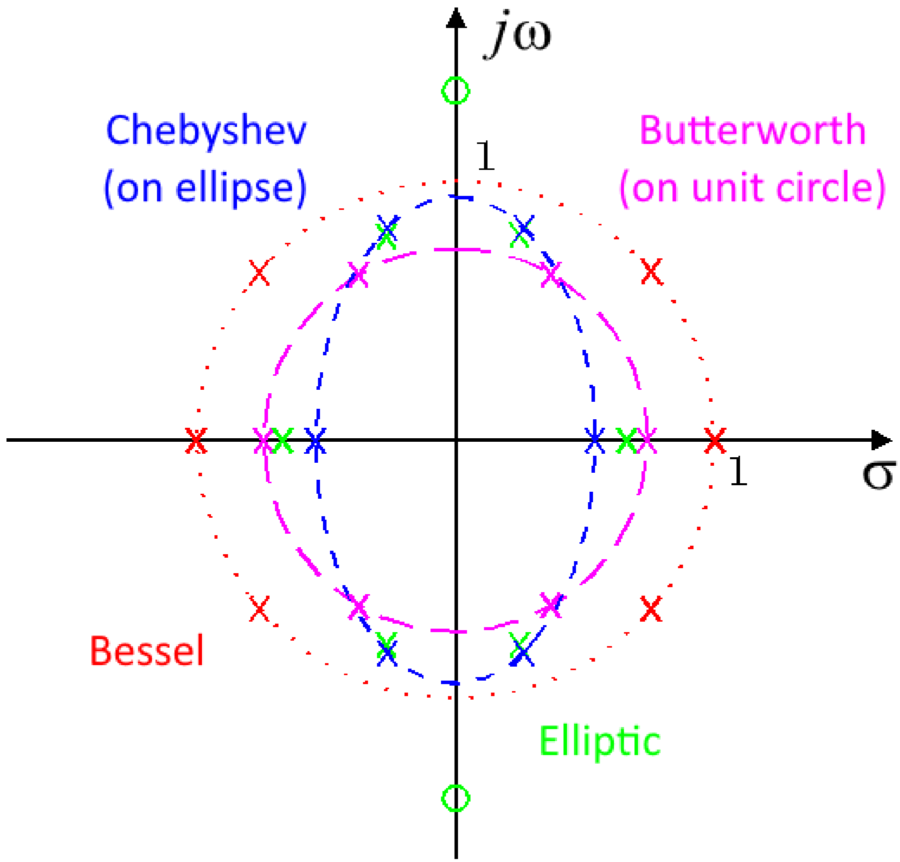
$\theta_n(s)$ is a reverse Bessel polynomial

- **Elliptic**

$$G_n(\omega) = \frac{1}{\sqrt{1+\epsilon^2 R_n^2(\xi,\omega)}}$$

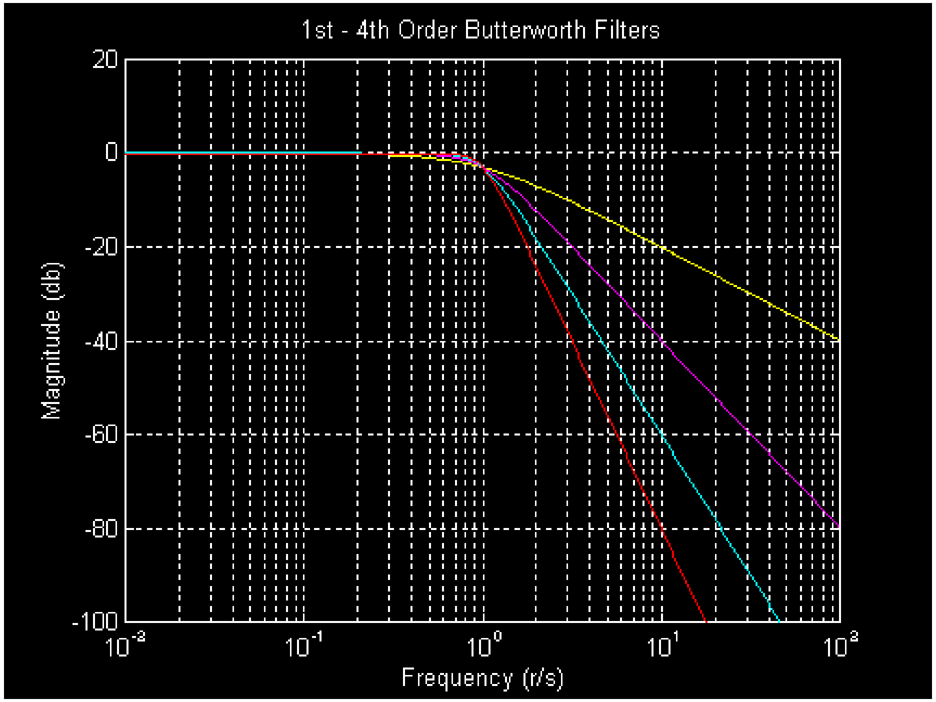
R_n is the n^{th} order elliptic rational function

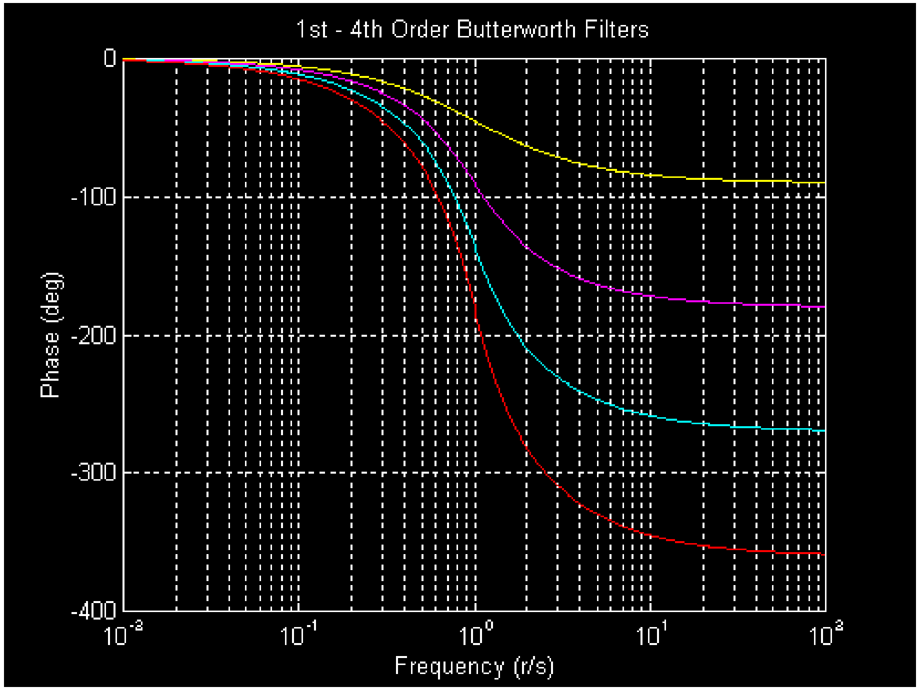
Pole (and Zero) Location for Common Filter Types



Why Choose Butterworth Filters for Data?

Butterworth Filters are used for filtering analog measurements because of their exceptional flat amplitude response in the pass band and linear phase response.





Determining the Characteristics of a Butterworth Filter

- You may find that the actual results of the characteristics of your filter are close, but not exactly what you have calculated.
- This is because the filter's characteristics are determined by physical components (resistors and capacitors) with values that can vary and can change with temperature. Therefore, the following equations are purely theoretical.

Magnitude Response of a Butterworth Filter

After some manipulation, the magnitude of the transfer function for a Butterworth filter is the following equation:

$$\boxed{G = |H(j\omega)| = \frac{1}{\sqrt{1 + (f/f_c)^{2n}}}}$$

where: G is the resultant gain at frequency *f*
n is the number of poles
f is the frequency of the input
f_c is the cutoff frequency of the filter

This equation is true for <u>all</u> Butterworth filters.

An Example of Finding Filter Gain

- If you have a 6-pole Butterworth filter with a cut off frequency of 14 Hz, how much gain will you see at 10 Hz?

$$G = |H(j\omega)| = \frac{1}{\sqrt{1 + (f/f_c)^{2n}}}$$

$$G = \frac{1}{\sqrt{1 + (10/14)^{2\cdot 6}}} = \frac{1}{\sqrt{1 + (0.714286)^{12}}}$$
$$= \frac{1}{\sqrt{1 + 0.01764}} = \frac{1}{\sqrt{1.01764}}$$
$$= 0.9912957$$

The output amplitude is 99.13% of the input amplitude at frequency *f* = 10 Hz.

What Does This Result Mean?

- The amplitudes of the signals which contain frequencies less than 10 Hz will be at worse 99.13% of what they actually are.

- So if a 10 V_p sine wave at 10 Hz was input to the filter, it would be a 9.913 V_p sine wave at 10 Hz at the output. Frequencies lower than 10 Hz will be closer to their actual input V_p (between 100% and 99.13% of V_p).

Magnitude Response of a Butterworth Filter

- The voltage magnitude response A, in decibels is:

$$G_{db} = 20\log(G_v)$$

$$G_{db} = 20\log\left(\frac{1}{\sqrt{1+(f/f_c)^{2n}}}\right)$$

$$G_{db} = 20\log\left((1+(f/f_c)^{2n})^{-1/2}\right)$$

$$\boxed{G_{db} = -10\log(1+(f/f_c)^{2n})}$$

Note that when $f = f_c$, G_{dB}= -3dB no matter how many poles the filter has.

Finding the Attenuation in dB

- For the filter we used in the example the gain, G, was found to be .9913 (99.13%) at 10 Hz. If we want to know what the gain is in dB, we use the equation:

$$\boxed{G_{db} = 20\log(G_v)}$$

$G_{dB} = 20 \log_{10} (0.9913)$
$G_{dB} = 20(-0.003795)$
$G_{dB} = -0.076$
$G_{dB} = -0.076$ dB

What Cutoff Frequency to Use?

- When a signal is to be recorded, the frequency response, *f* of the signal must be known in order to figure out what cutoff frequency to use on your pre-sample filter

- Also, the least gain G, acceptable within the frequency range of interest must be known (usually 99% or 0.99)

What Cutoff Frequency to Use?

Solve the equation for f_c to get:

$$G = \frac{1}{\sqrt{1 + (f/f_c)^{2n}}}$$

$$f_c = \frac{f}{\sqrt[2n]{\left(\frac{1}{G^2}\right) - 1}}$$

f_c is the cutoff frequency
f is the max frequency of interest
G is the minimum gain allowed at f
n is the number of poles

What Cutoff Frequency to Use?

To determine what cutoff frequency to use, you must first see what the requested frequency bandwidth (i.e. frequency response) is.

ITEM NO.	MEASUREMENT	RANGE	FREQUENCY RESPONSE	ACCURACY	SIGNAL SOURCE	DATA COLLECTION			
1	Noseboom Airspeed	0 - 600	10	1 kt	I	T	A		
2	Altitude	-2K/50K	10	50 ft	I/AS	T	A		
3	OAT	+ 50.0	5	1.0 °C	I	T	A	V	I
4	CG Normal Acceleration	-3.0/+7.0	20	0.02 G	I/AS	T	A		
5	CG Lateral Acceleration	+ 2.5	20	0.05 G	I/AS	T	A		
6	Roll Acceleration	+ 200.0	20	2.0 deg/s	I	T	A		
7	Angle of Sideslip (Noseboom)	+ 180.0	20	0.5 deg	I	T	A		
8	Noseboom Angle of Attack	-15 to 30	20	0.5 deg	I	T	A		
9	Bank Angle	+ 180.0	20	1.0 deg	I/AS	T	A		
10	Pitch Attitude	+ 90.0	20	0.5 deg	I/AS	T	A		

What Cutoff Frequency to Use?

Say you have a signal to record with the following characteristics

- Frequency components to 120 Hz
- 6-pole Butterworth filter
- Minimum gain acceptable is 99%

What should the cutoff frequency be?

What Cutoff Frequency to Use?

- *f* is the highest frequency of interest, 120 Hz
- G is the lowest allowable gain, 99%
- n is the number of poles in the Butterworth filter, 6

$$f_c = \frac{f}{\sqrt[2n]{\left(\frac{1}{G^2}\right) - 1}} = \frac{120}{\sqrt[2*6]{\left(\frac{1}{0.99^2}\right) - 1}}$$

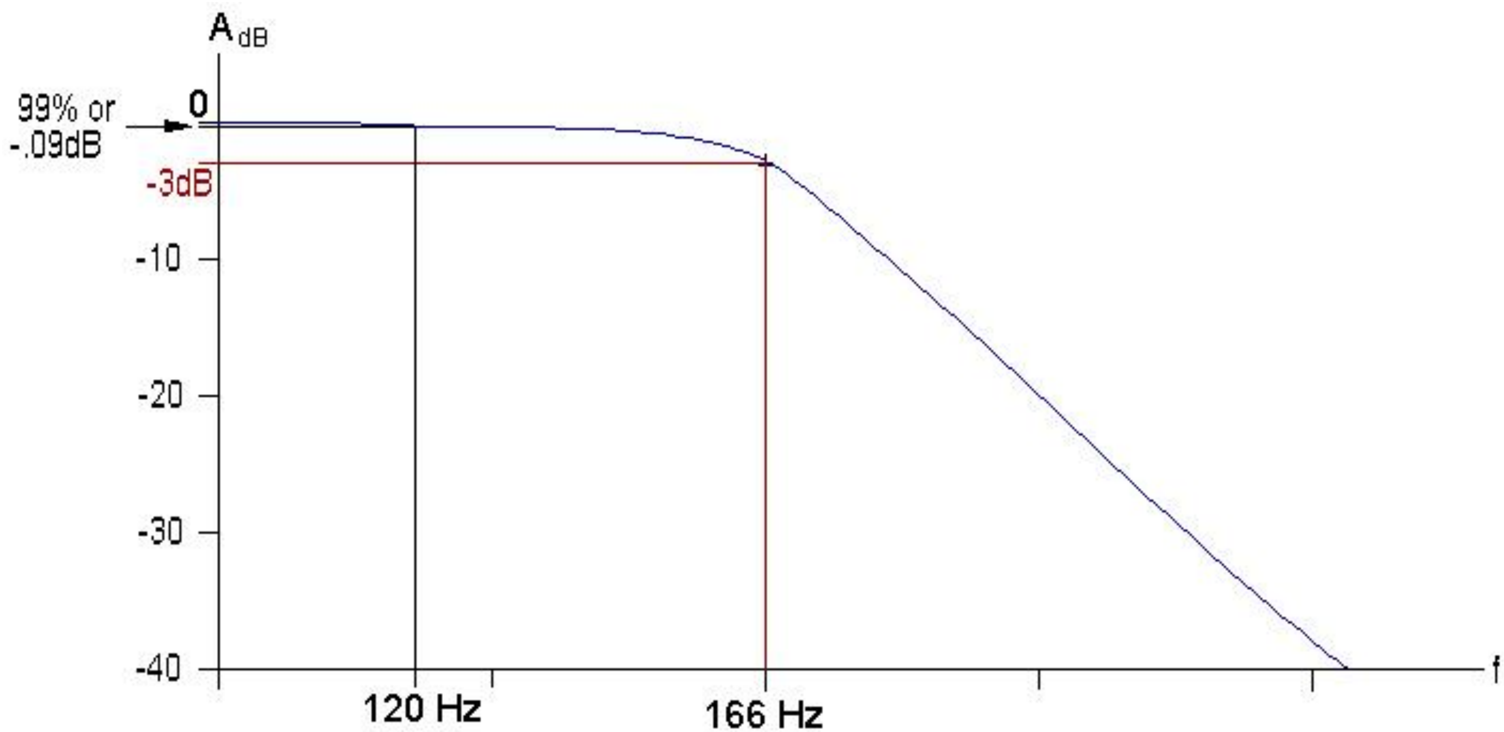
$$f_c = \frac{120}{\sqrt[12]{\left(\frac{1}{0.9801}\right) - 1}} = \frac{120}{\sqrt[12]{(1.02030405) - 1}}$$

$$f_c = \frac{120}{\sqrt[12]{(0.02030405)}} = \frac{120}{0.722712}$$

$$f_c = 166.04Hz$$

What Cutoff Frequency to Use?

By designing the 6-pole Butterworth filter with a cutoff frequency of 166 Hz, the gain of the signal is at least 99% at frequencies of 120 Hz or less.



What Cutoff Frequency to Use?

- A rule of thumb for the cutoff frequency of a 6-pole Butterworth filter, with a maximally flat response of 99% and a maximum frequency of interest *f*, should be:

$$\boxed{f_c = 1.383677f \cong \sqrt{2}f}$$

- So for our example, 99% data to 120 Hz, the cutoff frequency of the filter should be:

$$f_c \cong \sqrt{2}f = \sqrt{2} \cdot 120\ Hz = 169.7\ Hz$$

which is close to the exact calculation we made previously.

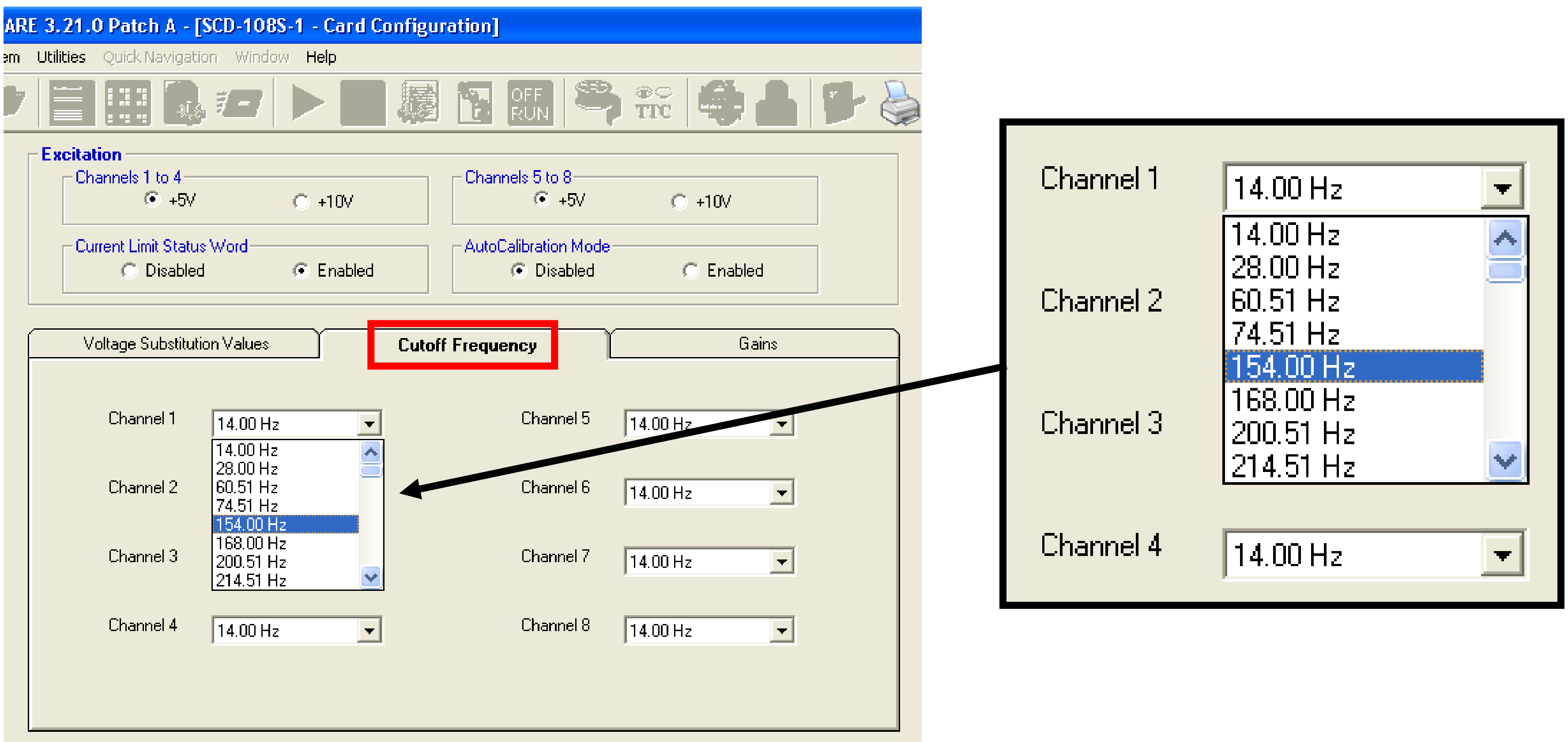
Selecting Filter Cutoffs Using a CDAU

- Programmable systems have made filtering less labor intensive. No longer do you have to select precise component values and solder them to a circuit board.

- With TTCWare, it is as simple as selecting the cutoff frequency – but as you can see, there is a lot of thought behind choosing the cutoff frequency.

Selecting the Cutoff Frequency on an SCD-108S

In the TTCWare card configuration screen for the SCD-108S, click on the cutoff frequency tab to select the filter cutoff for the channel's filter.

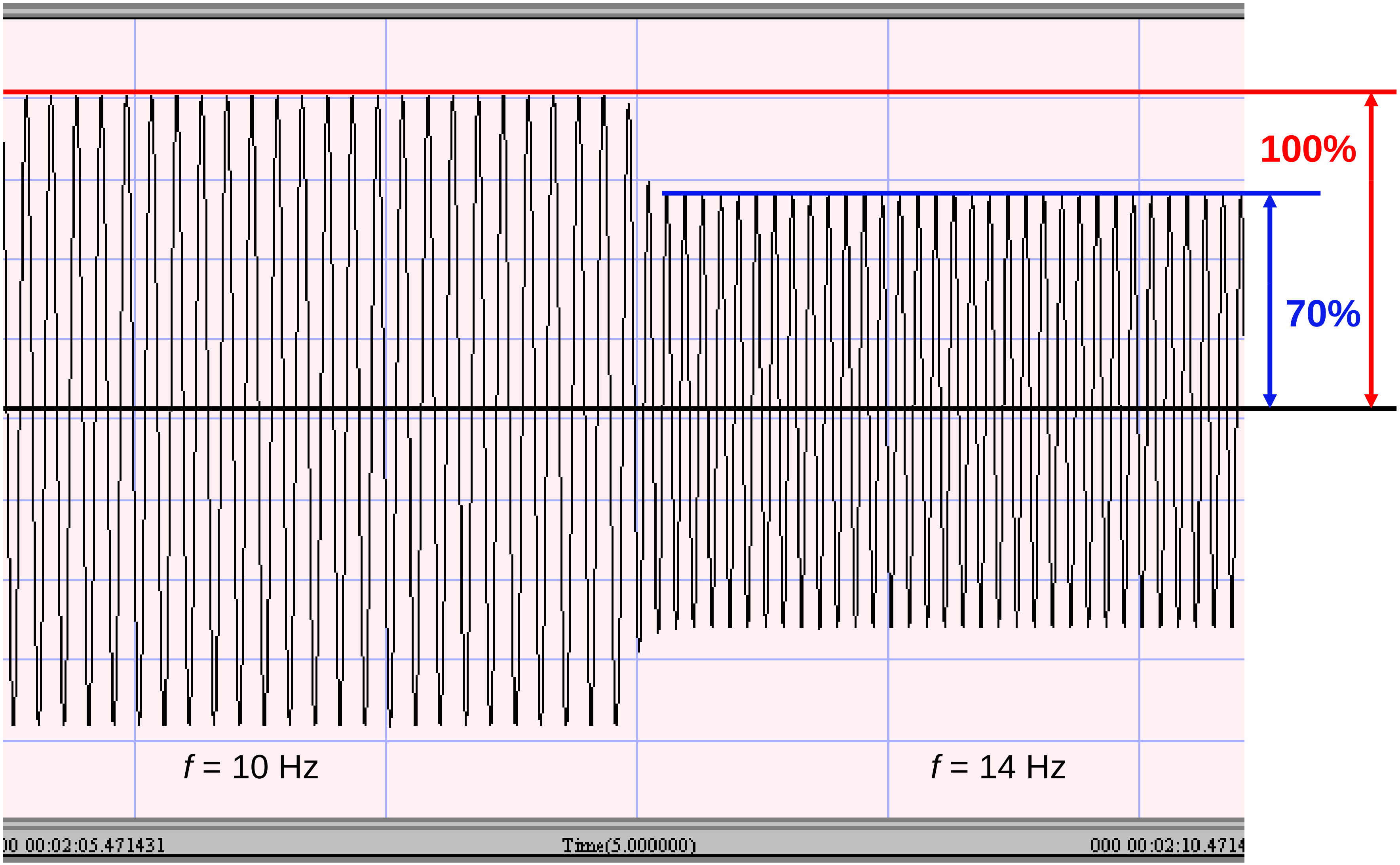


Verifying the Filter

- You can verify the cut-off of the programmable filter by injecting a sine wave into the channel, and monitoring the parameter on a strip chart.

- Note what the amplitude is at a low frequency, and verify that the amplitude reduces to approximately 70% (-3dB) when the frequency is increased to the filter’s cut-off frequency.

Verifying the Filter at f_c=14Hz



In Summary

- Filtering is a very important element of signal conditioning.
- *The object of pre-sample filtering is to attenuate unwanted frequency components such that they no longer contribute to the signal.*
- By not selecting a filter with the proper characteristics, a measurement can be *permanently* corrupted due to aliasing effects, noise, or attenuation of the amplitudes in the flight test engineer’s frequencies of interest.

Summary of Equations

Gain to dB Conversion

$$G_{db} = 10\log_{10} G_p$$
$$= 20\log_{10} G_v$$

dB to Gain Conversion

$$G_p = 10^{G_{db}/10}$$
$$G_v = 10^{G_{db}/20}$$

Butterworth Gain as a function of Frequency

$$\mathrm{G}(f) = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}}$$

Each pole contributes -20dB/decade
or -6dB/octave roll off in the stop band

Signal Conditioning

Lab Exercises

Viewing a 6-pole Butterworth Filter's Characteristics

- Back when AID built its own discrete component filters, each one had to be checked out to verify that the filter characteristics were correct.
- A misplaced component or a wrong valued component does not place the poles in the correct location and your filter characteristics will not be what is desired.
- The next slides will illustrate the characteristics of an actual 6-pole Butterworth Filter.

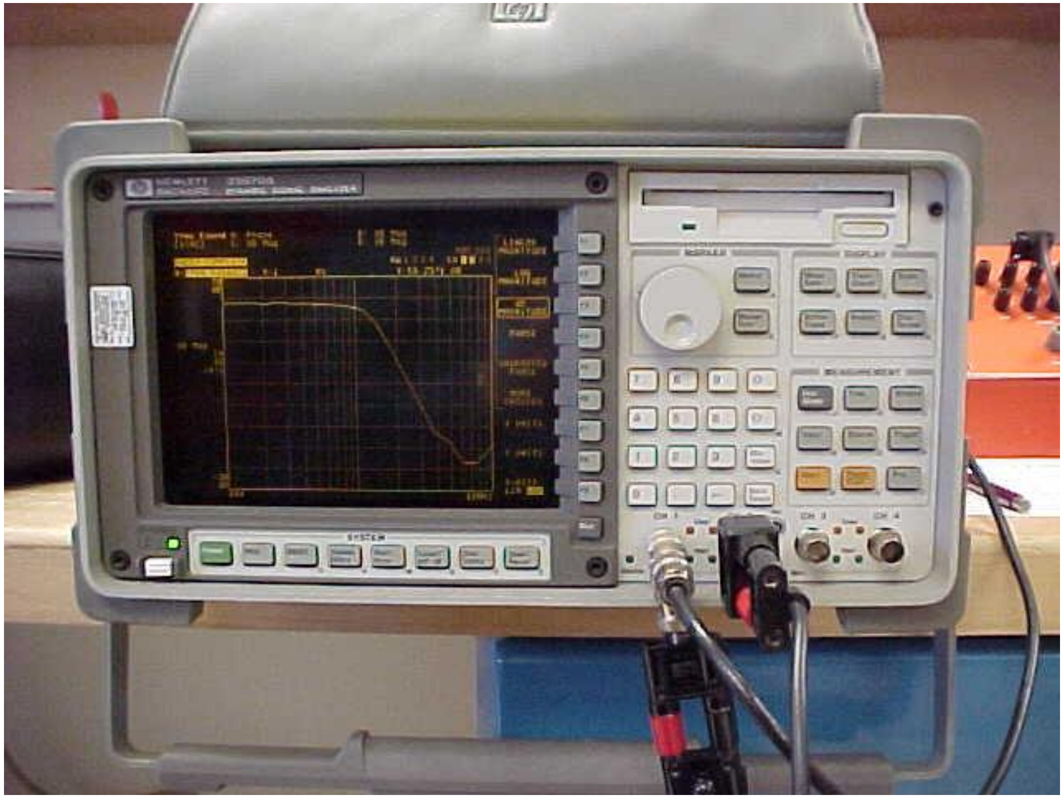
Viewing a 6-pole Butterworth Filter's Characteristics

We will now view the following characteristics of an actual Butterworth filter.

- **DC Gain**
- **-3dB cutoff frequency**
- **Attenuation in the frequency band of interest**
- **Proper roll off in the stop band**

Because we cannot access the output of the filtering in most of our modern signal conditioning, we cannot make these measurements on the CDAU or MCDAU.

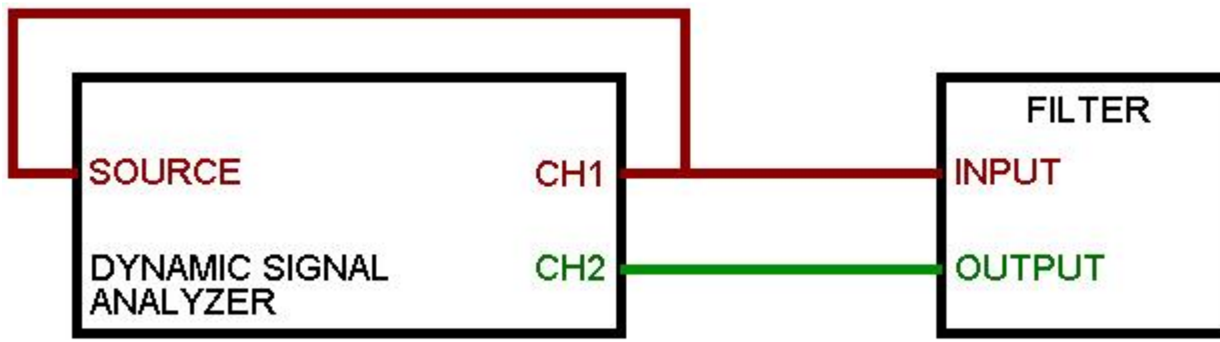
Dynamic Signal Analyzer



The Dynamic Signal Analyzer makes checking filters easy. The output is a Bode plot of the filter. All four of the filter’s characteristics can be seen and verified.

System Setup

The Dynamic Signal Analyzer generates its own swept sine wave that sweeps the frequency range of interest. That source is fed back into Ch1 and into the filter under test, and the filter output goes to Ch2.



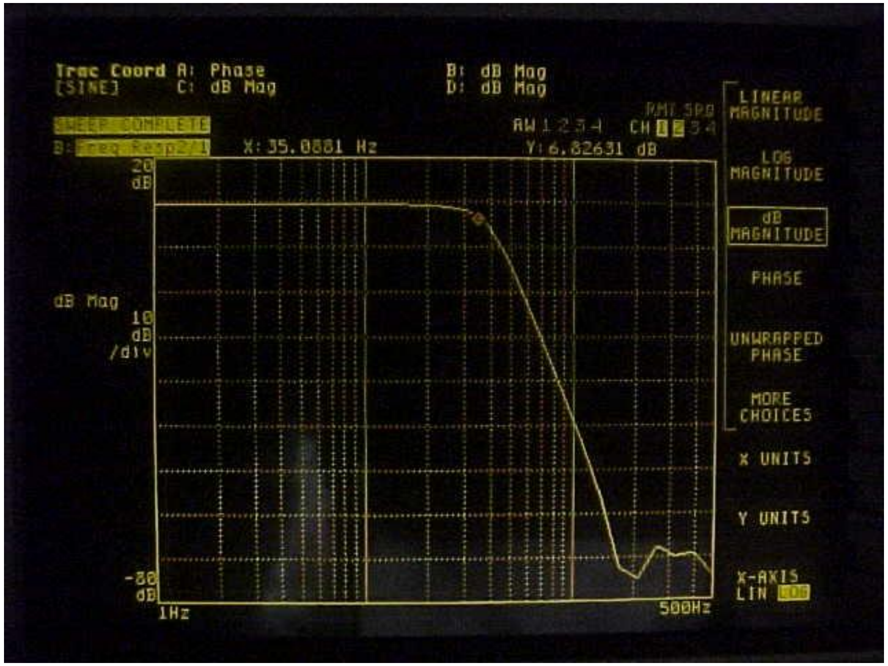
The configuration is Ch2/Ch1 which corresponds to V_{out} / V_{in}, the gain, A of the filter.

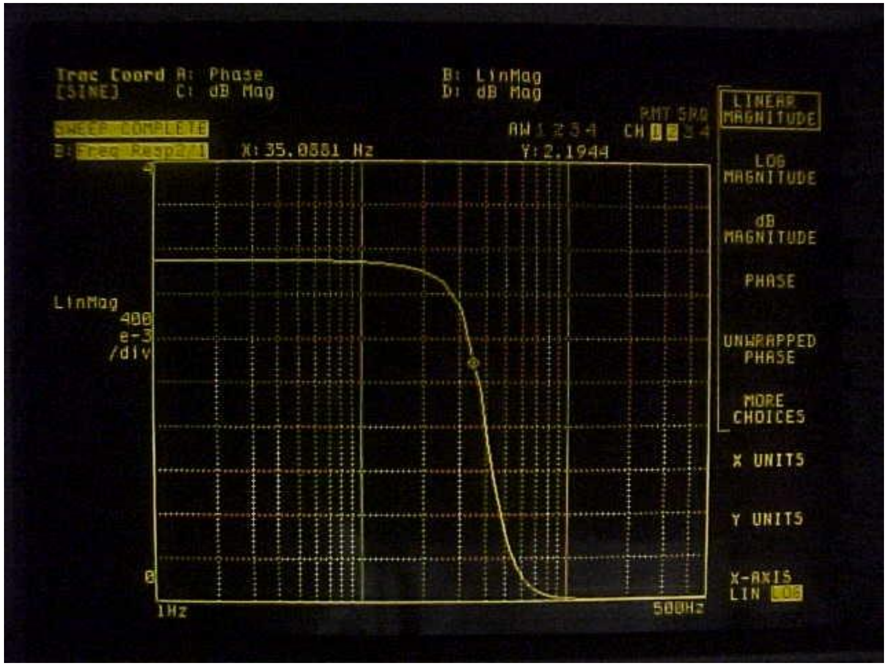
Viewing a Filter’s Characteristics

For this example, a 6-pole Butterworth Filter has been designed with a cutoff (-3dB down) frequency of 35 Hz. The gain on the card is designed to be 3.1 (9.83 dB). Here is how the characteristics are viewed as seen on the dynamic signal analyzer.

Bode Plot Display

Setting a frequency range of 1 - 500 Hz with a source input amplitude of 1V, the following Bode plot is generated. The left plot is in dB, the plot on the right is on a linear plot of gain. At 200 Hz, the gain is down to -70dB (.03%) – which is essentially below the noise floor.



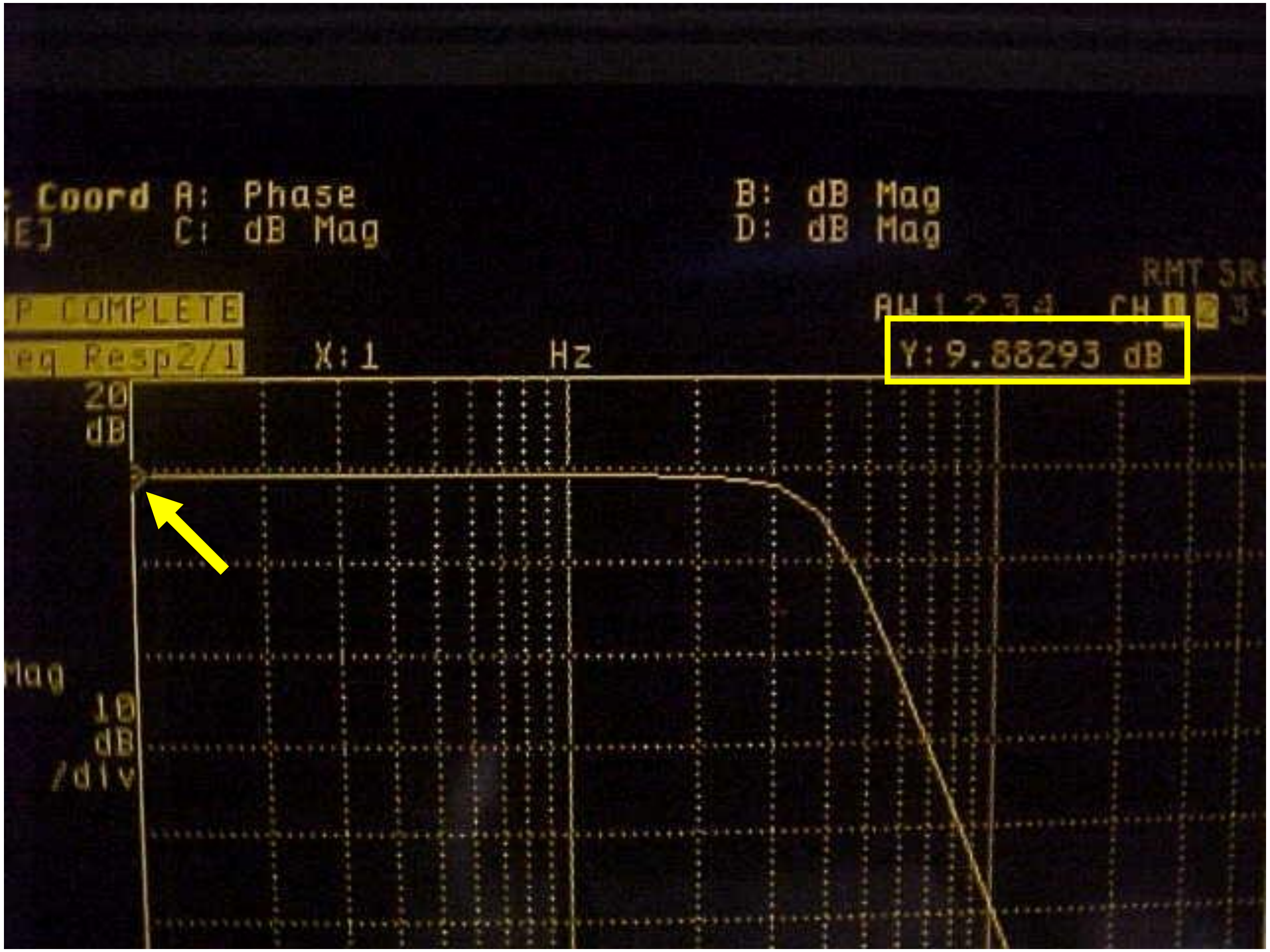


DC Gain

Moving the cursor to the far left (lowest frequency at 1Hz) you can get a measurement of the DC gain (3.1194) with the display in Linear Magnitude.

DC Gain

Changing the display to dB Magnitude, you will get a display of the DC gain in dB's (9.88293 dB)

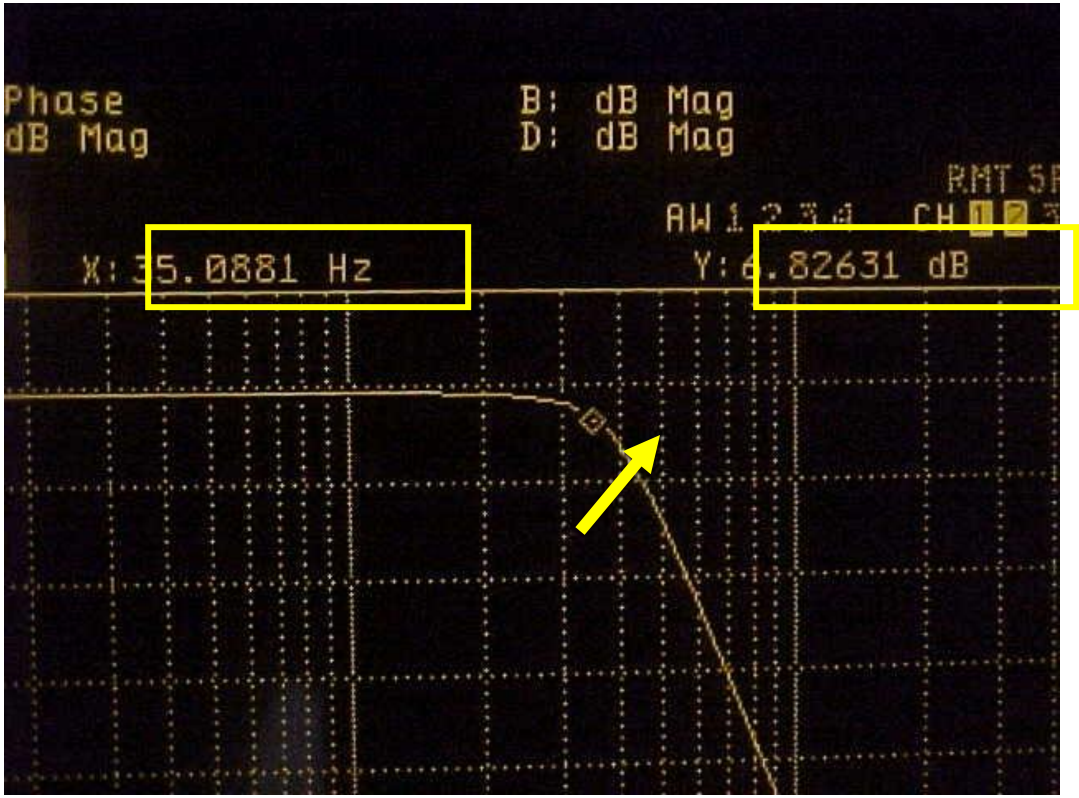


The -3dB Point

To find the frequency at the -3dB point, subtract 3dB from the DC gain in dB.

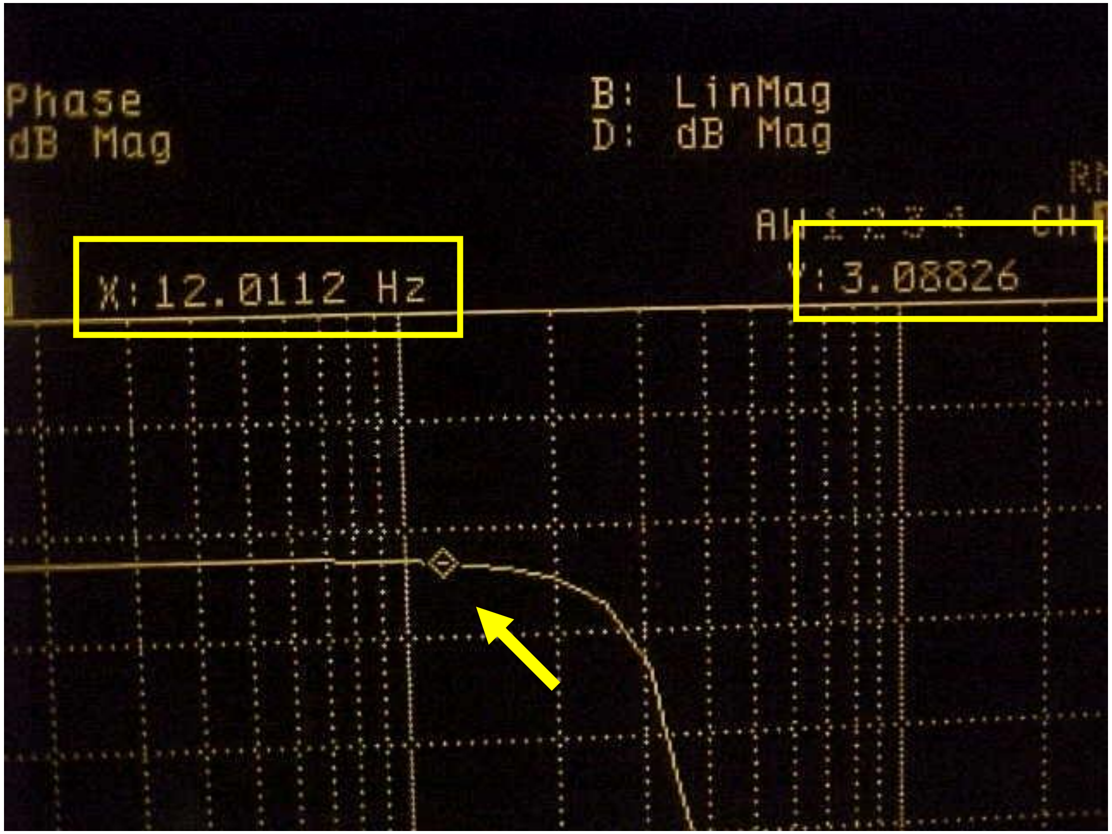
9.88293 dB - 3 dB = 6.88293 dB

Move the cursor to find the cutoff frequency (35.0881 Hz)



Attenuation in the Frequency Band of Interest

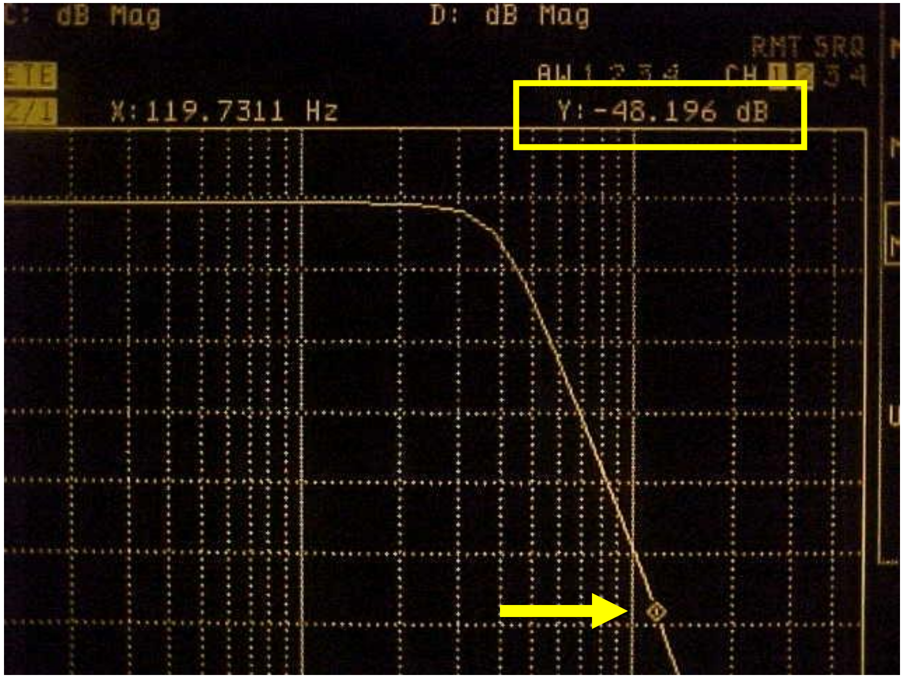
Changing back to Linear magnitude, take 99% of the DC gain 3.1194. That gain would be 3.088206. Move the cursor to that gain to find the 99% frequency (12.0112 Hz).

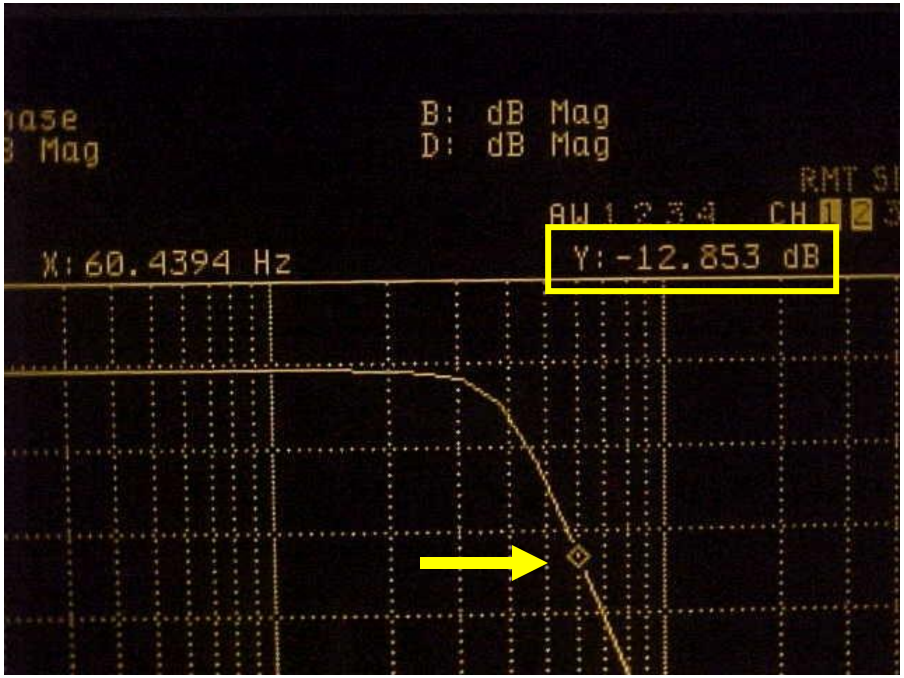


Proper Roll Off in the Stop Band

In the linear slope of the filter pick two frequencies that are octaves (60 Hz and 120 Hz). For a 6-pole filter you should see a gain change of -6dB/octave/pole (or -36dB)

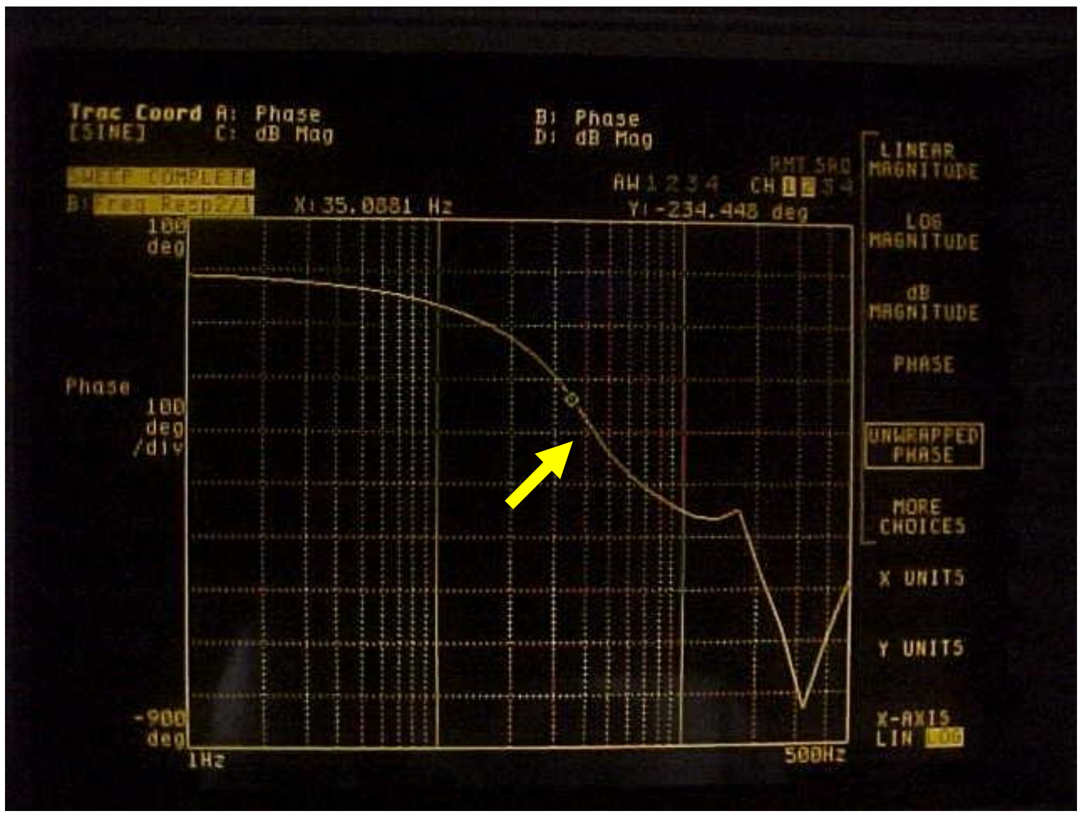
-48.196 dB - (-12.853 dB) = -35.353 dB





Phase Response of the Filter

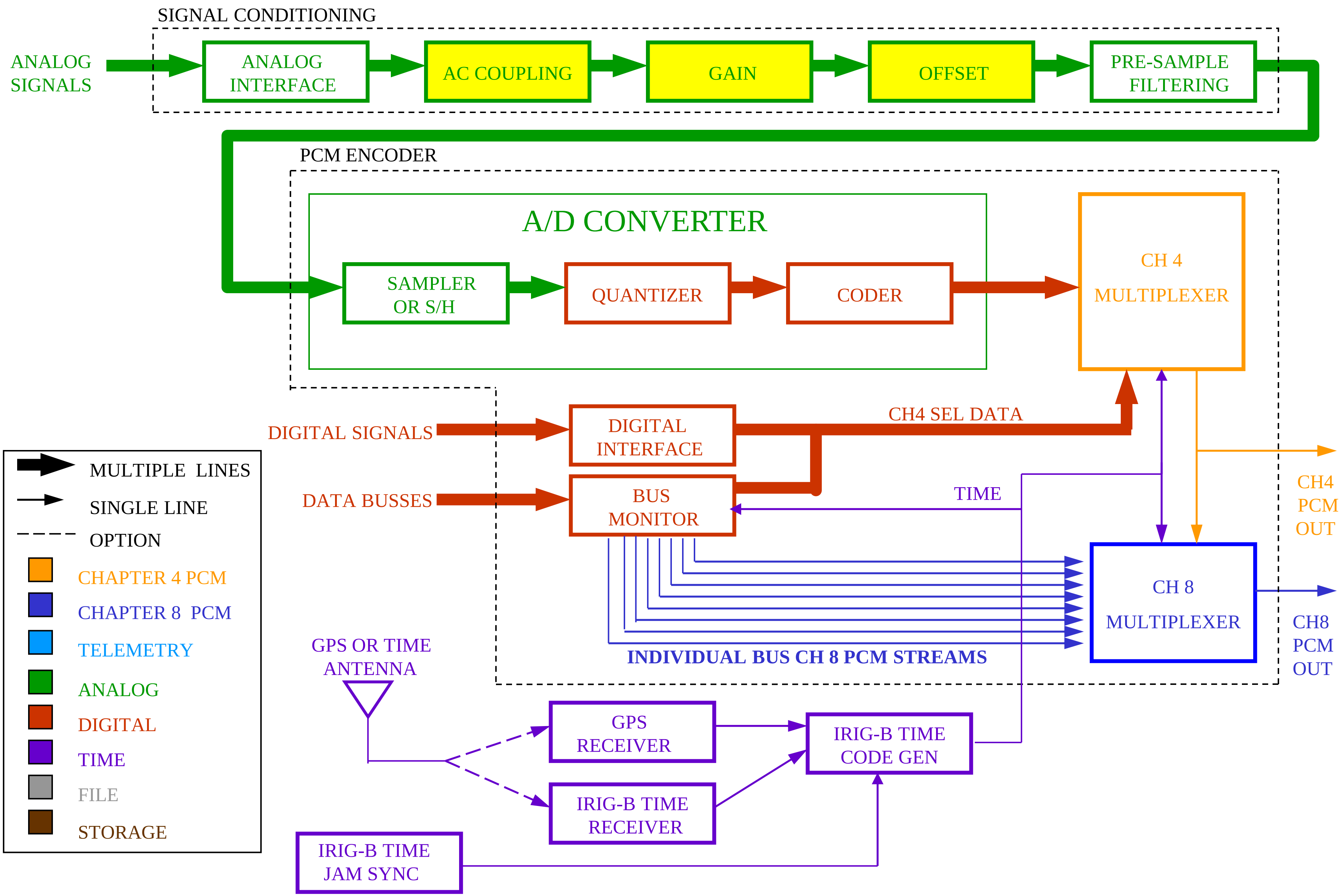
This shows the phase response of the filter. At DC the phase delay is 0º, at the filter cutoff it is -234.443º, and keeps increasing as the frequency increases. This would be seen as a delay in time at the output of the filter.



