Substituting $V_C = k_eQ/R$ into this expression and solving for V_D, we obtain

$$V_D = \boxed{\frac{k_eQ}{2R}\left(3 - \frac{r^2}{R^2}\right)} \qquad (\text{for } r < R) \quad \textbf{(25.26)}$$

At $r = R$, this expression gives a result that agrees with that for the potential at the surface, that is, V_C. A plot of V versus r for this charge distribution is given in Figure 25.20.

25.6 Electric Potential Due to a Charged Conductor

In Section 24.4 we found that when a solid conductor in equilibrium carries a net charge, the charge resides on the outer surface of the conductor. Furthermore, we showed that the electric field just outside the conductor is perpendicular to the surface and that the field inside is zero.

We now show that **every point on the surface of a charged conductor in equilibrium is at the same electric potential.** Consider two points A and B on the surface of a charged conductor, as shown in Figure 25.21. Along a surface path connecting these points, $\mathbf{E}$ is always perpendicular to the displacement $d\mathbf{s}$; therefore $\mathbf{E} \cdot d\mathbf{s} = 0$. Using this result and Equation 25.3, we conclude that the potential difference between A and B is necessarily zero:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

This result applies to any two points on the surface. Therefore, V is constant everywhere on the surface of a charged conductor in equilibrium. That is,

the surface of any charged conductor in electrostatic equilibrium is an equipotential surface. Furthermore, because the electric field is zero inside the conductor, we conclude that the electric potential is constant everywhere inside the conductor and equal to its value at the surface.

Because this is true, no work is required to move a test charge from the interior of a charged conductor to its surface.

Consider a solid metal conducting sphere of radius R and total positive charge Q, as shown in Figure 25.22a. The electric field outside the sphere is k_eQ/r^2 and points radially outward. From Example 25.8, we know that the electric potential at the interior and surface of the sphere must be k_eQ/R relative to infinity. The potential outside the sphere is k_eQ/r. Figure 25.22b is a plot of the electric potential as a function of r, and Figure 25.22c shows how the electric field varies with r.

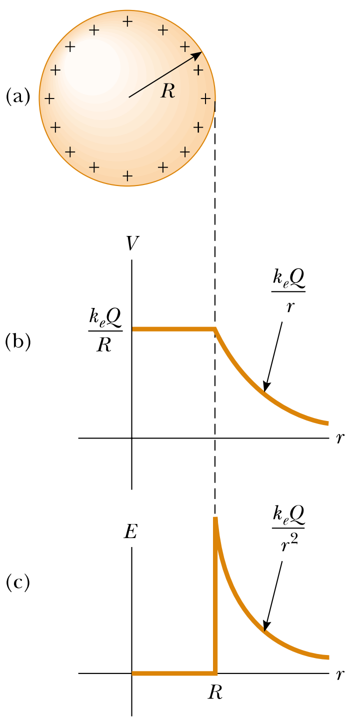


Figure 25.22 (a) The excess charge on a conducting sphere of radius R is uniformly distributed on its surface. (b) Electric potential versus distance r from the center of the charged conducting sphere. (c) Electric field magnitude versus distance r from the center of the charged conducting sphere.

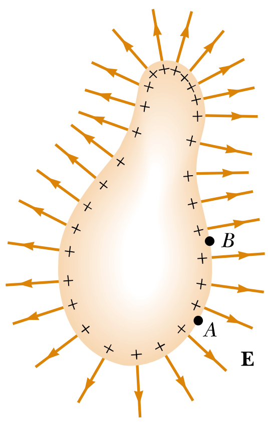


Figure 25.21 An arbitrarily shaped conductor carrying a positive charge. When the conductor is in electrostatic equilibrium, all of the charge resides at the surface, $\mathbf{E} = 0$ inside the conductor, and the direction of $\mathbf{E}$ just outside the conductor is perpendicular to the surface. The electric potential is constant inside the conductor and is equal to the potential at the surface. Note from the spacing of the positive signs that the surface charge density is nonuniform.

When a net charge is placed on a spherical conductor, the surface charge density is uniform, as indicated in Figure 25.22a. However, if the conductor is nonspherical, as in Figure 25.21, the surface charge density is high where the radius of curvature is small (as noted in Section 24.4), and it is low where the radius of curvature is large. Because the electric field just outside the conductor is proportional to the surface charge density, we see that **the electric field is large near convex points having small radii of curvature and reaches very high values at sharp points.** This is demonstrated in Figure 25.23, in which small pieces of thread suspended in oil show the electric field lines. Notice that the density of field lines is highest at the sharp tip of the left-hand conductor and at the highly curved ends of the right-hand conductor. In Example 25.9, the relationship between electric field and radius of curvature is explored mathematically.