Signal Conditioning

AC Coupling, Gains, Offsets

Data Acquisition System



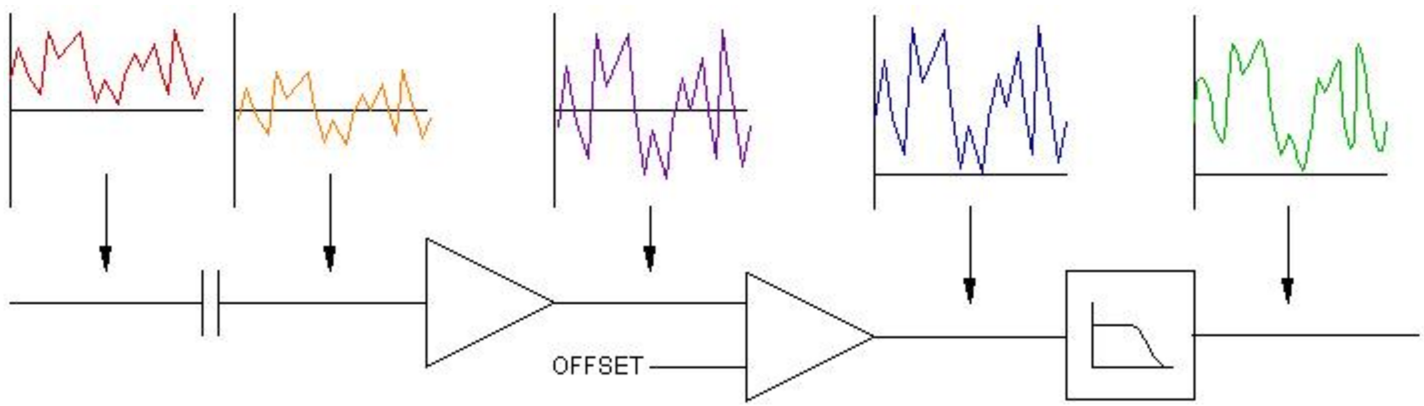
Introduction

- AC/DC Coupling, Gain, and Offset are used to condition almost every analog parameter.

- Each analog measurement in a PCM encoder is digitized, so the analog input range has to be set to most efficiently use the range of the Analog to Digital Converter (ADC)

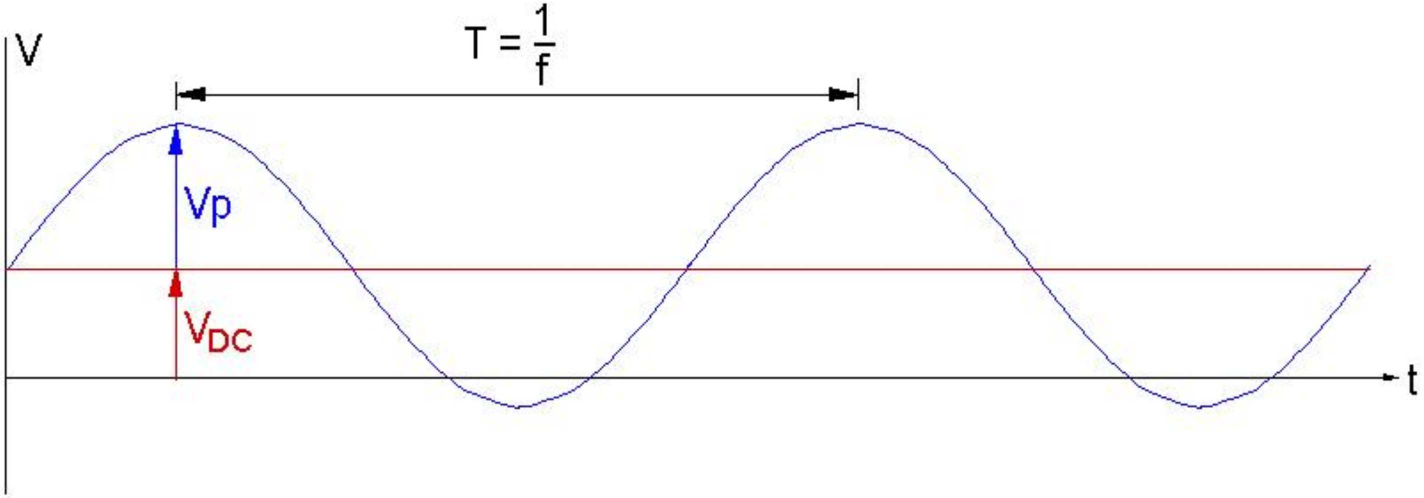
Purpose of Each Type of Conditioning

- <u>AC Coupling</u> - Used to remove any DC component (or offset) of a measurement that may not be desirable.
- <u>Gain</u> - Used to most efficiently utilize the input range of the ADC.
- <u>Offset</u> - Used to center the range of a measurement within the full range of the ADC.



AC and DC Component of a Signal

$$V = V_p \sin(2\pi f t) + V_{DC}$$



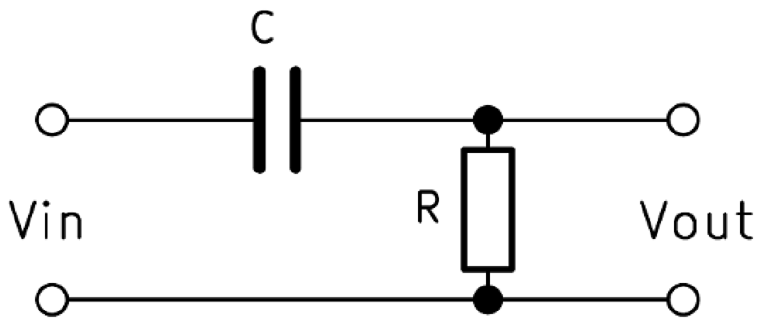
Signals not centered around 0 volts have a DC component, V_{DC}. The AC component of the signal has an amplitude of V_p.

AC Coupling

- AC Coupling is used to remove any DC component (or offset) of a measurement. This is done by placing capacitors in series with the signals. Capacitors do not pass DC signals.

- With no AC coupling (sometimes called DC coupling) both the AC and DC components of the signal are passed through to the encoder.

How AC Coupling Eliminates the DC Component of the Signal

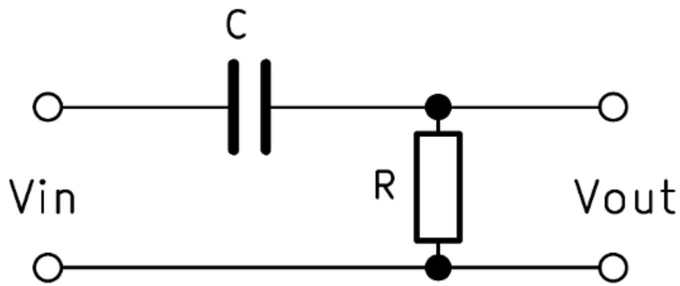


$$V_{out} = \frac{j(2\pi)fRC}{1 + j(2\pi)fRC} V_{in}$$

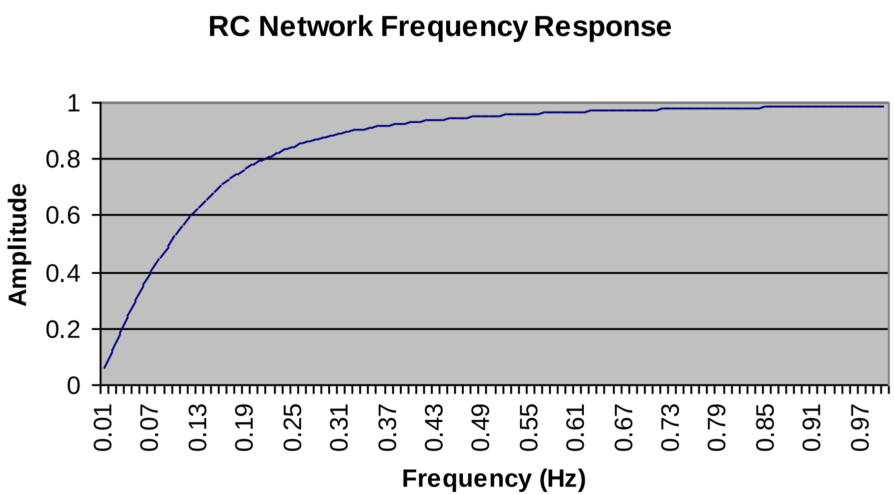
- The circuit shown above is a passive high-pass filter which uses a capacitor in series with the input signal. The DC portion of the signal does not have a frequency (*f* = 0). Substituting *f* = 0 into the equation yields V_{out}= 0. Therefore, no DC voltage passes through the capacitor.

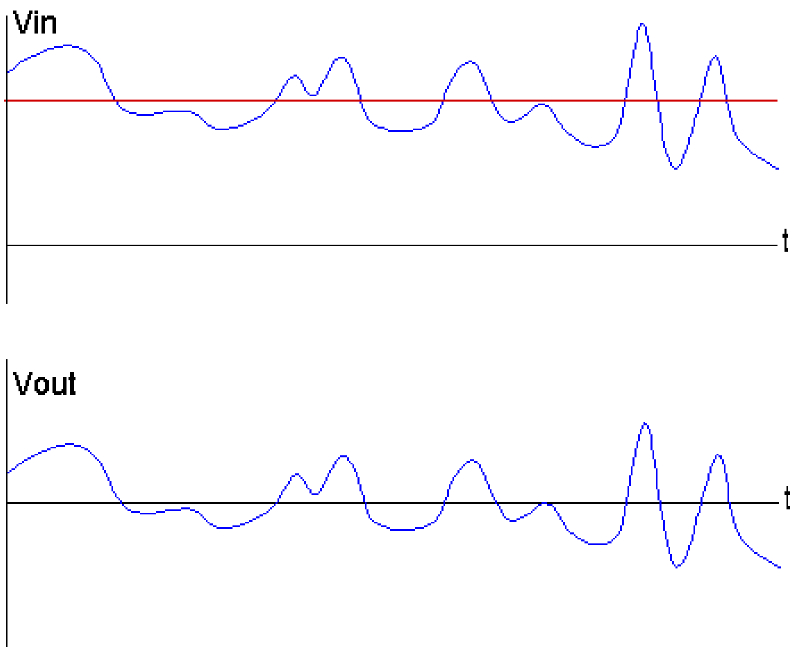
- The AC portion of the signal has a frequency. Therefore *f* does not equal 0, and V_{out} will have some value dependent upon the frequency.

Effect of AC Coupling



$$V_{out} = \frac{j(2\pi)fRC}{1 + j(2\pi)fRC} V_{in}$$

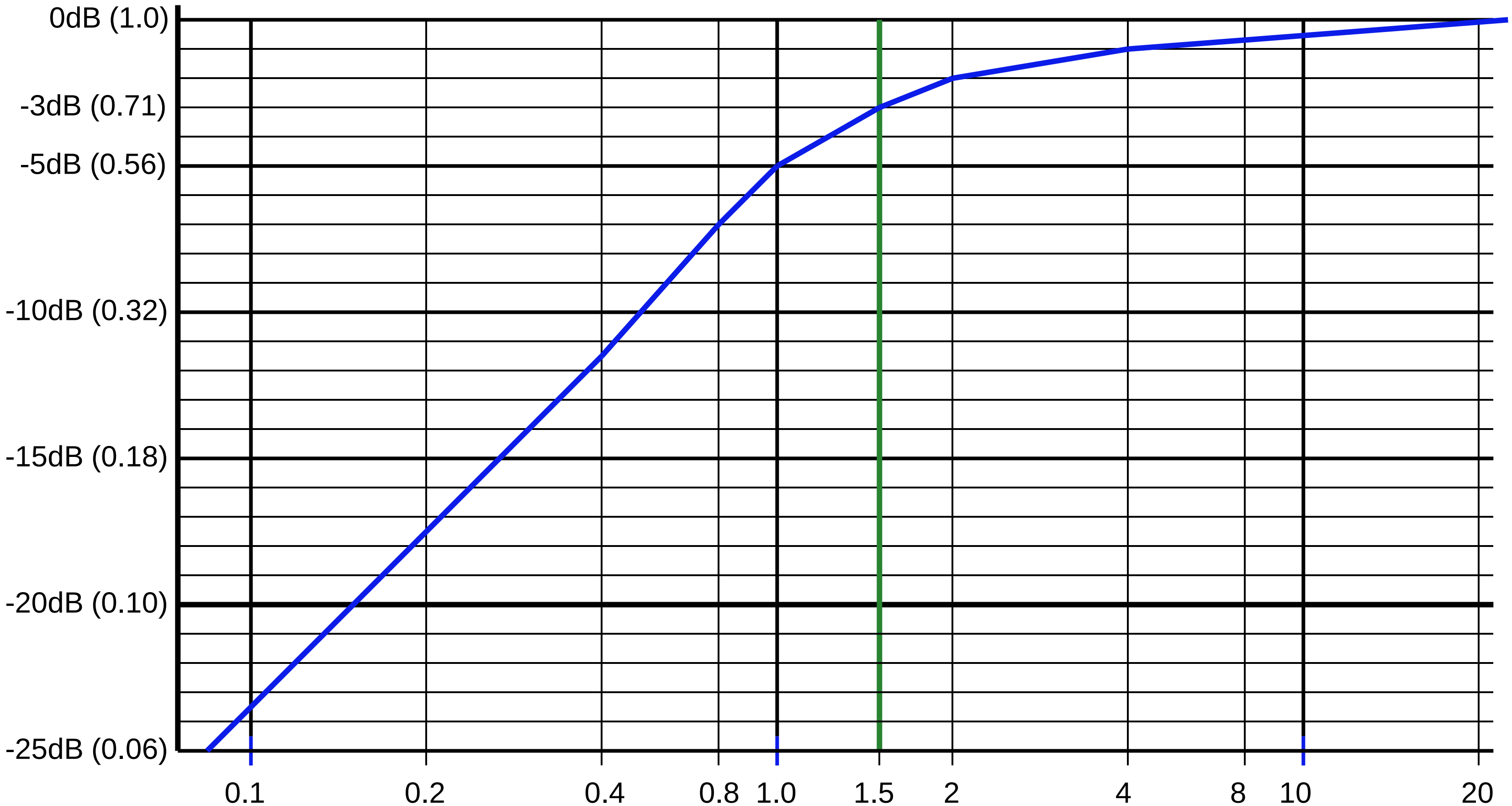




As the frequency increases, the filter's gain approaches 1.

Effect of AC Coupling

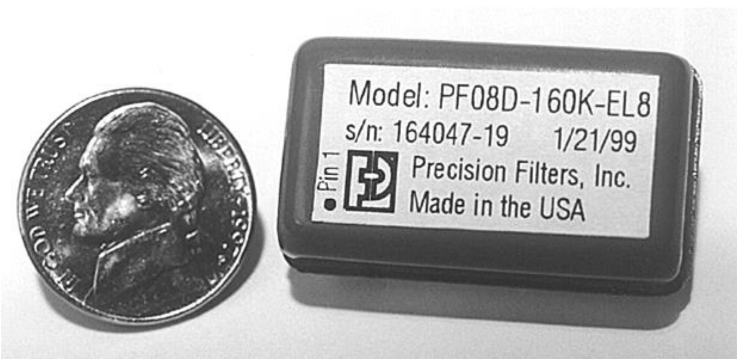
Be aware of the filtering effect of AC Coupling. Note the attenuation that occurs from 0 Hz to your lower data frequency.



This is the response for AC coupling on the SCD-608S card. It is a one-pole Butterworth with a cutoff frequency of 1.5 Hz. The attenuation does not reach 99% until 10 Hz, so frequencies below that are attenuated even more.

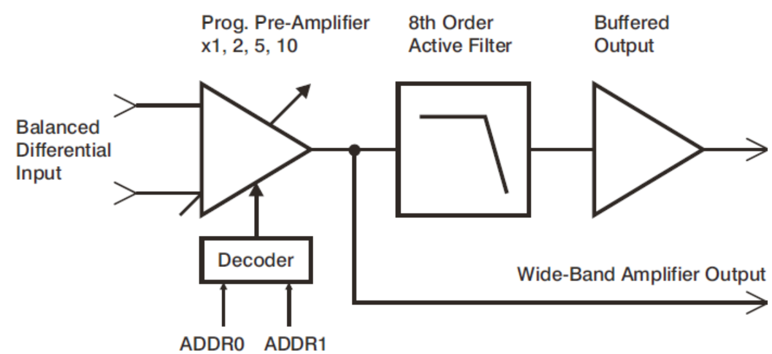
Special AC Coupling Applications

Sometimes customers want frequencies as low as 2 Hz to not be attenuated by the standard 1.5 Hz cutoff AC-Coupling. In these instances, we must use an external high pass filter.



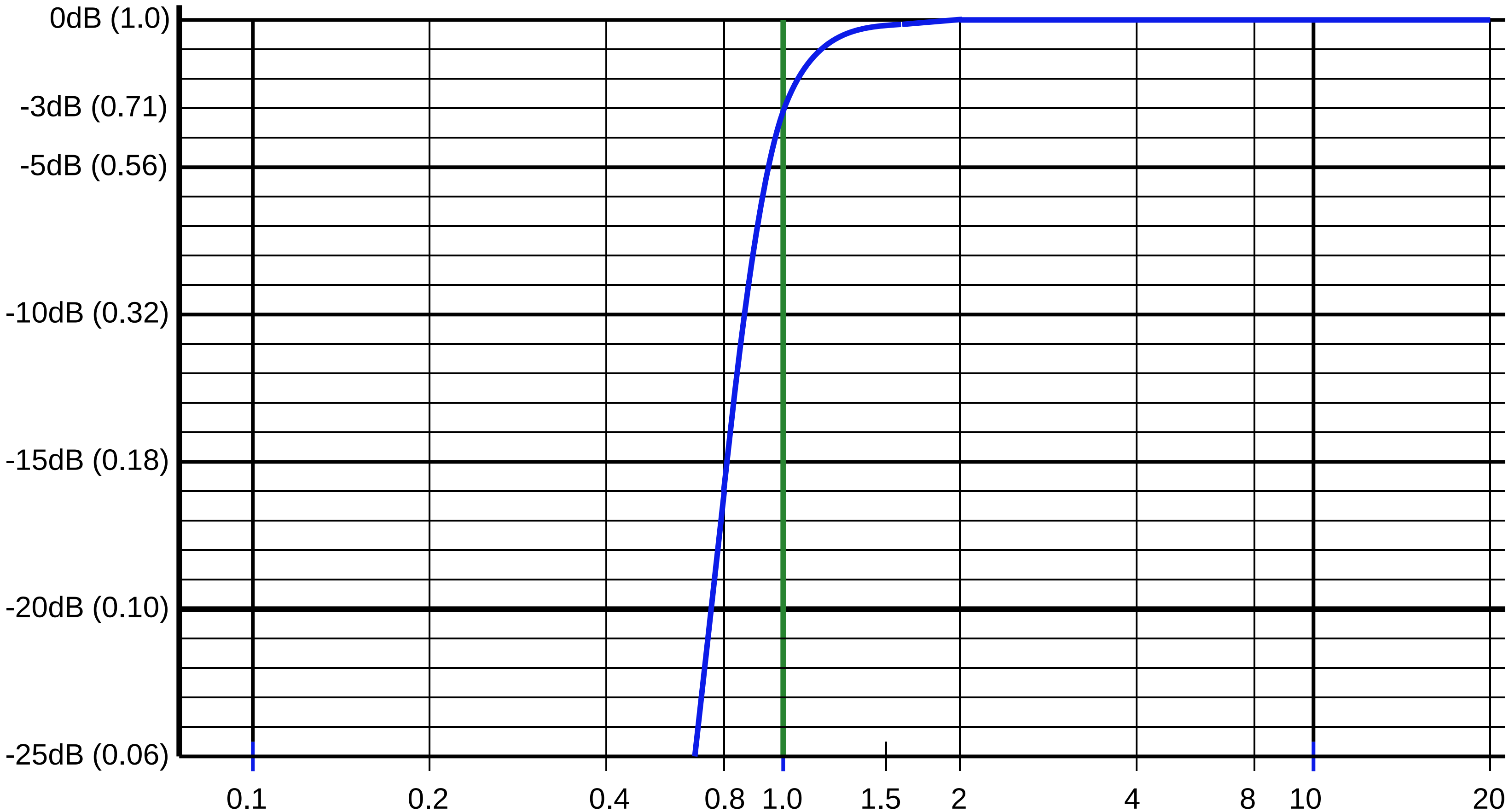
Precision Filters Inc. makes these filter modules that are custom made for your application. If you have a specific requirement, see the Advanced Systems Group.

- High Pass / Low Pass Filters
- 8 poles
- Butterworth, Elliptical
- Custom cutoff frequencies



Lower AC Coupling Frequencies

Response of the 8-pole High Pass Butterworth filter, fc=1Hz

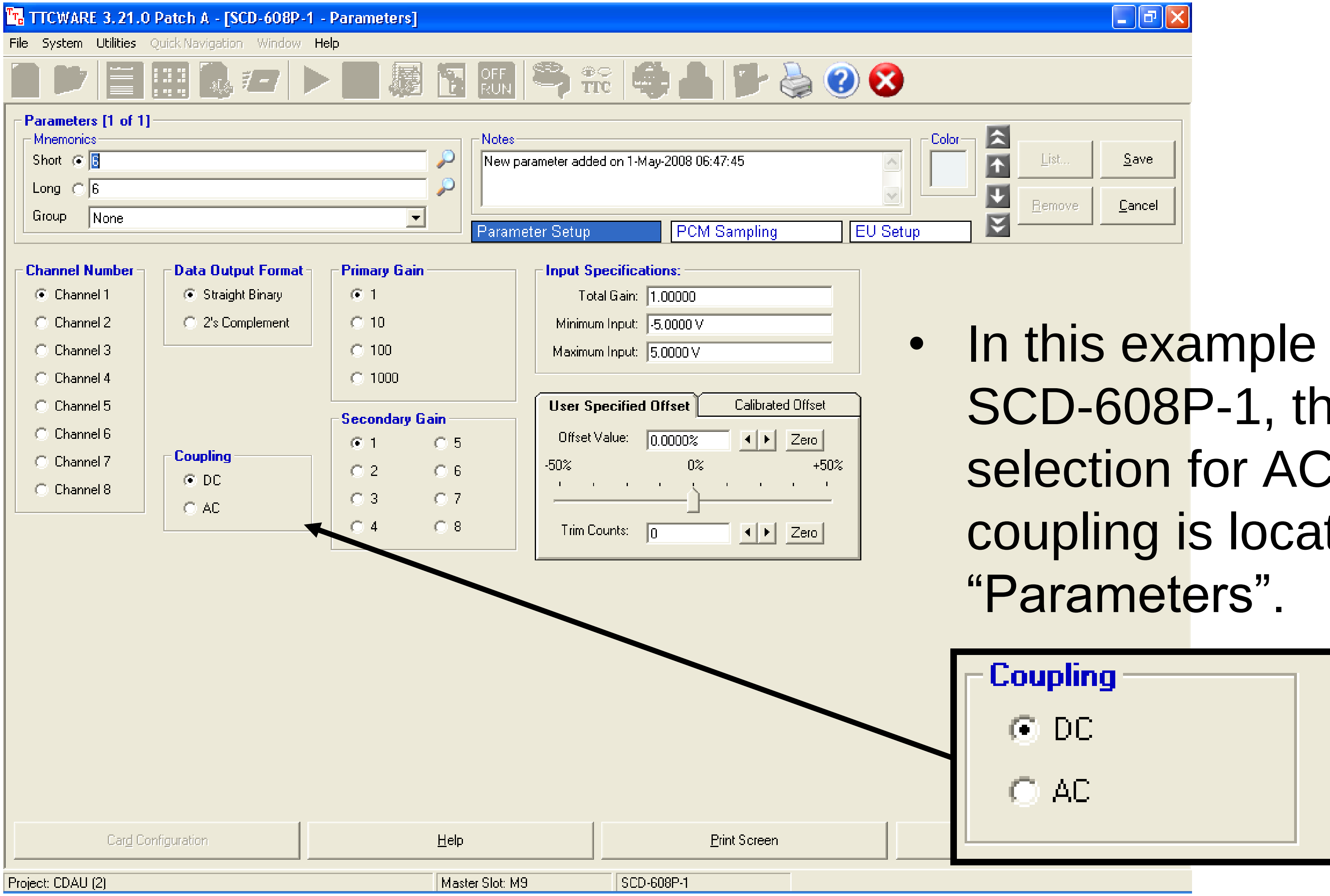


With the 8-pole high pass filter, frequencies at 2 Hz are not attenuated.

Why AC Couple a Signal?

- You are interested in the acceleration due to the movement of the wings during flight, but not in the 1 G acceleration due to gravity
- You are interested in the flutter or vibration response properties from a strain gage, not the static loading from the structure itself
- Sometimes a sensor has an output DC offset for now good reason

Selecting AC/DC Coupling



- In this example for the SCD-608P-1, the selection for AC/DC coupling is located in “Parameters”.

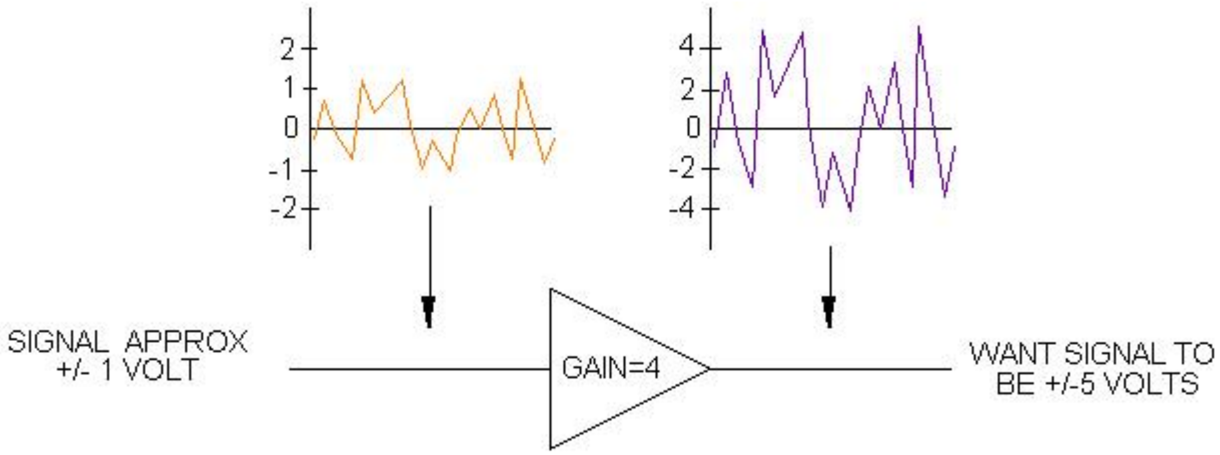
Important Note When AC Coupling

When a parameter is AC Coupled, no DC component of the signal is passed to the encoder. This poses a problem when performing calibrations where you are substituting a DC voltage to simulate the transducer.

Gain

- Gain is used in signal conditioning to expand or attenuate the measurement voltage range to maximize the usage of the range of the Analog to Digital Converter (ADC).
- In the CDAU and MCDAU, most of the input ranges of the ADC's input is ±5 volts.
- Select input signal gain such that when the ADC encodes the data (from 0-4095 counts for a 12-bit system) it is using as much of the 4095 count range as possible.

Selecting the Right Gain

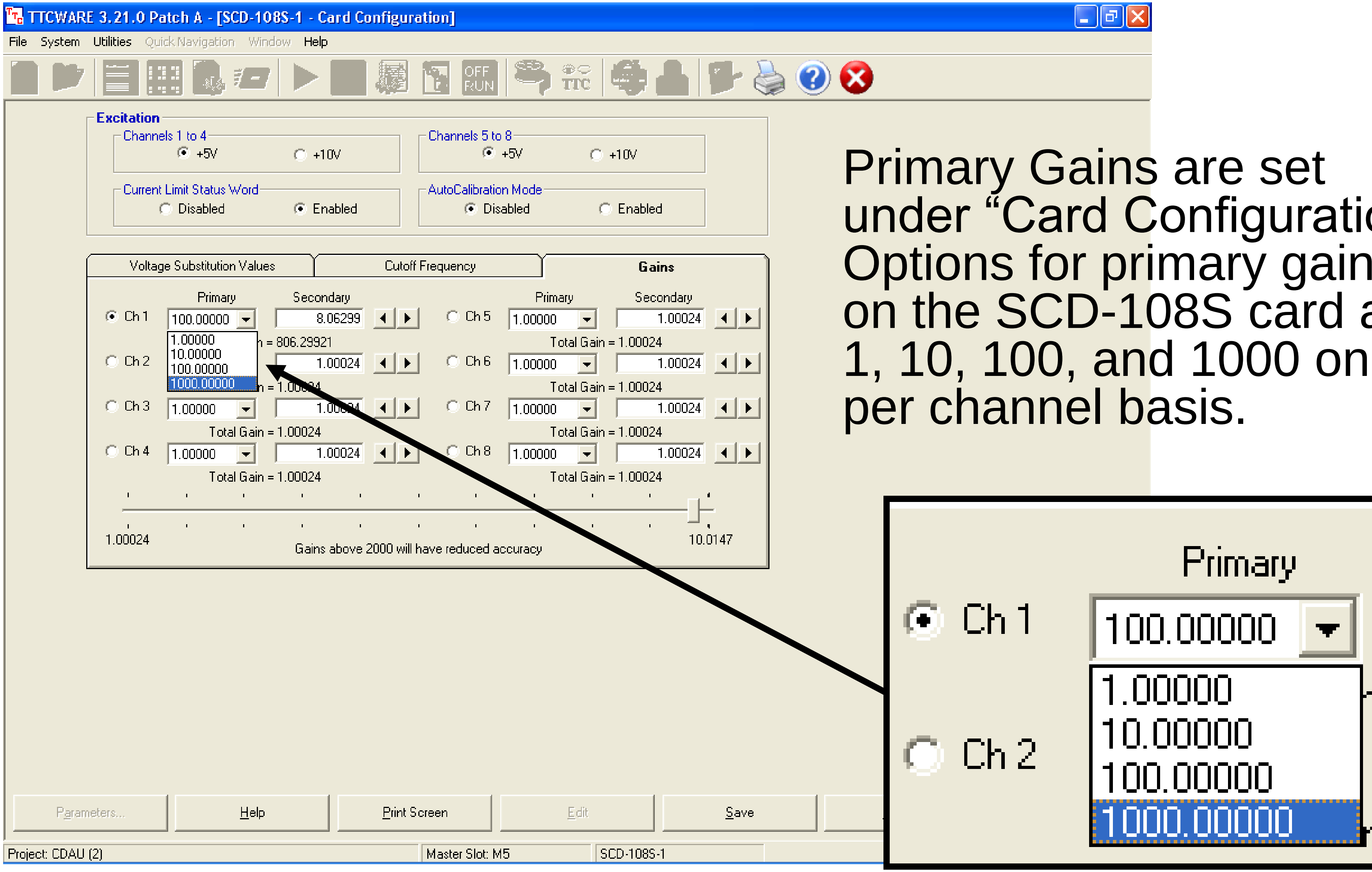


A programmable amplifier has gains of 1, 2, 4, and 8. If your input signal is approximately ±1 volts and the ADC wants to see ±5 volts, then a gain of 4 should be selected to maximally utilize the input range of ±5 volts.

Multiple Gain Stages

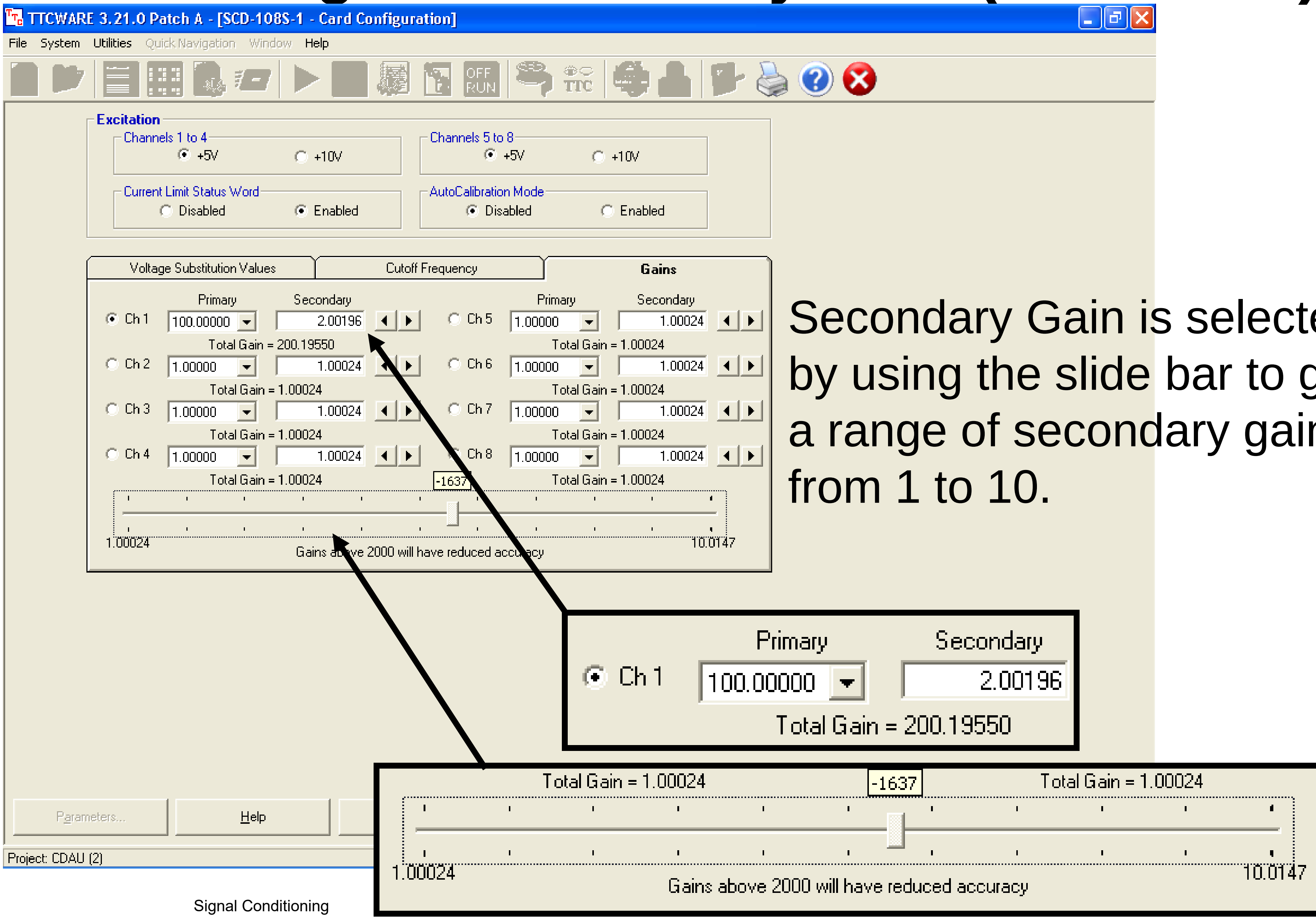
- Sometimes when you have measurements that have a very low output in the milli-volt range, you could have gains which are in the hundreds.
- Usually there are two stages of gains that must be selected in order to get the gain that you want.
- In series, the gains are multiplied.

Selecting the Primary Gain (SCD-108S)



Primary Gains are set under “Card Configuration”. Options for primary gains on the SCD-108S card are 1, 10, 100, and 1000 on a per channel basis.

Selecting the Secondary Gain (SCD-108S)



Secondary Gain is selected by using the slide bar to get a range of secondary gains from 1 to 10.

Calculating Gain

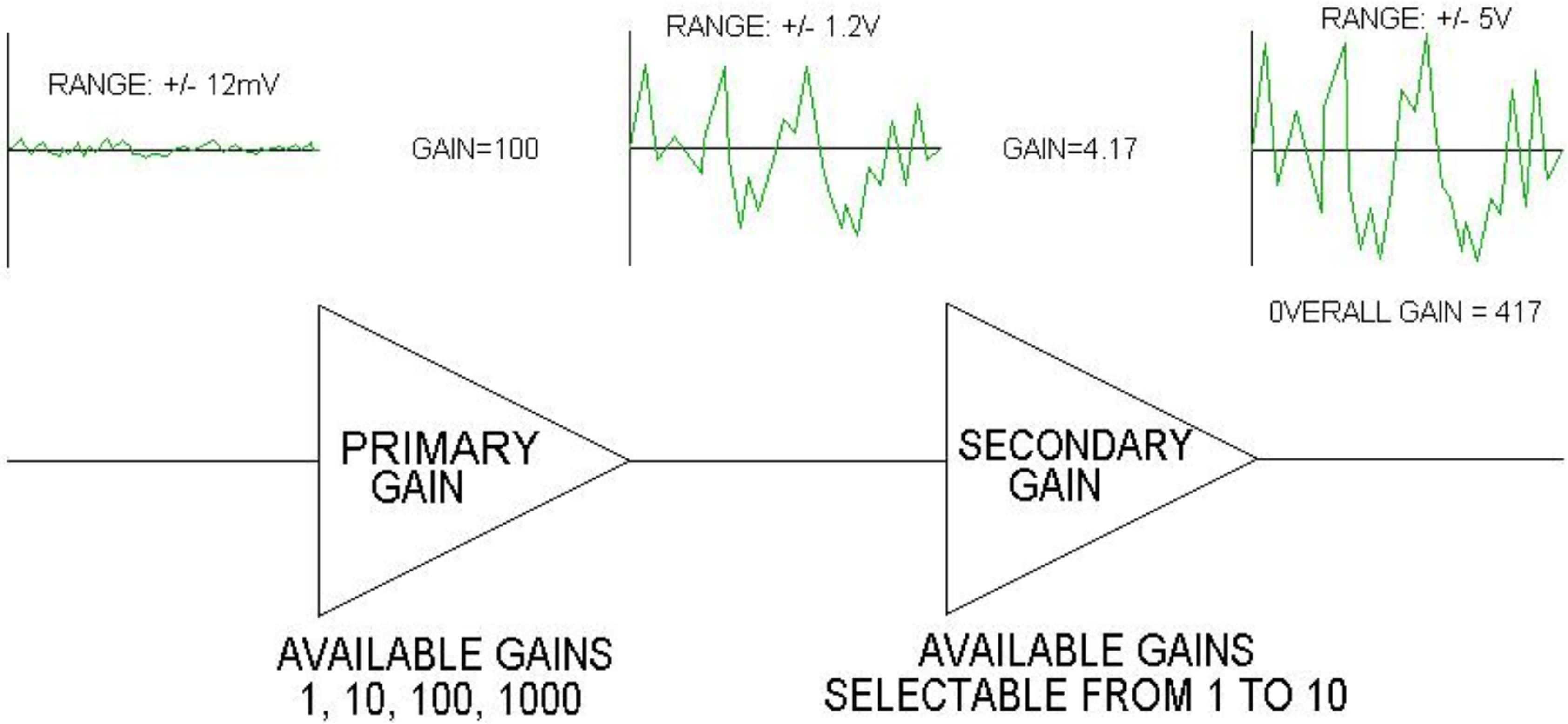
- If you have a measurement that has a ±12mV signal range, you need to gain the signal to the encoder's full scale input of ±5V.

- To calculate the total needed gain,

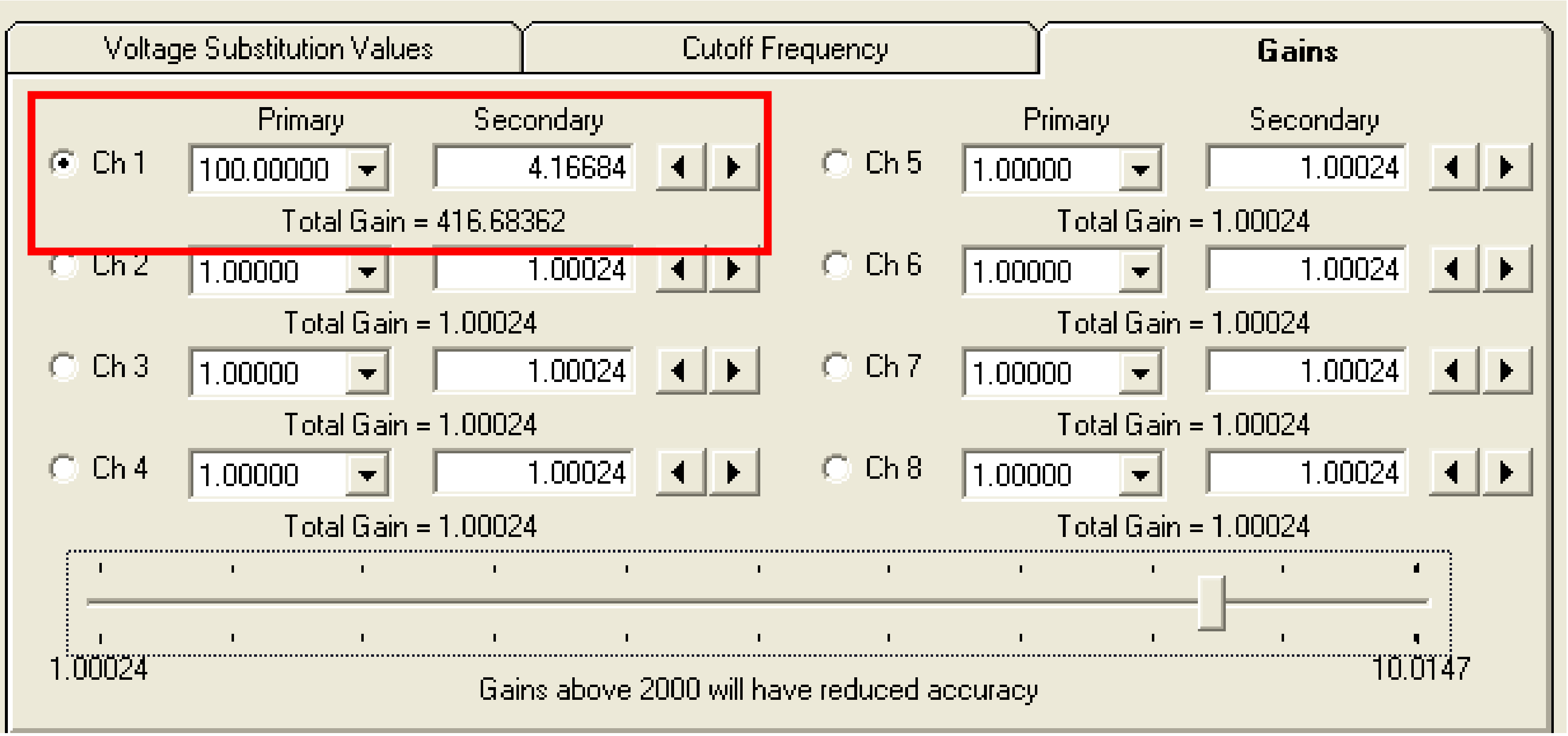
$$Gain = \frac{V_{OutputRange}}{V_{InputRange}}$$

$$Gain = \frac{5V - (-5V)}{12mV - (-12mV)} = \frac{10V}{24mV} = \frac{10V}{0.024V} = 416.67$$

Multiple Gain Stages

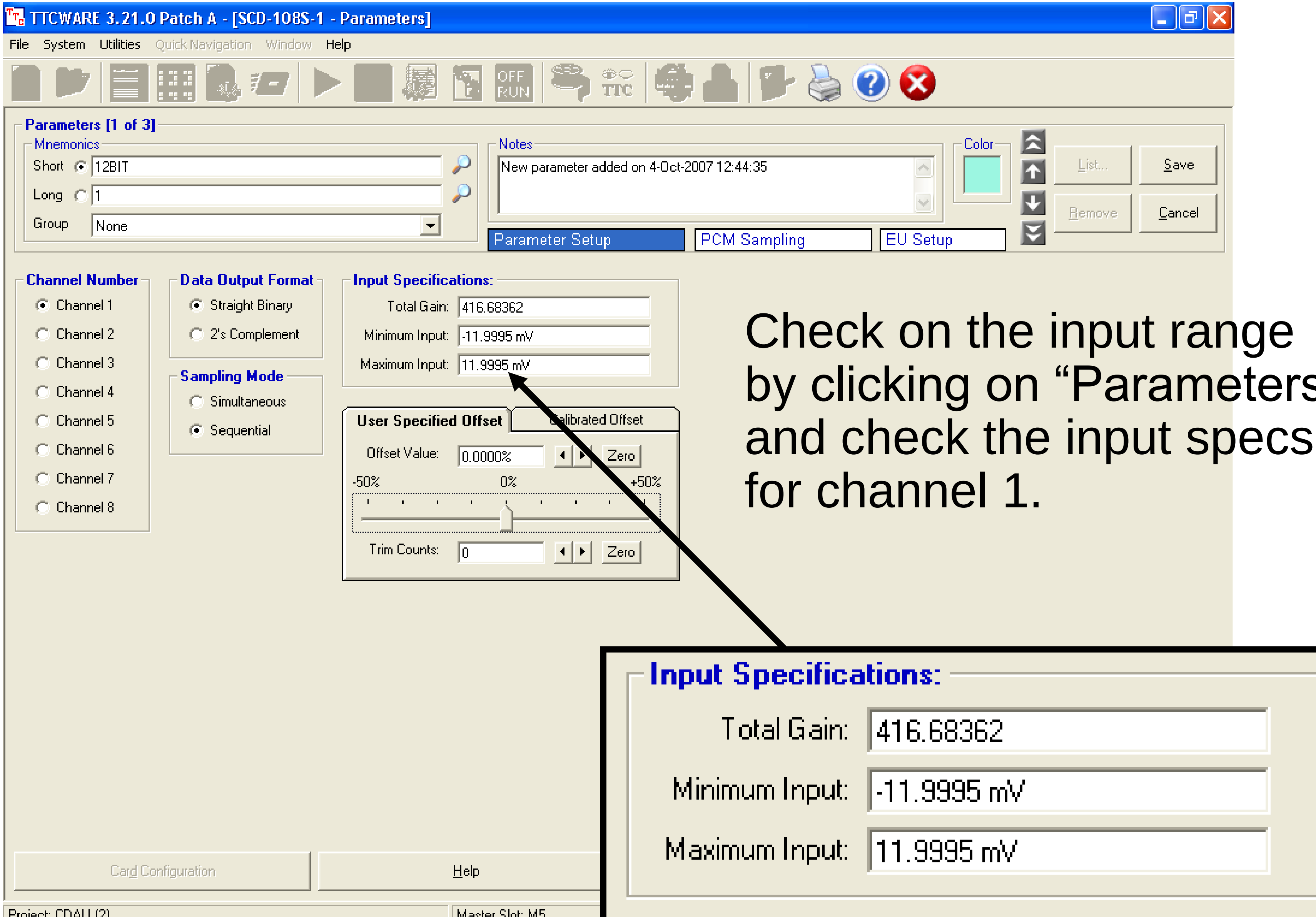


Overall Gain and Range (SCD-108S)



Under the Card Configuration, select a primary gain of 100 and a secondary gain of 4.16684 to get a total gain of 416.68.

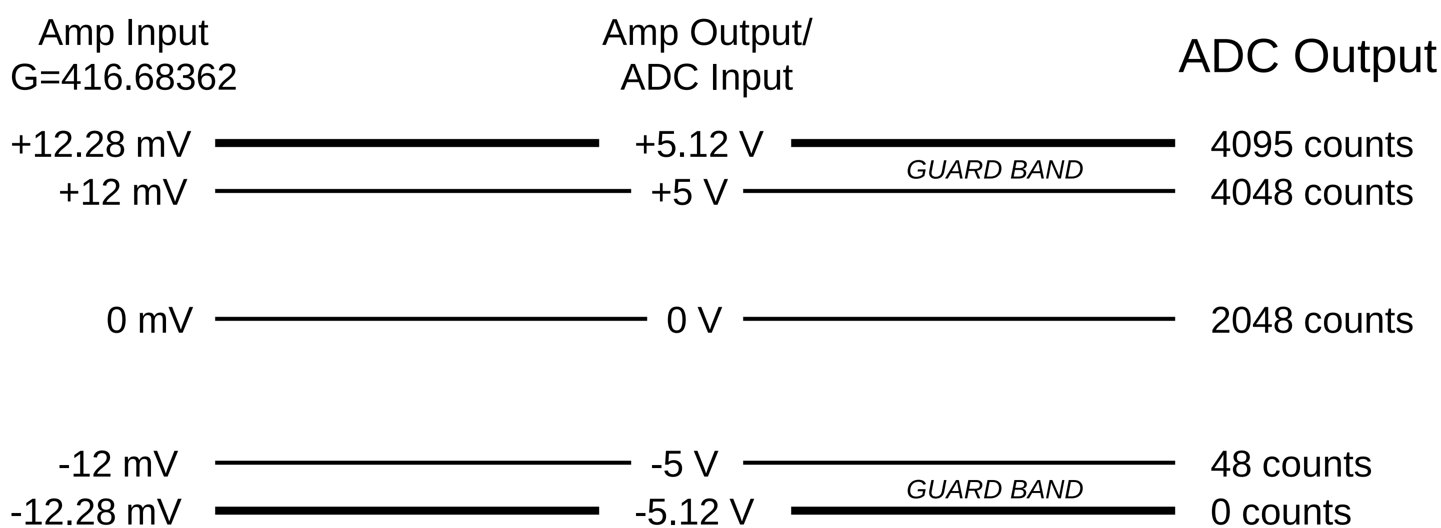
Overall Gain and Range (SCD-108S)



Check on the input range by clicking on "Parameters" and check the input specs for channel 1.

Guard Bands on ADC's

- Most analog to digital converters have guard bands such that if you exceed the ±5V, you will not max out or bottom out the count value. On TTC SCD cards, the voltage input and the count output of the ADC is set up like this:



Offsets

- Offsets are used when the voltage of a measurement is not centered around zero volts. Many measurements such as positions, strain gages, and some high output transducers have unwanted offsets in their output.
- Since the input to the ADC is ±5 volts (centered around zero volts), offset is added or subtracted from the signal to most efficiently use that range.

Signal with an Offset

- Say the output of a potentiometer varies from -0.5 to 2 volts for the EU range requested.

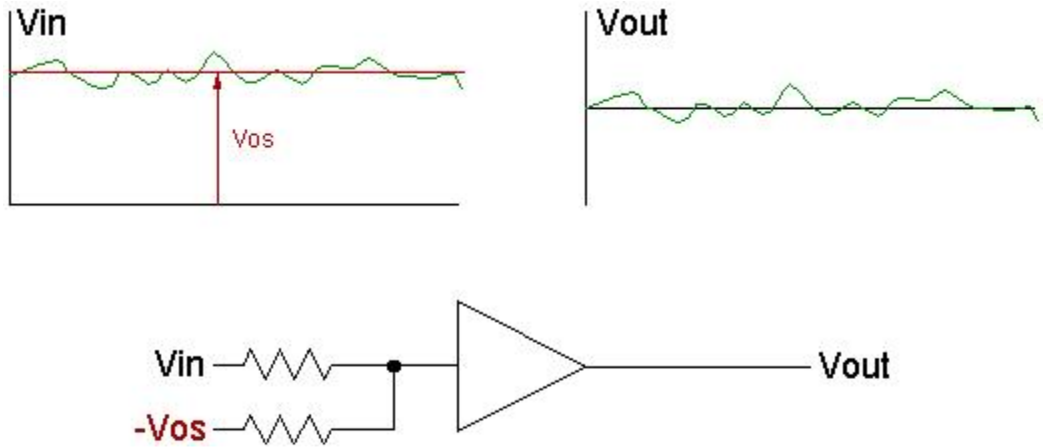
- First we will calculate the gain needed for this channel as we did before:

$$Gain = \frac{V_{OutputRange}}{V_{InputRange}}$$

$$Gain = \frac{5V - (-5V)}{2V - (-0.5V)} = \frac{10V}{2.5V} = 4$$

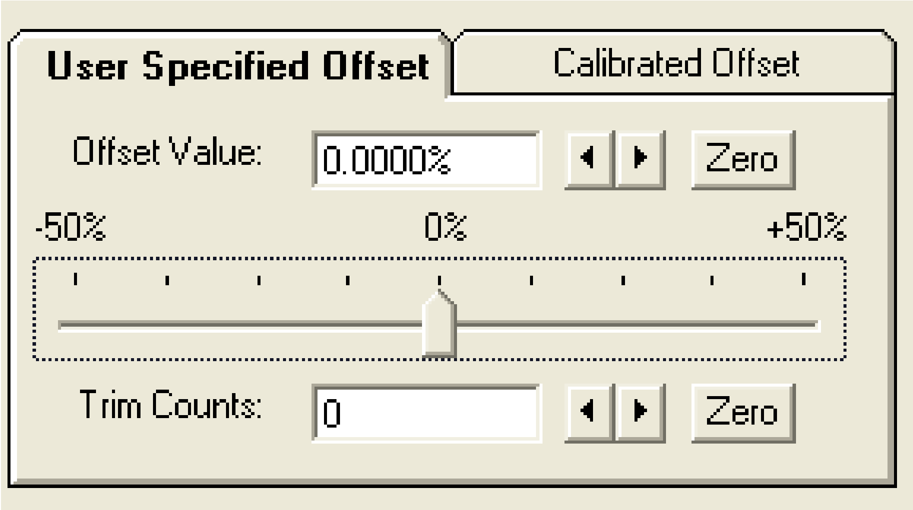
Calculating Offset

The signal range is from -0.5 to 2 volts. The middle of that range is the offset in the signal.



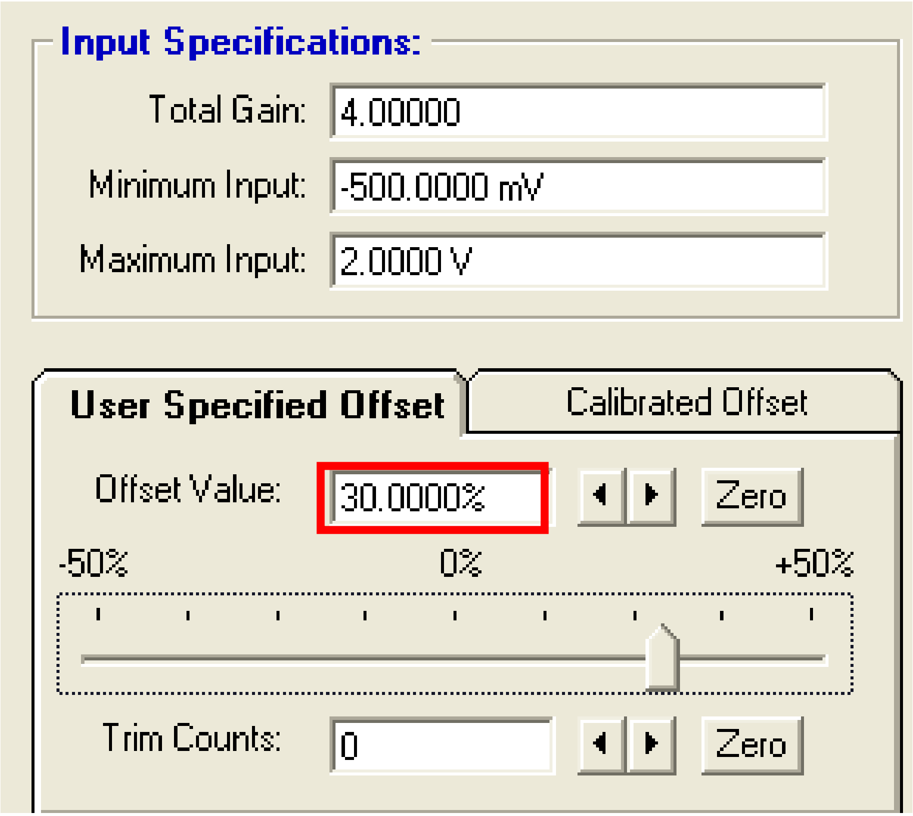
$$V_{offset} = \frac{1}{2}\left(V_{upper} + V_{lower}\right) = \frac{1}{2}\left(2V + (-0.5V)\right) = \frac{1}{2}(1.5V) = 0.75V$$

Selecting the Offset (SCD-108S)



- Offset is selected as a percentage of the full scale input range, not as the offset voltage.
- Trim Counts allow fine tuning of the offset in counts from the ADC.

Selecting the Offset for the Example



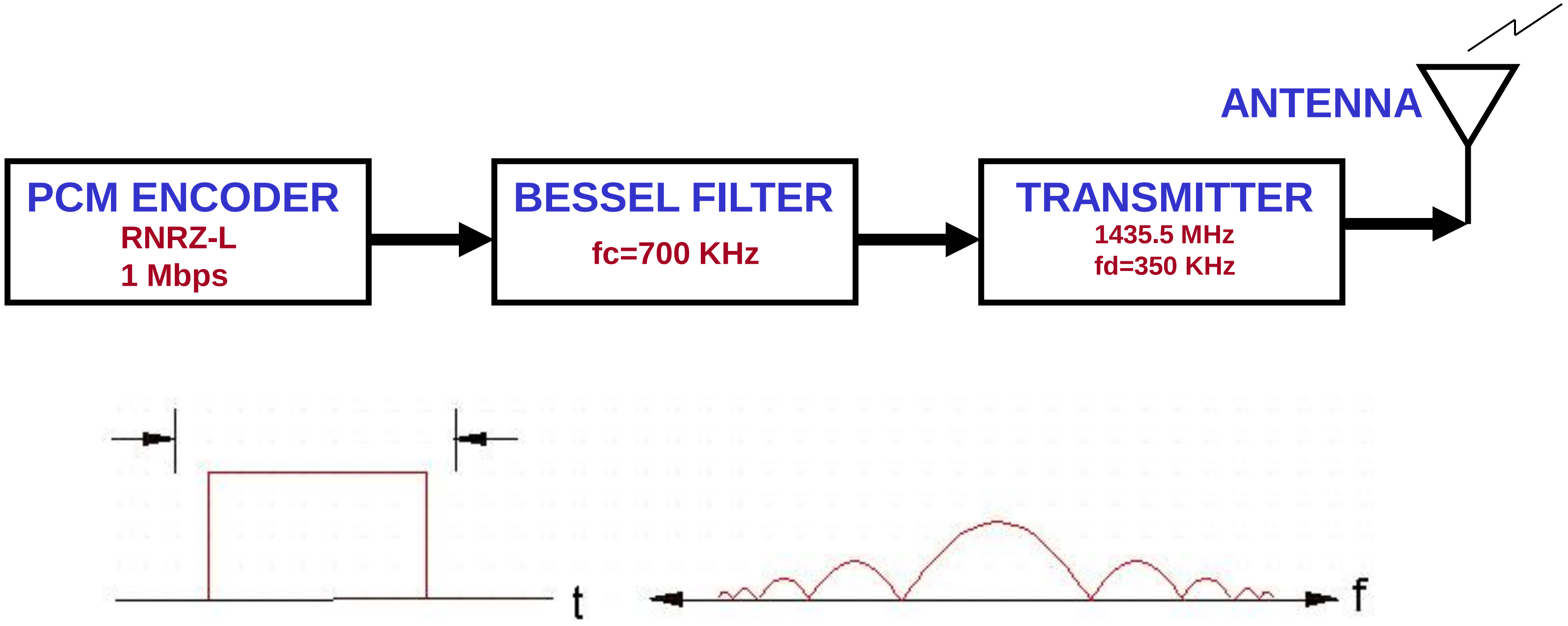
There is a 0.75V offset in our input range of -0.5 to 2V. We need to calculate the percentage offset.

$$Offset = \frac{V_{offset}}{V_{InputRange}}$$

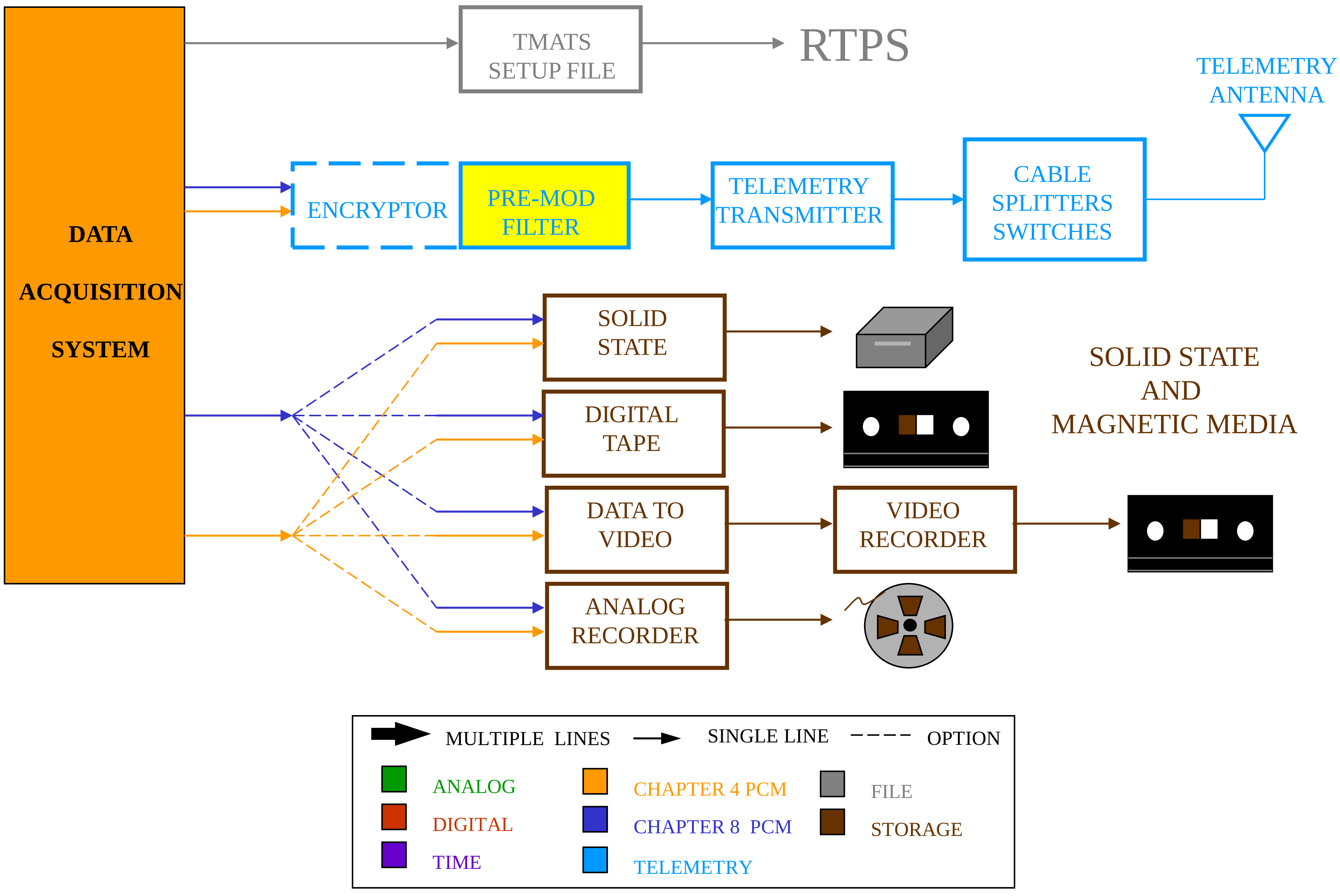
$$Offset = \frac{0.75V}{2V - (-0.5V)} = \frac{0.75V}{2.5V}$$

$$Offset = 0.3 \text{ or } 30\%$$

Pre-Modulation Filtering for PCM/FM Systems



Telemetry and Data Storage



Introduction

For real-time data acquisition, the PCM stream generated by our instrumentation systems is transmitted (via FM) to the ground station. The PCM stream has a frequency content which when transmitted occupies bandwidth within our allocated frequency spectrum.

Introduction

The data we transmit occupies bandwidth within the frequency spectrum. We must try to use as little of it as possible, such that many aircraft can transmit over the same band of frequencies.

This can be accomplished by reducing the frequency content of the PCM stream through filtering and proper selection of the frequency deviation of the transmitter.

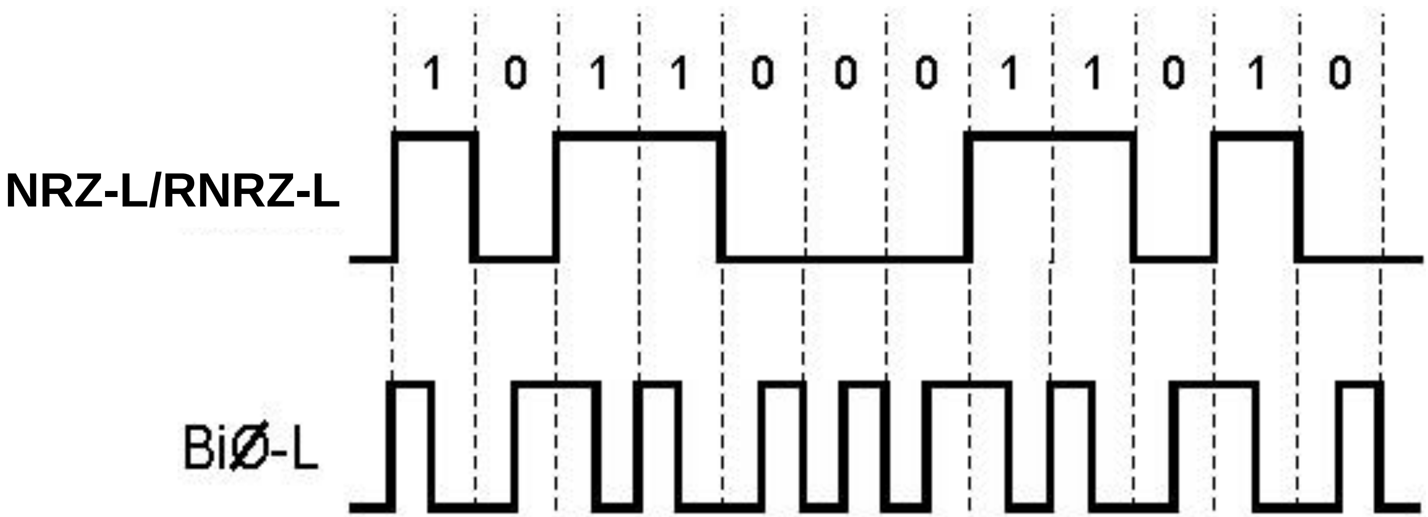
Introduction

This presentation will only deal with the effects of pre-modulation filtering on the bandwidth of a transmitted PCM stream.

Proper selection of the deviation frequency of the transmitter will be covered in *Telemetry System Design, Set-up and Optimization* training.

Frequency Content of PCM Streams

As a rule of thumb, Biϕ-L has twice the frequency content as NRZ-L or RNRZ-L. This is because Biϕ-L has more transitions than NRZ-L or RNRZ-L. This is why NRZ-L and RNRZ-L is used for transmitting data.



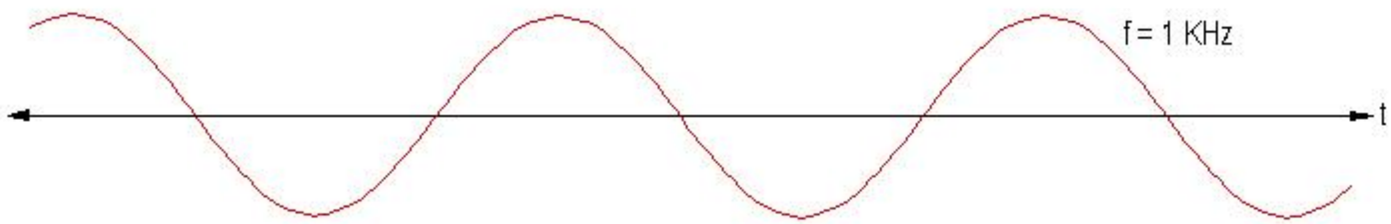
Frequency Content of PCM Streams

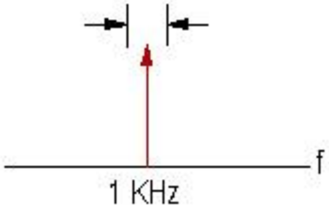
A PCM stream is simply a series of square pulses whose pattern follows the 1 and 0 pattern of bits. It takes many frequency components to make up a square pulse. Therefore the PCM stream is made up of many frequencies that are much higher than its bit rate.

Where do all of these frequencies come from?

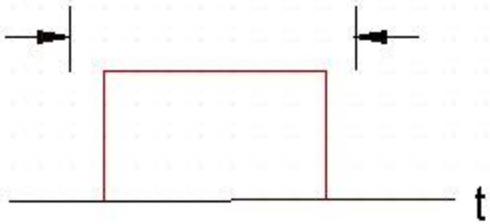
Time Limited/Band Limited Signals

Signals that are not time limited, are band limited in the frequency domain.



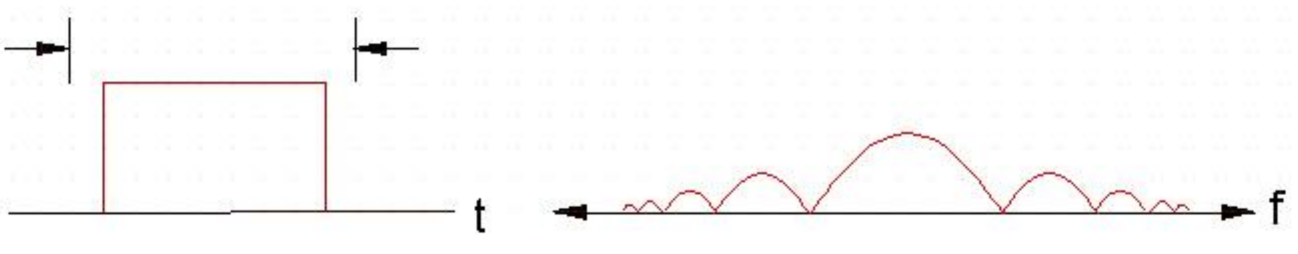


Signals which are time limited, are not band limited in the frequency domain.

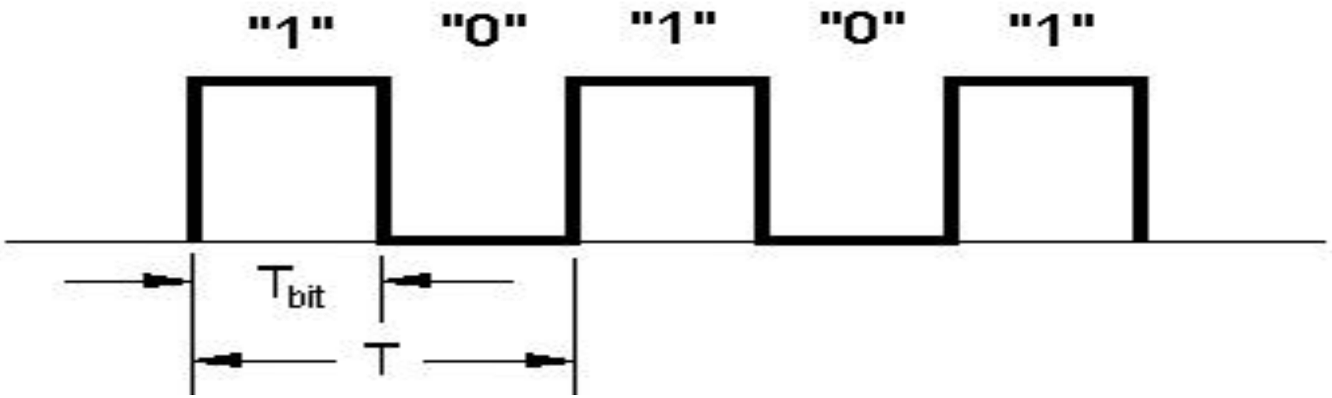


A Square Pulse is a Time Limited Signal

PCM streams are essentially a series of pulses. In the frequency domain, a PCM stream contains an infinite number of frequencies of decaying amplitudes. The higher frequencies are what gives the pulses its squared look.



Determining the Fundamental Frequency of an NRZ-L PCM Stream



Bit Period: $T_{bit} = \frac{1}{BitRate} \,{}^{sec}/_{bit}$

Period: $T = 2T_{bit}\ sec$

Frequency: $f = \frac{1}{T} = \frac{1}{2T_{bit}} = \frac{BitRate}{2} Hz$

The Fundamental Frequency of an NRZ-L PCM stream is half the bit rate.

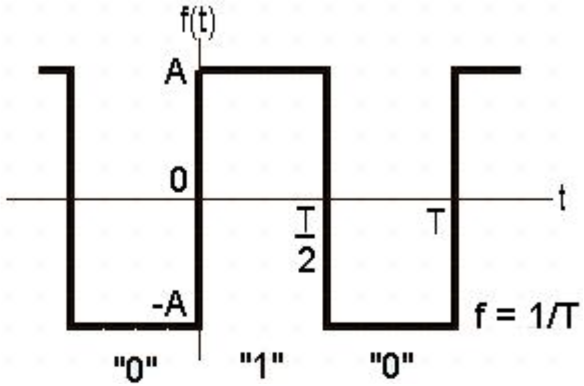
For a 1 Mbit/sec NRZ-L stream,

$$T_{bit} = \frac{1}{1x10^6 \frac{bit}{sec}} = 1\frac{usec}{bit} = 1usec$$

$$T = 2T_{bit} = 2\ usec$$

$$f = \frac{1}{2usec} = 500\ kHz$$

Determining the Harmonic Frequencies of NRZ-L



All square waves can be expressed mathematically as a sum of sine waves:

$$f(t) = \frac{4A}{\pi} \sum_{\substack{n=1 \\ odd}}^{\infty} \frac{1}{n} \sin(2\pi n f t)$$

For a 1Mbps NRZ-L PCM stream, T=2μsec, and f would be 500kHz

The result would be the following sum of sine waves:

$$f(t) = \frac{4A}{\pi}\left[\sin(2\pi f t) + \frac{1}{3}\sin(2\pi f t) + \frac{1}{5}\sin(2\pi 5 f t) + \frac{1}{7}\sin(2\pi 7 f t) + \cdots\right]$$

so the 1Mbps PCM stream would contain the frequencies:

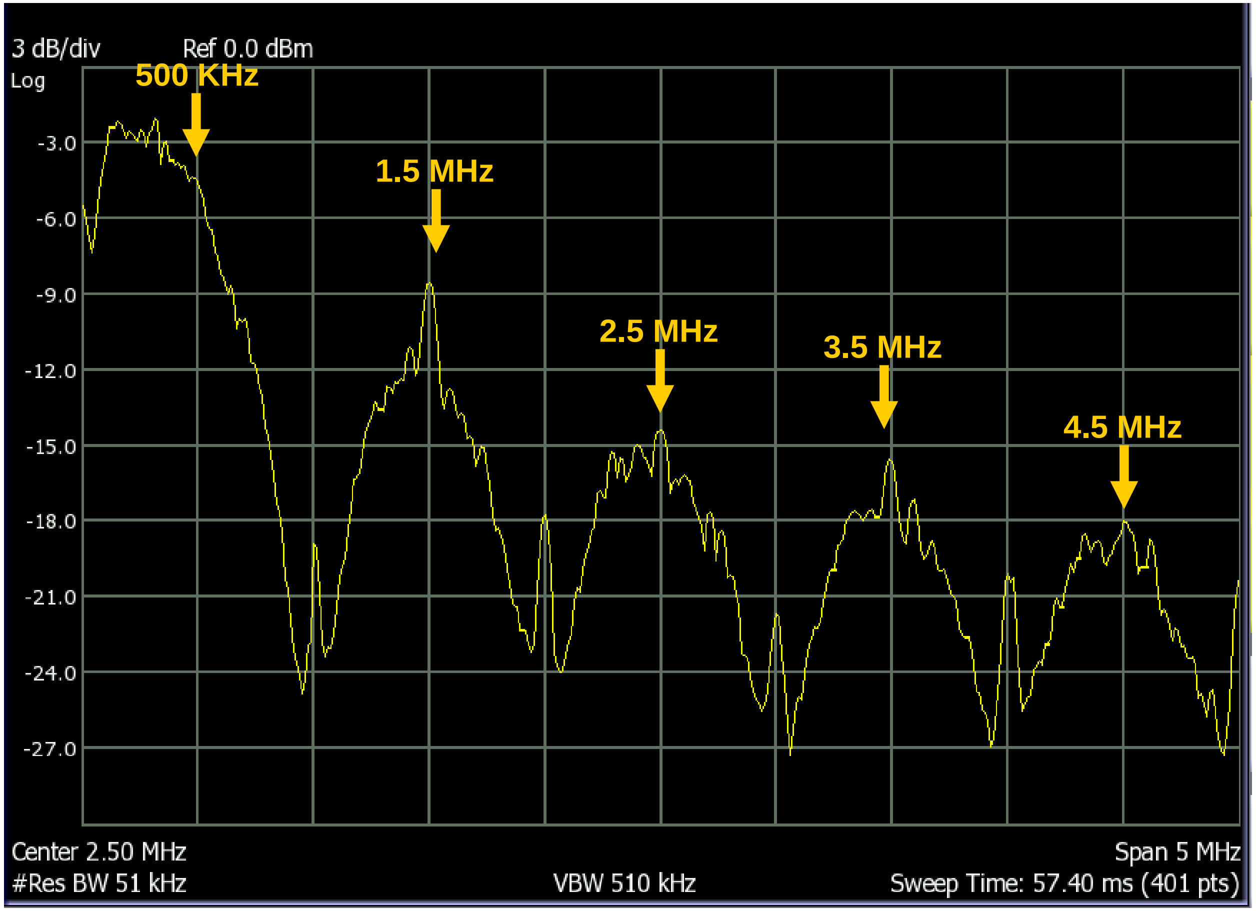
500 kHz, 1.5 MHz, 2.5 MHz, 3.5 MHz and so on . . . with amplitudes decreasing by a factor of 1/n.

Harmonics

Each of the frequencies that make up the NRZ-L PCM stream are called harmonics.

Because square waves are made up of the odd multiples (3f, 5f, 7f, . . .) of the base frequency (f), we say that square waves are made up of odd harmonics.

Frequency Content of 1 Mbps NRZ-L

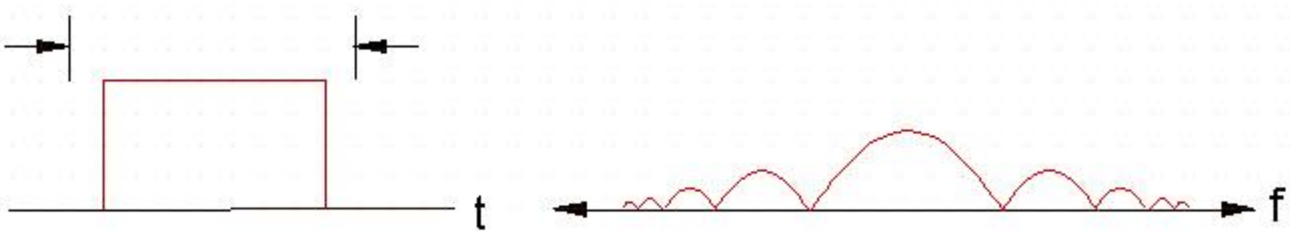


The screenshot shows the frequency content of the 1 Mbps NRZ-L PSM stream. The fundamental and harmonic frequencies can be easily identified.

Frequencies of a Square Pulse

The odd harmonic frequency components which give the pulses their squared look also consume bandwidth when transmitting the PCM stream.

A Pre-Modulation filter attenuates those higher frequencies such that the amount of bandwidth used during transmission is minimized.



The Pre Modulation Filter

Unlike analog signals which use Butterworth filtering, digital signals use Bessel filters due to their linear phase characteristic within the pass band.

Bessel filters have linear phase delay at the expense of a maximally flat response in the pass band, but this is not critical for digital data.

The linear phase allows for a near constant group delay for all frequencies in the pass band which make up the square wave, minimizing distortion.

Group Delay

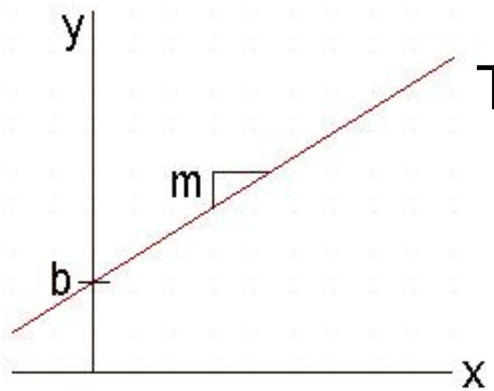
Group Delay measurements are based on phase measurements. It is similar to group velocity in wave-propagation theory.

The group delay, τ_{gr} is the negative derivative of phase delay. Or in equation form, the following:

$$\tau_{gr} = -\frac{1}{2\pi}\frac{\partial\varphi}{\partial f}$$

where φ is the phase delay with respect to frequency, f

Constant Group Delay from Linear Phase Delay



The derivative of a linear function is a constant.

$y = mx + b$ is a linear function

where *m* is the slope and is constant

b is the *y* intercept

Taking the derivative of y with respect to x,

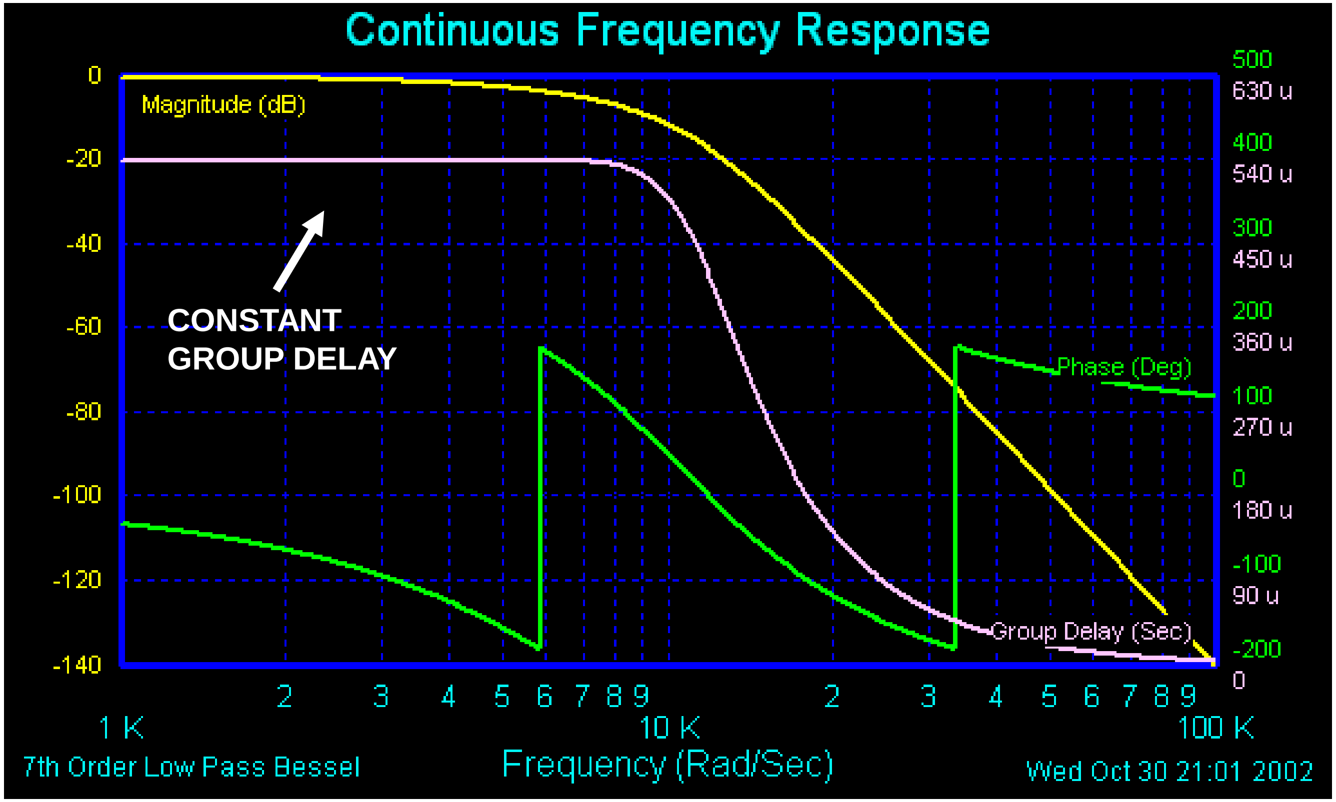
$$\frac{\partial y}{\partial x} = m$$

m is a constant, so $\partial y/\partial x$ is a constant

Therefore, if the phase delay, φ is linear, then the group delay, τ_{gr} is constant.

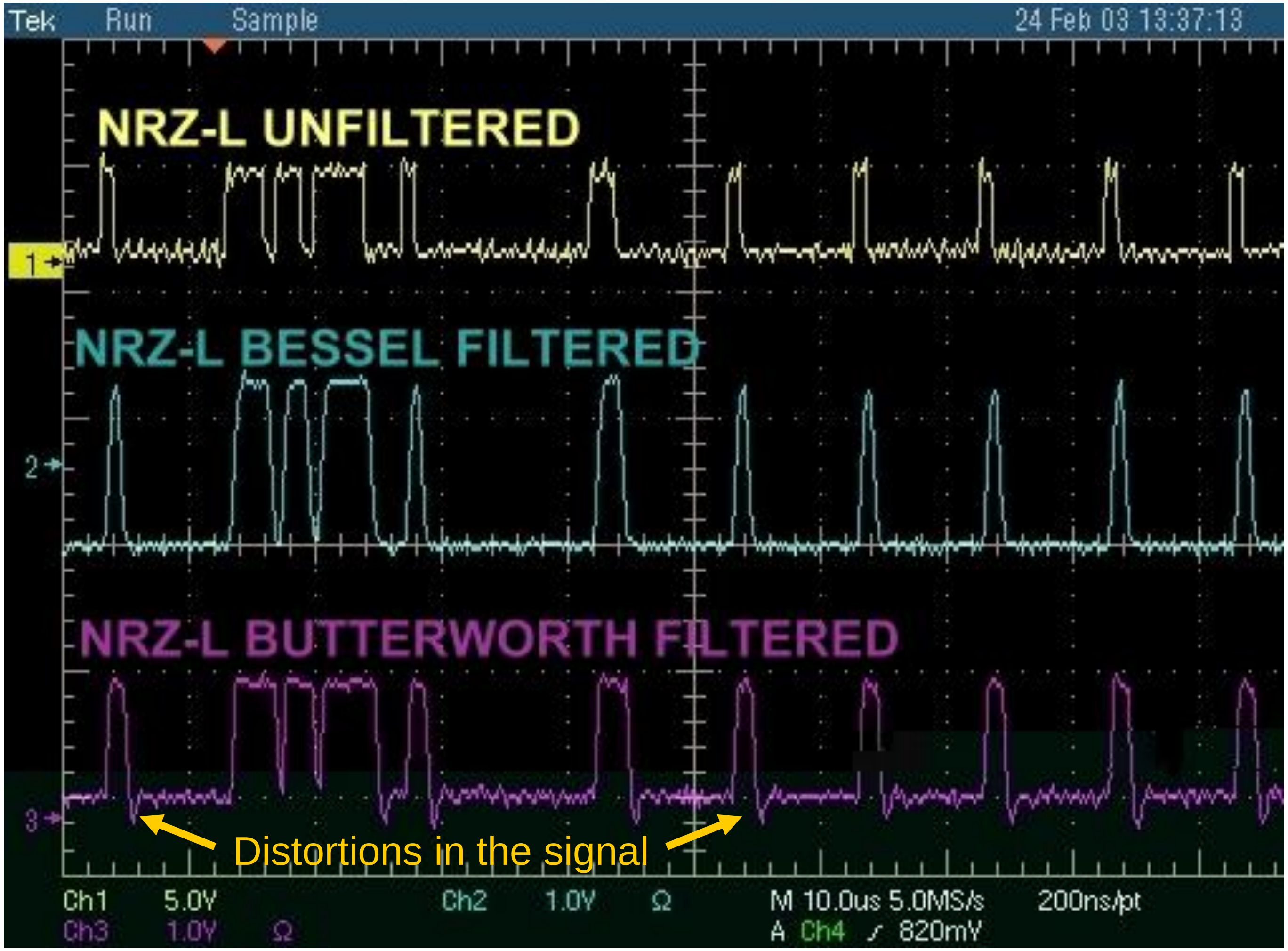
The Bessel Pre-Modulation Filter

The Bessel frequency response showing both magnitude and phase delays. Note the constant group delay within the pass band.



Group Delay

A constant group (linear phase) delay is desired to reduce the distortion of the PCM stream. The picture shows a NRZ-L stream filtered with a Bessel filter (linear phase) and a Butterworth filter (non-linear phase).



Filter Cutoffs for PCM Streams

- NRZ-L: $f_c = 0.7BR$
- RNRZ-L: $f_c = 0.7BR$
- Biϕ-L: $f_c = 1.4BR$

BR is the bit rate of the PCM stream.

Note that f_c for Biϕ-L is twice that for NRZ-L and RNRZ-L

These are the frequency cutoffs that were found to not cause a significant increase in the Bit Error Rate (BER) or degrade performance. Source: IRIG 119-06 Telemetry Applications Handbook, paragraphs 2.4.3 and 2.5.3.

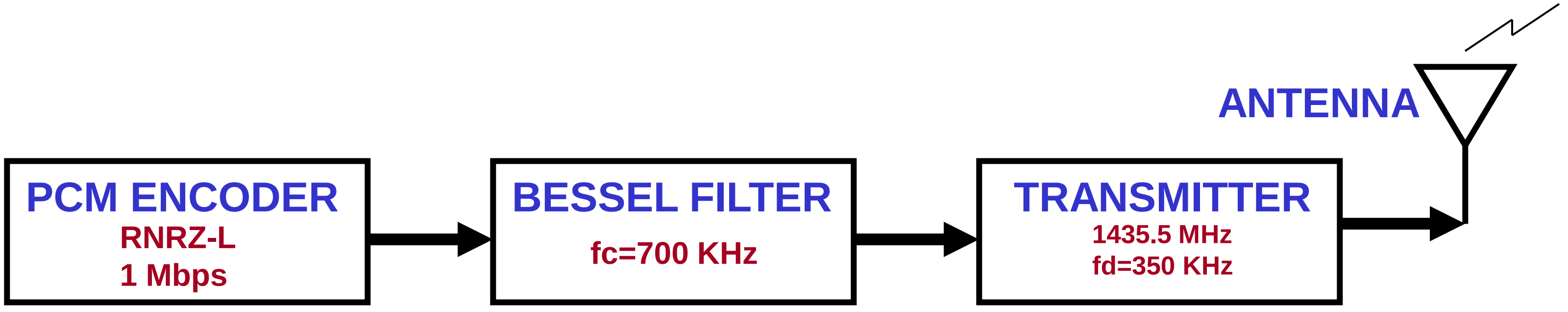
Filter Effects on Transmitted Bandwidth

This example will show the effects of pre-modulation filtering as it pertains to bandwidth of the transmitted spectrum (to the antenna).

The PCM stream is a 1 Mbps RNRZ-L stream with 12 bits/word and 24 words/minor frame.

The filter is a 6-pole Bessel filter with f_c=700KHz (.7BR).

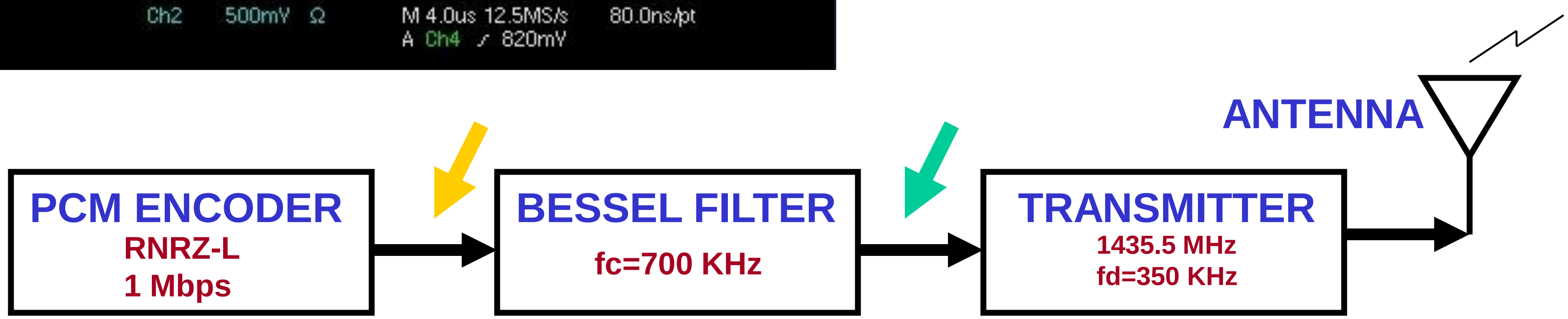
The transmitter is tuned to 1435.5 MHz with a deviation frequency f_d of 350 KHz.



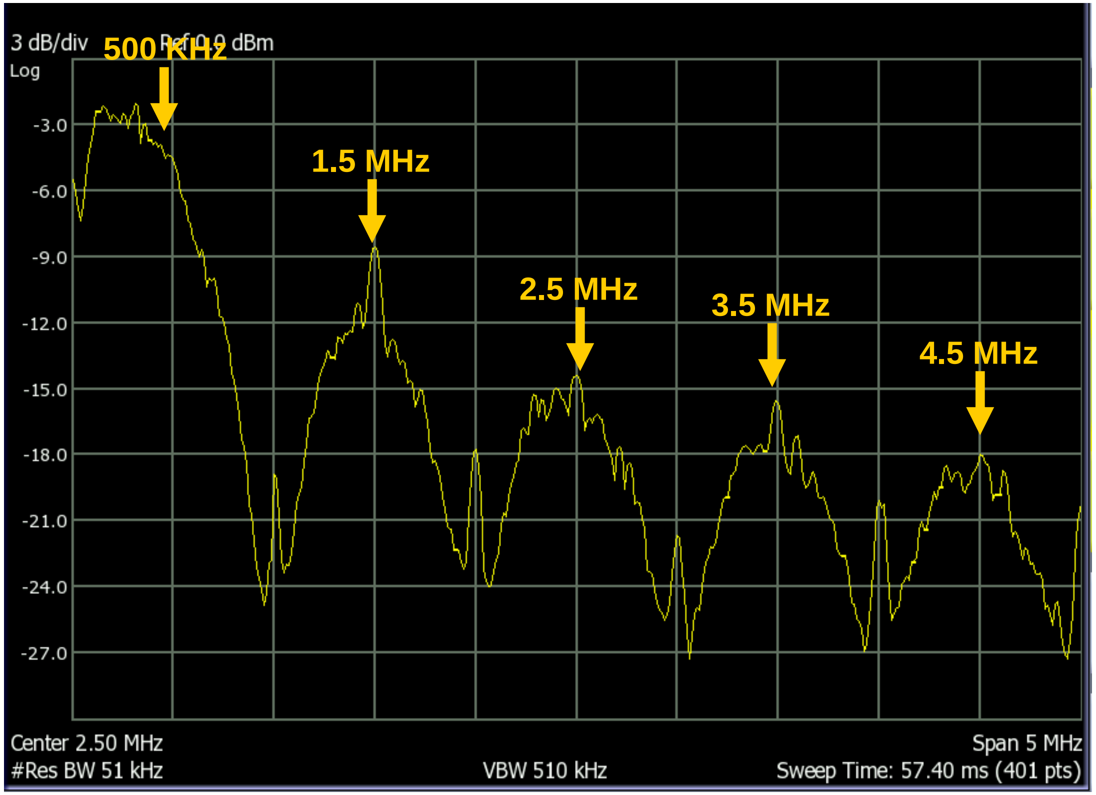
Filter Effects on Transmitted Bandwidth

O-Scope Display

This shows the input and the output of the Bessel filter. The edges of the pulses are rounded due to the absence of higher frequencies.



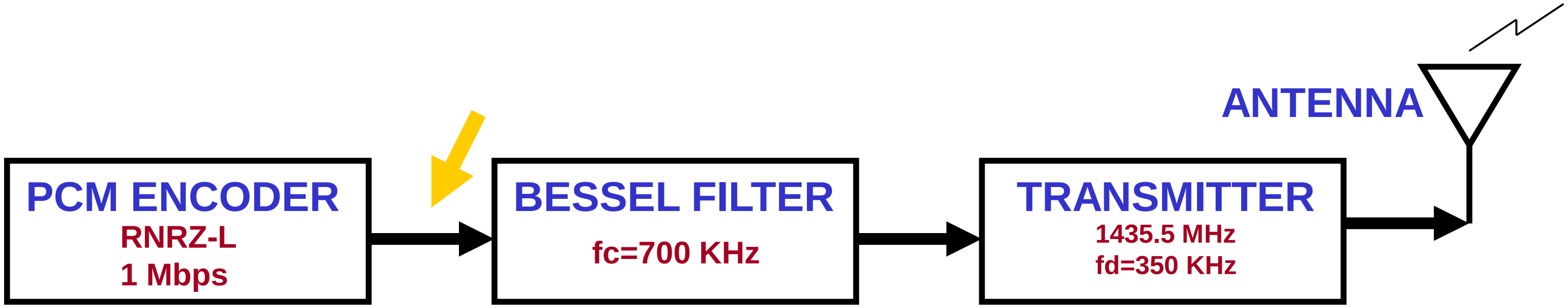
Filter Effects on Transmitted Bandwidth



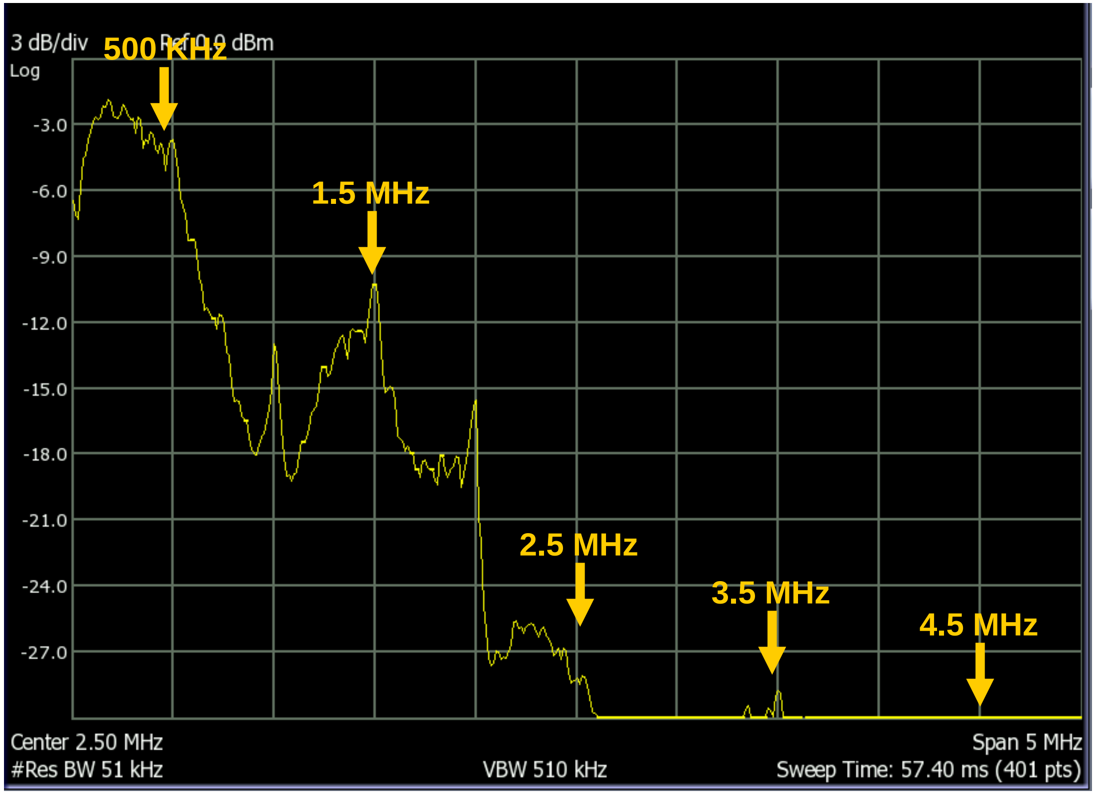
Spectrum Analyzer

This is the frequency spectrum of the <u>unfiltered</u> 1 Mbps PCM stream.

Note the numerous odd harmonic frequencies that make up the PCM stream.