Figure 25.24 shows the electric field lines around two spherical conductors: one carrying a net charge Q, and a larger one carrying zero net charge. In this case, the surface charge density is not uniform on either conductor. The sphere having zero net charge has negative charges induced on its side that faces the charged sphere and positive charges induced on its side opposite the charged sphere. The broken blue curves in the figure represent the cross sections of the equipotential surfaces for this charge configuration. As usual, the field lines are perpendicular to the conducting surfaces at all points, and the equipotential surfaces are perpendicular to the field lines everywhere.

Quick Quiz 25.10 Consider starting at the center of the left-hand sphere (sphere 1, of radius a) in Figure 25.24 and moving to the far right of the diagram, passing through the center of the right-hand sphere (sphere 2, of radius c) along the way. The centers of the spheres are a distance b apart. Draw a graph of the electric potential as a function of position relative to the center of the left-hand sphere.

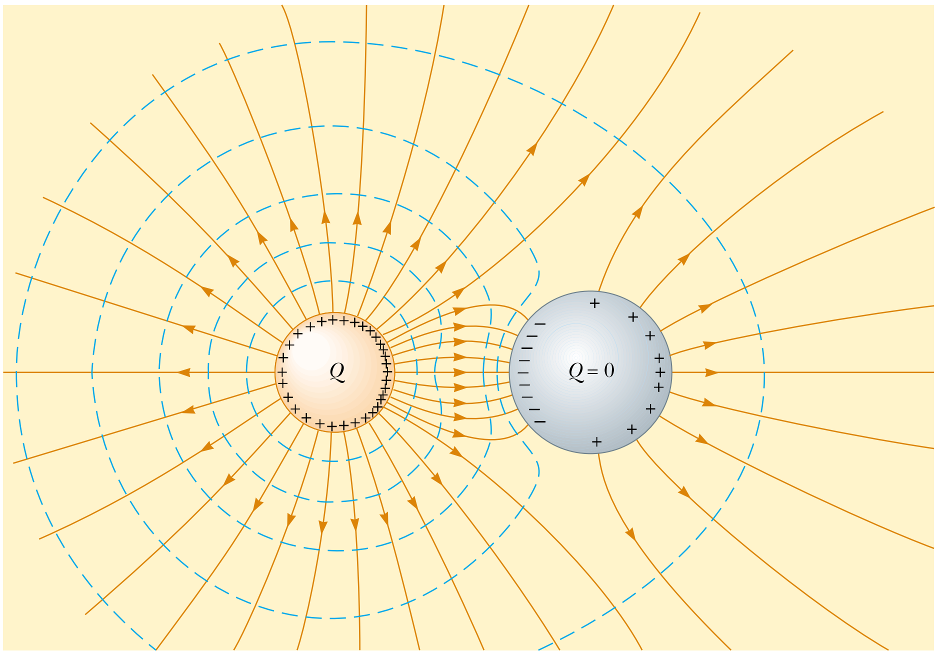
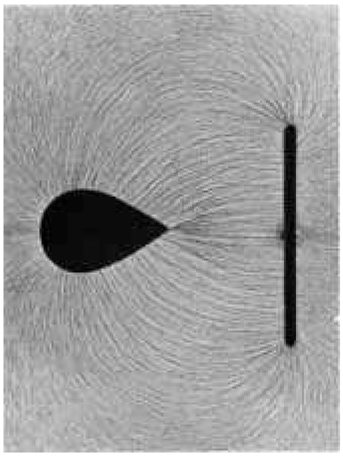


Figure 25.24 The electric field lines (in red-brown) around two spherical conductors. The smaller sphere has a net charge Q, and the larger one has zero net charge. The broken blue curves are intersections of equipotential surfaces with the page.

PITFALL PREVENTION

25.6 Potential May Not Be Zero

The electric potential inside the conductor is not necessarily zero in Figure 25.22, even though the electric field is zero. From Equation 25.15, we see that a zero value of the field results in no *change* in the potential from one point to another inside the conductor. Thus, the potential everywhere inside the conductor, including the surface, has the same value, which may or may not be zero, depending on where the zero of potential is defined.



Courtesy of Harold M. Waage, Princeton University

Figure 25.23 Electric field pattern of a charged conducting plate placed near an oppositely charged pointed conductor. Small pieces of thread suspended in oil align with the electric field lines. The field surrounding the pointed conductor is most intense near the pointed end and at other places where the radius of curvature is small.