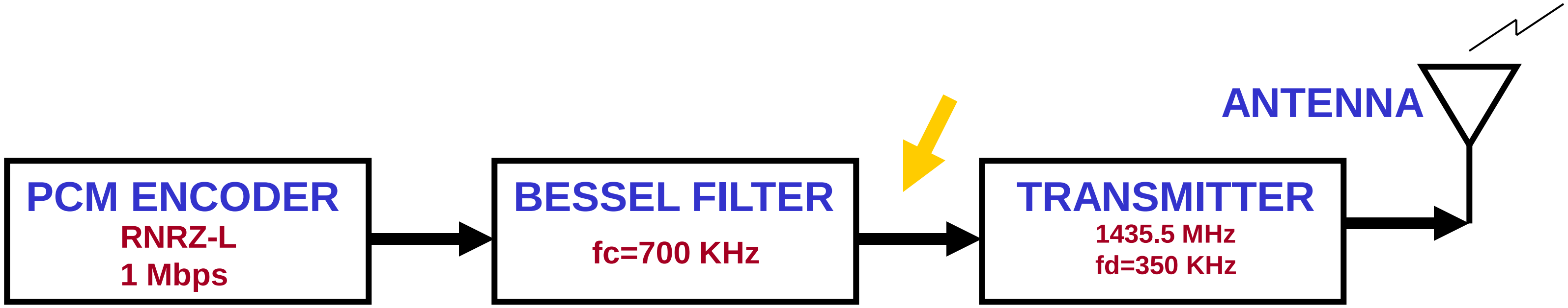


Filter Effects on Transmitted Bandwidth



Spectrum Analyzer

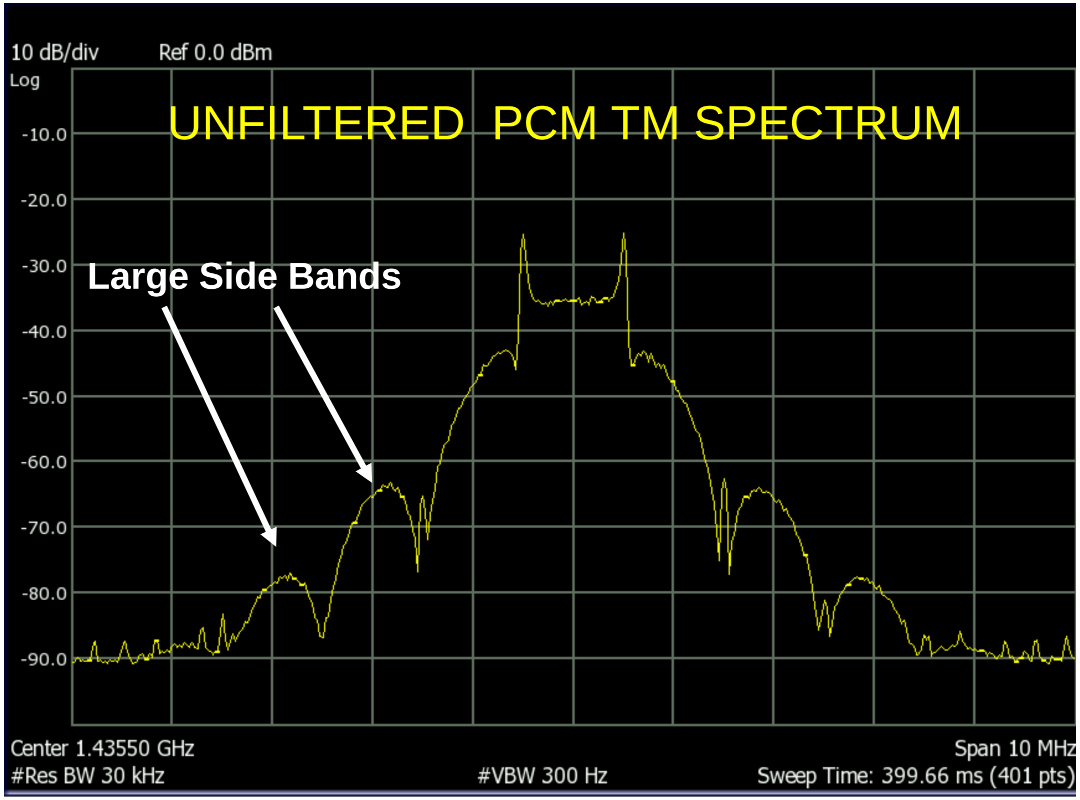
This is the frequency spectrum of the Bessel filtered 1Mbps PCM stream. Notice how much the harmonics above 1 MHz are now attenuated.



Filter Effects on Transmitted Bandwidth

The reduction in the bandwidth of the PCM stream will result in an overall reduction in the transmitted bandwidth of the PCM stream. This allows more aircraft to transmit data at the same time.

Filter Effects on Transmitted Bandwidth

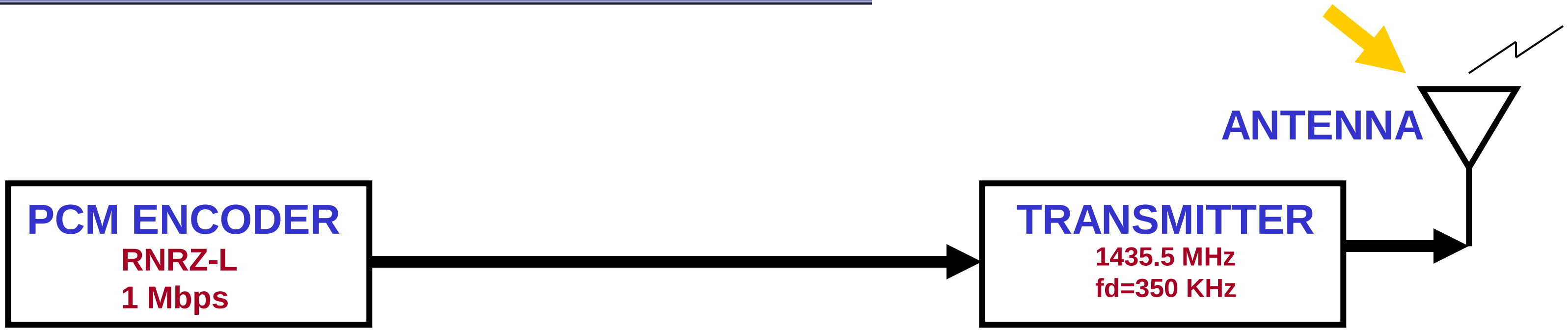


Spectrum Analyzer

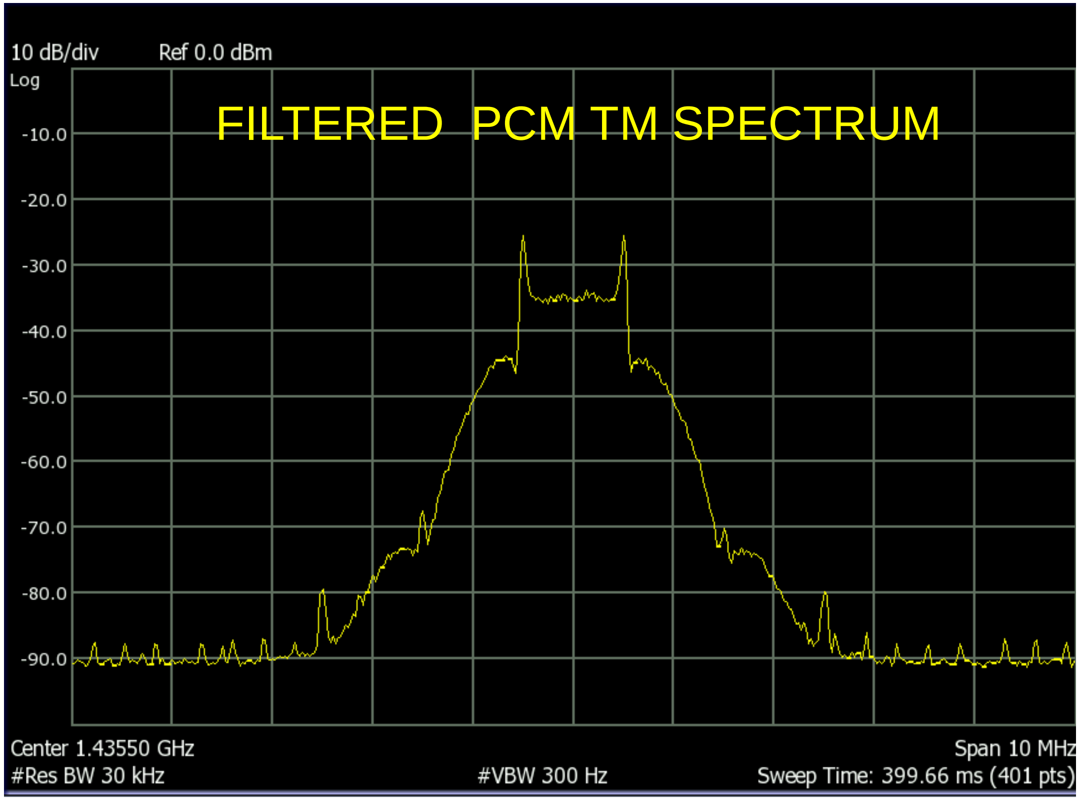
This is the RF spectrum of the transmitted <u>unfiltered</u> PCM stream.

1) The transmitter's power is wasted on the large sidebands (RTPS would filter that out).

2) More spectrum is being used than is needed.



Filter Effects on Transmitted Bandwidth

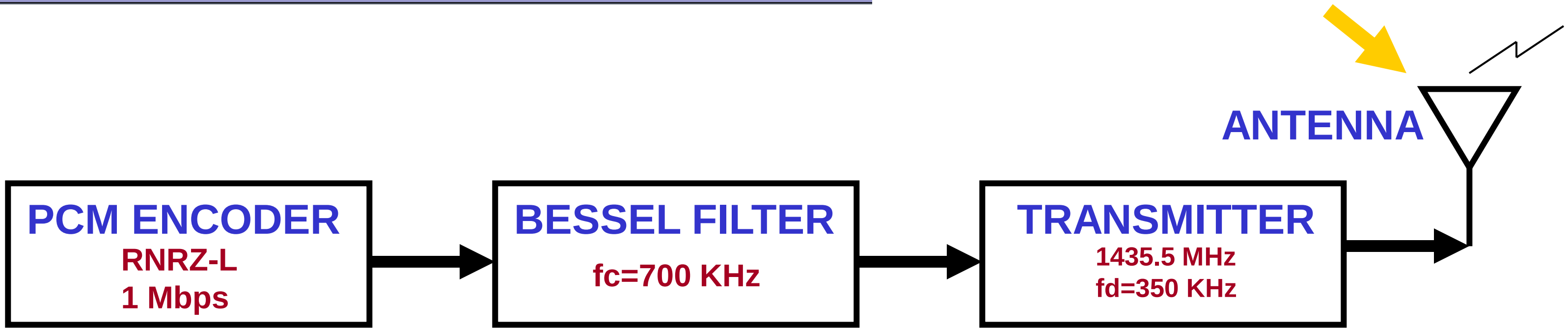


Spectrum Analyzer

This is the RF spectrum of the transmitted PCM stream that has been filtered.

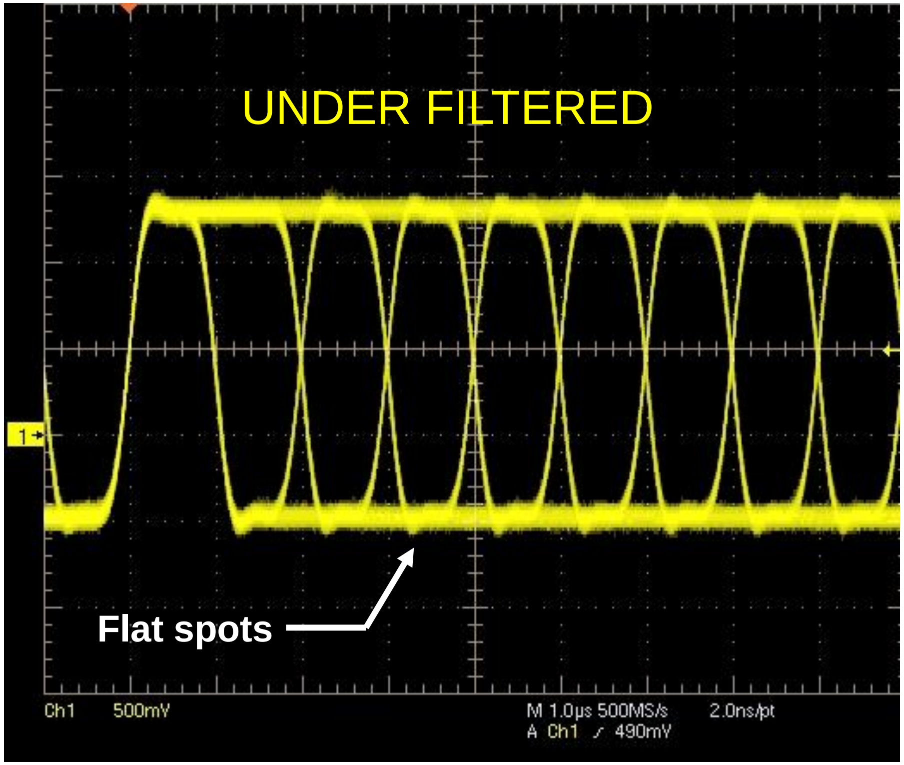
1) The transmitter's power is concentrated within the receiving frequency band.

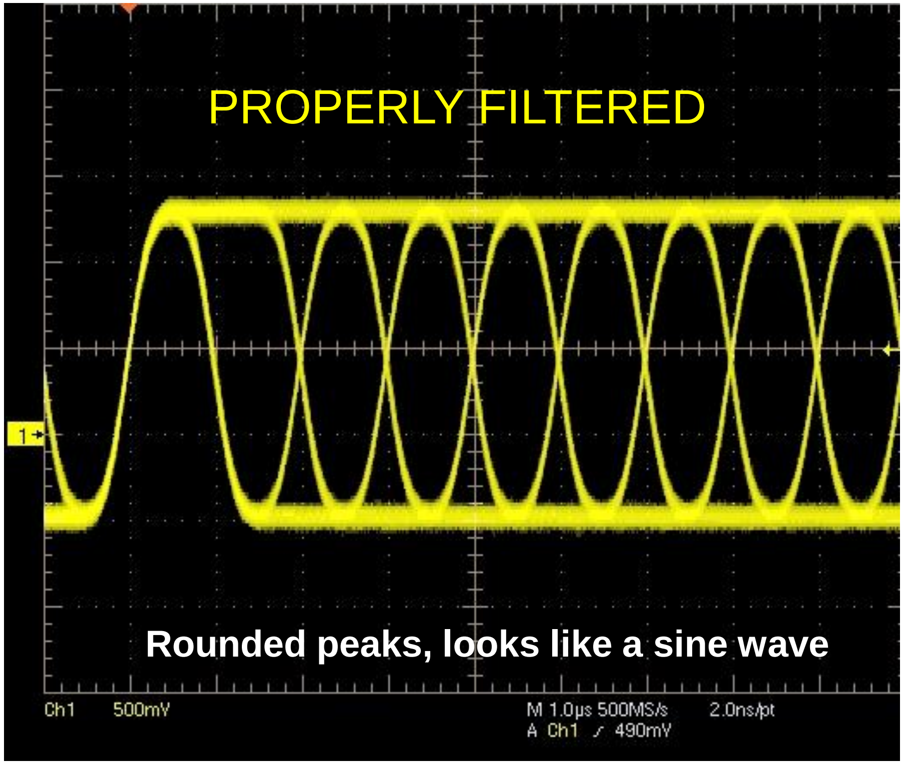
2) Less spectrum is being used such that aircraft TM spectrums can be located closer.



Be Careful when Changing Bit Rates

It is very easy to change your bit rate and forget that you need to replace your pre-modulation filter SIP also. Here are some things to look for that might indicate that your pre-modulation filter setting is incorrect.

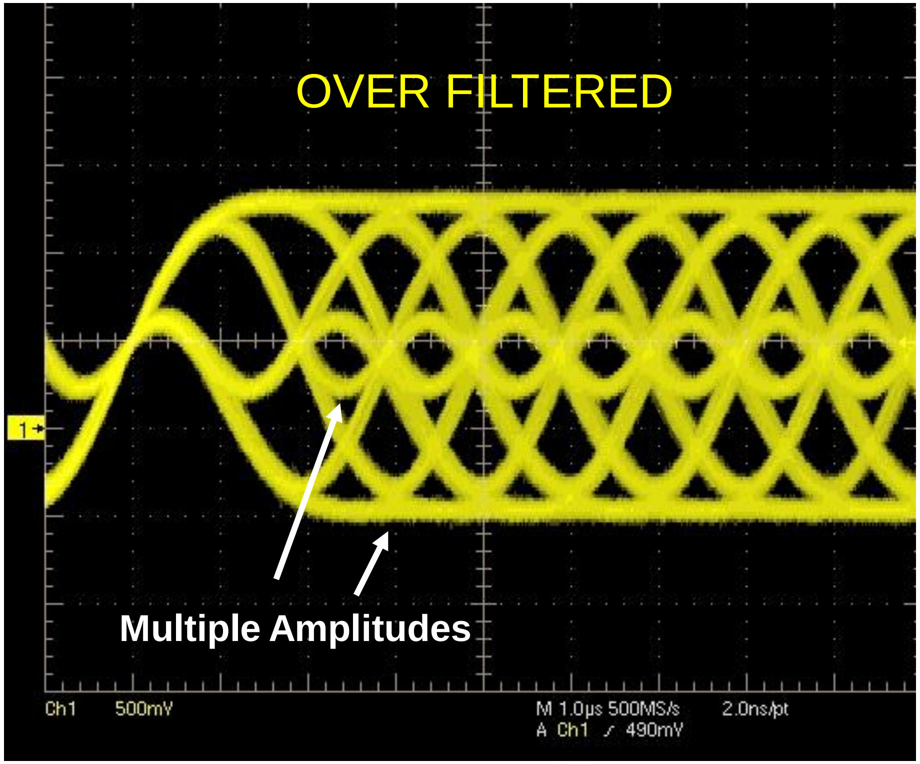


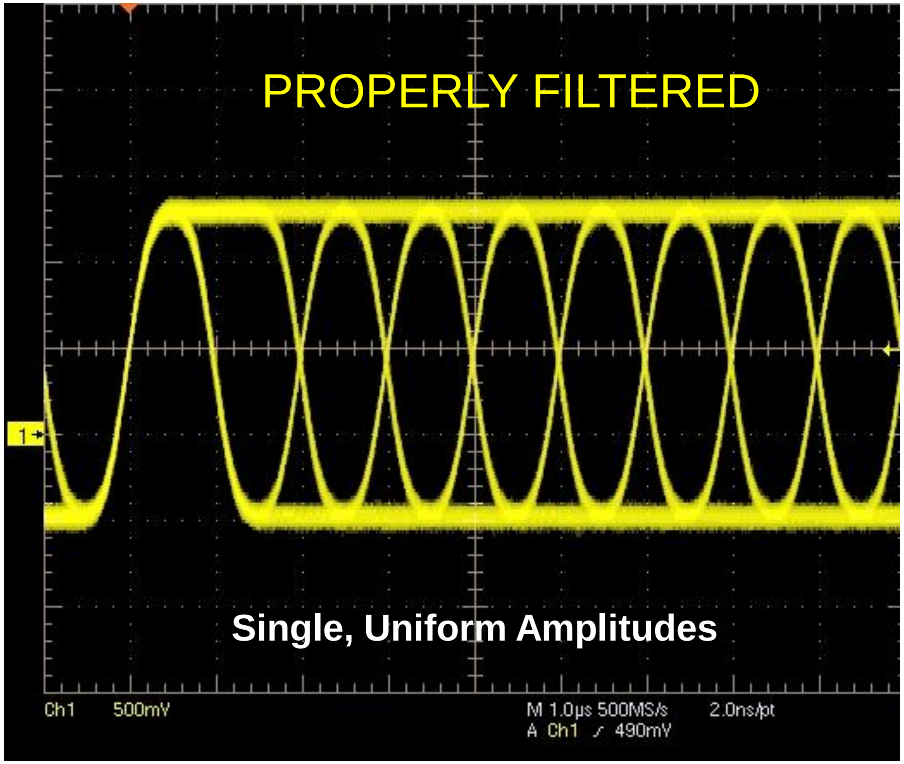


Screenshots are from the output of the filtered RNRZ-L on the MCI-105.

Be Careful when Changing Bit Rates

It is very easy to change your bit rate and forget that you need to replace your pre-modulation filter SIP also. Here are some things to look for that might indicate that your pre-modulation filter setting is incorrect.





Screenshots are from the output of the filtered RNRZ-L on the MCI-105.

One last note on filtering…

Our transmitters utilize AC-coupling (basically a high-pass filter) at the input. There is a rule of thumb regarding that cutoff frequency.

The cutoff frequency of the high pass filter in the transmitter should be no higher than the bit rate divided by 3000.

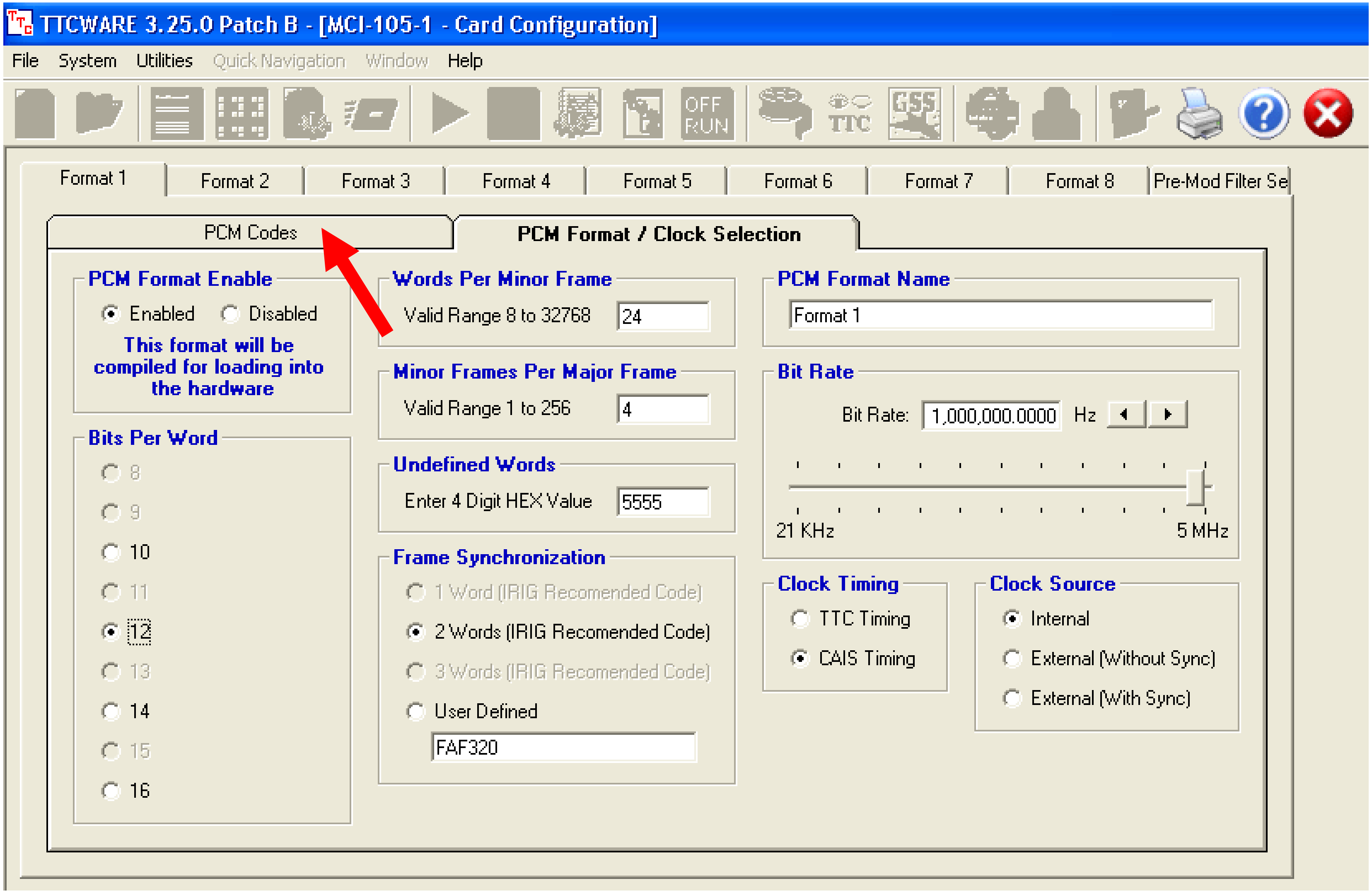
$$\text{high-pass filter cutoff frequency} \leq \frac{BitRate}{3000}$$

Summary

- PCM streams contain many different frequency components.
- These frequency components contribute to the bandwidth of a transmitted PCM stream.
- Filtering with a Bessel filter attenuates the higher frequencies such that the amount of bandwidth used is minimal.
- Bessel filters have linear phase delays such that there is a minimal amount of distortion.
- If you are transmitting PCM and the bit rate changes, remember to change the filter cutoff frequency!

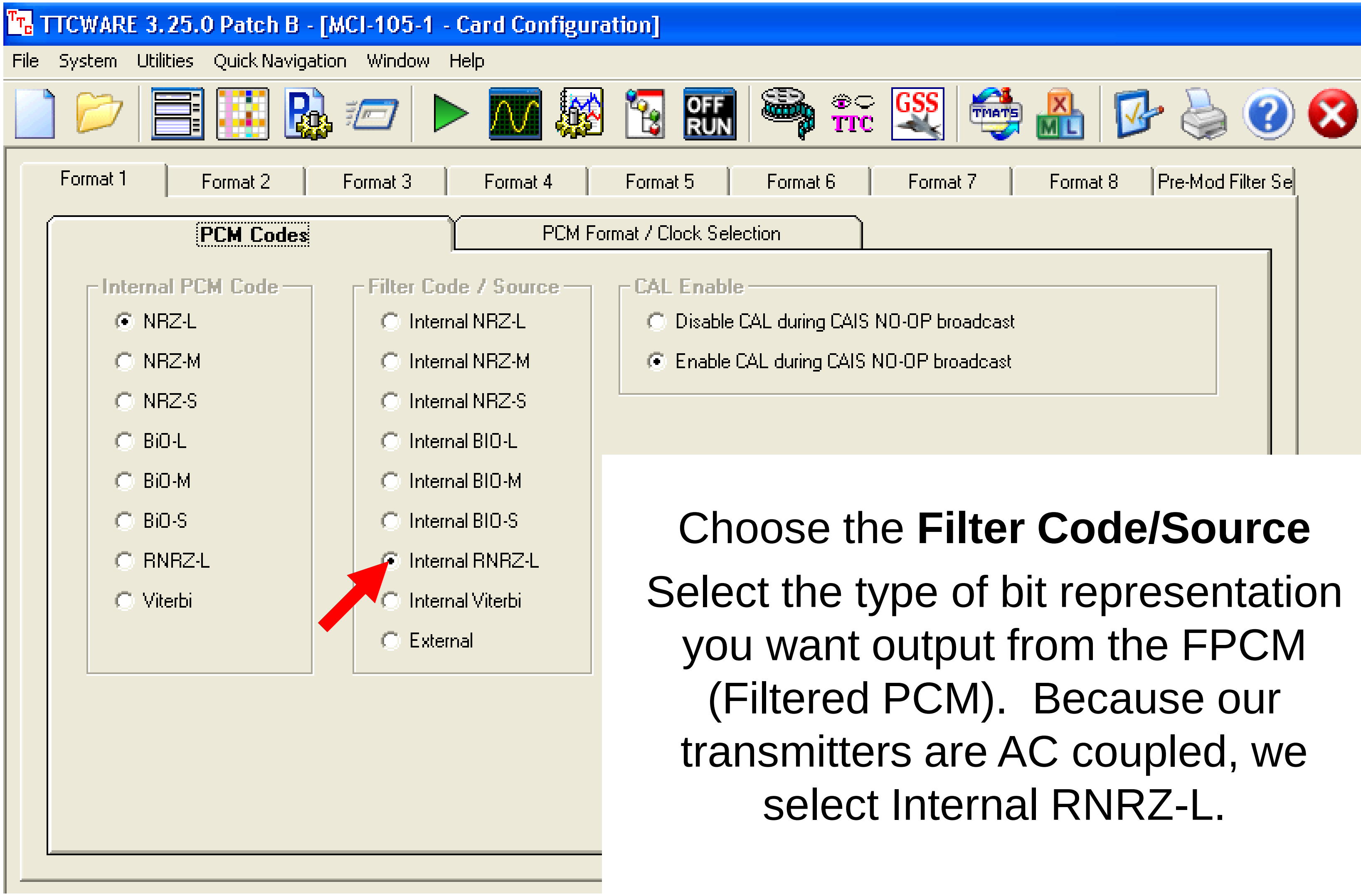
Hardware Setting Up

Filtering PCM on the CDAU & MCDAU



Click on the PCM Codes tab in the MCI-105 Card Configuration.

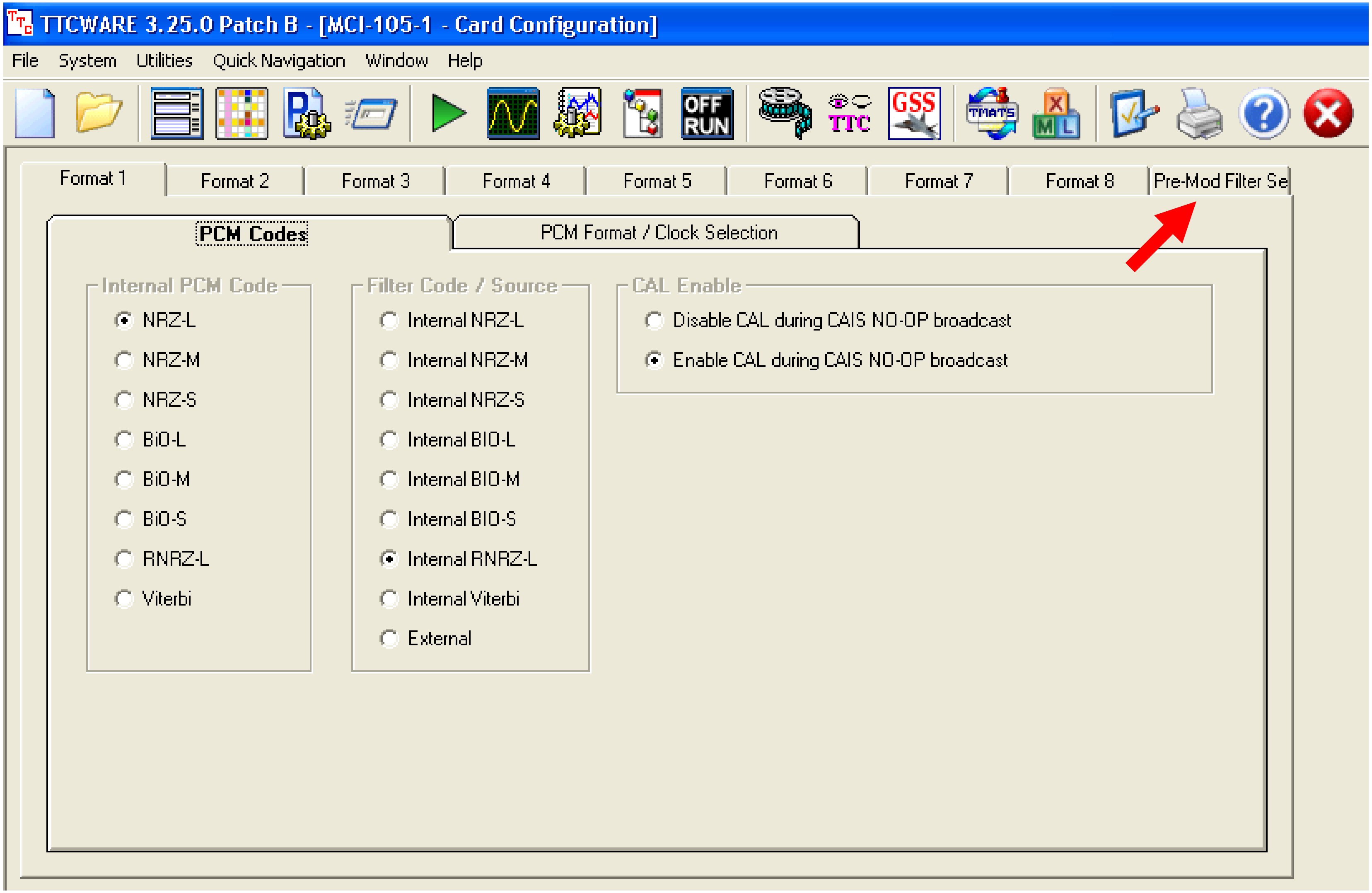
Filtering PCM on the CDAU & MCDAU



Choose the **Filter Code/Source**

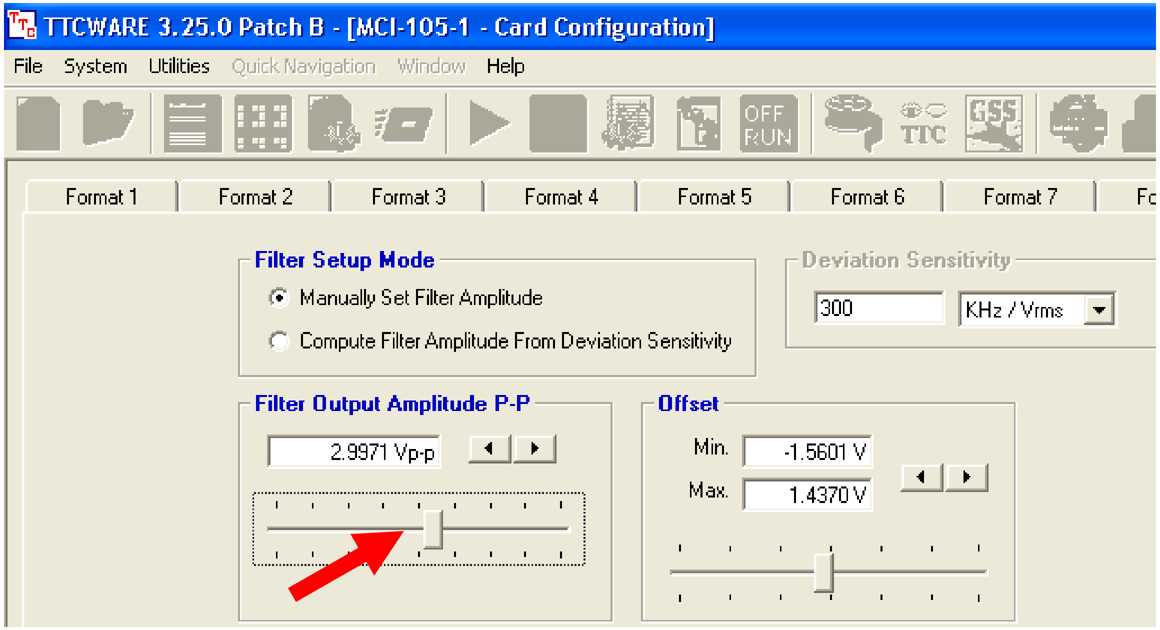
Select the type of bit representation you want output from the FPCM (Filtered PCM). Because our transmitters are AC coupled, we select Internal RNRZ-L.

Filtering PCM on the CDAU & MCDAU



Now click on Pre-Mod Filter Setup.

Filtering PCM on the CDAU & MCDAU



The amplitude of the filtered PCM stream is adjusted in the "Filter Output Amplitude P-P" section. This amplitude affects the frequency deviation of the transmitter. See the training on *Telemetry System Design, Set-up, and Optimization* to determine the amplitude setting.

Setting the Pre-Modulation Filter on the CDAU

- The CDAU’s MCI-105 has an integrated pre-modulation filter that is used to filter the PCM output.
- This filter is <u>not programmable</u>, but must be set using SIP’s (Single Inline Package), or using resistors mounted on a DIP (Dual Inline Package) by the user.
- The filtered PCM is output from the MCI-105 on pin 1 (FPCM) of connector J2.

Setting the Pre-Modulation Filter on the CDAU

Plug-in Filter DIP: The user can configure a 14-pin plug-in DIP to set the cutoff frequency of the premodulation filter. This DIP is installed in the "RN4" location in lieu of the SIP package. The DIP assembly is shown in Figure 9. All seven (7) resistors (value = "R") are identical. The value of R is based on the operating bit rate and the PCM coding used (NRZ or BiPhase) cutoff frequency and is governed by Equation 1 or Equation 2:

Equation 1. Premod Filter Cutoff Frequency (for NRZ).

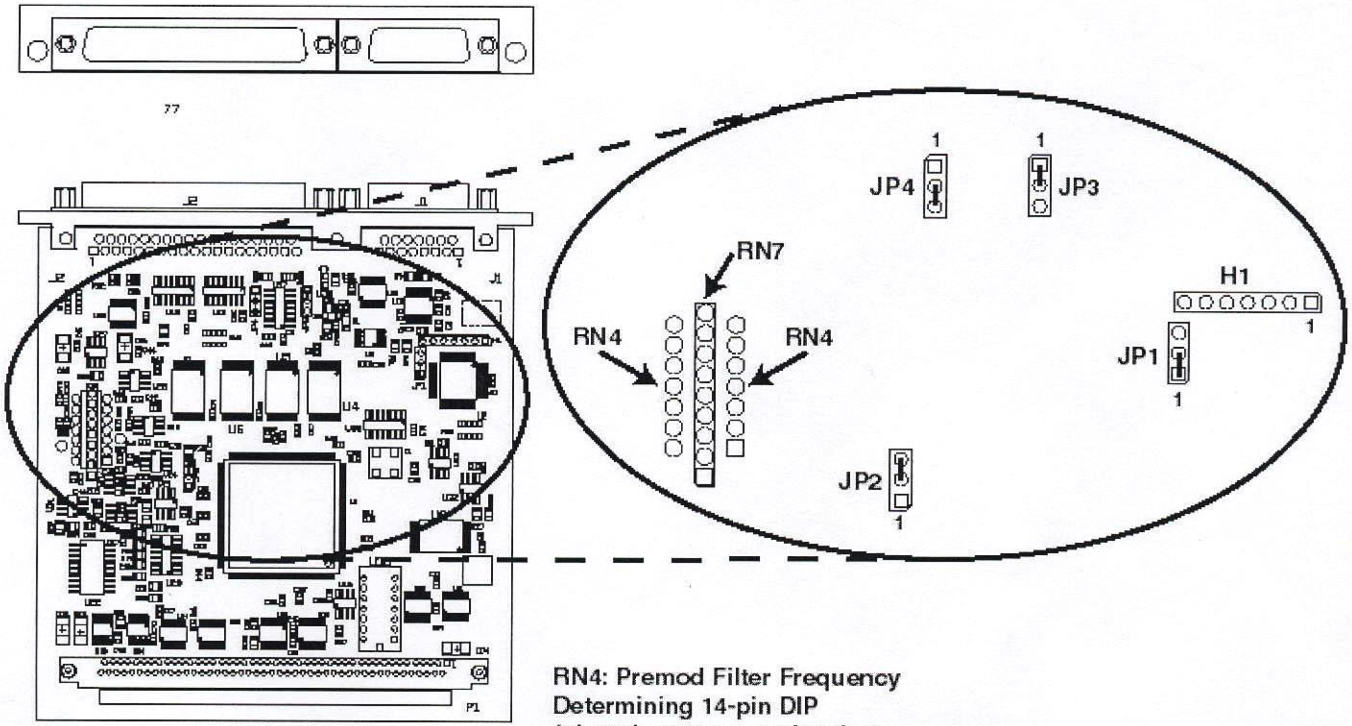
$$\text{R (in kilohms)} = \left[\frac{\text{1 Mbps}}{\text{Desired Bit Rate (in Mbps)}} \right] \text{x 1Kilohm}$$

Equation 2. Premod Filter Cutoff Frequency (for BiPhase).

$$\text{R (in kilohms)} = \left[\frac{\text{1 Mbps}}{\text{Desired Bit Rate (in Mbps)}} \right] \text{x .5 Kilohms}$$

Setting the Pre-Modulation Filter on the CDAU

This view shows the location of the RN4/RN7 resistor networks.



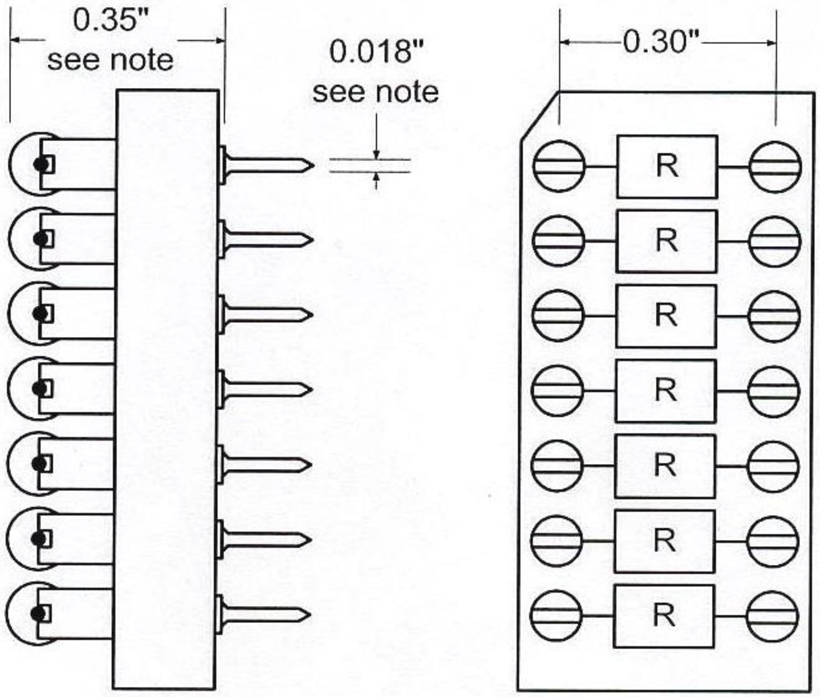
RN4: Premod Filter Frequency
Determining 14-pin DIP
(plugs into outer sockets), or

RN7: Premod Filter Frequency
Determining 9-pin SIP
(plugs into center socket).

Either RN4 or RN7 must be installed.
One SIP (2 MBPS NRZ-L or 1 MBPS
BiP-L) is provided from the factory.

Setting the Pre-Modulation Filter on the CDAU

Figure 9. Assembly, Premod Filter DIP.



Note: Do not exceed these dimensions.

This shows the DIP which contains the resistors used to configure the Bessel filter within the MCI-105 card.

Example of Determining the Resistor Value

Say we have a 3.5 Mbps NRZ-L PCM stream. What resistors should be used in the R4 resistor network?

Equation 1. Premod Filter Cutoff Frequency (for NRZ).

$$\text{R (in kilohms)} = \left[\frac{\text{1 Mbps}}{\text{Desired Bit Rate (in Mbps)}} \right] \times \text{1Kilohm}$$

Using the equation for the NRZ-L bit rate, substitute the bit rate into the equation.

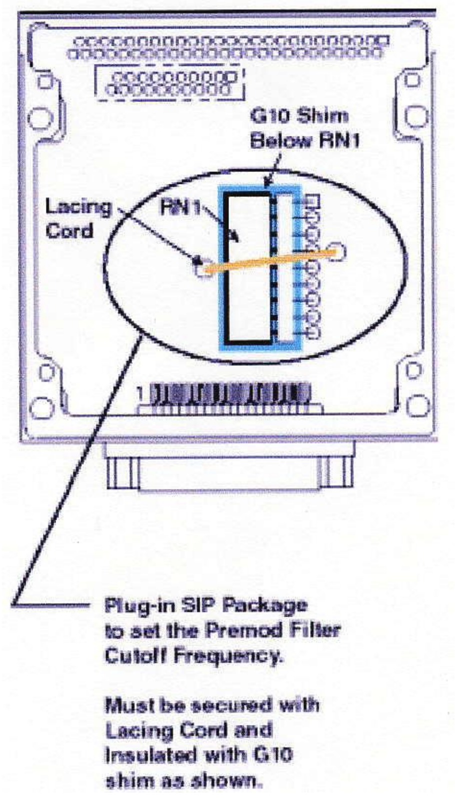
$$R(K\Omega) = (1\ \text{Mbps}/3.5\ \text{Mbps})(1\ K\Omega)$$

$$R(K\Omega) = (.2857)(1\ K\Omega)$$

$$R(K\Omega) = .2857\ K\Omega$$

R4 should be the closest valued resistor <u>lower</u> than 287.5 Ω . That would be a 274 Ω resistor which would be inserted in all seven resistor positions on the R4 DIP.

Setting the Pre-Modulation Filter on the MCDAU



The MMCI-105 has only factory SIP's installed, located as shown on the left. The SIP's can be used for NRZ-L.

So if you are transmitting your PCM from a MCDAU, you want to select a bit rate from the list of available SIP's.

SIP's Available for the CDAU/MCDAU

TTC Part Number	RNRZ-L Bit Rate
10740000-006	1.0 Mbps
10740000-015	2.0 Mbps
10740000-018	4.0 Mbps
10740000-021	312.5 Kbps
10740000-022	256 Kbps
10740000-023	4.5 Mbps
10740000-024	2.5 Mbps
10740000-028	5.0 Mbps
10740000-029	391 Kbps
10740000-030	500 Kbps

The SIPS can be obtained in the Special Flight Test Instrumentation Pool (SFTIP).

CDAUs Using the CMP-112

The CMP-112 Composite Output Card does not have a built-in pre-modulation filter.

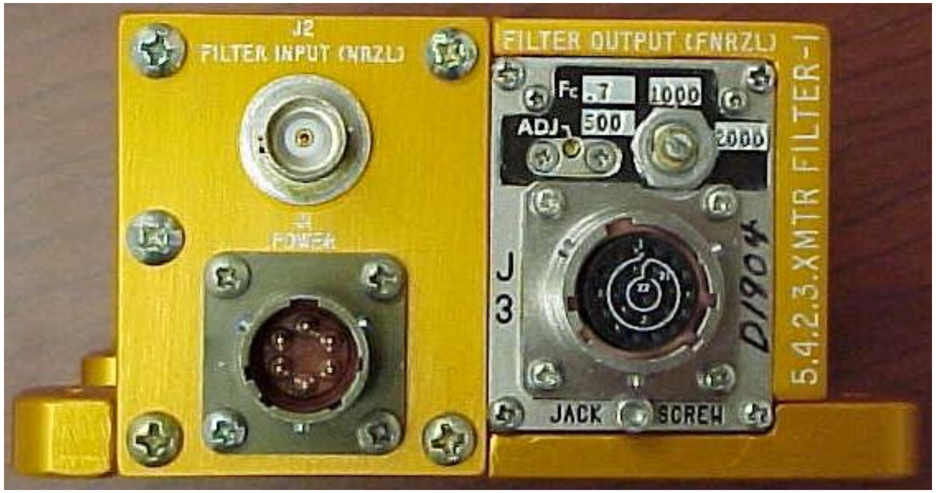
This card is normally used for combining the Chapter 8 (100% 1553) streams. Normally the combined stream is recorded only, but if the requirements call for transmitting the data, an external pre-modulation filter must be used.

Filtering PCM on Other Systems

For PCM systems that do not have pre-modulation filters, an external filter must be used. One easy way to insert a filter before transmitting is to utilize the PMF6-68 pre-modulation filter modules from the SP6-68 encryption units.

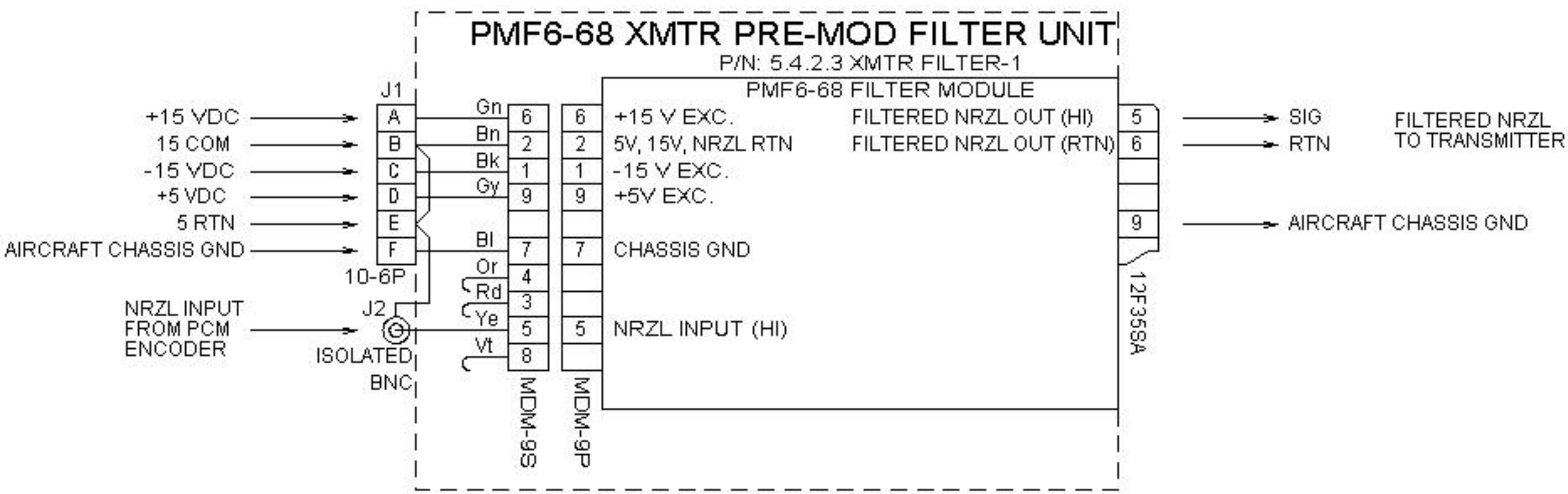
Filtering PCM on Other Systems

Housings for the filter modules have been manufactured in-house which have all of the power, input, and output pins readily available.



Filtering PCM on Other Systems

This is the wiring diagram of the filter module housing. The units require ±15V and 5V power, so an additional power supply will be needed.



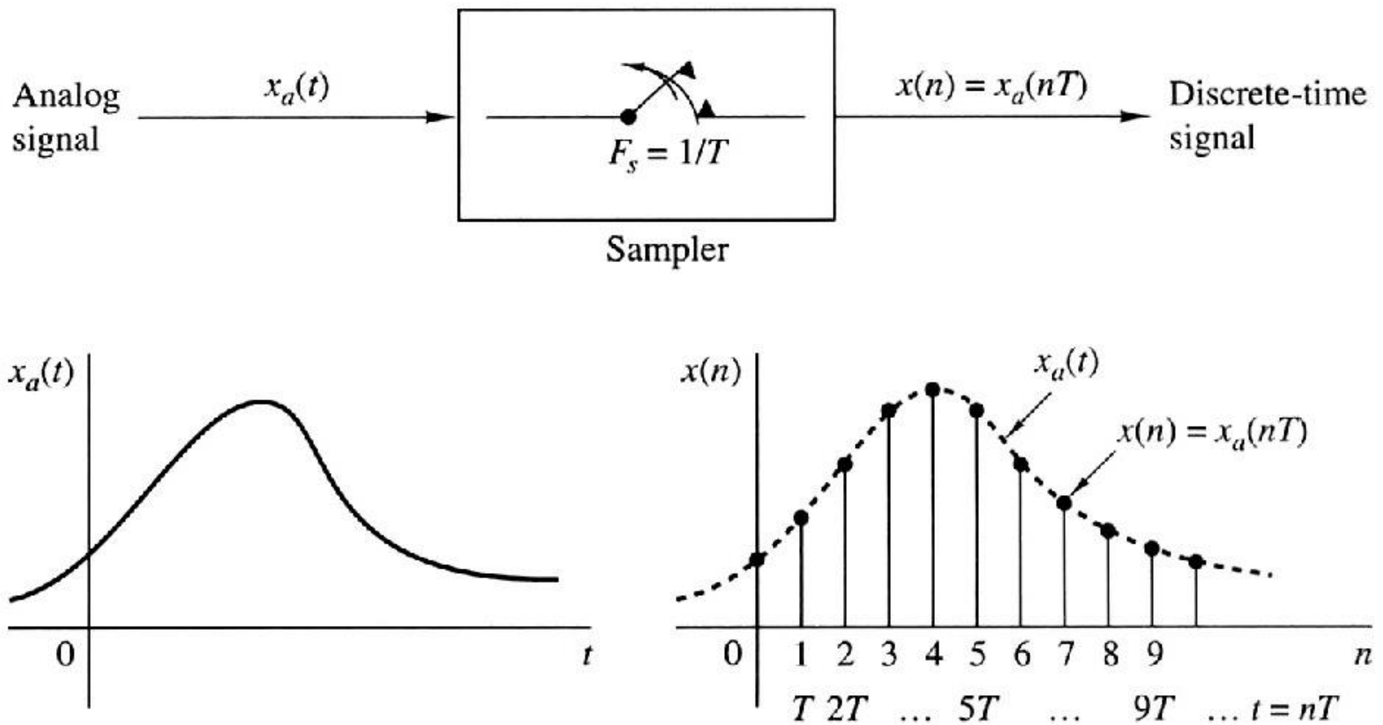
Filtering PCM on Other Systems

The filter module has two adjustments. One is the bit rate which is shown on the left as 500, 1000, and 2000 Kbps. Usually selections are BR, 2BR, and 4BR, but other choices exist. There is also an ADJ screw which varies the output amplitude (0 to 5V_{pp}) of the filtered NRZ-L PCM.

References

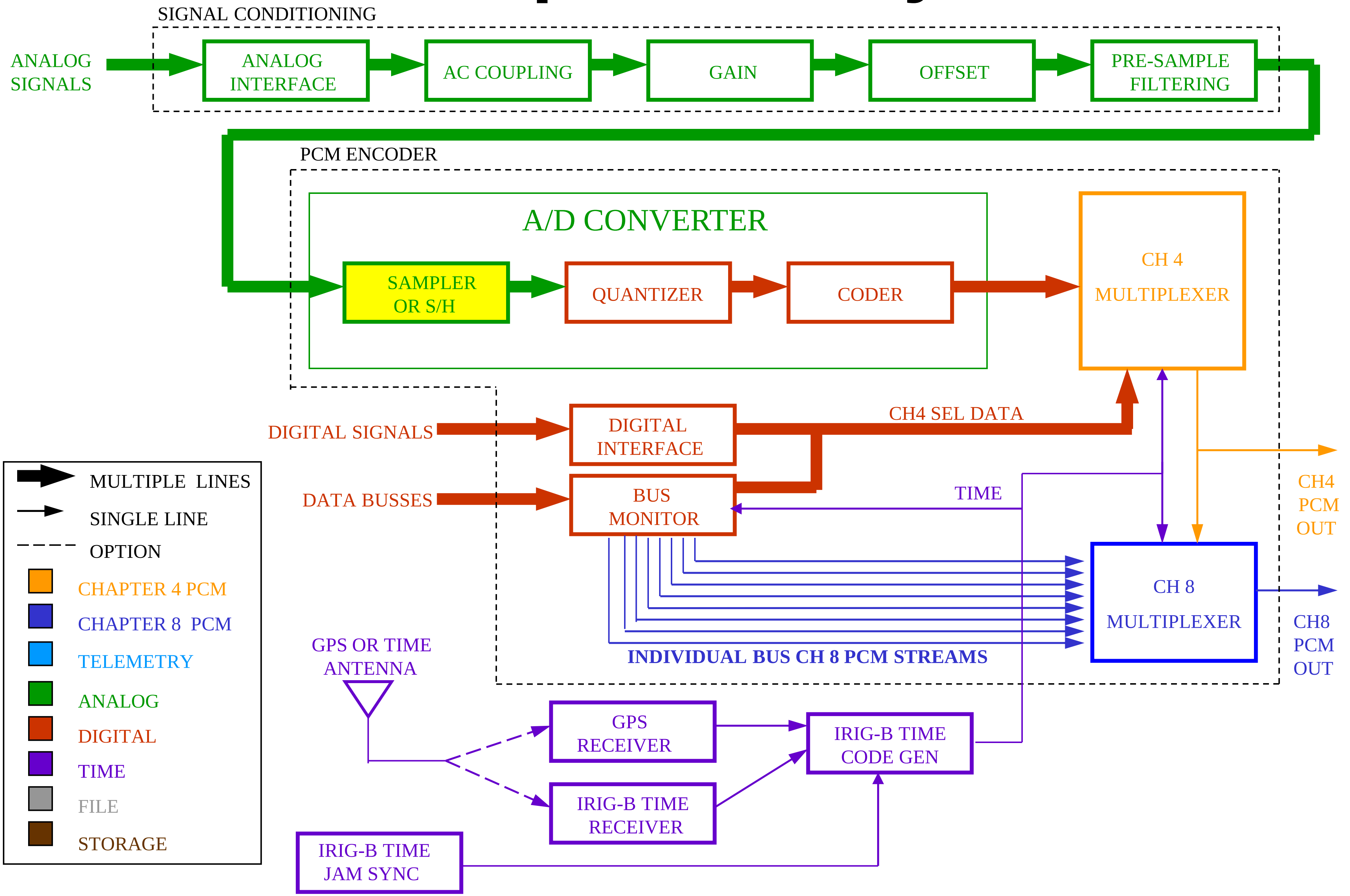
- IRIG 119-06: Telemetry Applications Handbook
- IRIG-106-09 Telemetry Standards
- IFT, 1995: Telemetry RF Signal Bandwidth; Definitions and Standards - Eugene L. Law
- CDAU Manual
- MCDAU Manual

Sampling Theory



Avoiding Aliasing and Minimizing the Error in Signal Replication

Data Acquisition System



Introduction

- A solid understanding of digital processing systems and sampling theory is very important when choosing sample rates to convert analog signals to digital signals.
- This training will illustrate the theory of sampling and illustrate the errors which can occur when you sample an analog signal.
- The amplitude errors which occur during sampling will then be discussed along with how to quantify and minimize the errors to a predetermined level.
- We will also briefly cover the sampling of digital signals.

Background

- Most real-world signals are continuous-time analog signals containing signal information for every instant of time.
- The majority of flight test data acquisition systems in service today are digital processing systems.
- The key step in digital processing of real-world analog signals is converting the analog signals into digital form.
- Continuous-time analog signals are sampled to create discrete-time signals.

Why Have Digitized Data?

- The quality of digital signals remains intact during storage, reproduction, transmission, etc. unless deliberately altered.
- Computers are based on bits (one’s and zero’s) and numbering systems, and require discrete digital information.
- Nearly all modern electronic devices use some digital data processing.
- Digital data systems are used to communicate (e.g., telemetry systems, cell phones, internet, fax machines, digital cameras, etc.).

You Must Always Filter First!!!

- Analog signals must be filtered or band-limited prior to being sampled. No matter how far we advance in data acquisition, there will always have to be an analog filter used to band limit the analog signal.
- The filter used to band-limit the analog signal is referred to as a pre-sample or an anti-alias filter
- Pre-sample filtering removes or attenuates unwanted frequencies and noise such that they no longer contribute to the signal of interest

What Cutoff Frequency to Use?

- When a measurement is requested, the frequency range, or highest frequency of interest f_d, must be known in order to figure out what cutoff frequency *fc* to use for your pre-sample filter.

- Also, the minimum gain (G), acceptable within the frequency range of interest must be known (usually 99%).

- The type of low-pass filter and number of poles must also be known (e.g., 6-pole low-pass Butterworth).

What Cutoff Frequency to Use?

The frequency response of a Butterworth filter is given by:

$$G = |H(j\omega)| = \frac{1}{\sqrt{1 + (f_d/f_c)^{2n}}}$$

Rearranging the equation for f_c, you get:

$$f_c = \frac{f_d}{\sqrt[2n]{\left(\frac{1}{G^2}\right) - 1}}$$

f_c is the cutoff frequency
f_d is the max frequency of interest
G is the minimum gain (99%) allowed at f_d
n is the number of poles in the filter

What Cutoff Frequency to Use?

To determine the cutoff frequency, you must first know what the maximum frequency of interest is. This can be found from the parameter list in the estimate request filled out by the customer. The “Frequency Range of Interest” indicates the band in which the flight test engineer wants minimal attenuation of the signal.

AIRCRAFT INSTRUMENTATION DIVISION AVMI, RANGE DEPARTMENT
PROJECT EFFORT ESTIMATE REQUEST

*** Signal Source ***
IT - Instrumentation Transducer
PT - Production Transducer
B - BUS Data
V-Video

**** Collection Method ****
V - Visual Indicator
A - Airborne Recorder
T - Telemetry
C - Camera

Measurement Description	MNEMONIC (Optional)	* Signal Sourc	Frequency Range of interest	Measurement Accuracy (Optional)	Eng Units	EU Range	** Collection Method	Remarks
Longitudinal Stick Position	LGSTKPOS	IT	0-30Hz	+/- 1Deg	% Percent	0-100	A, T	
NGB Oil Cool, Oil In Temp	TNGBOILN	IT	1 Hz	+/- 0.5 Deg	DEG C	40 - 200	A, T	
Airspeed	ASPD	B	N/A	See ICD	See ICD	See ICD	A, T	429 Bus rate
Pilot MFD	PiltMFD	V	N/A	N/A	N/A	N/A	A, T	RS-170 Connection on MFD
Pitch Rate	PITRT	PT	0-50Hz	+/- 0.5%	Deg/sec	+/- 30	A, T	
Altitude	ALT	*IT*	*0-20 Hz*	N/A	feet	-256/65535	*A, T*	
Angle of Attack	AOA	*IT*	*0-50 Hz*	N/A	Deg	+/- 45	*A, T*	
Angle of Sideslip	AOSS	*IT*	*0-50 Hz*	N/A	Deg	+/- 45	*A, T*	
Outside Air Temp	OAT	*IT*	*0-10 Hz*	N/A	DEG C	-40/120	*A, T*	
Bank Angle	BANKANG	*IT*	*0-30 Hz*	N/A	Deg	+/-180	*A, T*	

What Cutoff Frequency to Use?

- Say you are given a parameter request that has a frequency range of interest from 0 to 120 Hz.
- 120 Hz is the highest frequency component that the customer is interested in analyzing.
- The frequencies above 120 Hz must be attenuated so that we can select a sample rate
- The filter being used will be a 6-pole low-pass Butterworth filter. The maximum attenuation of the signal at the frequencies of interest is 99%.
- What should the cutoff frequency be?

What Cutoff Frequency to Use?

- *f* is the highest frequency of interest, 120 Hz
- G is the lowest allowable gain, 99%
- n is the number of poles in the Butterworth filter, 6

$$f_c = \frac{f_d}{\sqrt[2n]{\left(\frac{1}{G^2}\right) - 1}} = \frac{120}{\sqrt[2*6]{\left(\frac{1}{0.99^2}\right) - 1}}$$

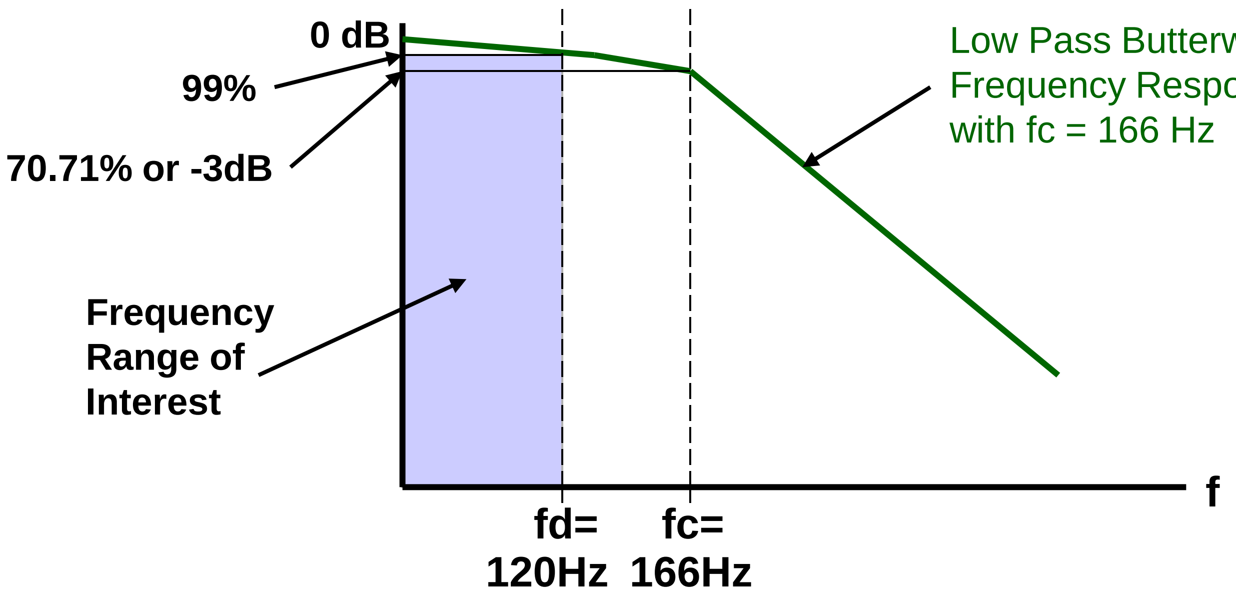
$$f_c = \frac{120}{\sqrt[12]{\left(\frac{1}{0.9801}\right) - 1}} = \frac{120}{\sqrt[12]{(1.02030405) - 1}}$$

$$f_c = \frac{120}{\sqrt[12]{(0.02030405)}} = \frac{120}{0.722712}$$

$$f_c = 166.04Hz$$

What Cutoff Frequency to Use?

By designing the 6-pole Butterworth filter with a cutoff frequency of 166 Hz, the gain of the signal is at least 99% at frequencies of 120 Hz or less.



What Cutoff Frequency to Use?

A rule of thumb for the cutoff frequency of a 6-pole Butterworth filter, with a maximally flat response of 99% and a maximum frequency of interest *f*, should be:

$$f_c = 1.383677f \cong \sqrt{2}f$$

So for our example, 99% data to 120 Hz, the cutoff frequency of the filter should be:

$$f_c \cong \sqrt{2}f = \sqrt{2} \cdot 120\ Hz = 169.7\ Hz$$

which is close to the exact calculation we made previously.

The rule of thumb should only be used when you have to verbally state an approximate cutoff frequency in a meeting. Keep in mind that using the rule of thumb equation may cause you to select a higher cutoff frequency than is necessary.

What Cutoff Frequency to Use?

For Butterworth filters of other orders, these are the multipliers to use when calculating the cutoff frequency or data frequency.

1-pole filter:	$f_c = 7.0175 \bullet f_d$	$f_d = 0.1425 \bullet f_c$
2-pole filter:	$f_c = 2.6491 \bullet f_d$	$f_d = 0.3775 \bullet f_c$
4-pole filter:	$f_c = 1.6276 \bullet f_d$	$f_d = 0.6144 \bullet f_c$
6-pole filter:	$f_c = 1.3837 \bullet f_d$	$f_d = 0.7227 \bullet f_c$
8-pole filter:	$f_c = 1.2758 \bullet f_d$	$f_d = 0.7838 \bullet f_c$

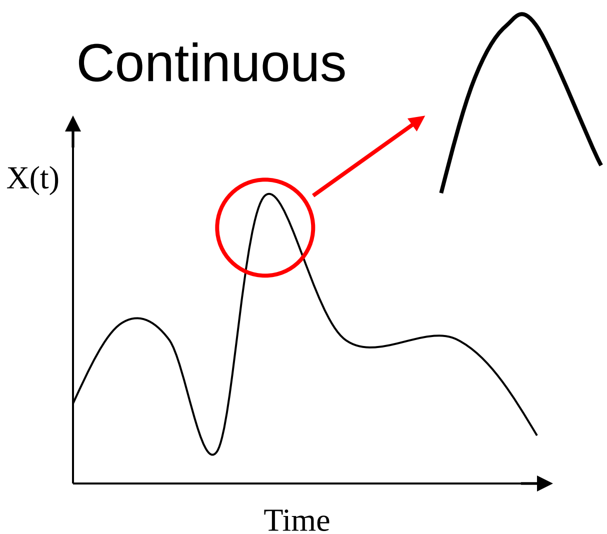
What Is Sampling?

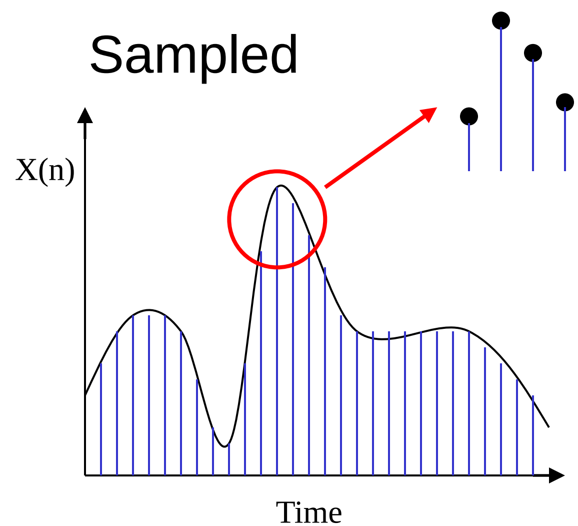
- Sampling is the act of taking a small part or quantity of something, as a sample for testing or analysis (i.e., accelerometer signal, voice, video, a piece of pie, etc.)

- The more samples you have, the more information about the signal you possess, the easier it is to determine what the original signal looked like.

What Is Sampling?

- Analog signals are continuous-time signals and have a value for every instant of time, whereas sampled signals occur at discrete intervals
- In digital data processing systems, samples of continuous-time signals are taken at discrete-time intervals to create a discrete-time signal.

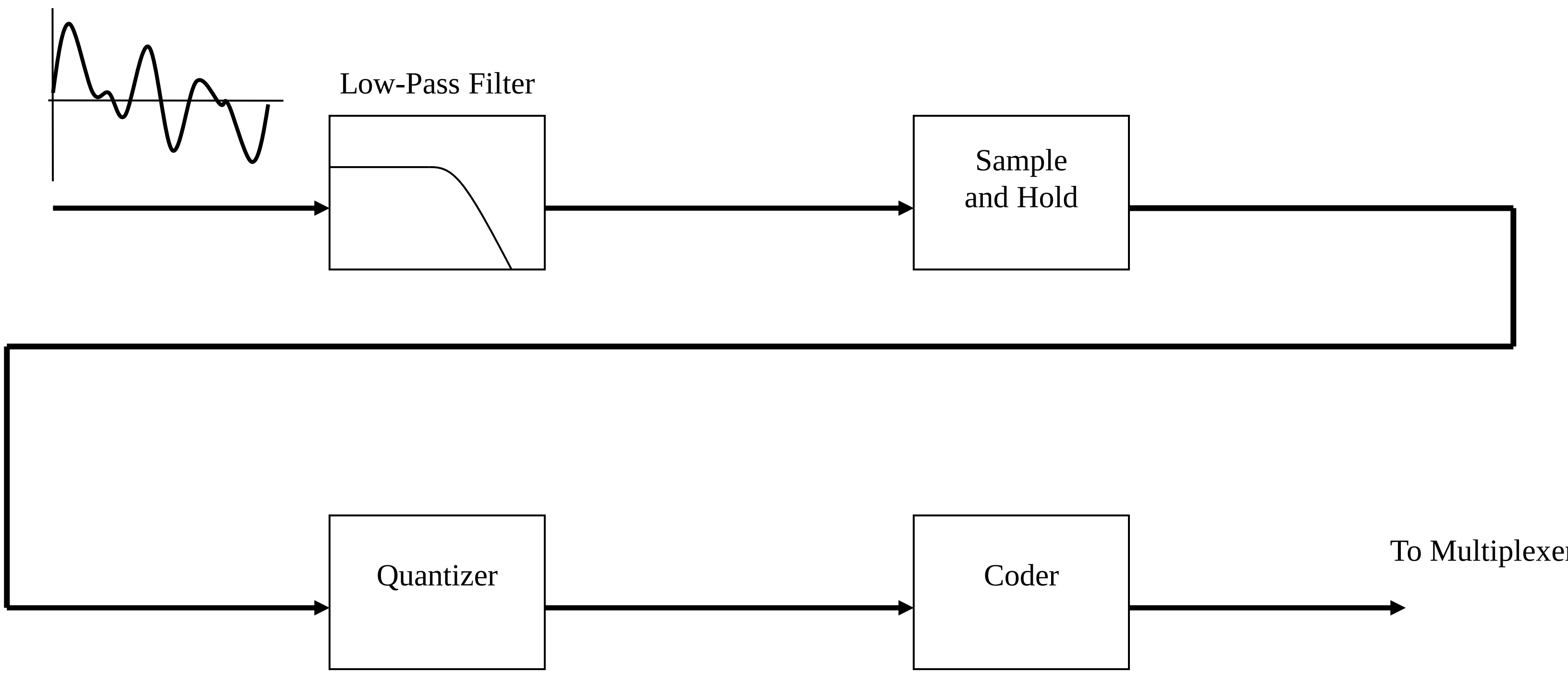




What Is Sampling?

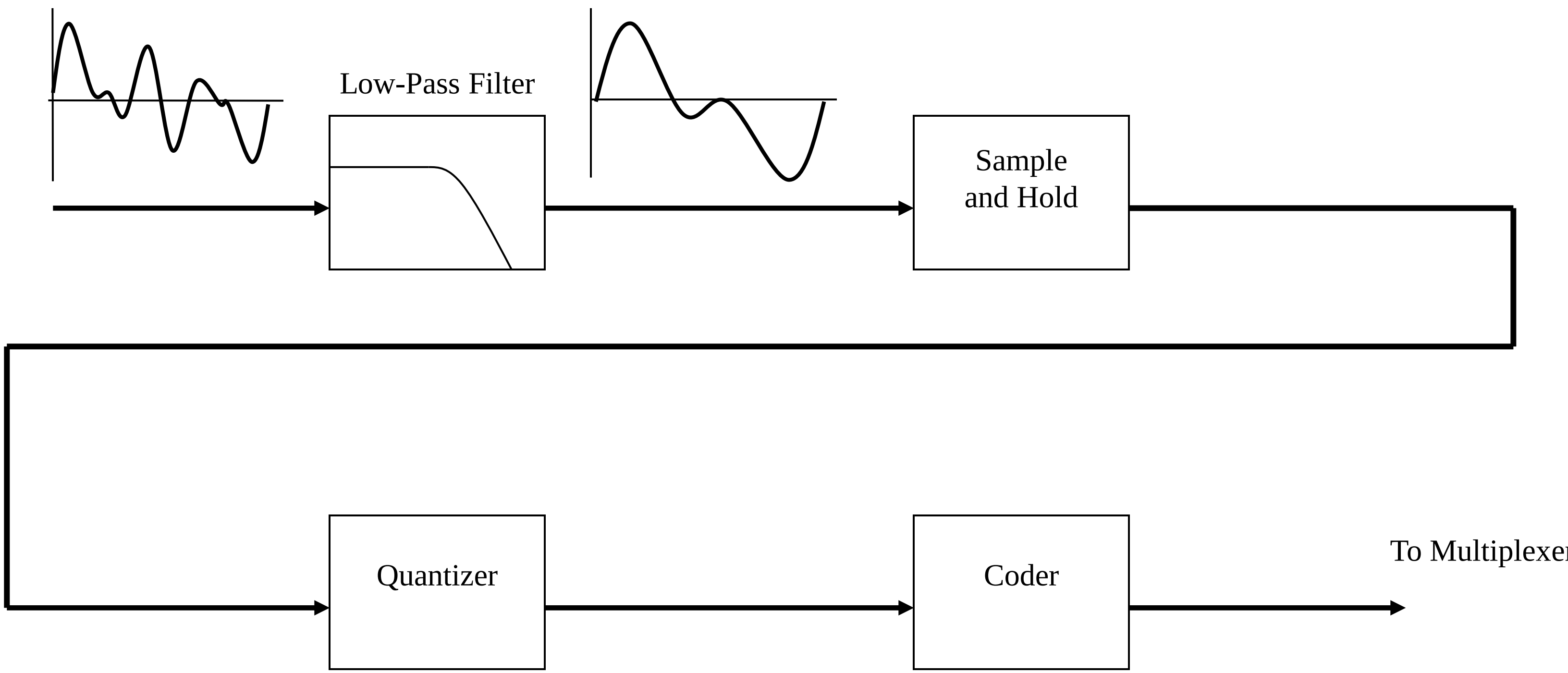
- It is important to remember that the sampling process produces a digital signal containing partial information from the original signal.
- Basic to all digital data systems is how many samples are necessary to reproduce the original signal accurately and with minimal distortion before selecting the number of samples.
- To determine the required number of samples, you must know the frequency range of interest of the continuous-time (analog) signal.

The Sampling and Digitization Process



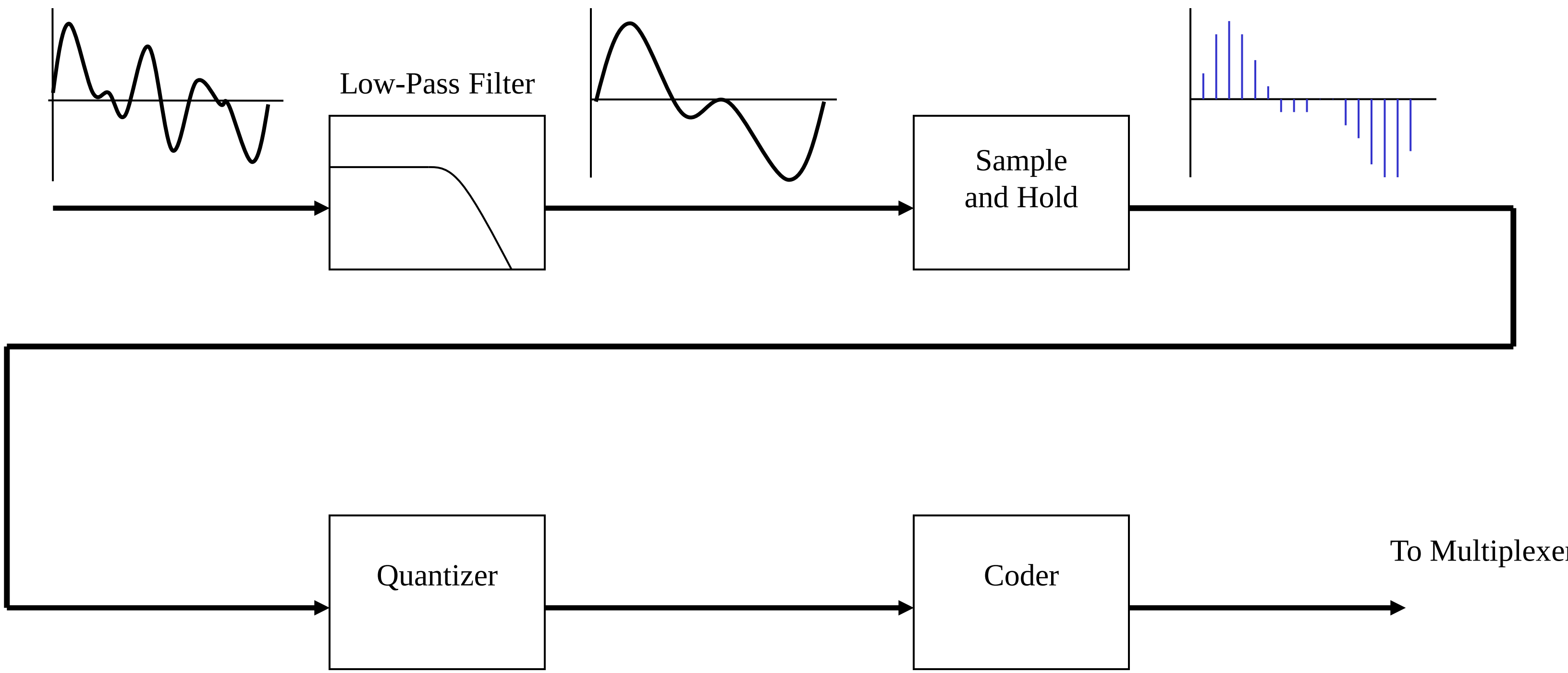
Remember that the first step before sampling is filtering. The input signal can have a wide range of frequencies – most of the times the highest frequency component won't be known.

The Sampling and Digitization Process



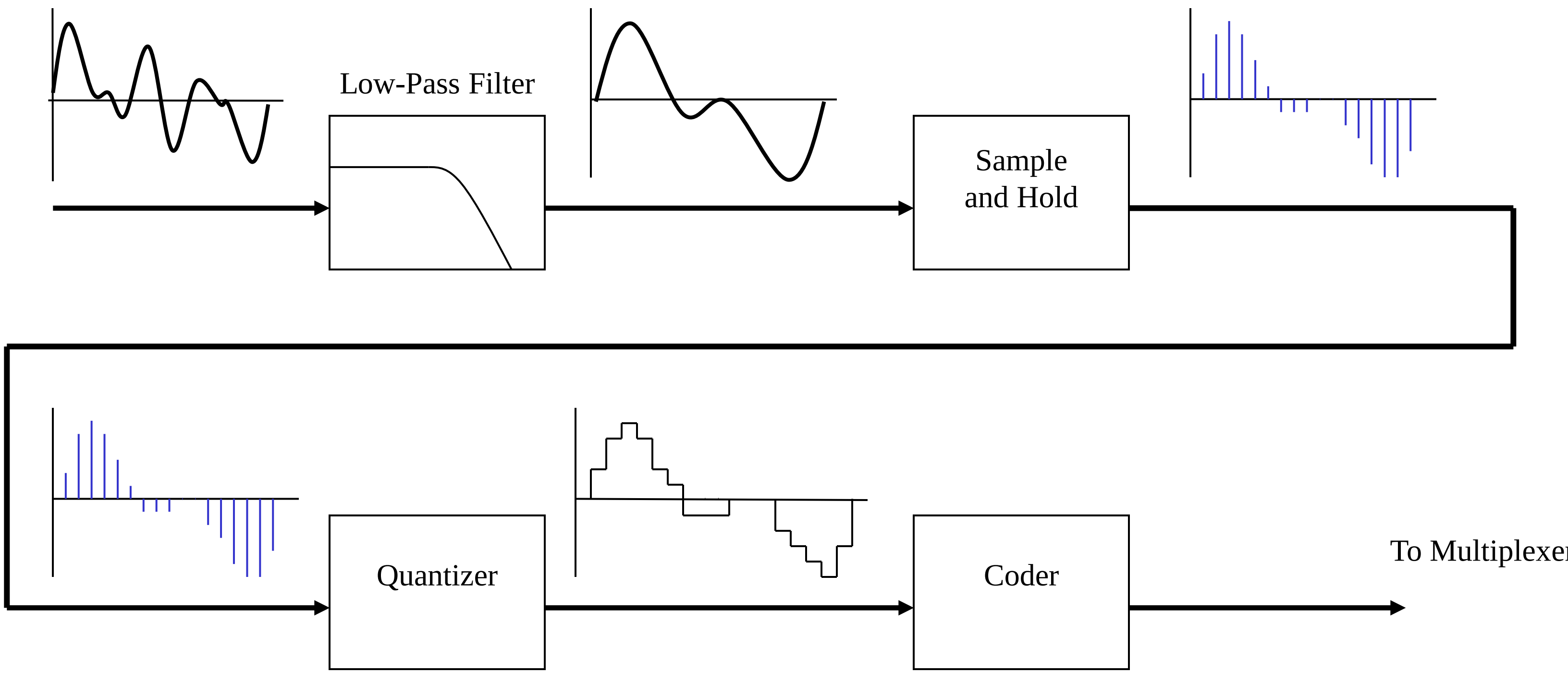
The signal is now band limited to a known range of frequencies. The sampling rate will depend on that highest frequency component. Filtering guarantees that you know what that maximum frequency will be and at what attenuation.

The Sampling and Digitization Process



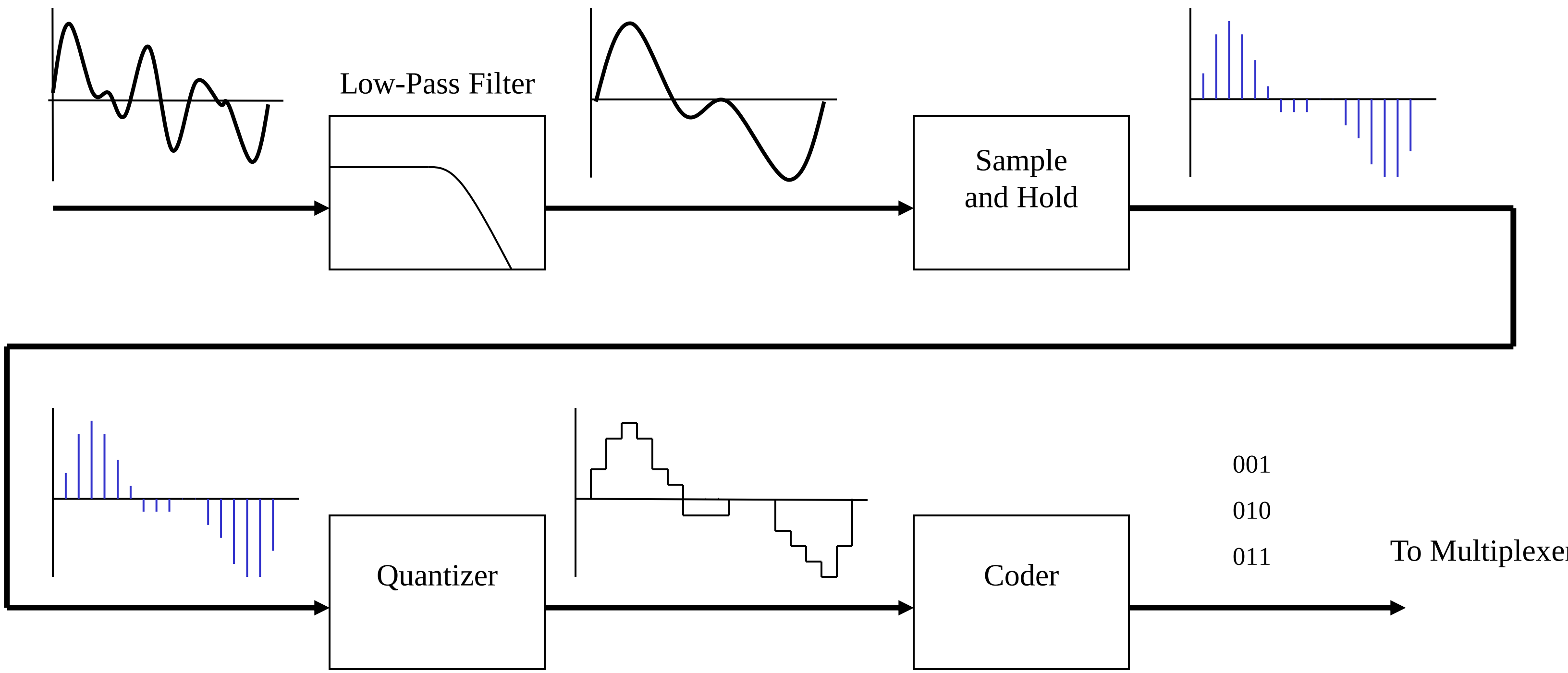
The signal is now sampled at a certain rate. For discrete instances in time, you know what the amplitude of the signal is. However, in between samples you don't. These analog amplitudes are held such that it can be digitized.

The Sampling and Digitization Process



The quantizer assigns a particular level to each of the signal amplitudes that are held. The level is determined by the amplitude surpassing a certain threshold level.

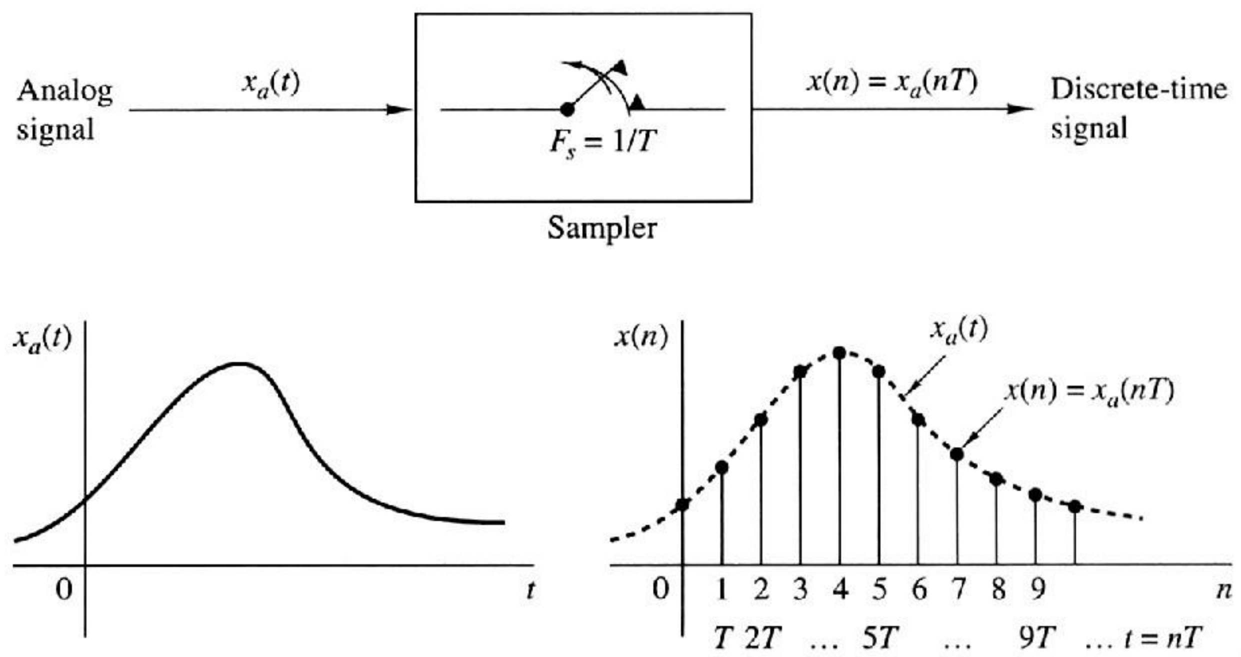
The Sampling and Digitization Process



The coder then assigned the appropriate code to each quantized level. This is the data that is multiplexed into the PCM stream.

Sampling of Analog Signals

- The discussion of sampling for this training will be limited to *periodic* or *uniform sampling*, which is the most common type of sampling used.

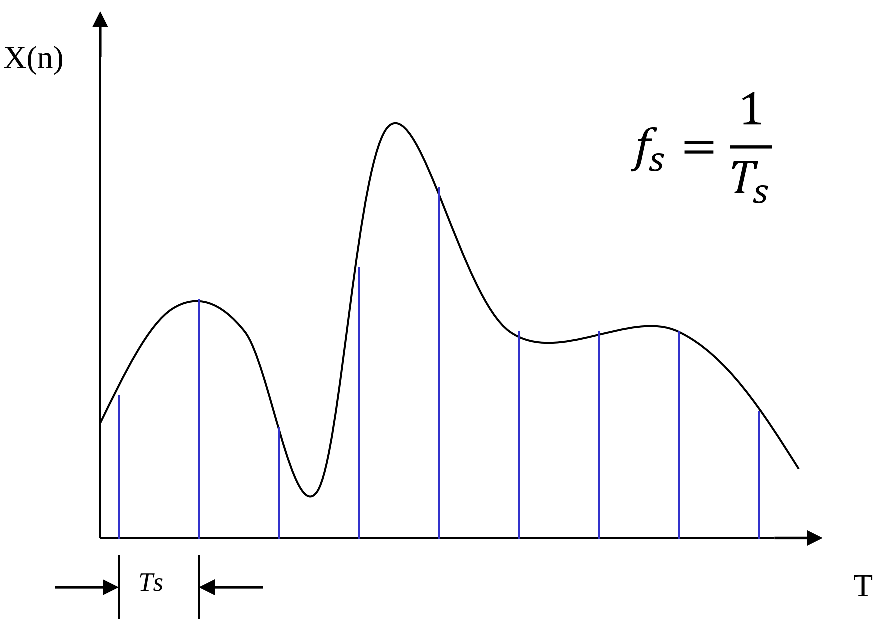


Sampling of Analog Signals

- The time interval *T* between successive samples is called the sampling period or sample interval T_s
- The reciprocal of the sampling period is called the sampling frequency (Hz) or sampling rate (samples per second).

$$T_s = 0.005\ sec$$

$$f_s = \frac{1}{T_s} = \frac{1}{0.005sec} = 200\ Hz$$

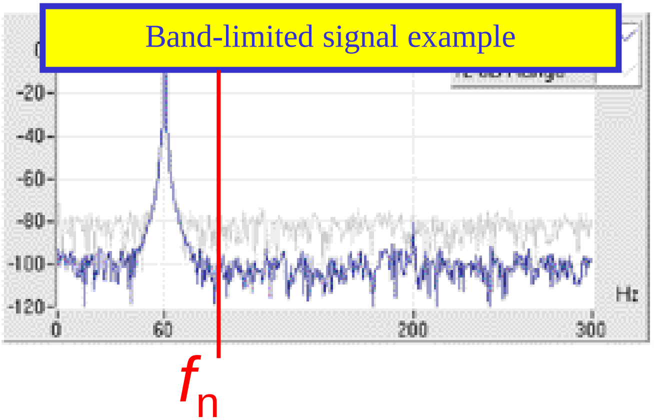


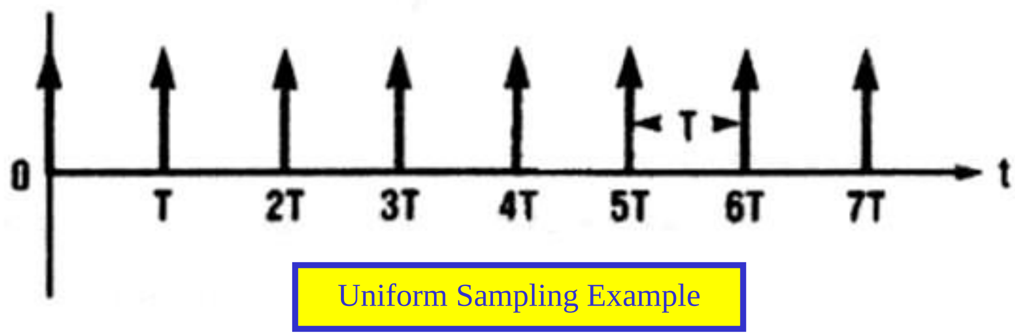
The Sampling Theorem

- There has been some debate about who to credit for the sampling theorem, Nyquist or Shannon, (most people speak about sampling in terms of Nyquist).
- For this training we will refer to the theorem as the Sampling Theorem, however terms like *Nyquist frequency*, *Nyquist rate*, and *Nyquist factor* are often used.
- There is a wealth of information about the sampling theorem and how it came about, in textbooks and on the internet if you would like to read more about it.

The Sampling Theorem

- A band-limited (filtered) signal that has no spectral components above f_n Hz , can be determined uniquely by values sampled at uniform intervals of T_s seconds, if $T_s \le \frac{1}{2f_n}$ (*the uniform sampling theorem*).





- In other words, the sample rate (f_s = 1/T_s) must be at least twice the Nyquist frequency f_n.

$$f_s \ge 2f_s$$

The Sampling Theorem

- The sampling theorem simply states that in order to recover a signal function f(t), the signal must be sampled at a rate that is at least twice as high as the highest frequency component (we refer to this as the *Nyquist Factor*, where $NF=f_s/f_n$).
- We <u>must</u> use a *Nyquist Factor* greater than 2 to successfully reconstruct the original signal (this will be discussed in detail in upcoming slides)
- Sophisticated systems that employ digital signal processing and interpolation techniques can recover signals with Nyquist factors slightly higher than 2
- Sampling higher than twice the Nyquist frequency is referred to as *over-sampling*
- Sampling below twice the Nyquist frequency is referred to as *under-sampling,* and will result in <u>*aliasing*</u> .

Aliasing Is A Bad Word!

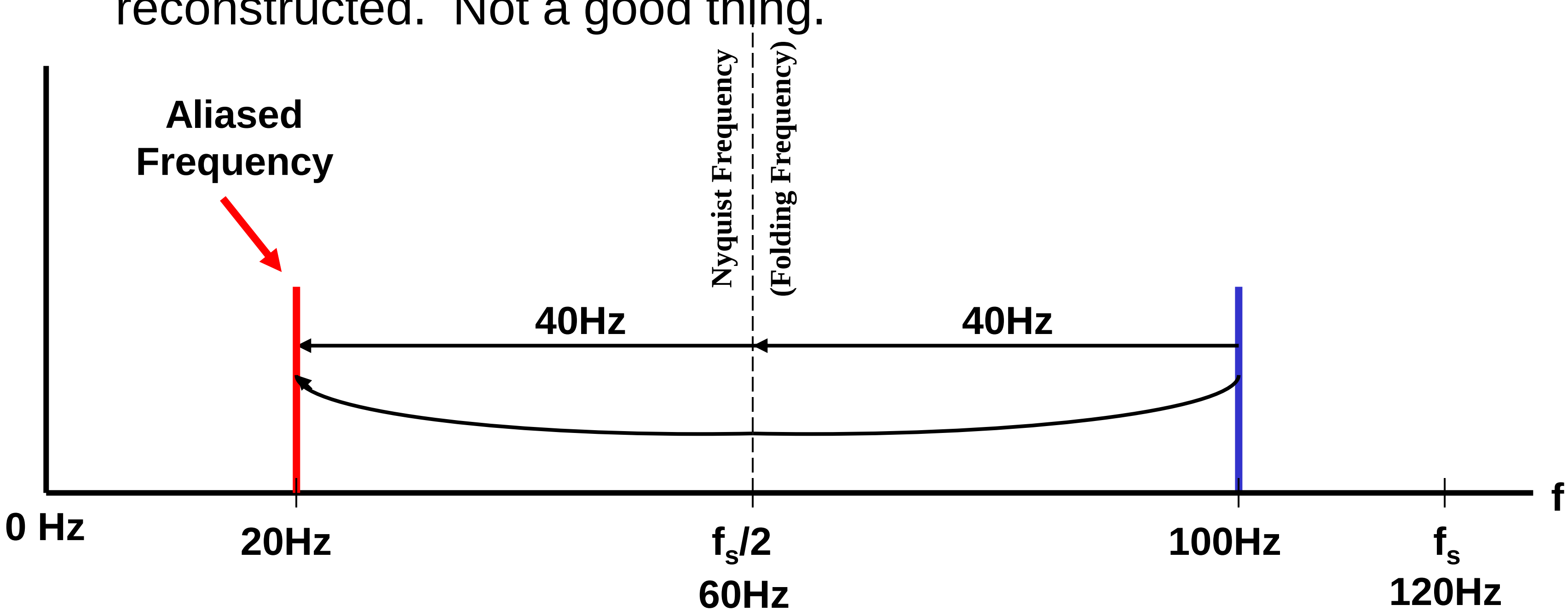
- *<u>Alias</u>* – A false name, otherwise known as, or Latin for, at another time
- In digital systems, it is defined as the phenomenon where high frequencies take on the identity of lower-frequencies, is called *aliasing*
- The *Nyquist Frequency* is known as the frequency which is equal to one half of the sampling frequency

$$f_{nyquist} = {}^{1}/_{2}\, f_s$$

- For example: if the sampling frequency is 120 Hz, then the *Nyquist frequency* is 120 Hz / 2 = 60 Hz
- Frequencies above 60 Hz will alias or fold into the 0 – 60 Hz frequency range and corrupt the data

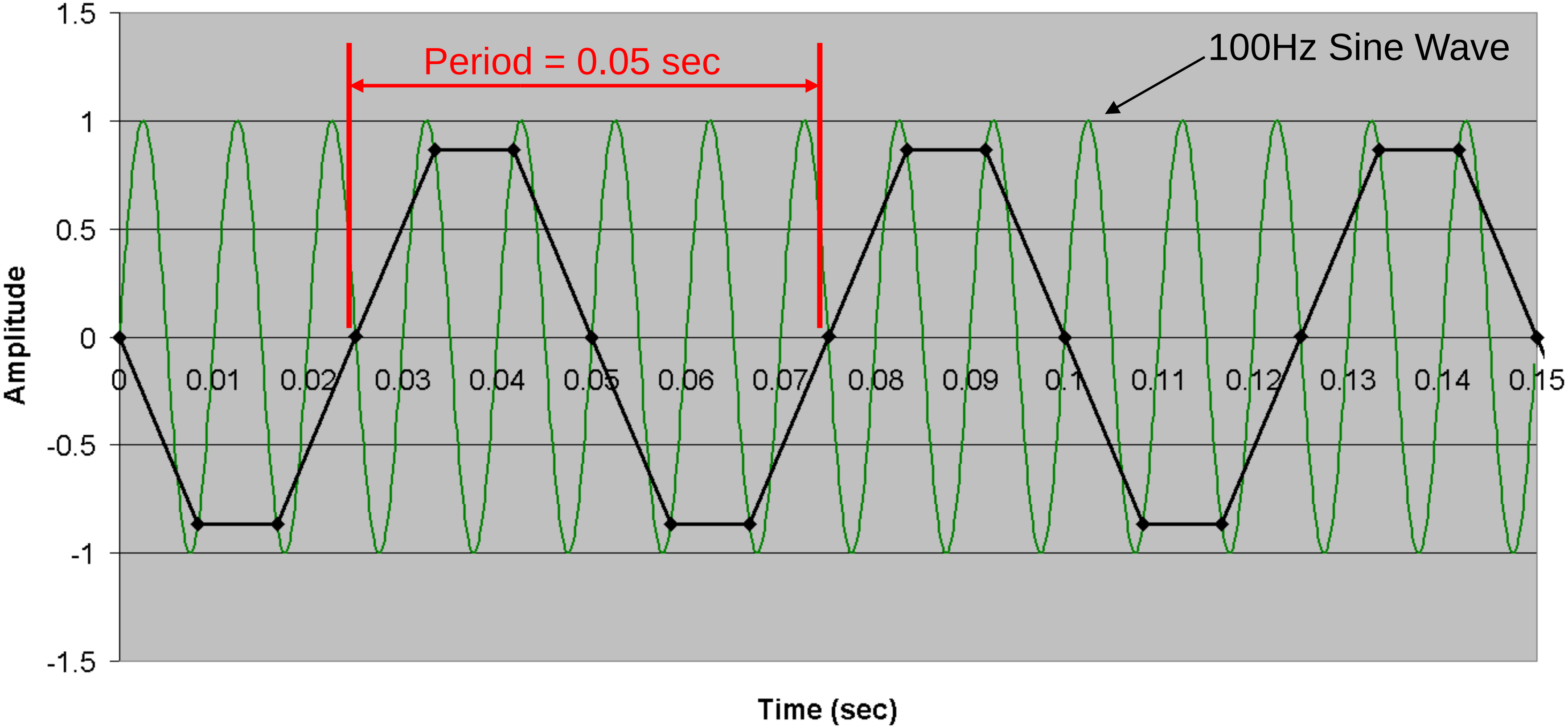
Visualization of how a Frequency is Aliased in the frequency domain

- If the sampling rate is 120 sps, then the *Nyquist frequency* is 120 Hz/2 = 60 Hz
- Frequencies above 60 Hz will alias or fold about the Nyquist frequency into the 0 – 60 Hz frequency range and corrupt the data, making it appear that there is a frequency component of the signal when one really does not exist.
- Say we have a 100 Hz sine wave sampled at 120 sps. It will appear as a 20 Hz sine wave when the sampled signal is reconstructed. Not a good thing.



Visualization of how a Frequency is Aliased in the time domain

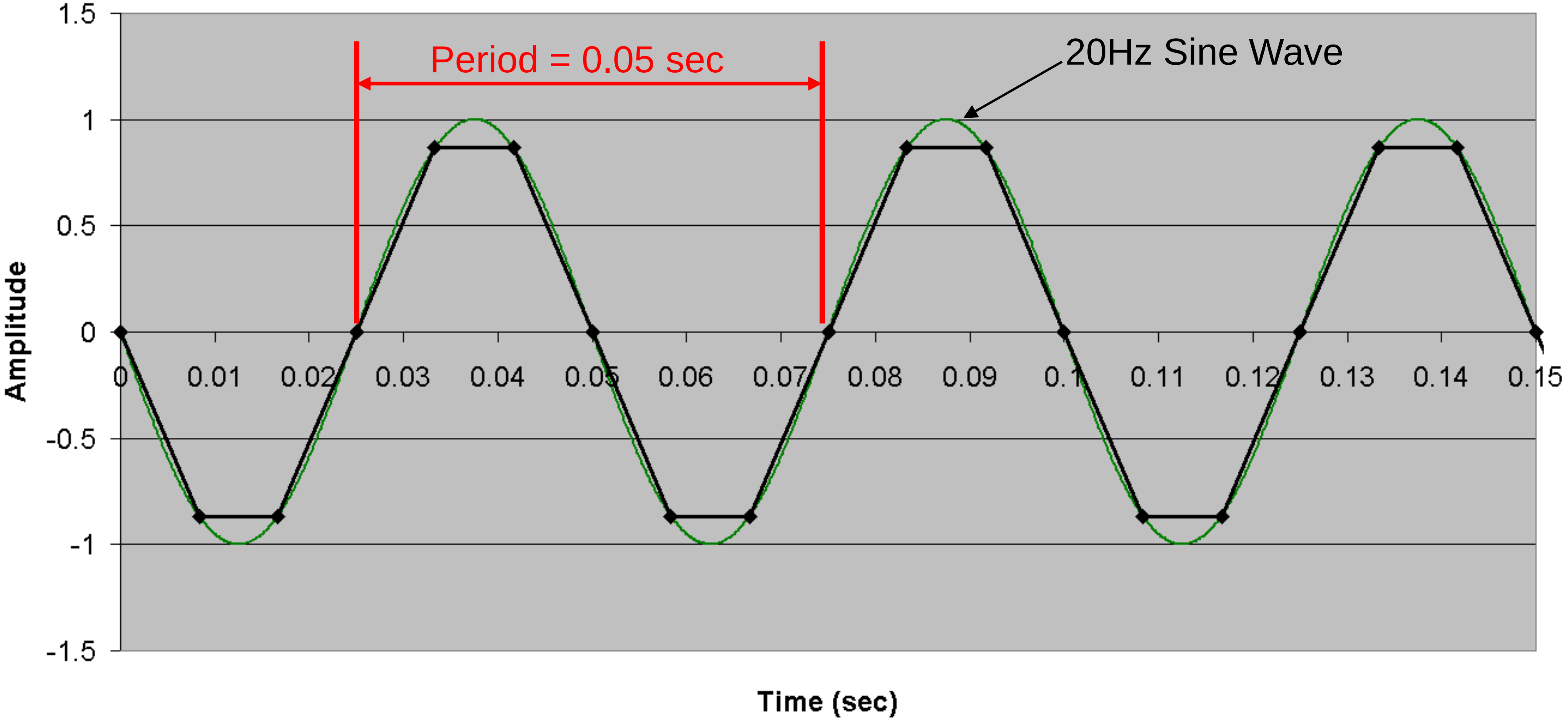
In the time domain, the reconstructed analog signal would look like a 20 Hz sine wave.



$$Frequency = \frac{1}{Period} = \frac{1}{T} = \frac{1}{0.05sec} = 20\ Hz$$

Visualization of how a Frequency is Aliased in the time domain

A flight test engineer analyzing the data would assume that a 20 Hz sine wave was present, when in actuality it was 100 Hz. This could have devastating effects if this was a test where frequencies over 75 Hz were known to cause damage to the aircraft being tested.



Aliasing

- Any frequencies present in our sampled data above the *Nyquist frequency* will fold back into our lower frequency data (below the *Nyquist frequency*) and appear as lower frequencies as shown in the example.
- Aliasing corrupts the signal in a way that cannot be fixed after sampling, it is irreversible!
- Aliased data can appear to be real data, most of the time you won't know that the data is corrupted.
- Aliasing must be dealt with in the analog world, which is why filtering/band-limiting is so important.
- Aliasing does not discriminate, it corrupts digital information of any type (i.e., PCM, video, audio, etc.).

A Classic Aliasing Example

- The wagon wheel is one of the classic examples used to help demonstrate the concept of aliasing and its effects.
- If you've watched a movie or TV show where a wagon wheel appears to being speeding up, slowing down, or going backwards, you have witnessed aliasing.
- At certain speeds, the frame-rate isn't fast enough to reproduce the actual rotational frequency of the wheel.
- Although the wheel is still moving forward, you observe aliasing by thinking that the wheel is going backwards.
- Things aren't always what they seem!!!

Sampling Sine Waves

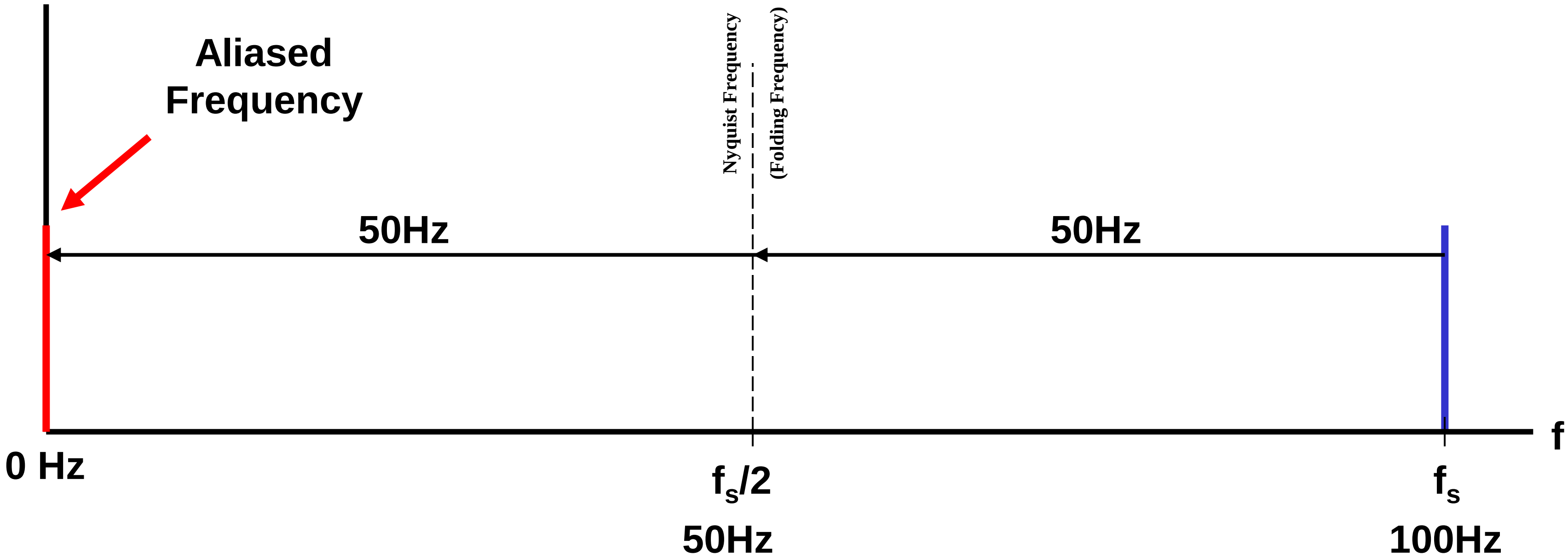
- Sampling sine waves is one of the best ways to demonstrate sampling rate and aliasing effects.
- This is basically what the sampling theorem is based on and is purely *theoretical*, meaning that if you are lucky enough to take 2 samples of a sinewave at the right time, then theoretically you should be able to reasonably reconstruct the original sine wave.
- The majority of the signals we measure are not pure sinusoids, they are more complex and are made up of multiple frequencies.
- This equates to the need for pre-sample filters and higher sampling rates (over sampling) to be able to reconstruct and best represent the original analog signal.

Sampling Sine Waves

- The following slides demonstrate the effect of increasing the sample rate of a pure 100 Hz sinusoid in the time domain and where applicable, the frequency domain.
- The sample rates will start off at 100 sps to illustrate the aliased frequencies.
- As the sample rate increases, the reproduced signal contains more information and approximates the original signal more closely
- The green trace is the original signal and the black traces are used to connect the individual samples with straight lines.

f_s=100 sps
In the Frequency Domain

Signal Freq: 100 Hz	fs: 100 sps Ts=0.01 sec	NF =100sps/100Hz = 1

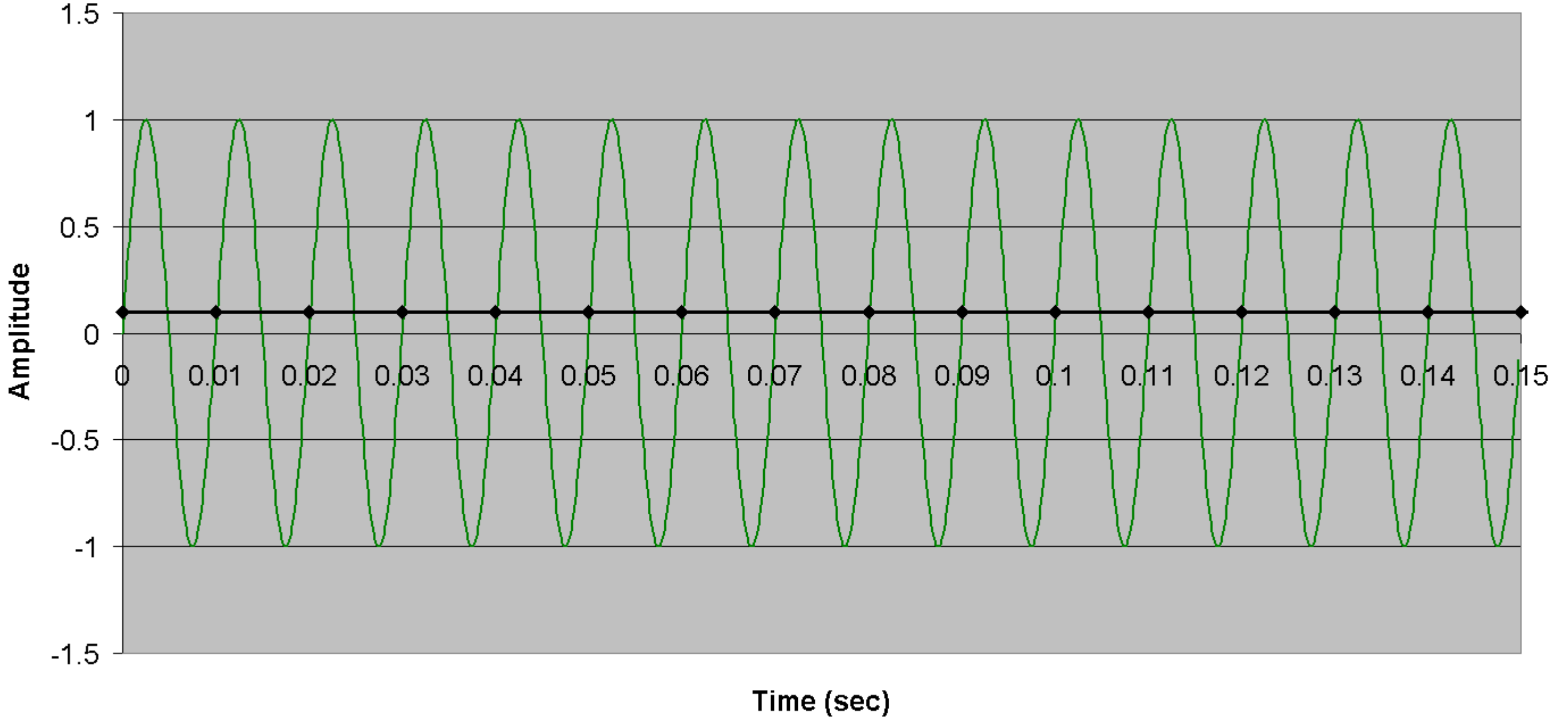


The 100 Hz sine wave will look like a 0 Hz sine wave when sampled at 100 sps. Or in other words, it will look like a DC voltage depending on where the samples happen to catch the sine wave.

Time (sec)

f_s=100 sps
In the Time Domain

Signal Freq: 100 Hz fs: 100 sps Ts=0.01 sec NF =100sps/100Hz = 1

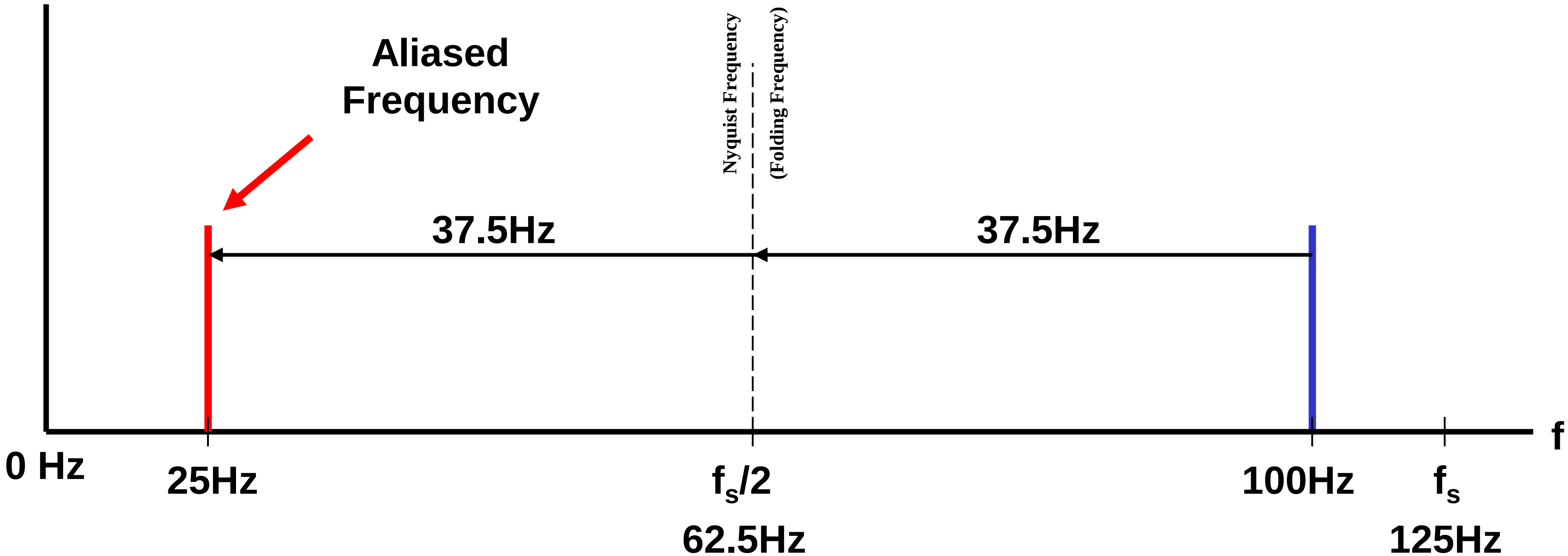


In the time domain, signal has a frequency of 0 Hz. The signal looks like a DC level voltage.

f_s=125 sps

In the Frequency Domain

Signal Freq: 100 Hz fs: 125 sps Ts=8 msec NF =125sps/100Hz = 1.25

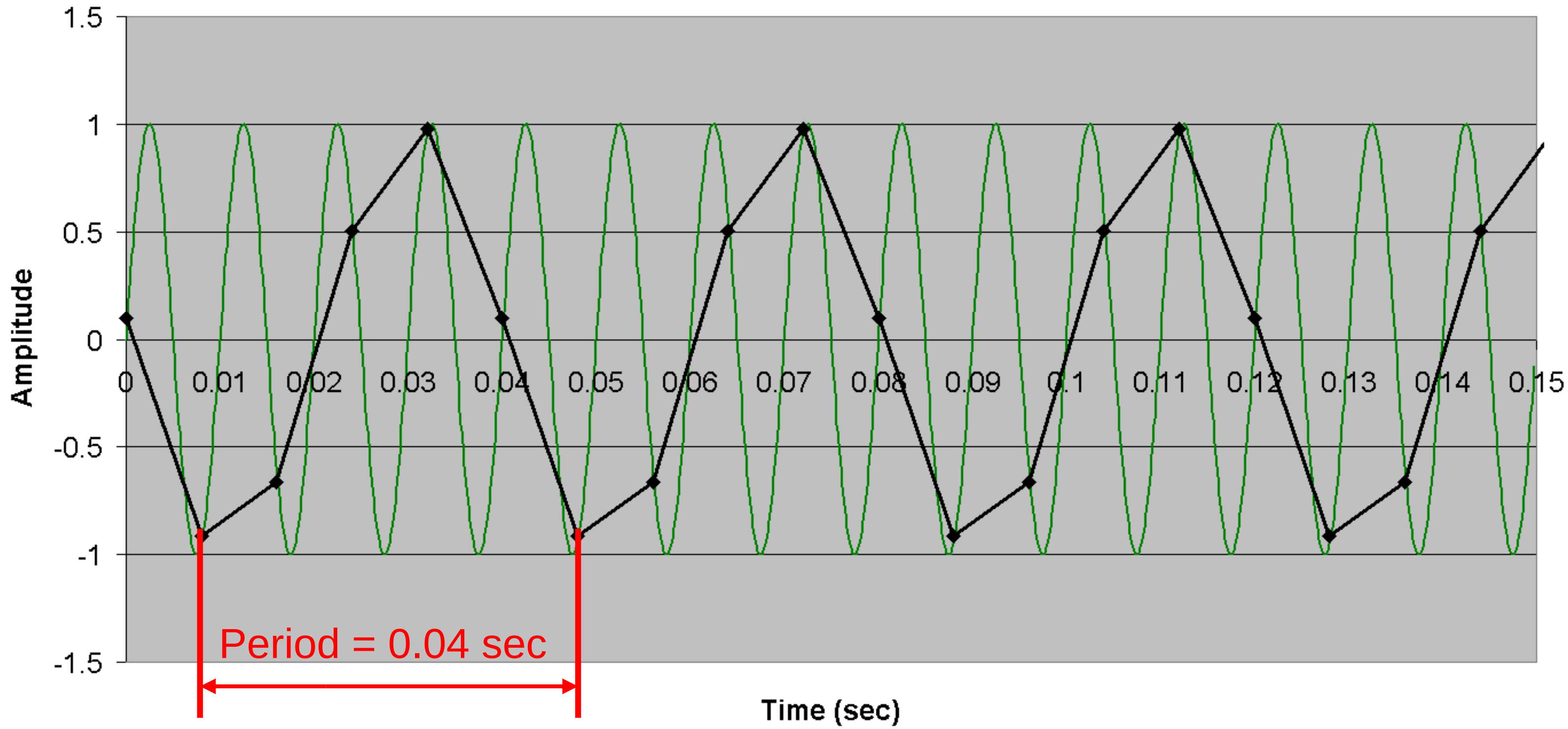


The 100 Hz sine wave will look like a 25 Hz sine wave when sampled at 125 sps.

f_s=125 sps

In the Time Domain

Signal Freq: 100 Hz	fs: 125 sps Ts=8 msec	NF =125sps/100Hz = 1.25



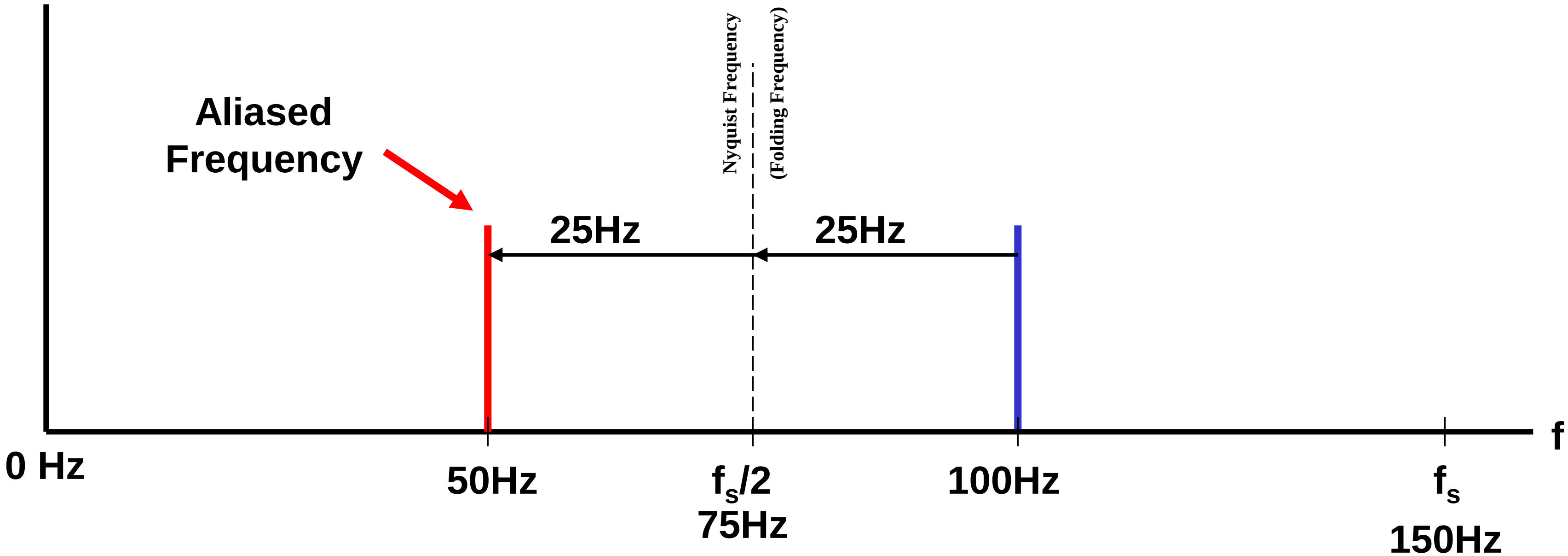
$$Frequency = \frac{1}{Period} = \frac{1}{T} = \frac{1}{0.04sec} = 25\ Hz$$

Measuring the frequency of the reproduced signal, it is determined to have a frequency of 25 Hz.

f_s=150 sps

In the Frequency Domain

Signal Freq: 100 Hz   fs: 150 sps Ts=6.67 msec   NF =150sps/100Hz = 1.5

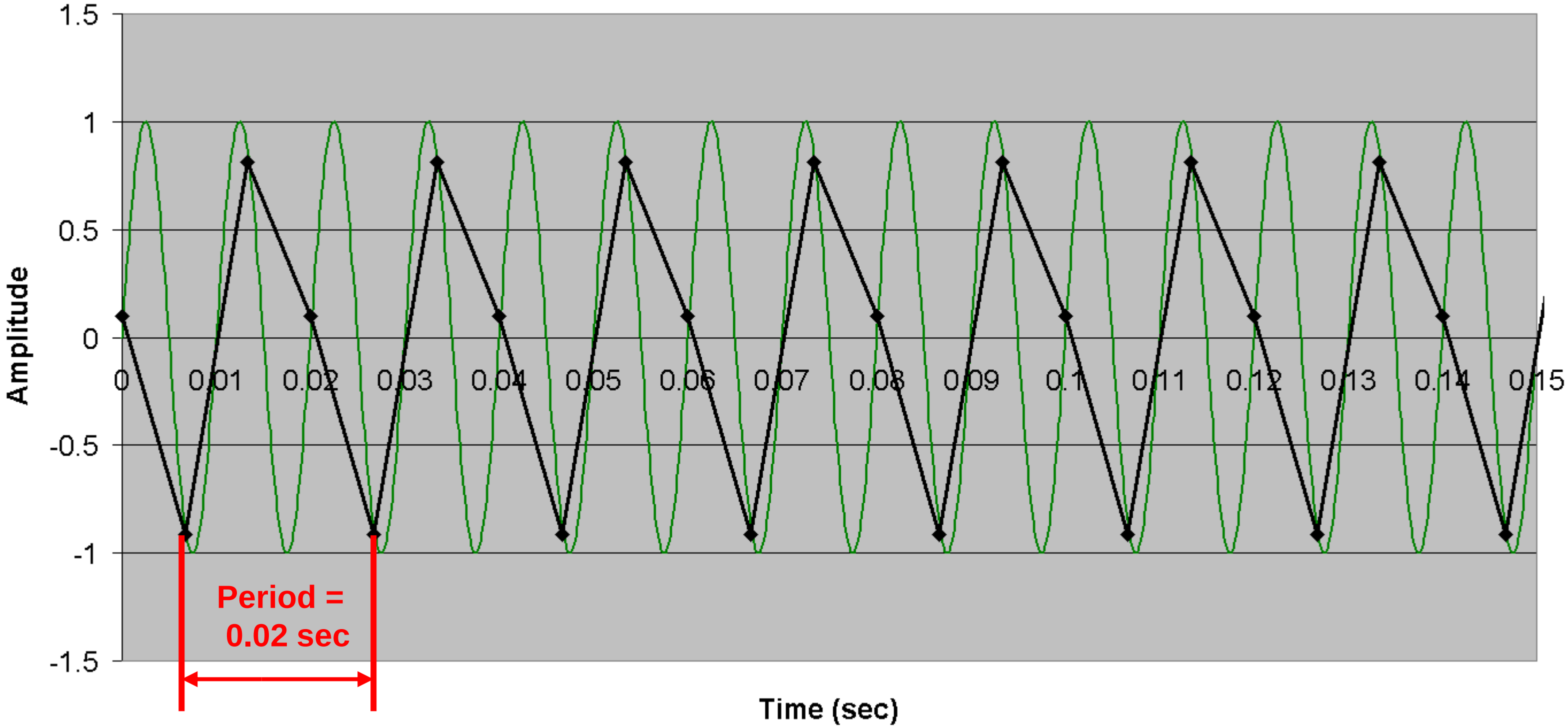


The 100 Hz sine wave will look like a 50 Hz sine wave when sampled at 150 sps.

f_s=150 sps

In the Time Domain

Signal Freq: 100 Hz fs: 150 sps Ts=6.67 msec NF =150sps/100Hz = 1.5



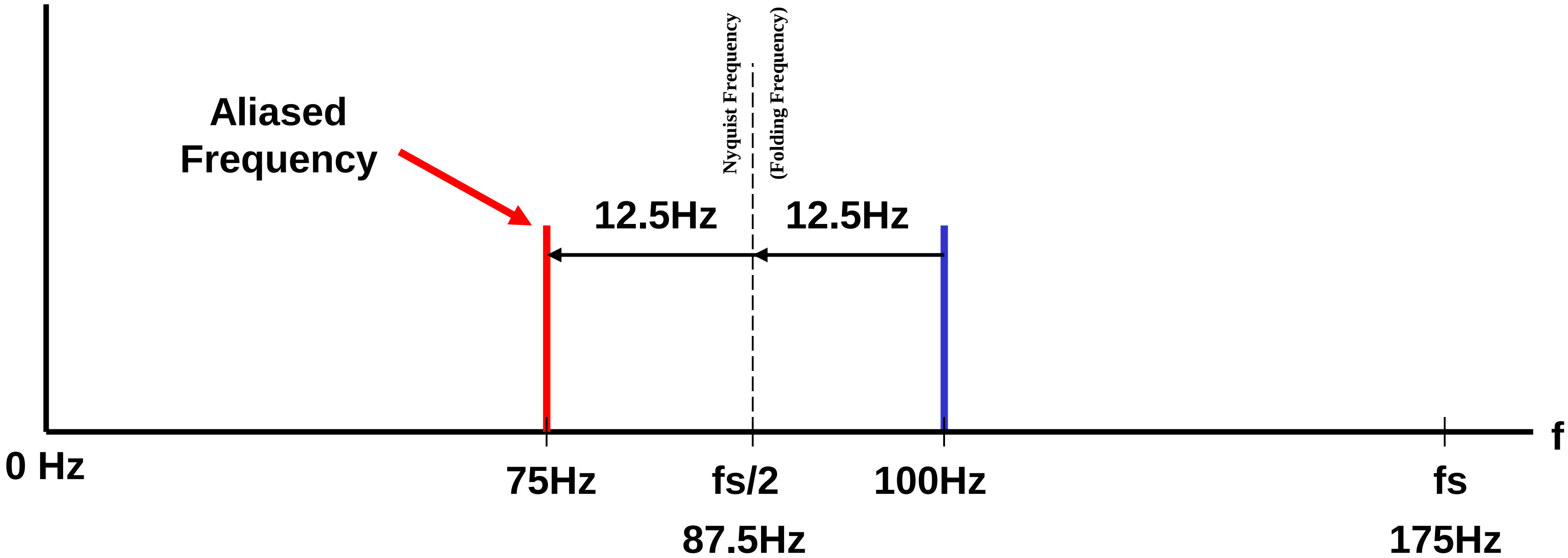
$$Frequency = \frac{1}{Period} = \frac{1}{T} = \frac{1}{0.02sec} = 50\ Hz$$

Measuring the frequency of the reproduced signal, it is determined to have a frequency of 50 Hz.

f_s=175 sps

In the Frequency Domain

Signal Freq: 100 Hz	fs: 175 sps	Ts=5.71 msec	NF =175sps/100Hz = 1.75

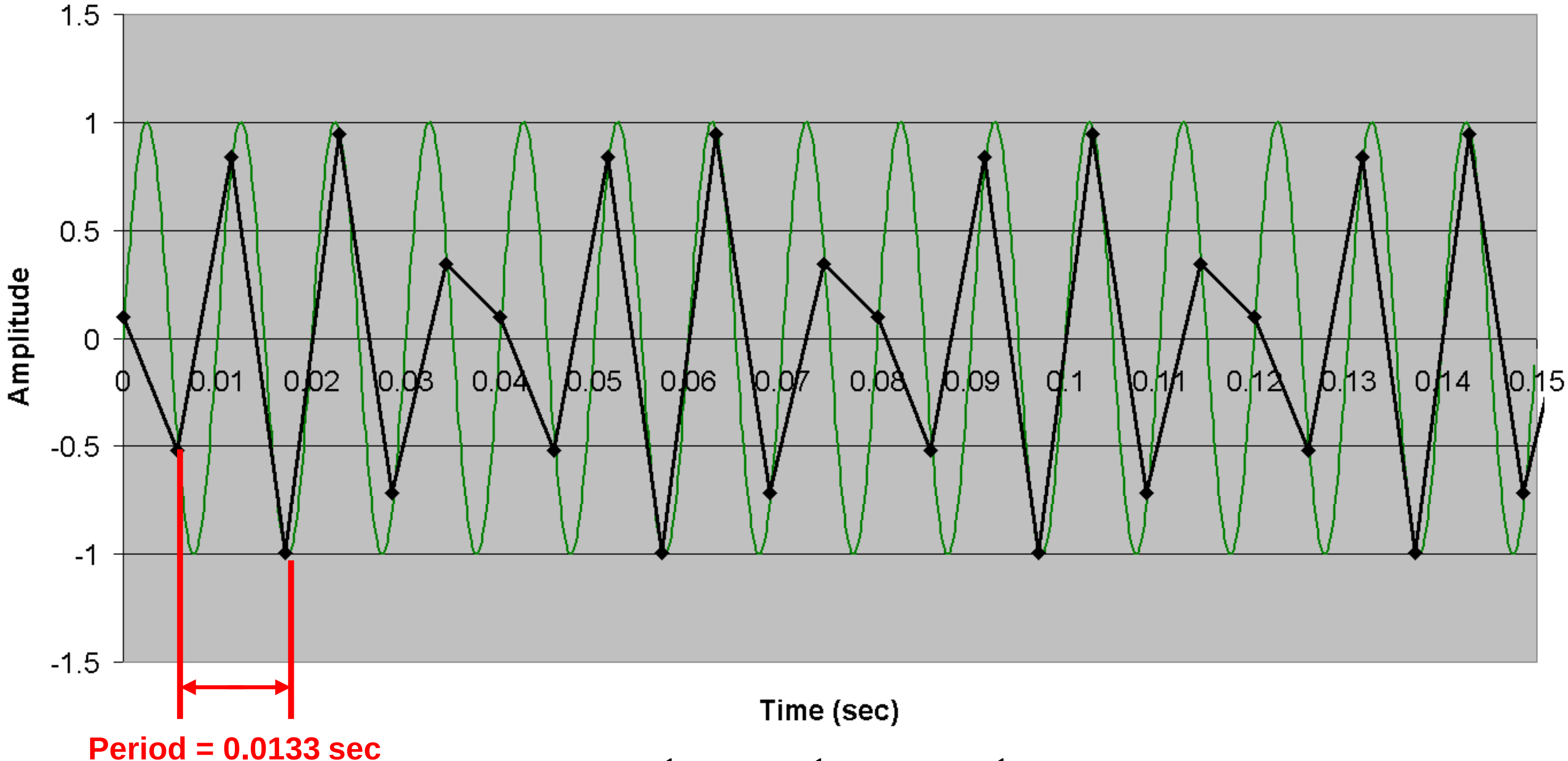


The 100 Hz sine wave will look like a 75 Hz sine wave when sampled at 175 sps.

f_s=175 sps

In the Time Domain

Signal Freq: 100 Hz fs: 175 sps Ts=5.71 msec NF =175sps/100Hz = 1.75



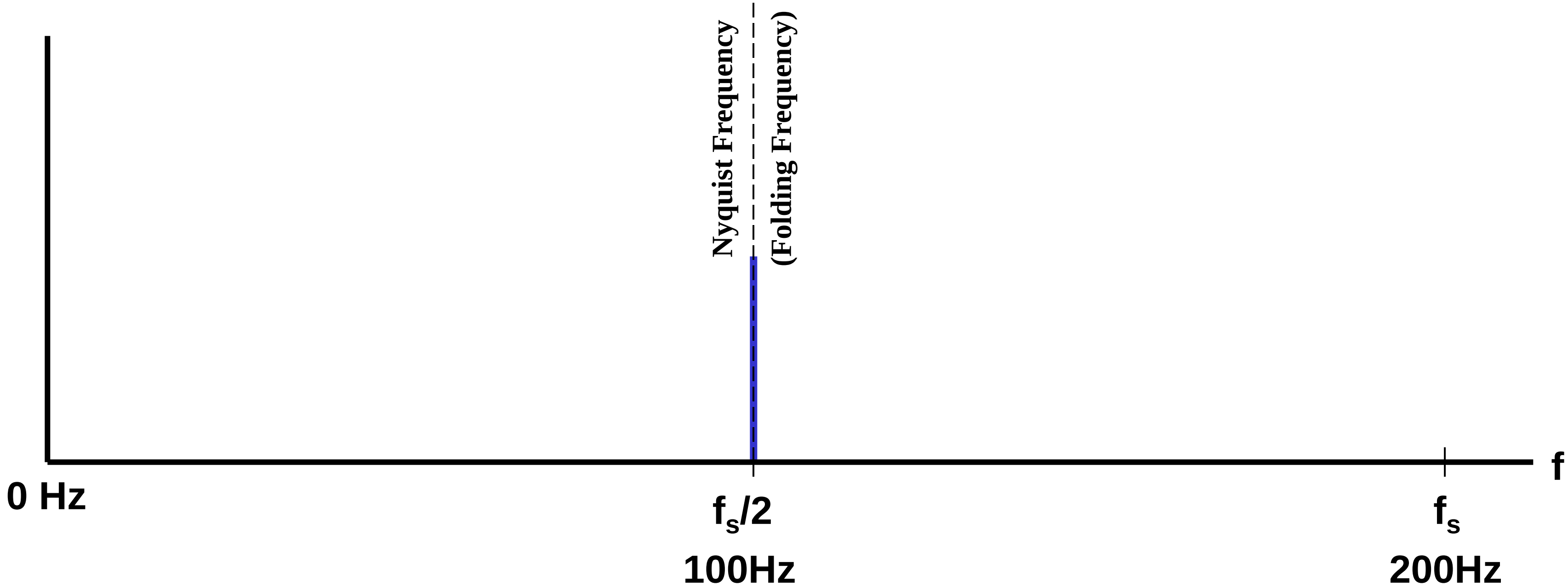
$$Frequency = \frac{1}{Period} = \frac{1}{T} = \frac{1}{0.0133sec} = 75\ Hz$$

Measuring the frequency of the reproduced signal, it is determined to have a frequency of 75 Hz.

f_s=200 sps

In the Frequency Domain

Signal Freq: 100 Hz	fs: 200 sps Ts=0.05 sec	NF =200sps/100Hz = 2

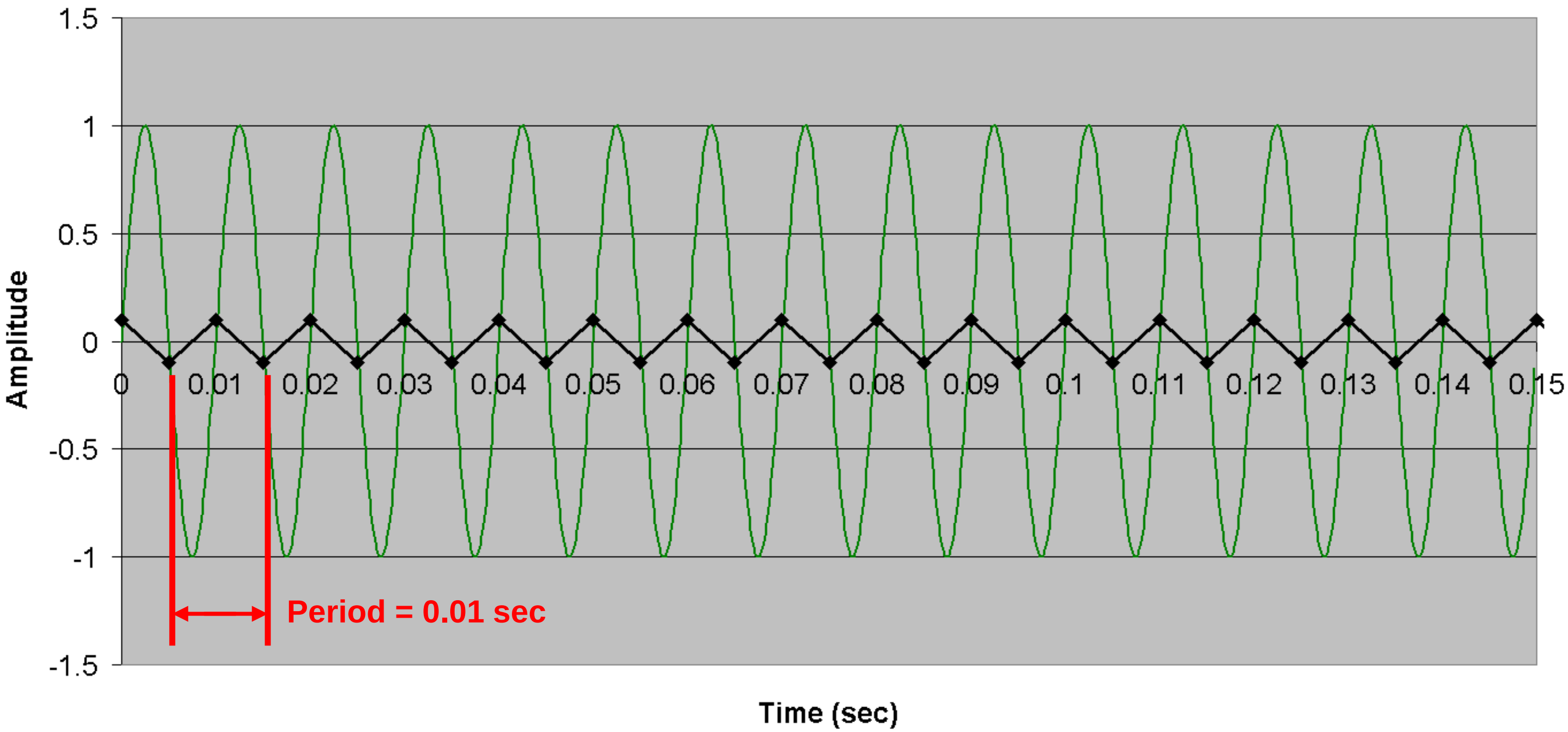


Two samples per period of a 100 Hz sine wave would yield a sample rate of 200 sps. The Nyquist Frequency will be half that or 100 Hz. The sine wave's frequency falls right on the Nyquist frequency, so theoretically you will be able to detect the frequency correctly.

f_s=200 sps

In the Time Domain

Signal Freq: 100 Hz	fs: 200 sps Ts=0.05 sec	NF =200sps/100Hz = 2



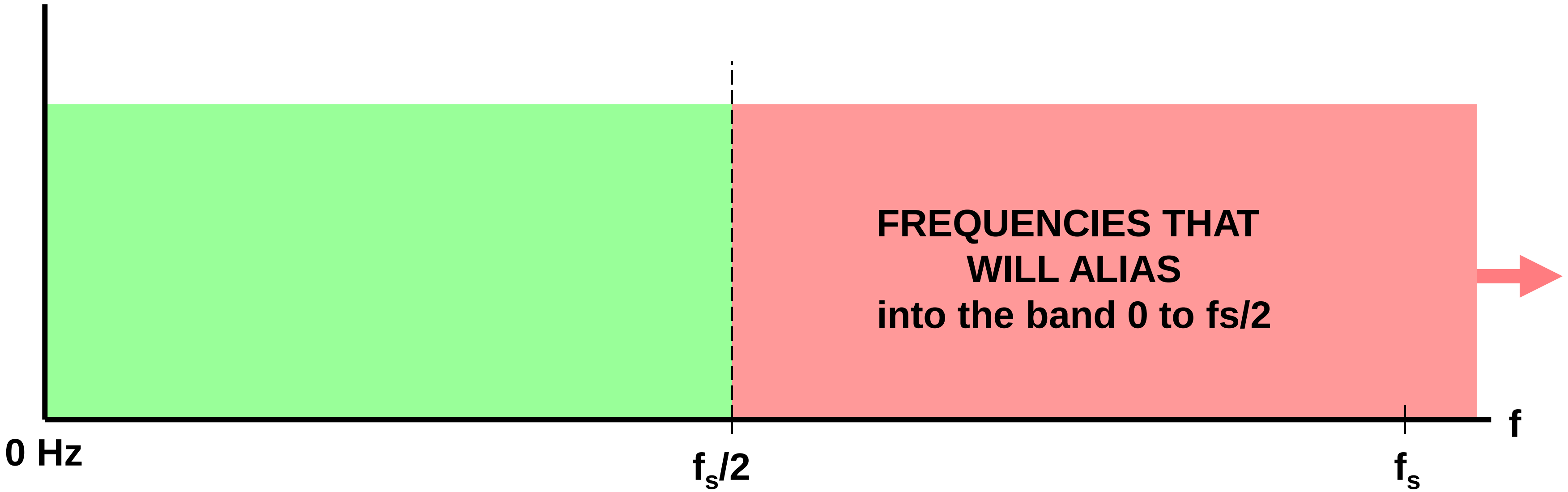
$$Frequency = \frac{1}{Period} = \frac{1}{T} = \frac{1}{0.01sec} = 100\ Hz$$

In the time domain, the frequency of the signal is correctly replicated. The Sampling Theory is mainly concerned with avoiding aliasing at the minimum rate of $f_s = 2f_n$.

Sampling to Avoid Aliasing in the real world

- Theoretically, to avoid aliasing, a sample rate of $f_s \geq 2f_n$ is needed for a signal band limited to a frequency of f_n.
- To band limit the signal, it is filtered such that unwanted frequency components of the signal are attenuated and no longer contribute to the signal.
- A frequency component is said to not contribute to a signal when it is <u>below the threshold of toggling the LSB weighting of an ADC</u> (analog to digital converter) within the band from 0 to f_c Hz (where f_c is the cutoff of the filter).
- The following slides will illustrate how the number of bits in the encoder, and the number of poles in the filter effects the sample rate needed to avoid aliasing.

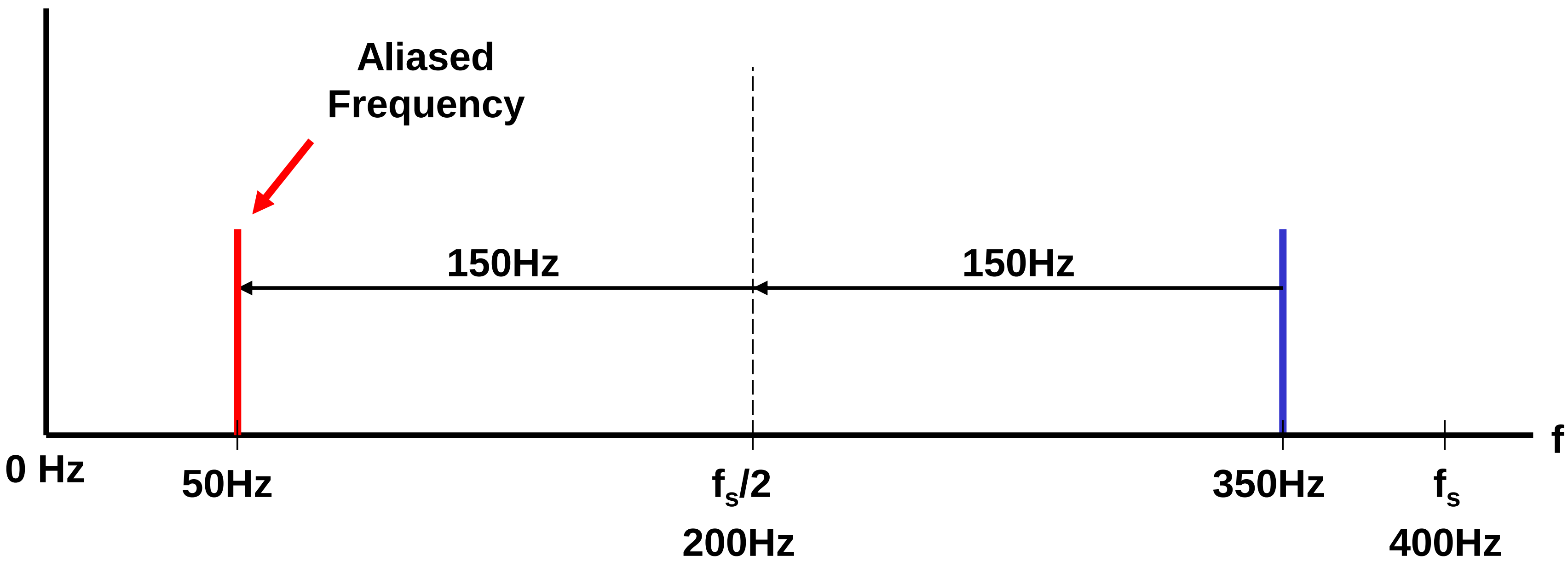
Illustration of Avoiding Aliasing by Pre Sample Filtering



For some sample rate f_s, frequencies greater than $f_s/2$ Hz will appear (alias) as frequencies between 0 and $f_s/2$ Hz.

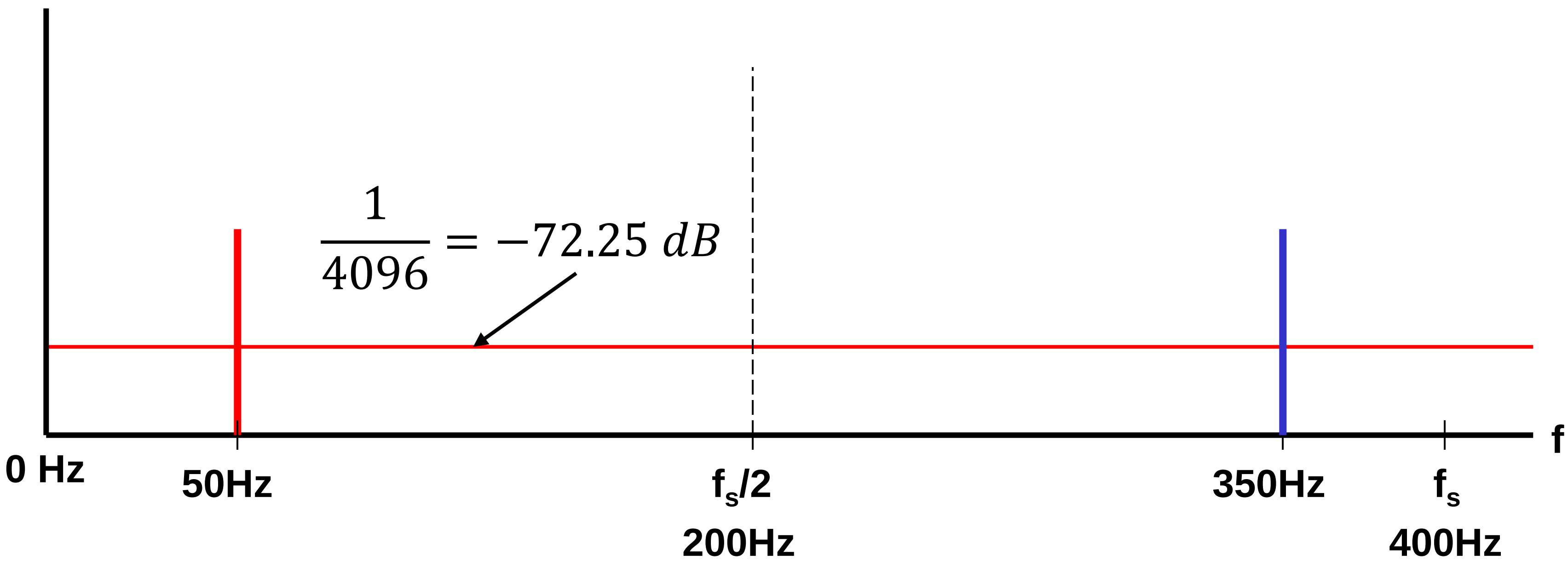
Sometimes it is said that the frequencies above $f_s/2$ will "fold" about $f_s/2$ into the band of frequencies below $f_s/2$.

Illustration of Avoiding Aliasing by Pre Sample Filtering



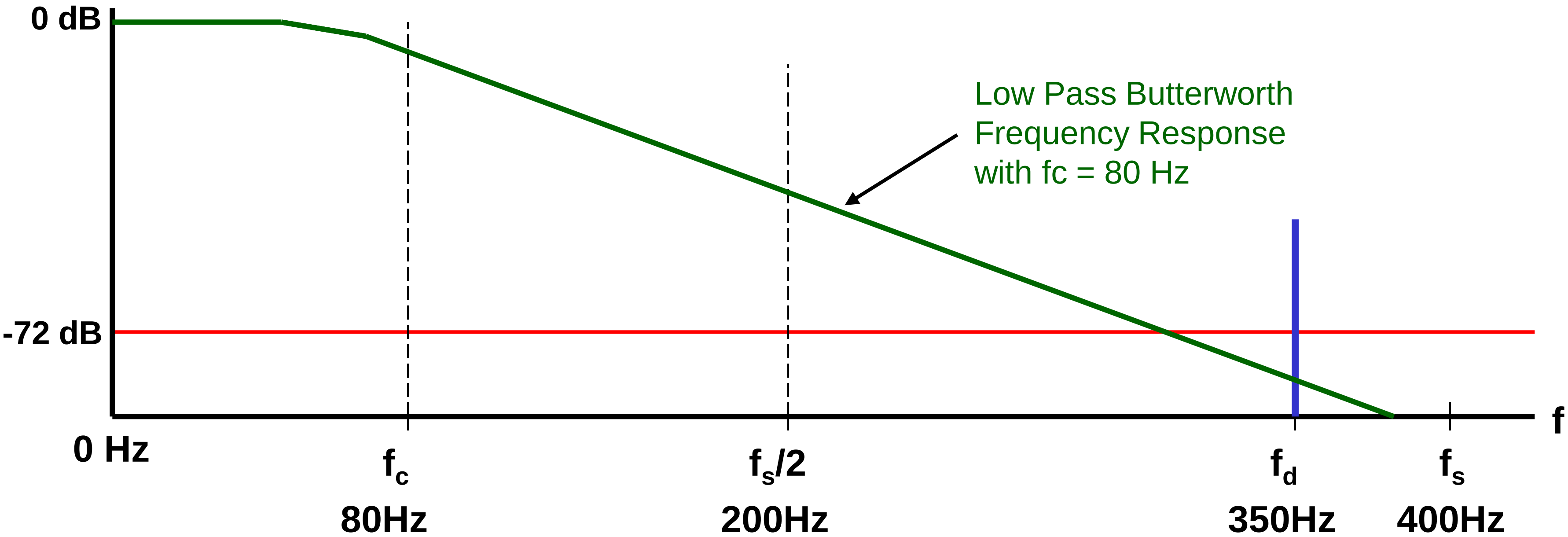
If f_s = 400 sps (Hz), and if a frequency component of a signal exists that is at 350 Hz, the 350 Hz component will fold about the Nyquist frequency of $f_s/2$ = 200 Hz. The 350 Hz component will appear as a frequency component at 50 Hz (the alias of 350 Hz) when the data is reconstructed.

Illustration of Avoiding Aliasing by Pre Sample Filtering



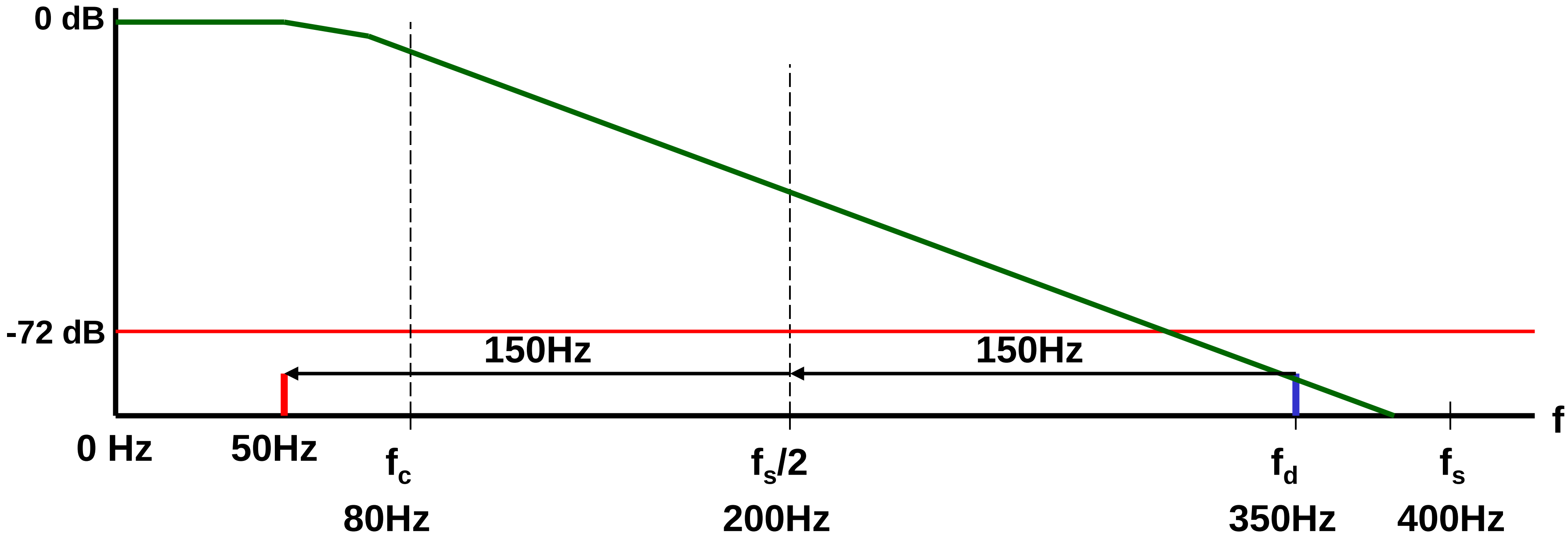
If the 350 Hz component can be attenuated (which will in theory attenuate the aliased 50 Hz component in the data) such that it cannot be detected by the ADC, then the aliasing issue will "go away". For a 12-bit system, the lowest threshold in the quantizer which will toggle the LSB is 1/4096 of the full scale input (or -72.25 dB since filter gains are described in dB).

Illustration of Avoiding Aliasing by Pre Sample Filtering



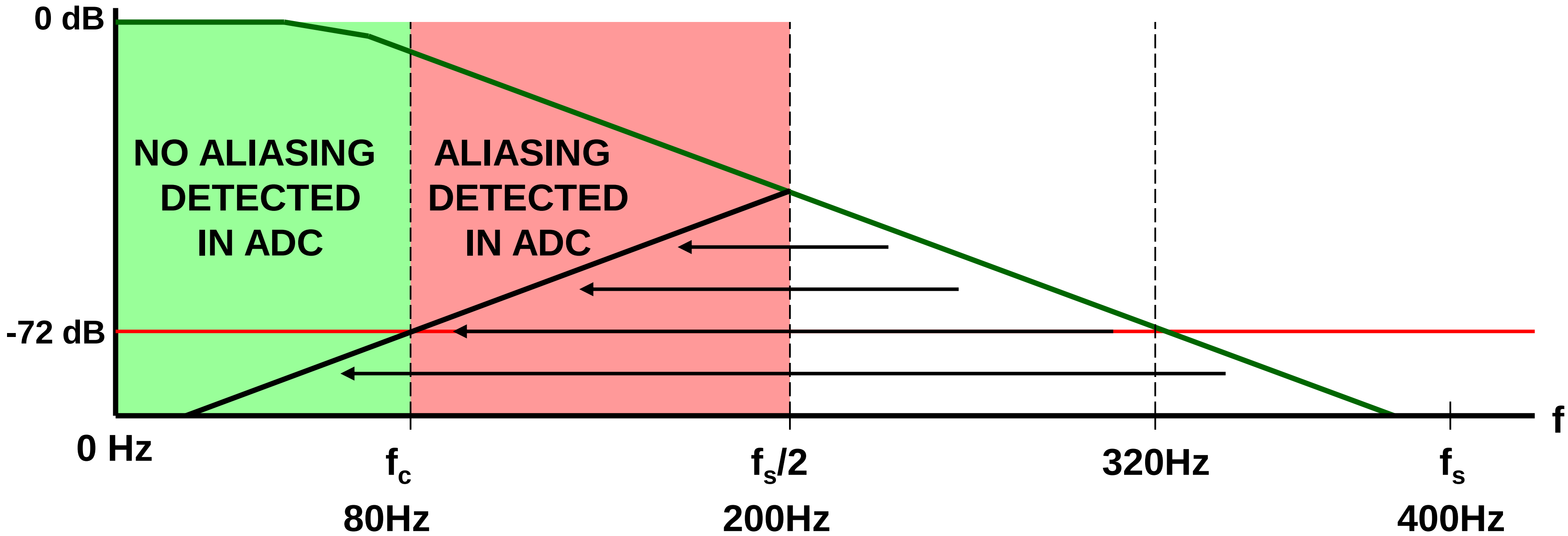
The best way to accomplish this is to use a low pass Butterworth filter to attenuate frequencies which will alias as frequencies in the band from 0 to f_c Hz.

Illustration of Avoiding Aliasing by Pre Sample Filtering



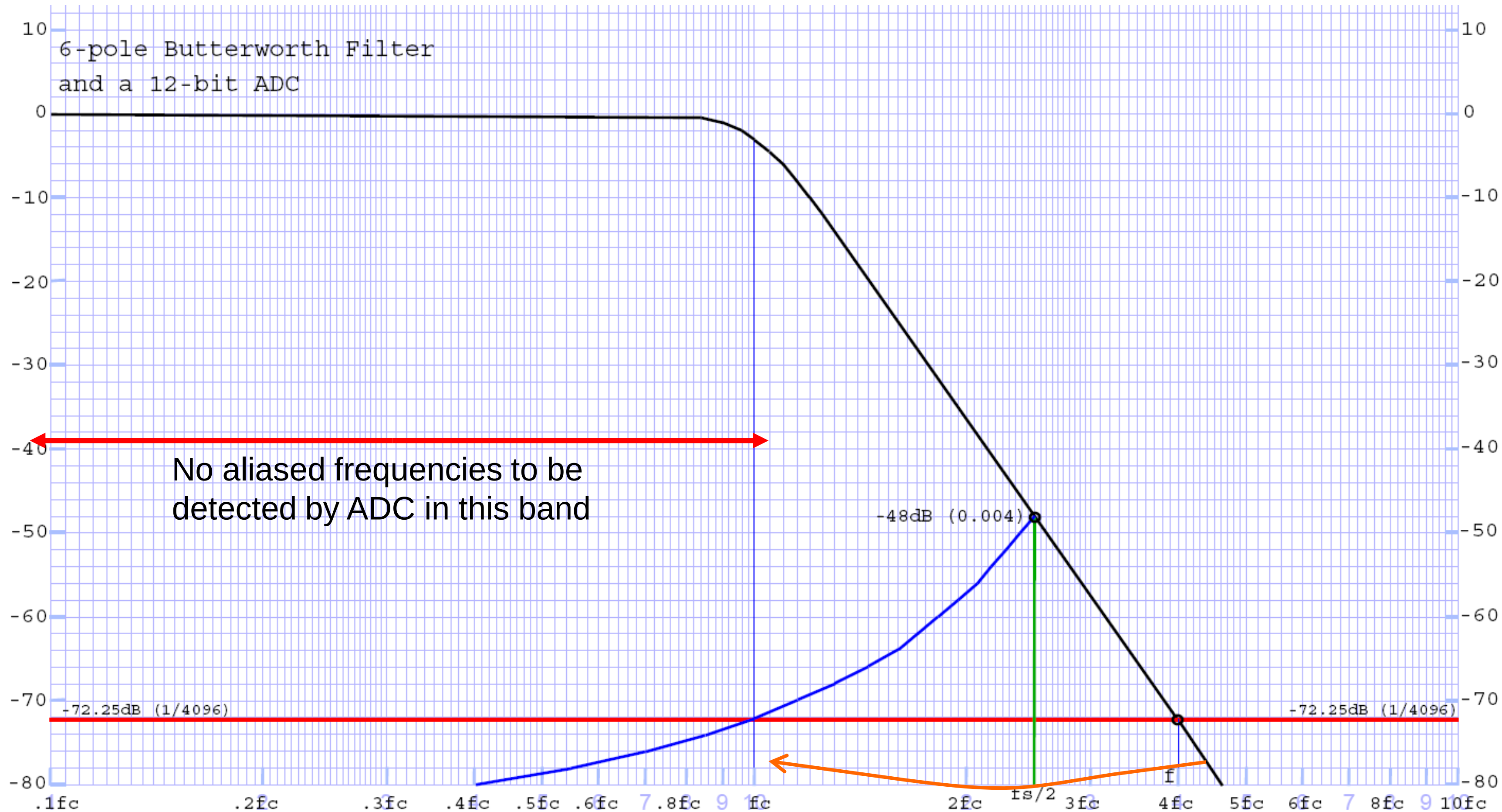
After filtering, the attenuated frequency component at 350 Hz (and therefore the aliased frequency at 50 Hz) is less than -72.25 dB and will not be detected by the ADC.

Illustration of Avoiding Aliasing by Pre Sample Filtering



In fact, all frequency components above 320 Hz will be below -72.25 dB. When folded about the Nyquist frequency of $f_s/2$ = 200 Hz, they will not be detected by the ADC in the frequency band of 0 to f_c = 80 Hz.

Sampling a 12-bit ADC channel with a 6-pole Butterworth Filter to Avoid Aliasing



According to the Nyquist theory, frequencies will fold about half the sampling frequency, $\frac{1}{2}f_s$. For the frequency f to alias as f_c or greater, $\frac{1}{2}f_s$ must be <u>at least</u> the midpoint of f_c and $f=4f_c$. Looking at the x-axis, that midpoint is $2.5f_c$.

Sampling a 12-bit ADC channel with a 6-pole Butterworth Filter to Avoid Aliasing

Solving for the sampling frequency, fs…

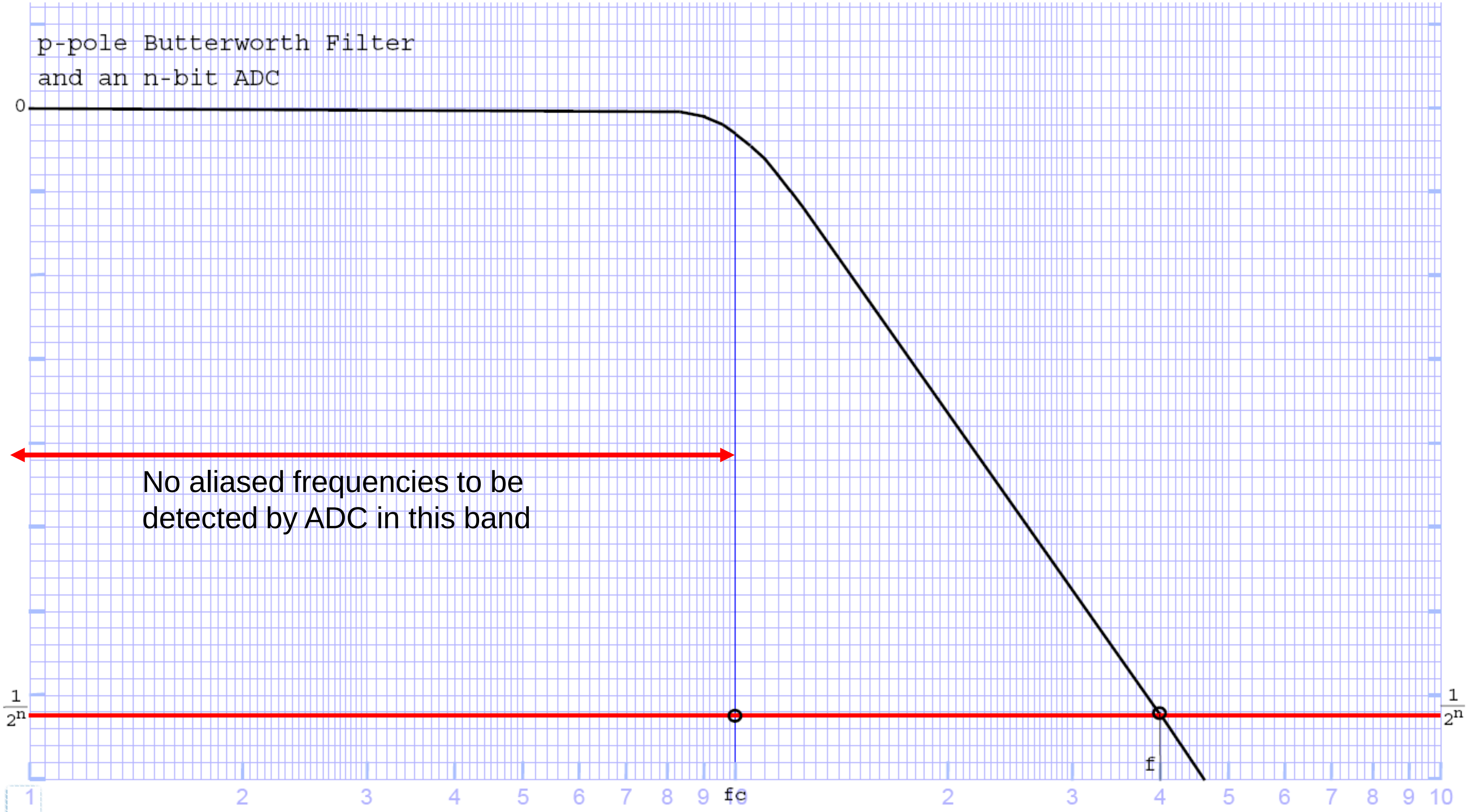
$$\frac{f_s}{2} \geq 2.5 f_c$$

$$\boxed{f_s \geq 5 f_c}$$

This leads to the sampling rule that you must sample at least five times the cutoff frequency for a channel with a 6-pole Butterworth filter and a 12-bit ADC.

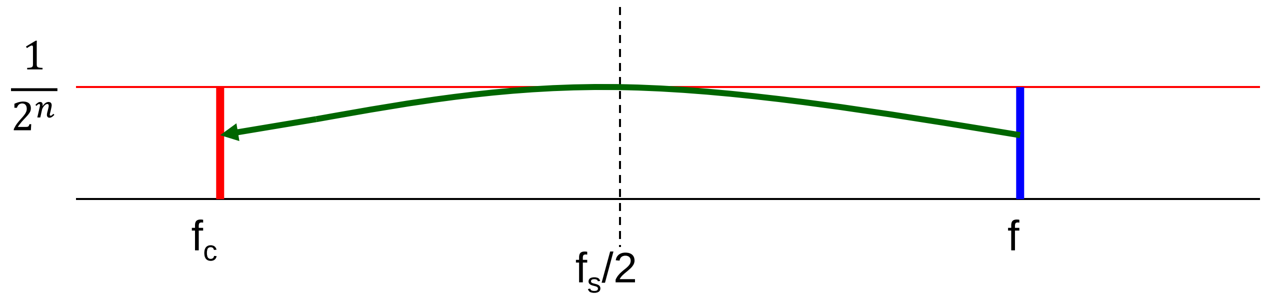
What would be the sampling rule for a p-pole Butterworth filter and an n-bit ADC?

Sampling Requirements for a p-pole Butterworth Filter and an n-bit ADC



Again we define corrupted data as an aliased frequency in the band from 0 to f_c Hz being detected by the ADC. The thresh hold of detection for an n-bit ADC, is $1/2^n$.

Sampling Requirements for a p-pole Butterworth Filter and an n-bit ADC



A signal at some frequency f will alias as a signal at frequency f_c. It is desired that the filter attenuate that signal by at least $20\log\left[\frac{1}{2^n}\right]$ dB in order to not be detected by the n-bit ADC.

The gain of the filter at frequency f is: $G_{db} = 20\log\left[\frac{1}{\sqrt{1+(f/f_c)^{2p}}}\right]$

We will solve for f in terms of fc by equating the above two gain equations.

$$20\log\left[\frac{1}{\sqrt{1+(f/f_c)^{2p}}}\right] = 20\log\left[\frac{1}{2^n}\right]$$

Sampling Requirements for a p-pole Butterworth Filter and an n-bit ADC

$$\frac{1}{\sqrt{1+(f/f_c)^{2p}}} = \frac{1}{2^n}$$

$$\sqrt{1+(f/f_c)^{2p}} = 2^n$$

$$1+(f/f_c)^{2p} = 2^{2n}$$

$$(f/f_c)^{2p} = 2^{2n}-1$$

$$f/f_c = (2^{2n}-1)^{\frac{1}{2p}}$$

$$f = (2^{2n}-1)^{\frac{1}{2p}} f_c$$

Because n is normally 10 or greater
$2^{2n} >> 1$
so the 1 can be ignored.

$$f = (2^{2n})^{\frac{1}{2p}} f_c$$

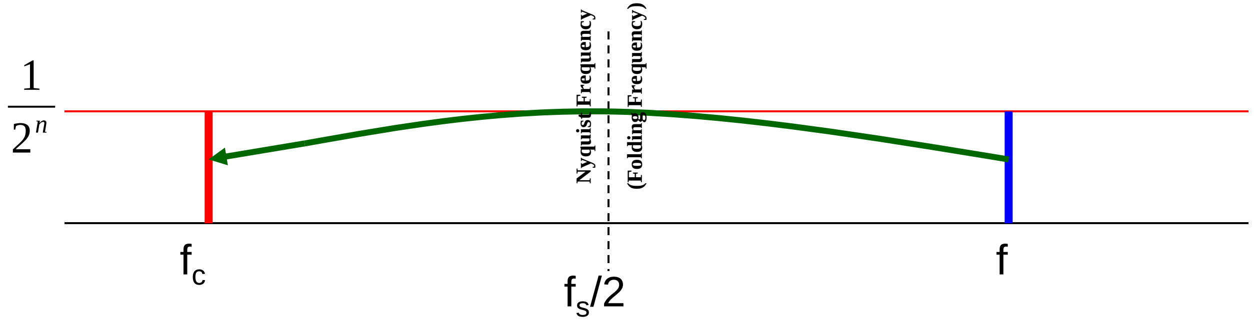
$$f = 2^{\frac{2n}{2p}} f_c$$

$$f = 2^{\frac{n}{p}} f_c$$

but we still need to find f_s

Sampling Requirements for a p-pole Butterworth Filter and an n-bit ADC

For a signal of frequency f to alias as a signal at frequency fc or greater, $f_s/2$ must be <u>at least</u> midway between f and f_c.



$f_s/2$ must be at least the midpoint of f and fc

This statement in equation form is:

$$\frac{f_s}{2} \geq \frac{f + f_c}{2}$$

$$f_s \geq f + f_c \quad \text{and} \quad f = 2^{\frac{n}{p}} f_c \quad \text{derived earlier}$$

$$f_s \geq 2^{\frac{n}{p}} f_c + f_c$$

$$f_s \geq \left(2^{\frac{n}{p}} + 1\right) f_c$$

Sampling Requirements for a p-pole Butterworth Filter and an n-bit ADC

For a p-pole Butterworth filter and an n-bit ADC, the sample rate needed to avoid aliasing in the band from 0 to fc Hz is:

$$\boxed{f_s \geq \left(2^{n/p} + 1\right) f_c}$$

Where, fs is the sample rate
n is the number of bits in the ADC
p is the number of poles in the Butterworth filter
fc is the cutoff frequency of the Butterworth filter

Example: minimum sampling rate for a 4-pole Butterworth and 12-bit ADC

$$f_S \geq \left(2^{12/4} + 1\right) f_C$$
$$f_S \geq (2^3 + 1) f_C$$
$$f_S \geq (8 + 1) f_C$$
$$f_S \geq 9 f_C$$

The Minimum Sampling Rate to Avoid Aliasing

- In the past, the minimum sampling rate was 7 times the highest frequency of interest. With our programmable pre-sample filters with discrete selections for the cutoff frequency, this is no longer valid.
- Now, the minimum sampling rate is calculated by
  - first calculating the appropriate filter cutoff frequency.
  - Then select the next highest cutoff frequency in the signal conditioning.
  - The minimum sampling rate to avoid aliasing will be

$$f_S \geq \left(2^{n/p} + 1\right) f_C$$

The Minimum Sampling Rate to Avoid Aliasing

- For a measurement with a highest frequency of interest of 500 Hz, using a 6-pole filter and a 12-bit ADC in the data acquisition system.
- f_c = 1.3837 x 500 Hz = 691.85 Hz
- The next highest selection for the SCD card being used is f_c =728.29 Hz
- The minimum sampling rate is then

  f_{sMIN}= 5 x 728.29 Hz = 3641.45 sps

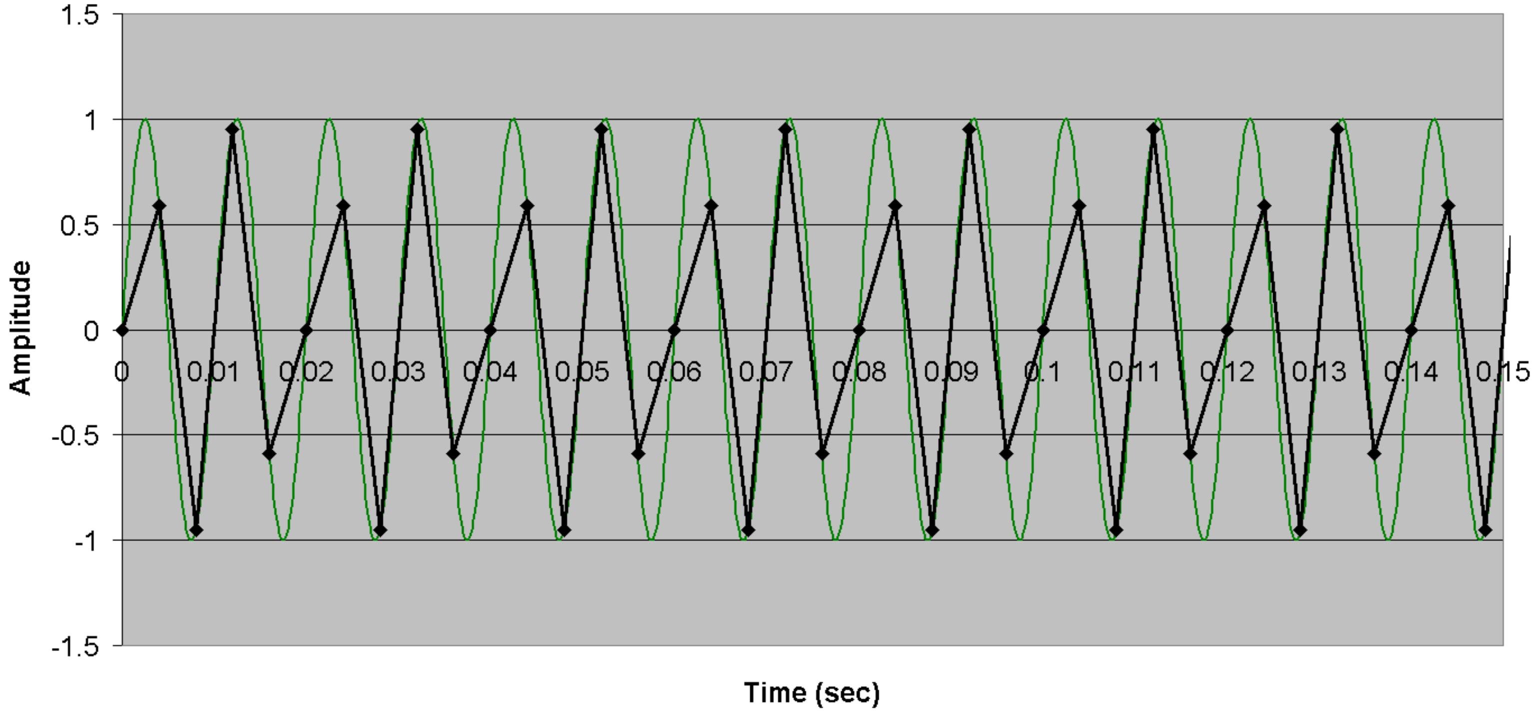
Signal Replication and Peak Detection

- The previous slides all dealt with sampling sufficiently to avoid aliasing. However we also want to accurately replicate the signal and capture the peaks.
- We will next illustrate how increasing the sample rate past $2f_n$ will better replicate the shape and amplitude of a 100 Hz sine wave.
- We will not illustrate the signal in the frequency domain because we are sampling enough not to have aliased frequencies.

f_s=250 sps

In the Time Domain

Signal Freq: 100 Hz	fs: 250 sps Ts=4 msec	NF =250sps/100Hz = 2.5

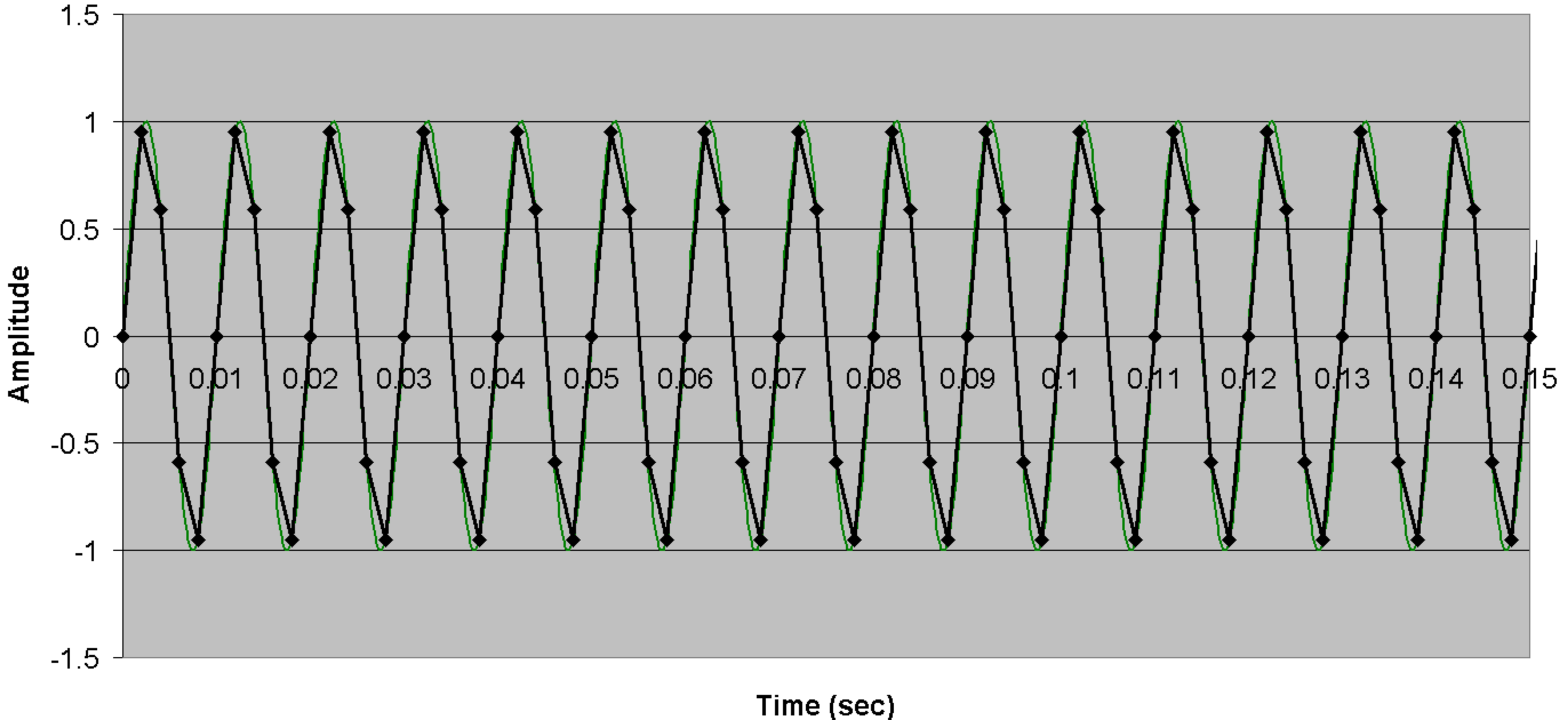


You can see the distortion of the amplitude, but the frequency content of the signal is preserved.

f_s=500 sps

In the Time Domain

Signal Freq: 100 Hz fs: 500 sps Ts=2 msec NF =500sps/100Hz = 5

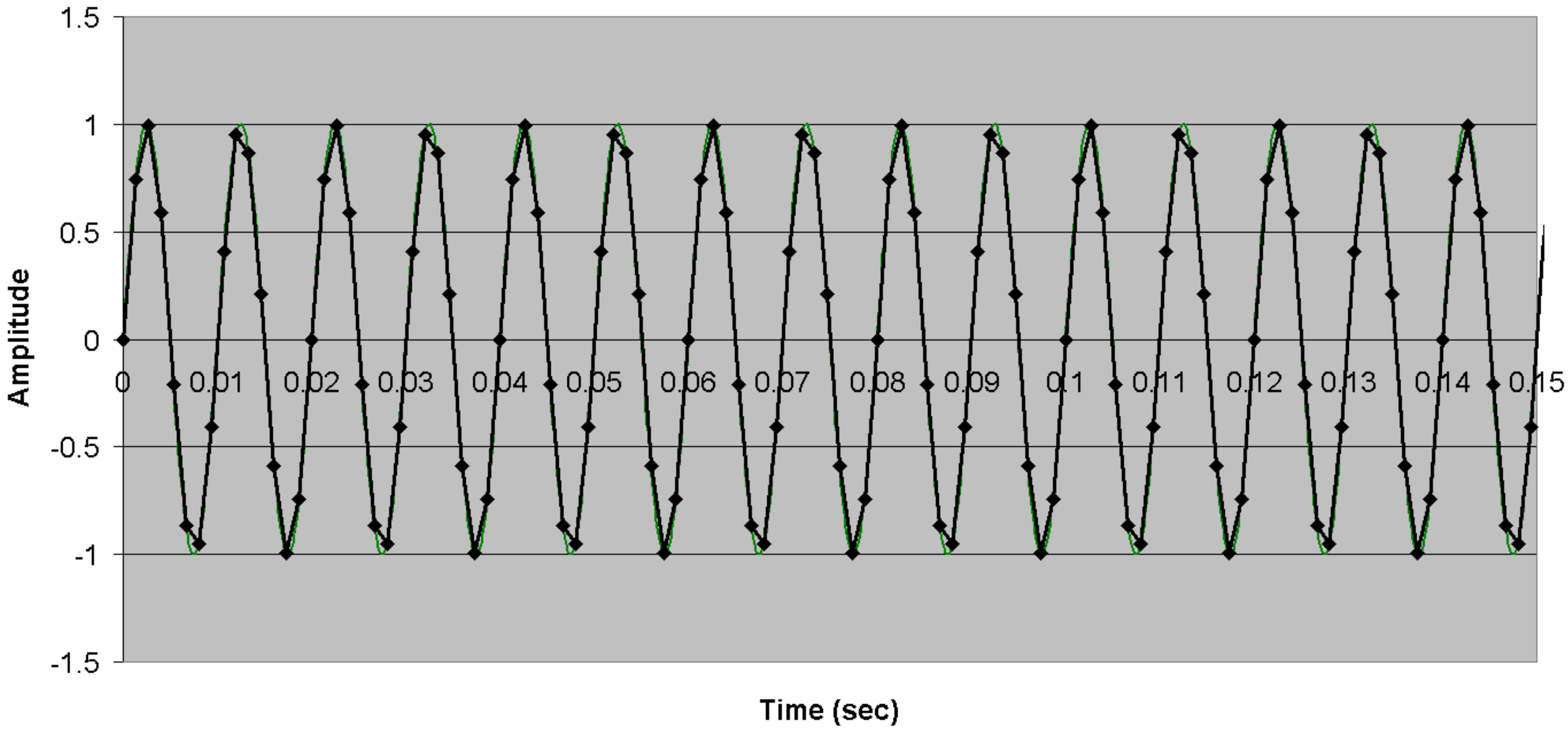


The replication of the signal is more improved as the Nyquist Factor increases to 5.

f_s=750 sps

In the Time Domain

Signal Freq: 100 Hz fs: 750 sps Ts=1.33 msec NF =750sps/100Hz = 7.5

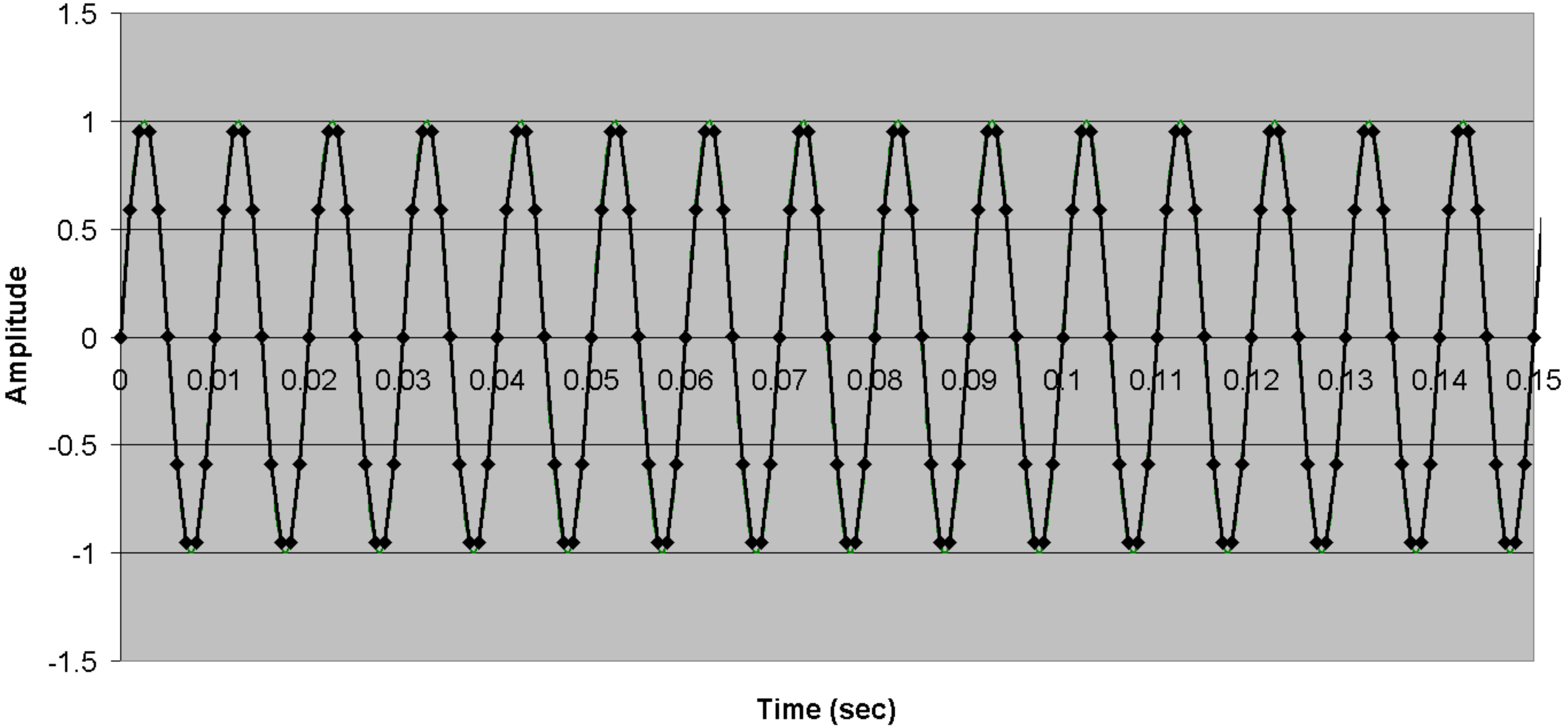


Even more improvement, but some of the peaks are being missed.

f_s=1000 sps

In the Time Domain

Signal Freq: 100 Hz	fs: 1000 sps Ts=1 msec	NF =1000sps/100Hz = 10

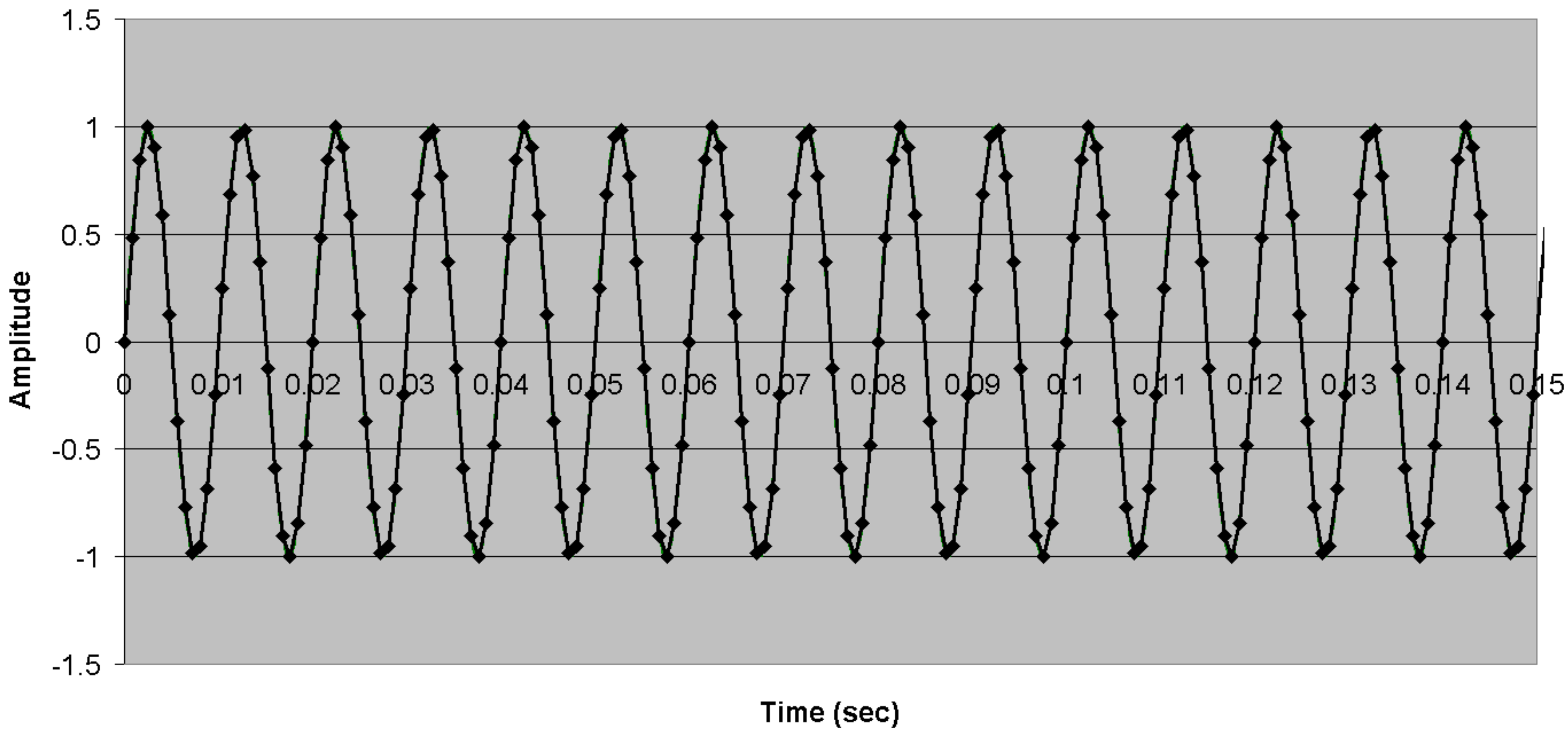


The peaks are being captured better, but not perfectly.

f_s=1250 sps

In the Time Domain

Signal Freq: 100 Hz   fs: 1250 sps Ts=0.8 msec   NF =1250sps/100Hz = 12.5



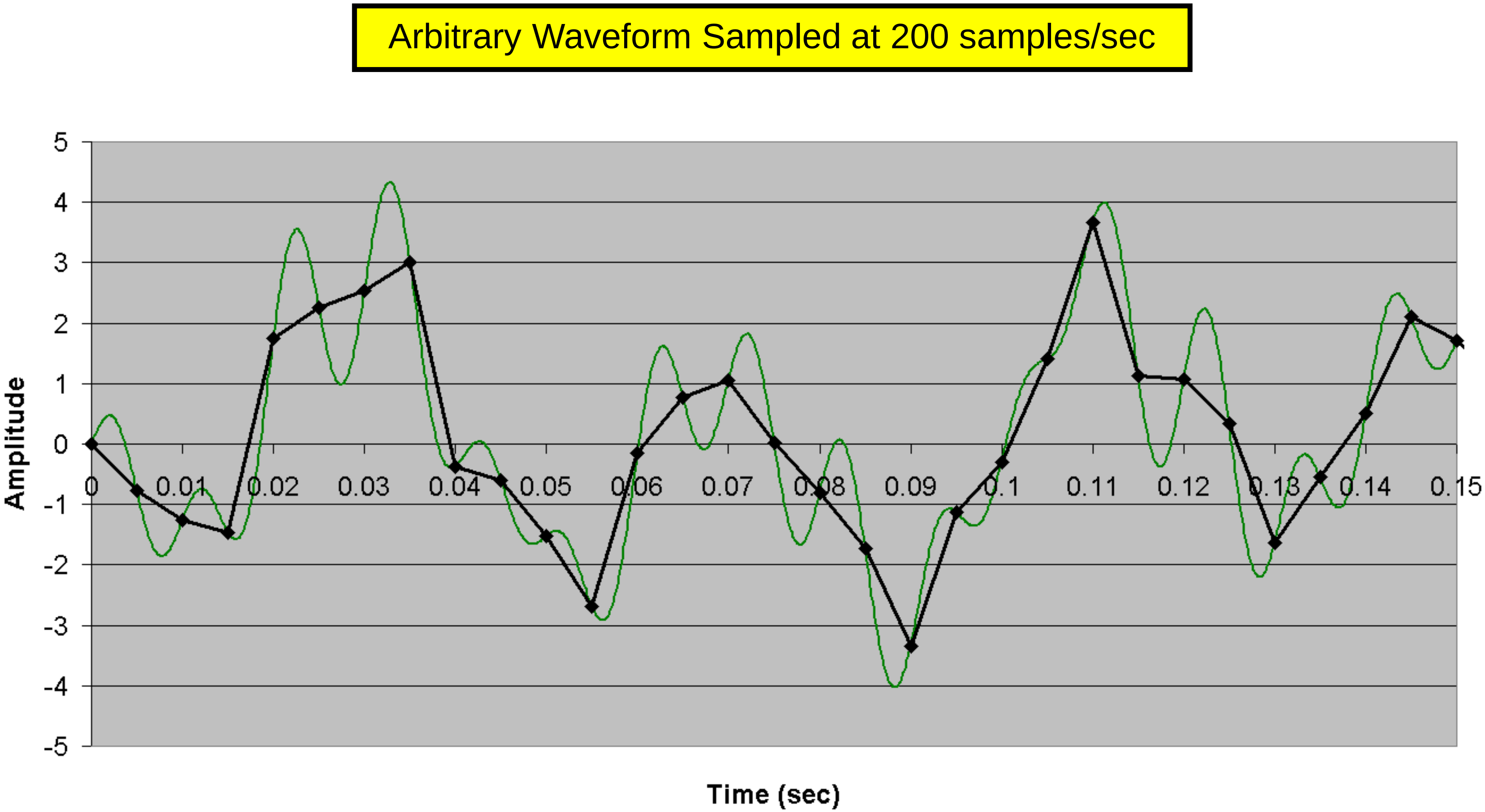
At 12.5 times the signal frequency, the peaks and shape of the signal are replicated rather well.

Arbitrary Waveform Example

- A more realistic view of sampling to replicate a signal is to look at an arbitrary signal.
- The following slides demonstrate different sample rates of an arbitrary waveform.
- The arbitrary waveform is more representative of a signal that we might measure (we rarely see pure sinusoidal signals).
- The green trace is the original waveform.
- The black trace represents the reconstructed signal.

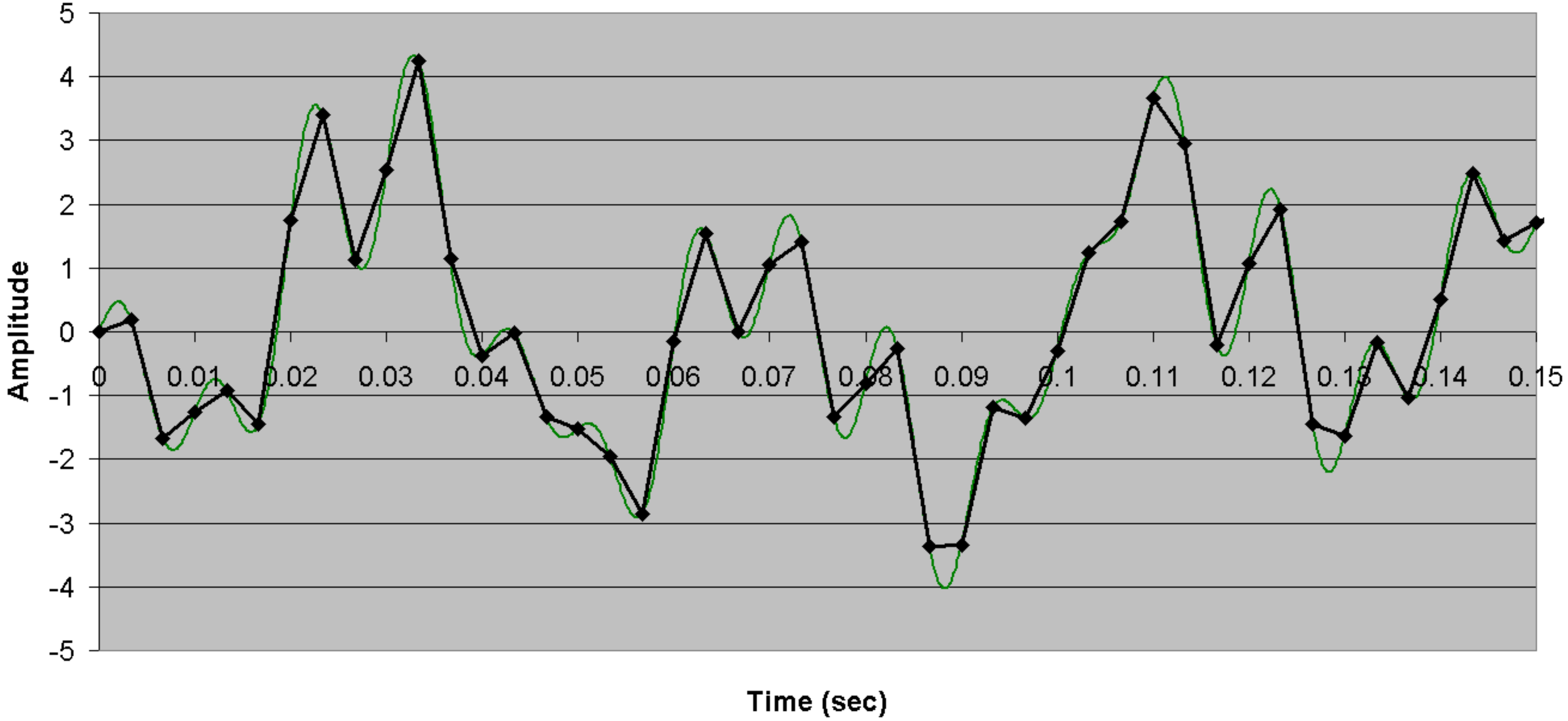
Arbitrary Waveform Example

- As the sample rate increases in each slide, the black trace becomes more representative of the original signal.
- The last slide does not contain a green trace because it is obvious that with the number of samples, the green trace would look identical to the sampled signal.
- This arbitrary signal is known to have a maximum frequency component of 100 Hz (band limited), so we will begin sampling at least twice that frequency.

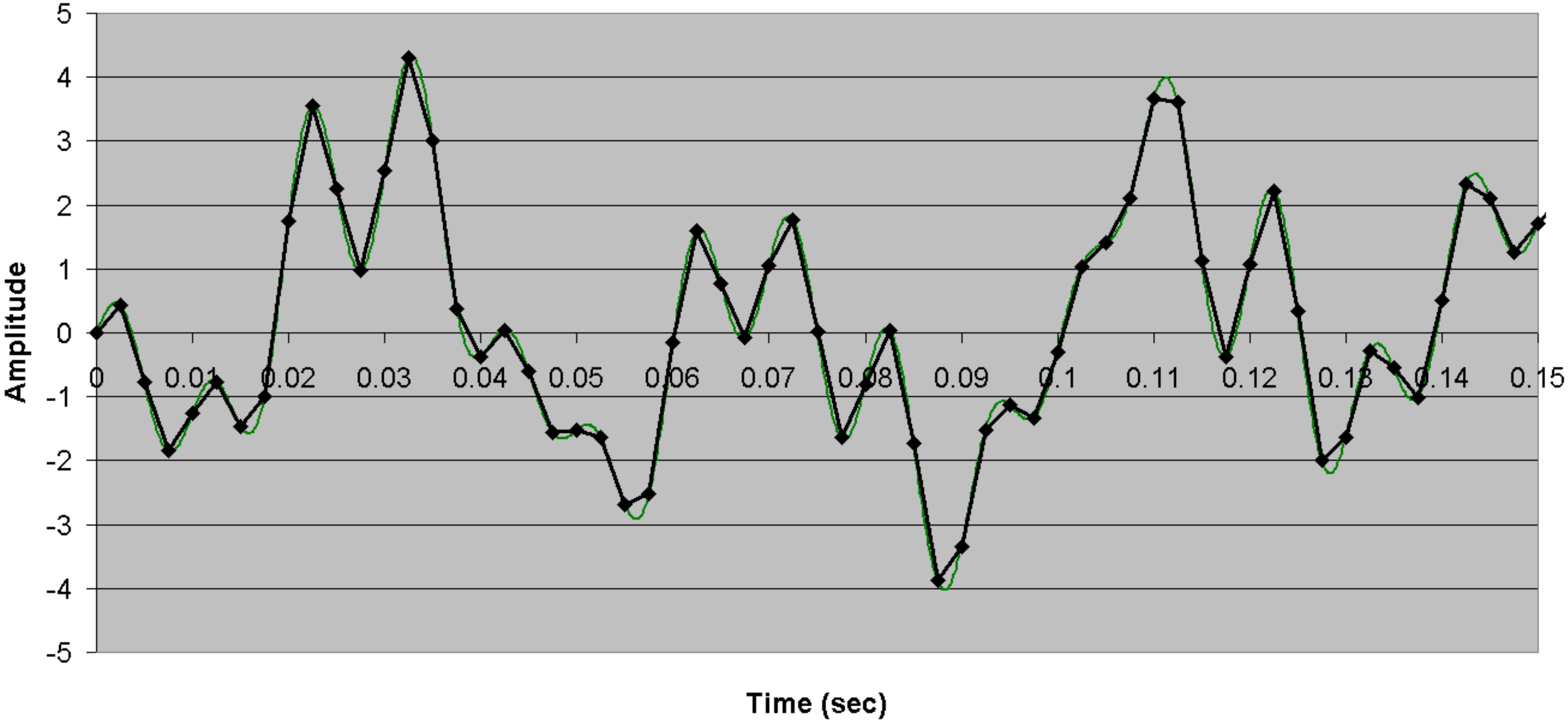


Arbitrary Waveform Sampled at 200 samples/sec
5
4
3
2
1
0
-1
-2
-3
-4
-5
Amplitude
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
Time (sec)

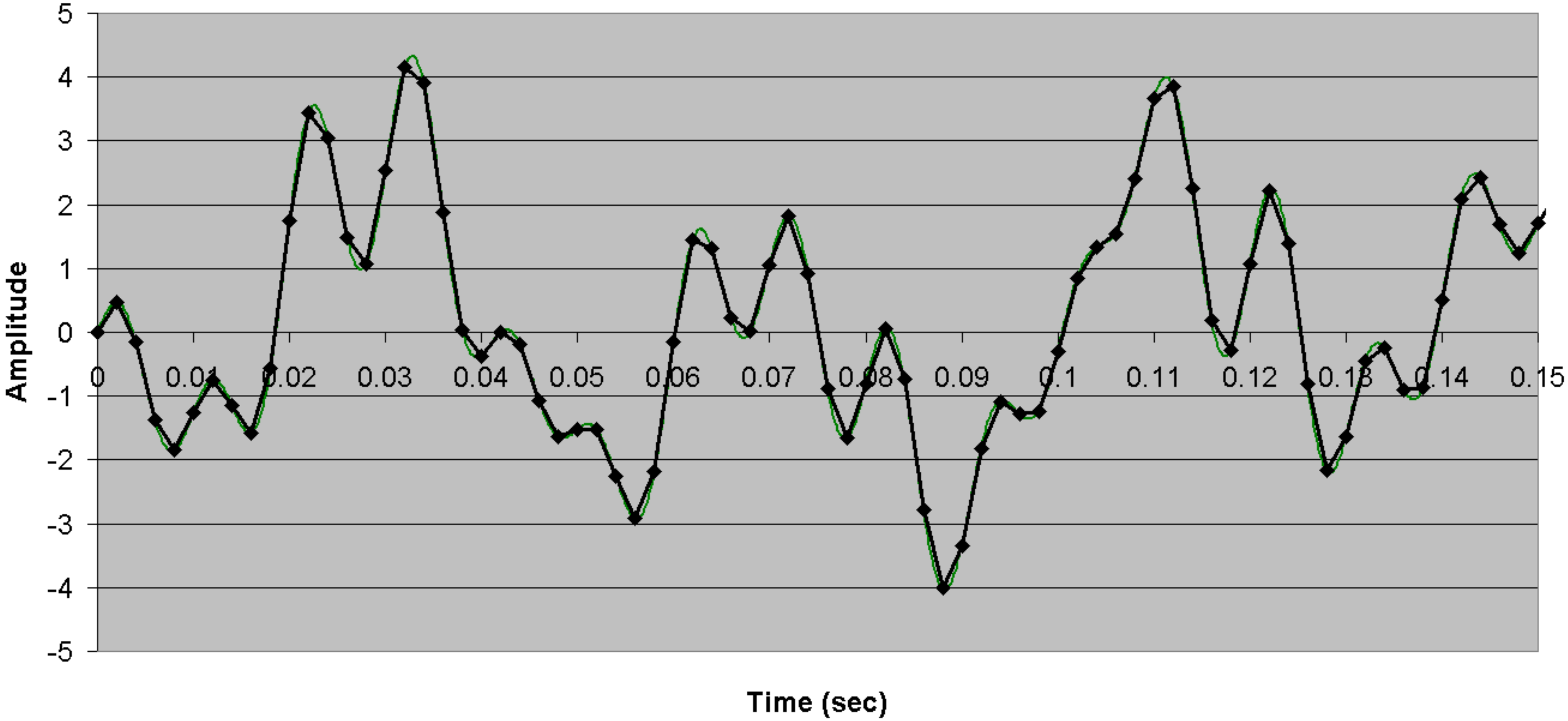
Arbitrary Waveform Sampled at 300 samples/sec



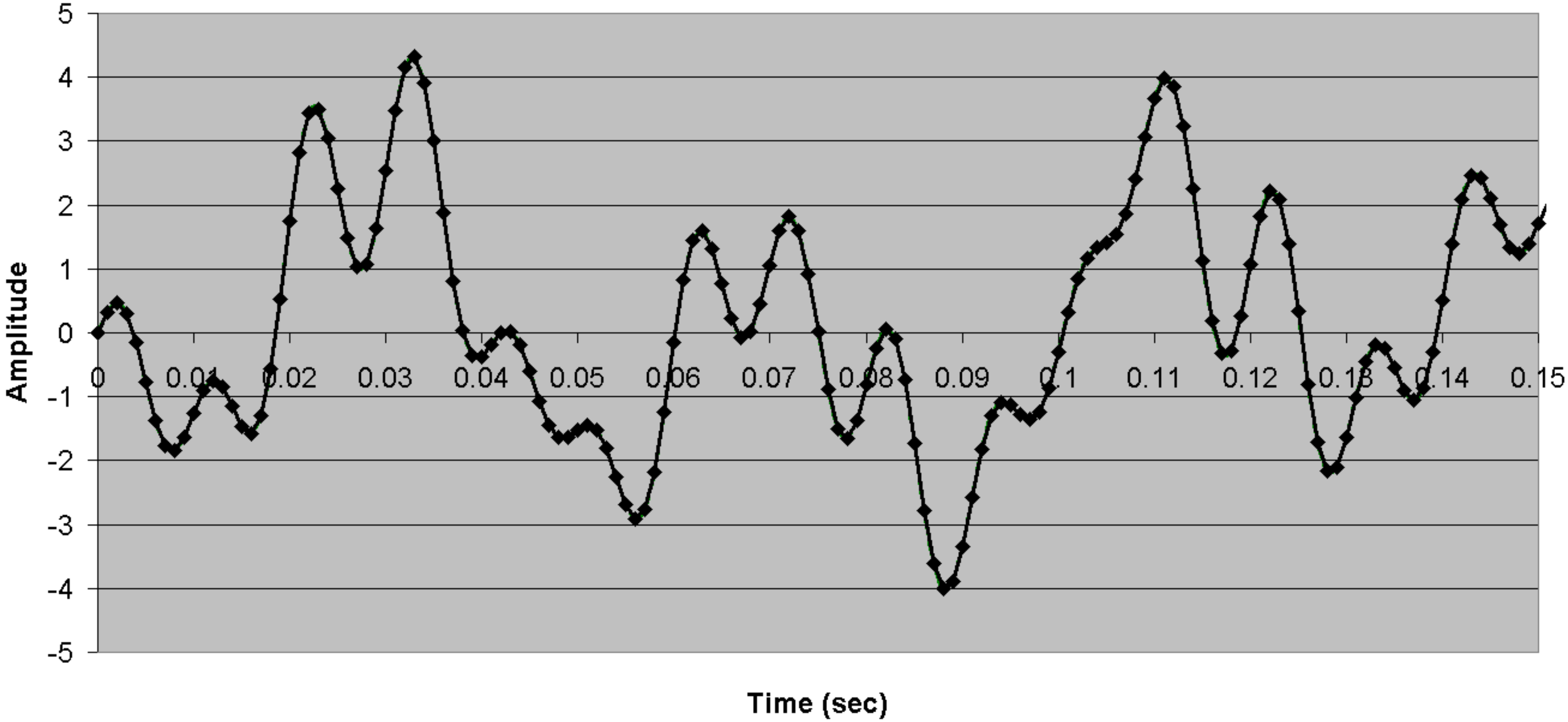
Arbitrary Waveform Sampled at 400 samples/sec



Arbitrary Waveform Sampled at 500 samples/sec



Arbitrary Waveform Sampled at 1000 samples/sec

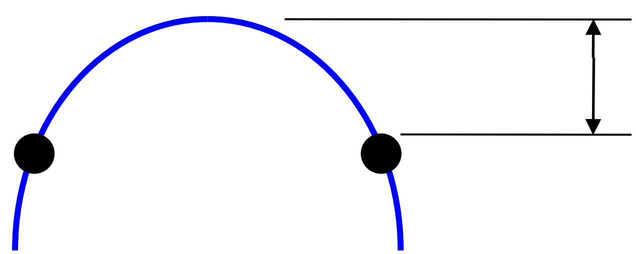


How Fast is Fast Enough?

- How fast must one sample to capture the peaks of a signal?

- The amount of error can be calculated such that if the maximum allowable error is known, then the sample rate can be derived from that.

Errors when Peak Sampling



When attempting to capture the amplitude of a signal, there will always be some maximum error directly related to the sample rate. The maximum error occurs when two consecutive samples are equidistant from the peak of the signal. This error is known and can be calculated as shown below.

$$AE = 100\left[1 - \sin\left\{90^\circ\left(1 - \frac{2}{n}\right)\right\}\right] \quad \text{where} \quad n = \frac{f_s}{f_d}$$

AE is the amplitude error in percent
f_s is the sample rate
f_d is the maximum frequency component of the signal

Example of Calculating the Maximum Amplitude Error

What is the maximum amplitude error when measuring a signal that has a maximum data frequency of 100 Hz, and the sample rate of that channel is 700 sps?

$$n = \frac{f_s}{f_d} = \frac{700}{100} = 7$$

$$\begin{aligned} AE &= 100\left[1 - \sin\left\{90°\left(1 - \frac{2}{n}\right)\right\}\right] \\ &= 100\left[1 - \sin\left\{90°\left(1 - \frac{2}{7}\right)\right\}\right] \\ &= 100\{1 - \sin(\,64.2°)\} \\ &= 9.9\% \end{aligned}$$

So the maximum amplitude error is 9.9%

This may seem high, but consider this, components below 100 Hz will have less error. Also, the 9.9% error only occurs when two consecutive samples are equidistant from the peak of a 100 Hz component.

Calculating a Sample Rate

Going in reverse, if you know what amplitude error should not be exceeded, and you want to calculate the sample rate, then use the equation below.

$$n = \frac{180°}{90° - \sin^{-1}\left(1 - \frac{AE}{100}\right)}$$

where AE is the percent of amplitude error

Then calculate $$f_s = (n)(f_d)$$

Example of Calculating the Sample Rate

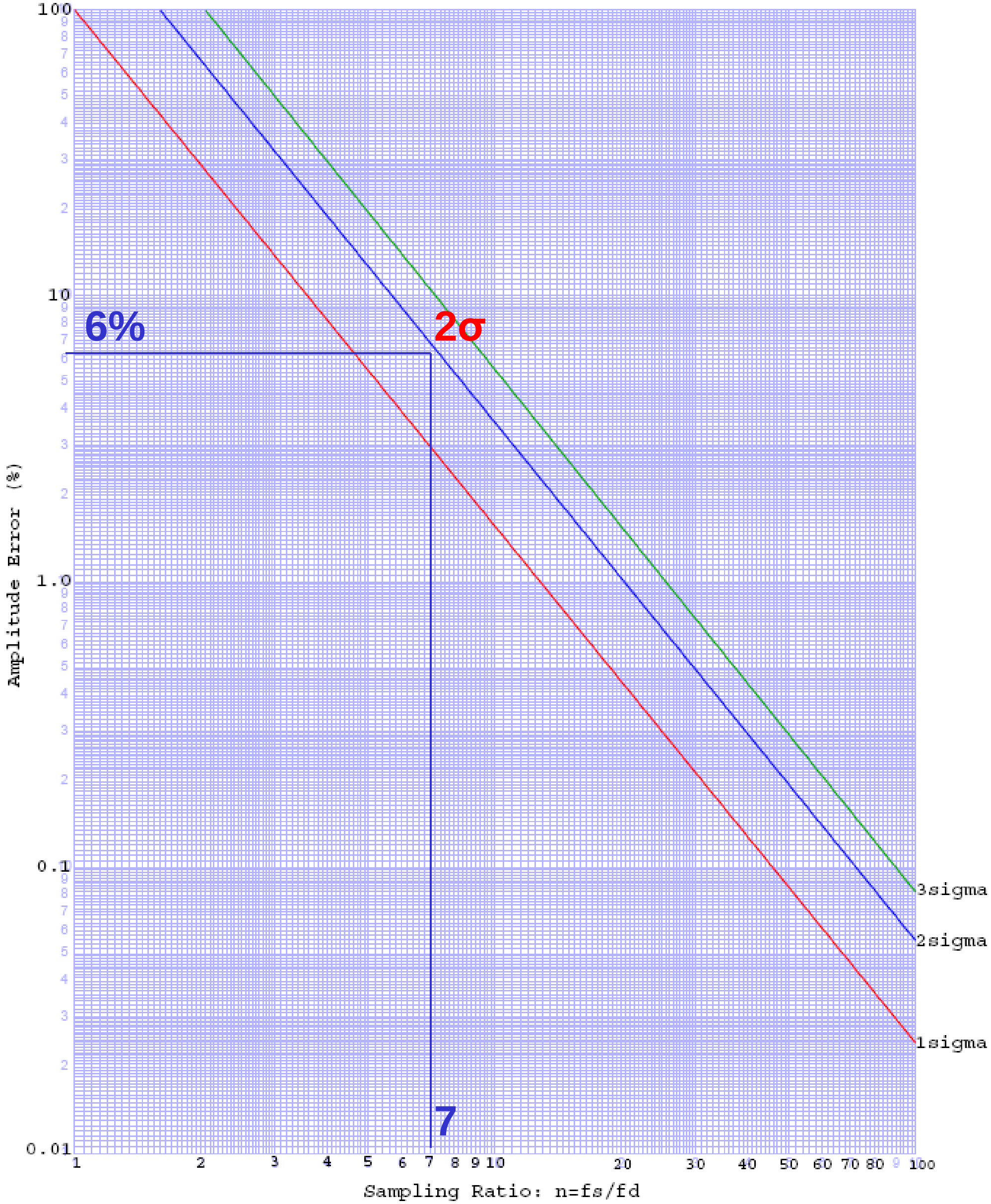
There is a requirement such that the maximum amplitude error is 3.5%. The maximum frequency content of the signal being measured is 1000 Hz, what should the sample rate be?

$$n = \frac{180°}{90° - \sin^{-1}\left(1 - \frac{AE}{100}\right)}$$
$$= \frac{180°}{90° - \sin^{-1}\left(1 - \frac{3.5}{100}\right)}$$
$$= \frac{180°}{90° - \sin^{-1}(0.965)}$$
$$= \frac{180°}{15.2°} = 11.84$$

$$f_s = (n)(f_d) = (11.84)(1000) = 11{,}840 sps$$

The sample rate must therefore be 11.84 Ksps. Again, frequency components below 1000 Hz will have less error.

A More Realistic Approach



- Instead of reporting an absolute maximum error, it is best to describe the error in most of the peak data. Using the graph shown on the left, 95% of the peak data can be characterized.
- To determine the 95% confidence interval of the amplitude error, first determine *n*, then read the amplitude error on the y-axis from the 2σ line.
- For the example we did earlier where *n*=7 we get an amplitude error of approximately 6% vice the maximum of 9.9% calculated before.

This graph was adapted from the Measurement Systems Engineering Course by Charles P. Wright of The Aerospace Corporation. A larger version of this graph is included in TSOP-013

Sampling Example

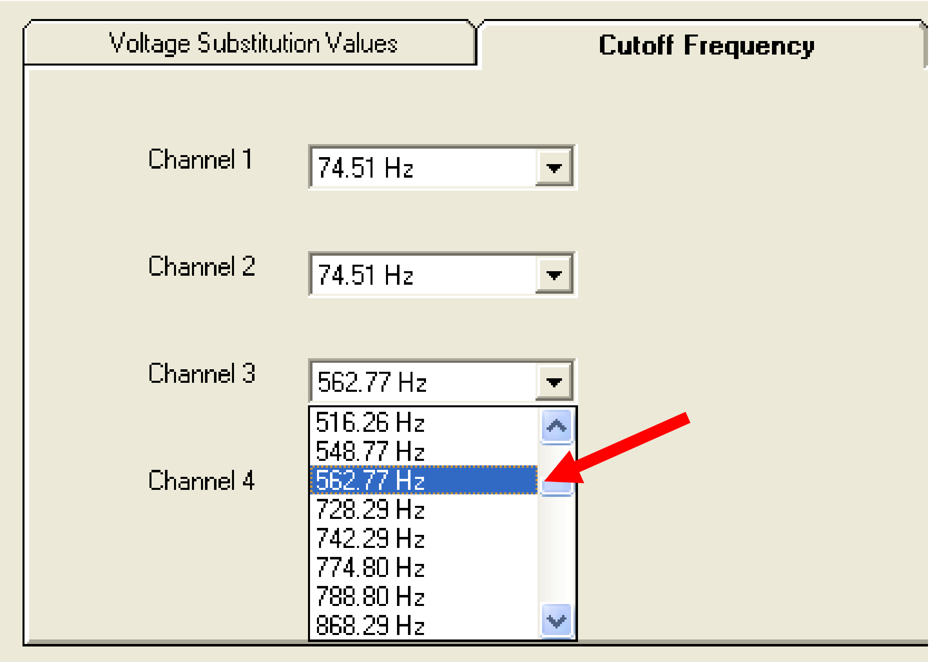
- The customer is interested in an accurate reproduction of a lateral lug vibration. To conserve bandwidth for solid-state record time, a 4% amplitude error is determined to be acceptable by the customer.
- The frequency range of interest is 0-400 Hz.
- We will be using a CDAU with an SCD-108S card to acquire, condition, and digitize this signal.

Measurement Description	MNEMONIC	* Signal Sourc	Frequency Range of interest	Measurement Accuracy	Eng Units	EU Range	** Collection Method	Remarks
LEFT WING VIBRATION	LWNGVIB	IT	0 -2000 HZ	+/- 2%	g's	+/- 50	A, T	
Roll Rate	Rollrate	PT	0-50 Hz	+/- 5%	Deg/sec	+/- 30 I14	A, T	
Airspeed	ASPD	B	N/A	See ICD	See ICD	See ICD	A, T	1553 Bus rate 20Hz (Avionics 1)
Pilot MFD	PiltMFD	V	N/A	N/A	N/A	N/A	A	RS-170 Connection on MFD
Lug Vibration - NX	LUGVIBNX	IT	0-400	5%	G's	+/-250	A,T	
Lug Vibration - NY	LUGVIBNY	IT	0-400	5%	G's	+/-250	A,T	
Lug Vibration - NZ	LUGVIBNZ	IT	0-400	5%	G's	+/-500	A,T	
Angle of Attack	AOA	IT	10	5%	Degrees	+/-60	A,T	
Angle of Sideslip	AOSS	IT	10	5%	Degrees	+/-45	A,T	

Sampling Example

- **<u>Calculating the filter cutoff</u>**
  - **The SCD-108S card has 6-pole low-pass Butterworth filter and a 12-bit ADC. Use the formula f_c = 1.3837 f_d to find f_c.**

$$f_c = 1.3837 f_d = (1.3837)(400Hz) = 553.48Hz$$



In TTCWare, the next highest cutoff frequency selection is 562.77Hz.

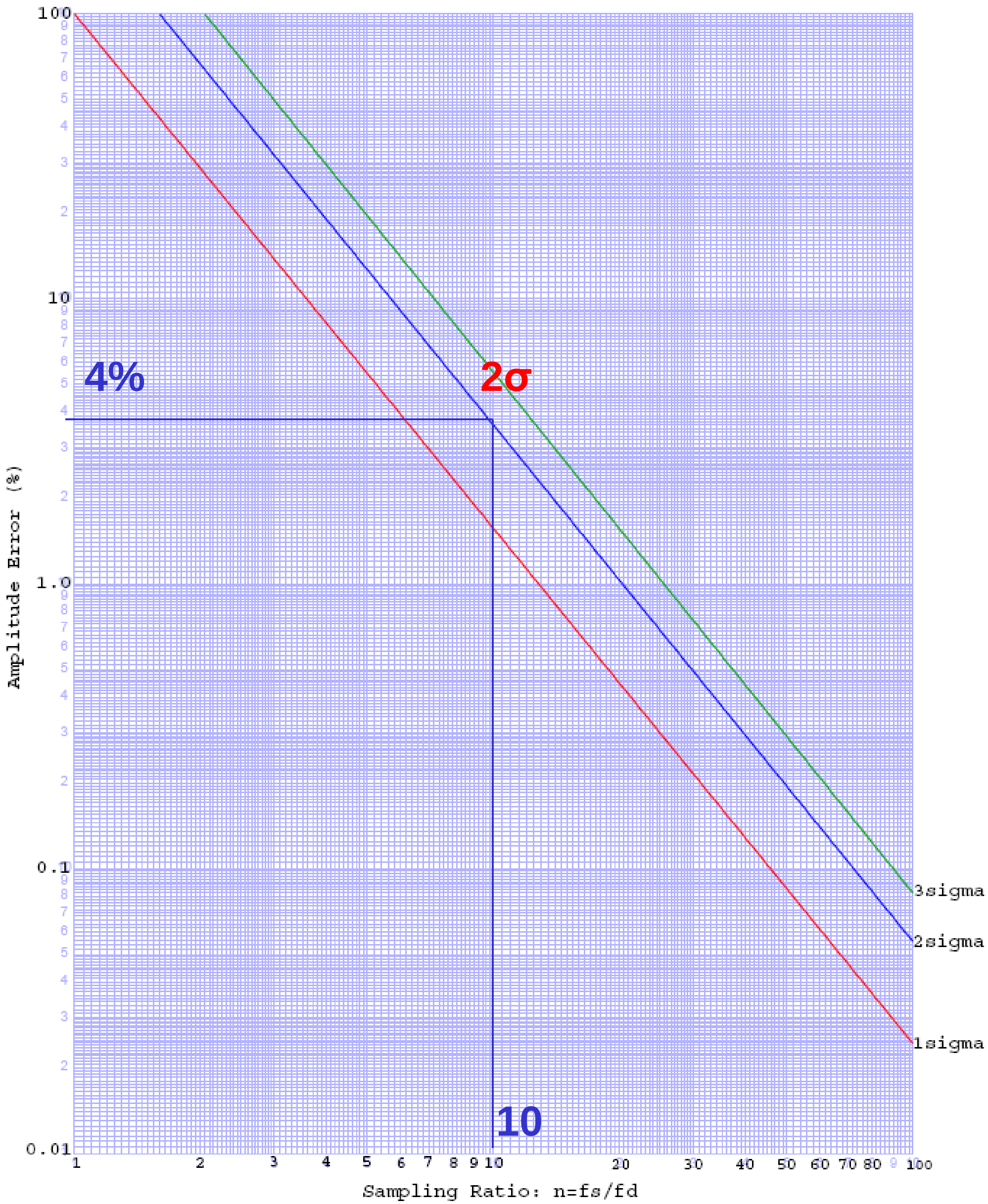
Note that if the "rule of thumb" had been used, $f_c = \sqrt{2} f_d$ the cutoff frequency chosen would have been 728.29 Hz.

Sampling Example

- Calculating the minimum sample rate to avoid aliasing
  - It was determined that to avoid aliasing with a 6-pole Butterworth filter and a 12-bit ADC, you sample at minimum, 5x the cutoff frequency.

$$f_s = 5f_c = (5)(562.77Hz) = 2813.85sps$$

Sampling Example



- <u>Calculating the minimum sample rate to have a 4% amplitude error</u>
  - Read from the 4% on the left and go horizontally to the 2σ line.
  - Read down from that point to determine the multiplier. In this case it is n=10.
  - To determine the sample rate,

$$f_s = nf_d$$
$$f_s = 10(400Hz)$$
$$f_s = 4000sps$$

Sampling Example

- Sampling the lateral lug vibration measurement at 4000 sps will avoid aliasing and have 4% amplitude error.
- Now that the sample rate is determined, follow the training in IRIG-106 Chapter 4 PCM Maps and Structures to achieve the appropriate sample rate.

Sampling for non-Sinusoidal Signals

- When the signal is not sinusoidal, the rules for sampling will change.

- Some of these signal types are the following:
  - Abrupt signal change – relay closure
  - Slow-response transducer with a known time constant – thermocouple
  - Half-sine – shock pulse

Sampling an Abrupt Signal Change

When sampling to detect an abrupt signal change, you normally care about when it occurred and not what the voltage levels are. These types of signals are not filtered and normally captured on a BLS-148 card.

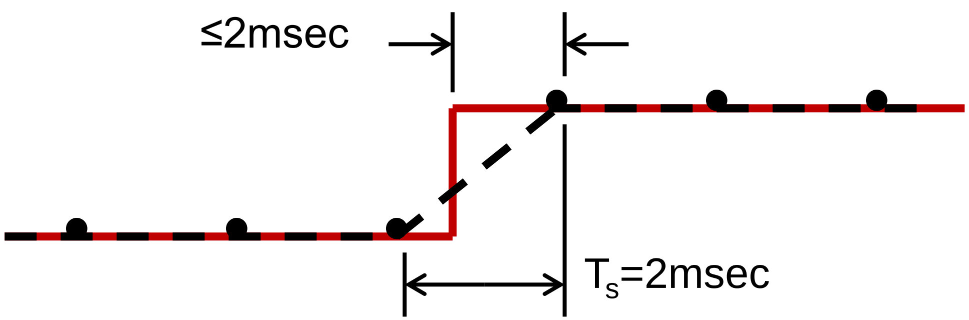
The sample rate for these signals should be $f_S \geq \frac{1}{T}$

Where T is the time within where the pulse is to be detected (ex. to within 20 µsec)

If you are to detect when a firing pulse occurs by monitoring a relay closure, how fast must you sample? The closure is to be detected to within 2 msec.

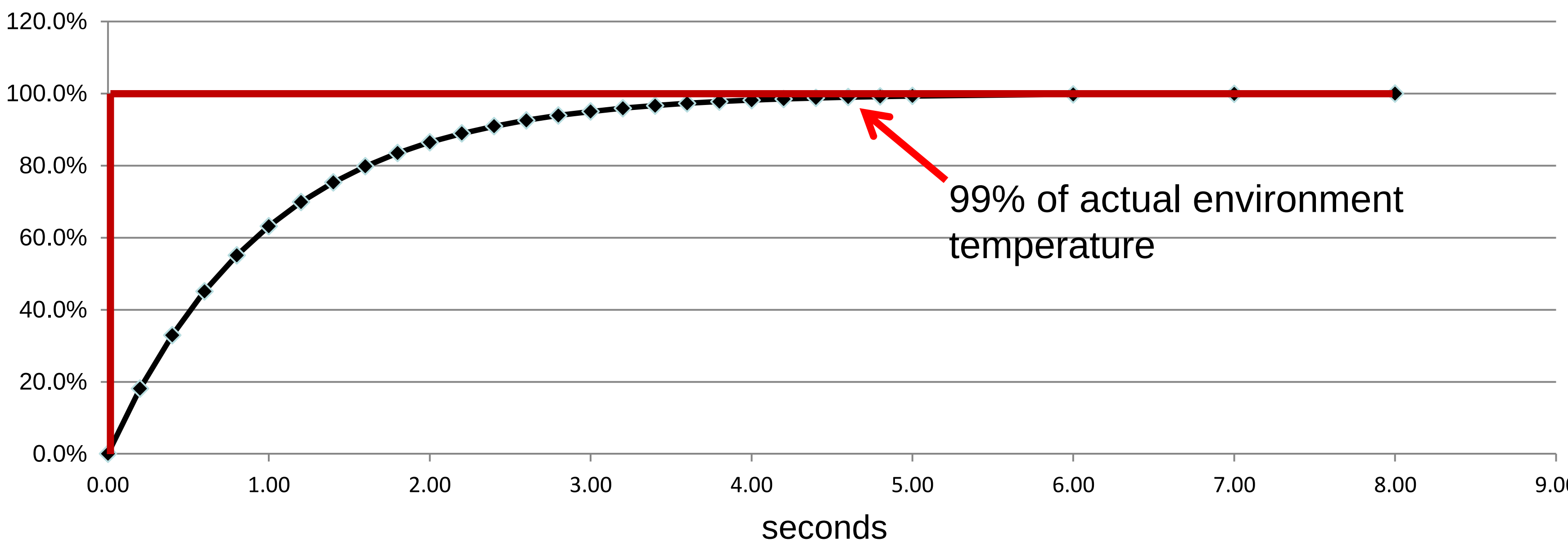
$$f_S = \frac{1}{T} = \frac{1}{2msec}$$

$$T_s = 2msec$$



Sampling Slow-Response Transducers with a Known Time Constant

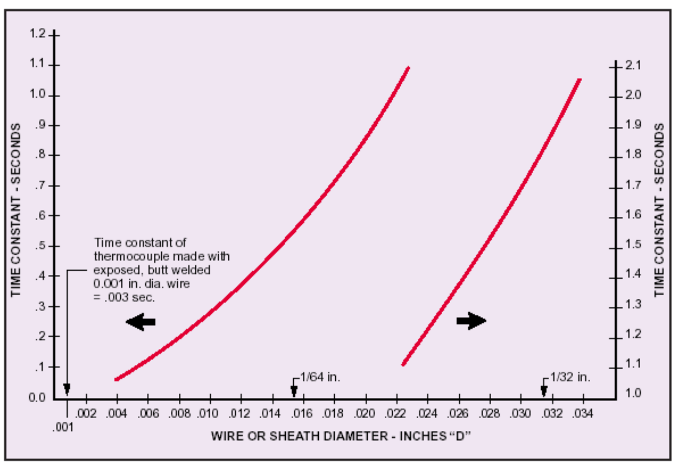
- Transducers which have a defined time constant take time to reach the measured EU value of the environment.
- A thermocouple with a time constant of 1 second, will take 4.6 seconds to reach 99% of the actual temperature when a stepped increase in temperature occurs. This would be worst case, because in reality, temperatures do not instantaneously change.



Time Constant of a Thermocouple

- The response of a thermocouple (or other similar transducer) is determined by its time constant. The size and shielding type of the thermocouple determine the time constant.

- Manufacturers such as Omega have time constant curves which give the value for various thermocouple types and wire sizes.



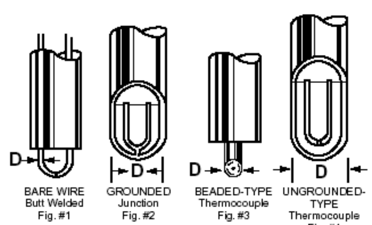
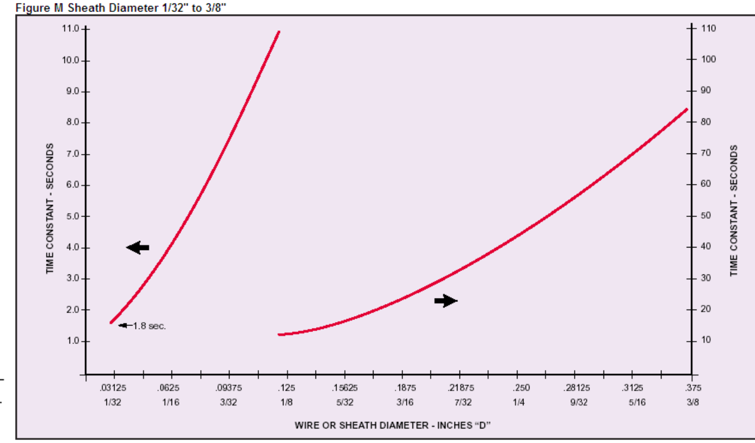


Figure M Sheath Diameter 1/32" to 3/8"



Response of a Thermocouple

The increasing sensed temperature of a thermocouple follows the equation

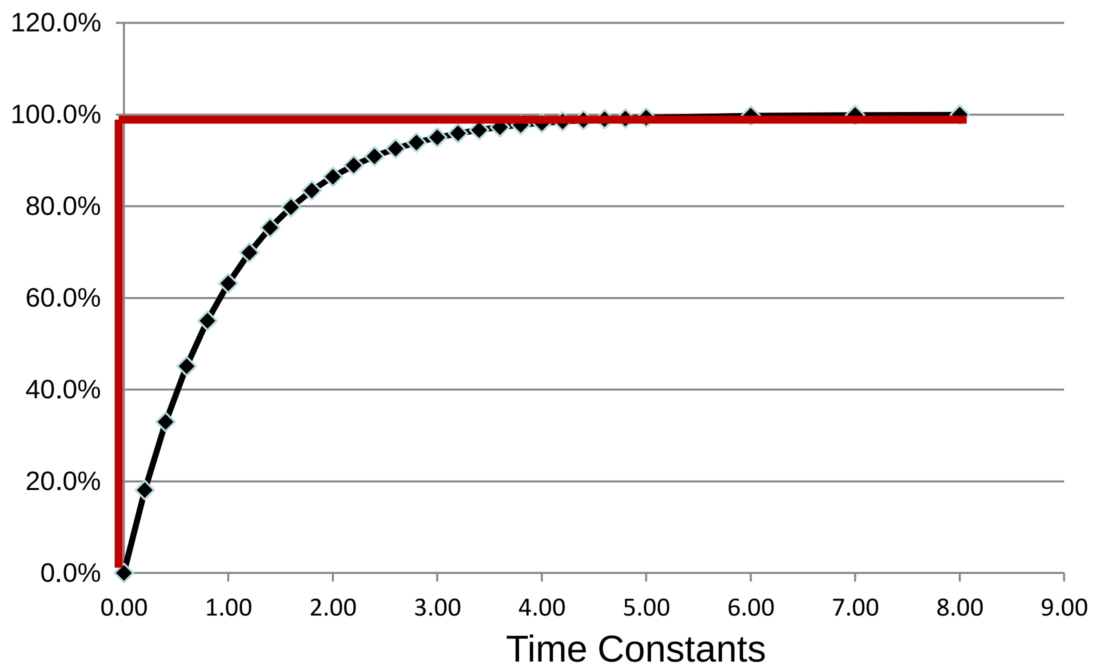
$$T = T_0\left(1 - e^{-t/\tau}\right)$$

T_0 is the environment temperature

t is the time

τ is the time constant of the thermocouple

Time	% Full Temp
0	0%
1τ	63.2%
2τ	86.5%
3τ	95.0%
4τ	98.2%
5τ	99.3%



Response of a Thermocouple

A decrease in sensed temperature of a thermocouple follows the equation

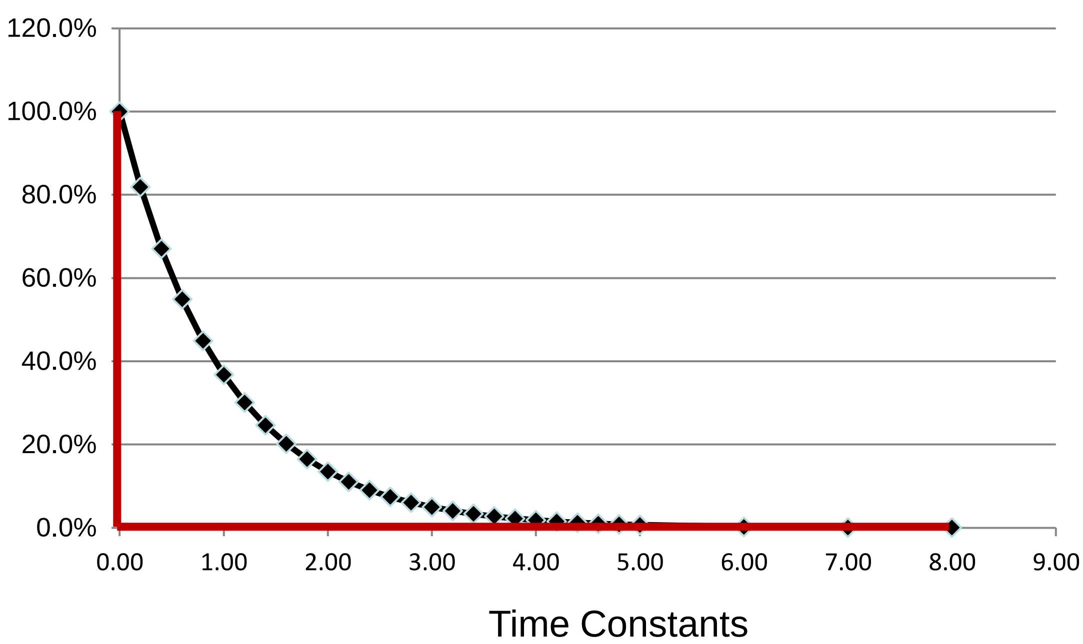
$$T = T_0 e^{-t/\tau}$$

T_0 is the environment temperature

t is the time

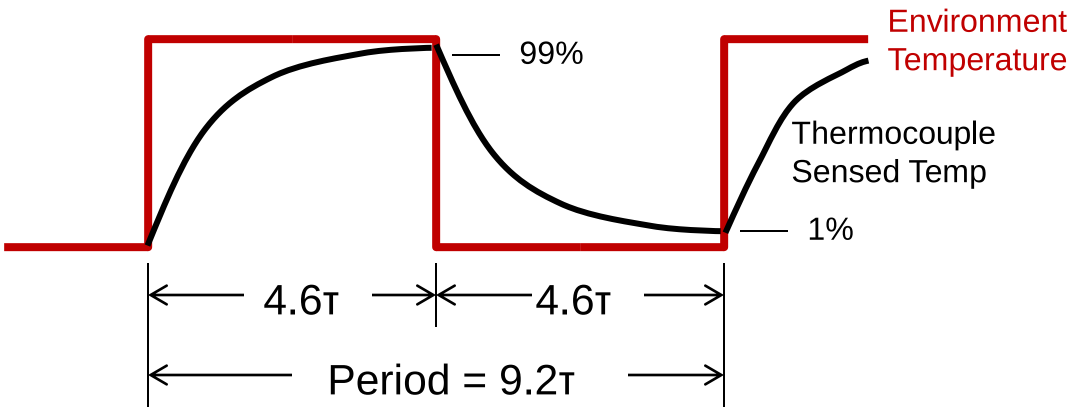
τ is the time constant of the thermocouple

Time	% from 0%
0	100%
1τ	36.8%
2τ	13.5%
3τ	5.0%
4τ	1.8%
5τ	0.7%



Frequency Response of a Thermocouple

We can define the frequency response range as the band where the amplitude is within 1% of the actual value. This concept will be applied to the thermocouple response.

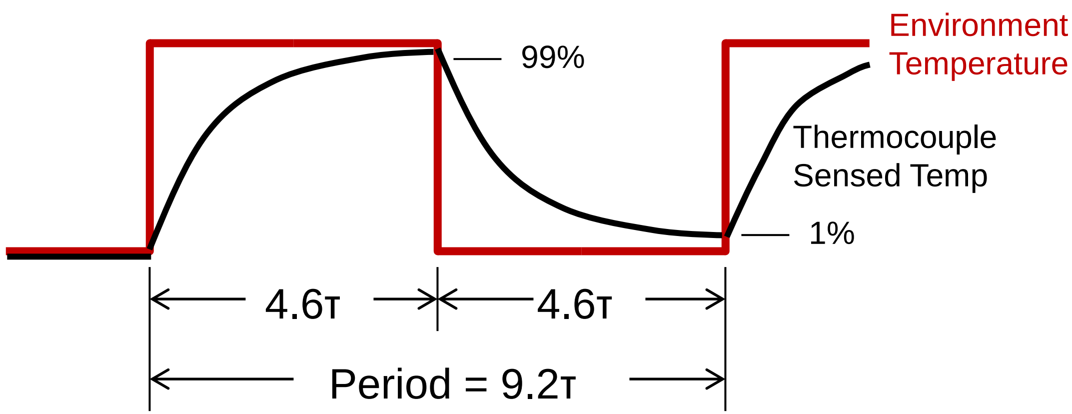


The high and low temperature levels take 4.6τ each for the thermocouple to sense 99% and 1%. A full cycle or period would be twice that, or 9.2τ. The frequency is the inverse of the period.

The maximum data frequency response of the thermocouple is therefore,

$$f_D = {}^{1}/_{9.2\tau}$$

Minimum Sample Rate Required for a Thermocouple Measurement



To have a 95% confidence that you only have a 1% error in amplitude, you must sample at least 20 times the highest frequency (n≥20).

$$n = {}^{f_S}\!/_{f_D} \geq 20$$

$$ {}^{f_S}\!\Big/_{\left(\frac{1}{9.2\tau}\right)} \geq 20$$

$$f_S \geq {}^{20}\!/_{9.2\tau}$$

$$f_S \geq {}^{2.174}\!/_{\tau}$$

Minimum Sample Rate Required for a Thermocouple Measurement

For a wire or sheath thermocouple with a diameter of 0.030", the time constant is approximately 1.7 seconds. What should the minimum sample rate be?

Time to reach 99% of temp environment:

$$4.6\tau = 4.6(1.7) = 7.82sec$$

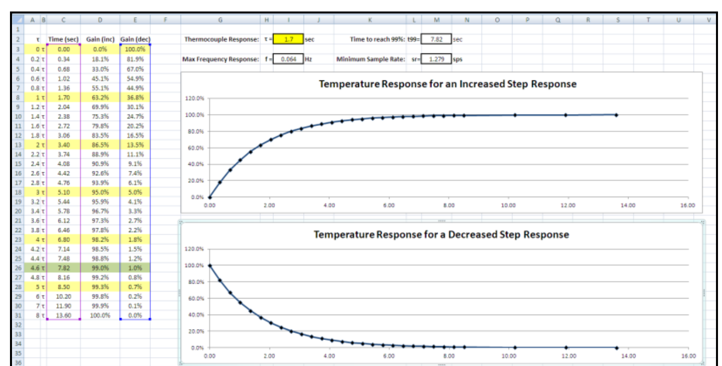
It's maximum frequency response is:

$$f_D = {}^{1}\!/_{9.2\tau} = {}^{1}\!/_{(9.2 \times 1.7)} = 0.064Hz$$

The minimum sampling rate is:

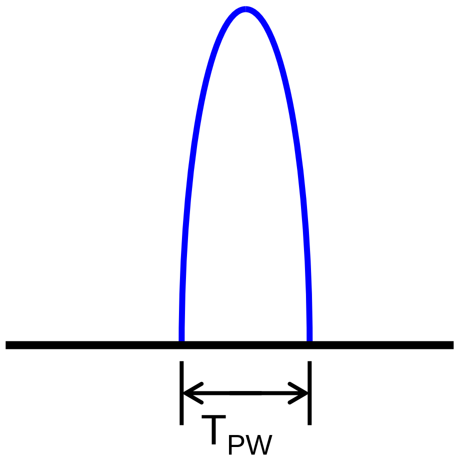
$$f_S = {}^{2.174}\!/_{\tau} = {}^{2.174}\!/_{1.7} = 1.279sps$$

An Excel spreadsheet which computes These calculations is on MyNAVAIR under "Tools and Applications".
Thermocouple Response.xls



Sampling a Half-sine Pulse

The half-sine pulse best describes a shock type signal. So the sampling rate can be determined once the maximum data frequency is defined.



The pulse width is defined to be from zero-crossing to zero-crossing. To determine the maximum data frequency, just double the pulse width and calculate the frequency.

$$f_D = \frac{1}{2T_{PW}}$$

Now that the frequency is known, use the chart to determine the sampling rate for a desired error bound. For example, to have a 95% confidence that the peak is captured within 5% (n=8.5 sampling ratio), find the sample rate if the pulse width is 40msec.

$$n = {}^{f_S}\!/_{f_D} = 8.5 \qquad f_D = \frac{1}{2T_{PW}} = \frac{1}{2(40m\,\text{sec})} = \frac{1}{80msec}$$

$$f_S = 8.5 f_D = 8.5(12.5) = 106.25sps$$

Digital Data (Data Bus) Sampling

- Digital data or data bus data is data that has already been digitized
- Because this data is already in a digital format, the sampling rules for analog signals <u>do not apply</u>
- The digital data words from a data bus (1553, ARINC-429, RS-232, etc.) are available on the bus at a specified update rate or transmission rate.
- The same follows for temperature measurements on the TCD-116 card. The card samples at a fixed rate, sample the word in the PCM map at a slightly higher rate.
- To ensure that no data is lost, you must sample these digital parameters at a rate that is at least slightly higher than the update rate

Digital Data (Data Bus) Sampling

- Example: suppose that airspeed is available on a 1553 bus at an update rate of 25 Hz, we would want to sample this parameter at a rate higher than 25 samples per second.
- Sampling at or below the update rate can result in lost data – this can be detected by observing the overflow bits in the status word for certain busses (e.g., 1553).
- It is the responsibility of the person integrating and checking out the data acquisition system to ensure that the proper data rates have been selected by observing data bus status information during a complete end-to-end system checkout.

Steps To Successful Sampling

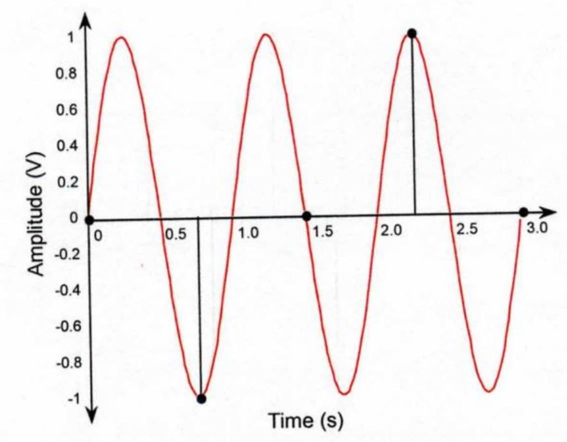
1. Understand the requirements of the person(s) that will be reviewing and using the data.
   - Frequency range of interest
   - Signal reproduction requirements
     - Basic frequency and amplitude
     - Max peak amplitude
     - FFT
2. Filter first – band-limit all analog signals so that they contain only the frequency range of interest, using a pre-sample/anti-aliasing filter.
3. For a p-pole Butterworth filter and an n-bit ADC, sample at $f_S \geq \left(2^{n/p} + 1\right) f_C$
4. Sample at higher rates to obtain a better amplitude reproduction of the signal. Use the chart to determine the multiplier n for a particular amplitude error. $f_s = n f_d$

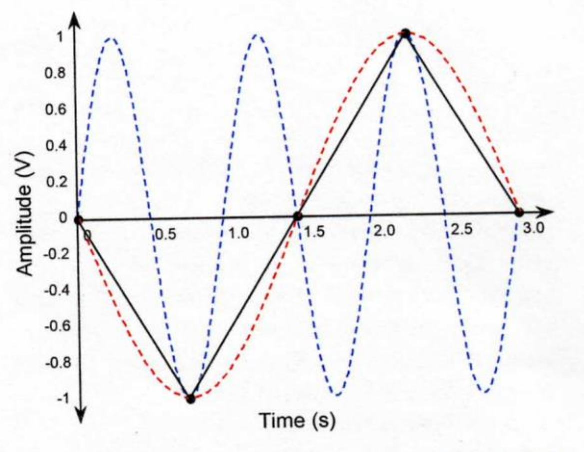
Steps To Successful Sampling

5. Sampling at a rate greater than or equal to the Nyquist frequency will prevent aliasing and begin to capture frequency content of the signal
$$f_s \geq 2f_n$$
6. Sampling at a rate much higher than the twice the Nyquist frequency provides a better reconstruction of the original signal
$$f_s \gg 2f_n$$
7. Peak sampling requires sampling at a rate many times higher than twice the Nyquist frequency
$$f_s \ggg 2f_n$$

Effects of Under-Sampling

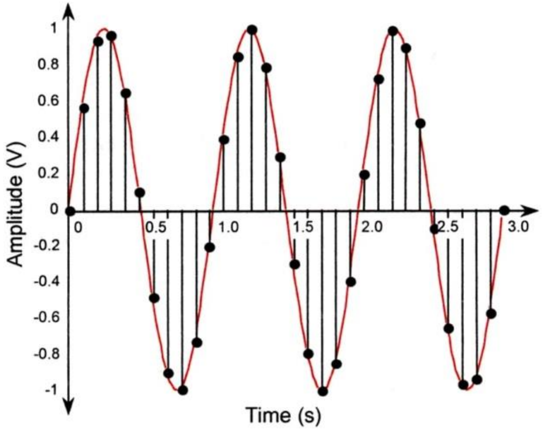
- Aliasing
- If data is aliased, it is <u>unusable</u>
- Lost data (information)
- Missed peaks (amplitude) in signal
- Difficult to reconstruct original signal accurately and with minimal distortion

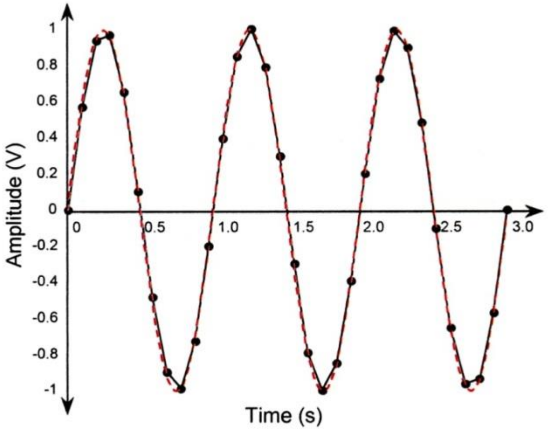




Effects of Over-Sampling

- No aliasing <u>if pre-sample filter is set correctly</u>
- More accurate data
- More data (information)
- Reconstructed signals will be more accurate with minimal distortion
- Requires more bandwidth for higher data rate
- Requires more recorder space or record time (this can be a problem with solid-state recorders)

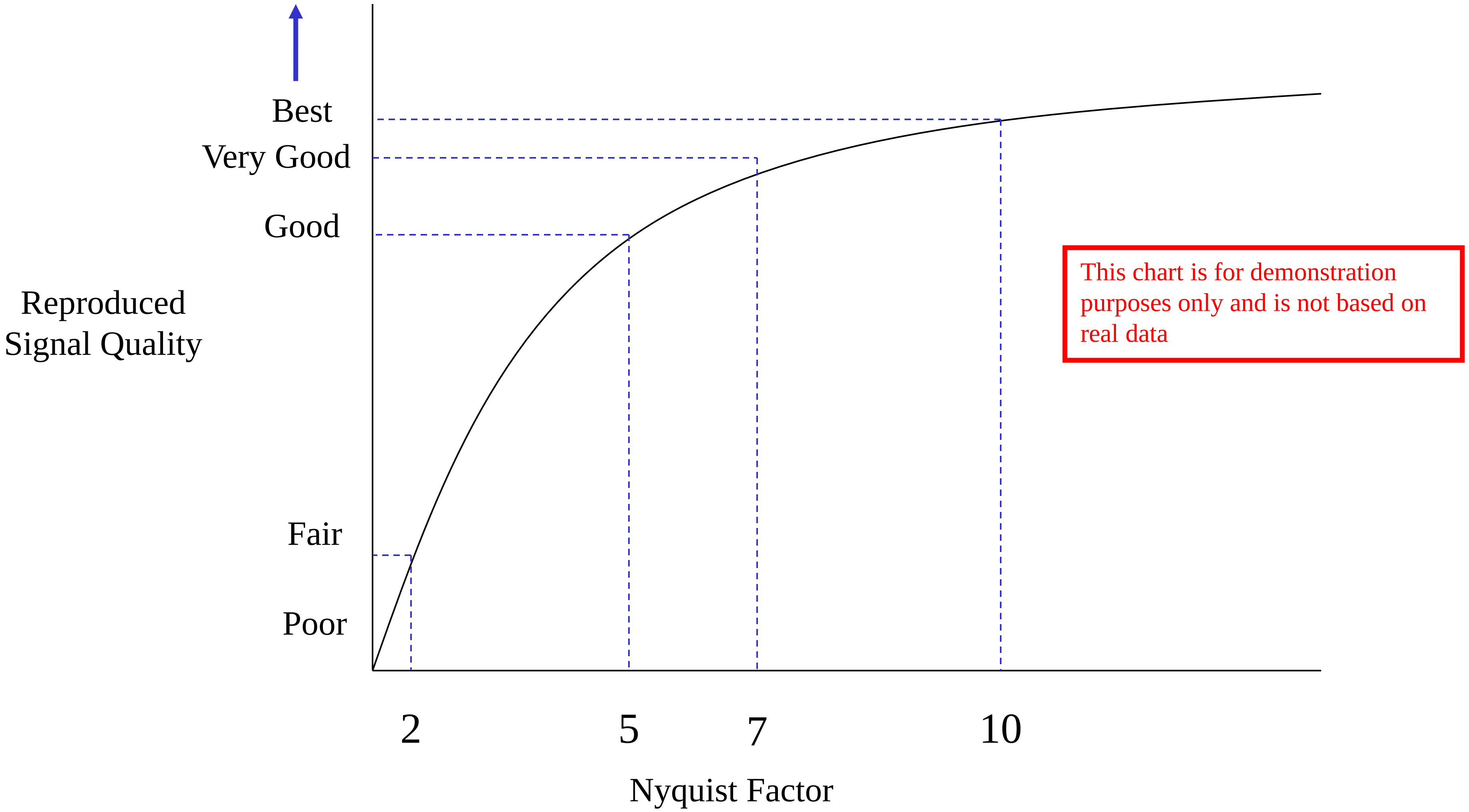




Over-sampling Bang For the Buck

- An important concept to be aware of is the fact that the relationship between Nyquist rates (over-sampling) and data quality is exponential
- Increasing the Nyquist factor from 2 to 5 provides a higher quality reproduced signal at a cost of 2.5 times the bit rate or bandwidth
- Increasing the Nyquist factor from 5 to 10 provides an even higher quality reproduced signal but the improvement for 2 times more bandwidth is not as good of a buy as increasing from 2 to 5 (bang for the $)
- The following slide illustrates this concept

Over-Sampling Bang For the Buck



References

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