PHIL131 Study

▼ Sites

Tree Proof Generator

Enter a formula of standard propositional, predicate, or modal logic. The page will try to find either a countermodel or a tree proof (a.k.a. semantic tableau). Examples (click!): If you want to test an argument with premises and conclusion, use |= to separate the premises from the conclusion, and use commas to separate the

⊨ https://www.umsu.de/trees/

Truth Table Generator

This page contains a JavaScript program which will generate a truth table given a well-formed formula of truth-functional logic. You can enter multiple formulas separated by commas to include more than one formula in a single table (e.g. to test for entailment).

https://mrieppel.net/prog/truthtable.html

Logic Calculator: Truth Tables

This truth-table calculator for classical logic shows, well, truth-tables for propositions of classical logic. Featuring a purple munster and a duck, and optionally showing intermediate results, it is one of the better instances of its kind. Use the buttons below (or your keyboard) to enter a proposition, then gently touch the duck

https://www.erpelstolz.at/gateway/TruthTable.html

Argument Structure

Argument Structure

What is an **Argument::**

An argument is a **sequence of statements** of which one is intended as a conclusion and the others, the premises, are intended to **prove** or at least **provide some evidence** for the conclusion.

Conclusion Indicators:

- Therefore
- Thus
- Hence
- So
- For this reason
- Accordingly
- Consequently
- This being so
- It follows that
- The moral is
- Which proves that
- Which means that
- From which we can infer that
- As a result

- In conclusion

Premise Indicators:

- For
- Since
- Because
- Assuming that
- Seeing that
- Granted that
- This is true because
- The reason is that
- For the reason that
- In view of the fact that
- It is a fact that
- As shown by the fact that
- Given that
- Inasmuch as
- One cannot doubt that

Formal Logic:: is the study of argument forms, abstract patterns **common to many different arguments.**

Informal Logic:: is the study of **particular arguments in natural language** and the **contexts** in which they occur

What is the difference between Formal vs Informal Logic :: Whereas formal logic emphasizes **generality and theory,** informal logic concentrates on **practical argument analysis.**

Natural Language:: Natural Language is any language that has evolved naturally in humans through use and repetition without conscious planning or premeditation.

Deductive Logic:: (Necessary Consequence)

Inductive Logic :: (Probable Consequence)

Premise::

Conclusion::

Arguments::

Complex Arguments:: Premises intended as conclusions from previous premises are called nonbasic premises or intermediate conclusions, otherwise basic premises or assumptions.

Principle of charity:: The primary constraint governing the interpolation of premises and conclusions is the principle of charity: in formulating implicit statements, give the arguer the benefit of the doubt; try to make the argument as strong as possible while remaining faithful to what you know of the arguer's thought.

Argument Diagram::

Dependent/Conjunction Argument::

Convergent/Independent Argument:: An argument is convergent when different statements function as independent reasons for the conclusion.

Implicit Statements:: In an incomplete argument when statements are premises suggest the implicit conclusion.

When to use ' ' (single quotation mark)?:

When something is mentioned

Use and Mention:

> An expression that is mentioned rather than used should be enclosed in **single quotation marks. ' '**

Argument Evaluation

Four Argument Evaluation Criteria

1. Check are all the **premises true?**
2. is the **conclusion at least probable,** given the (assumed) truth of the premises?
3. are the **premises relevant** to the conclusion?
4. is the conclusion **vulnerable to new evidence**?

ergo:: ∴ Therefore

Deductive Argument::

Whose conclusion follows **necessarily** from its basic premises. In other words, if its **impossible for its conclusion to be false** while its basic premises are all true.

Inductive Argument::

Whose conclusion is **not necessary** relative to the premises: there is a certain probabilty that the conclusion is true if the premises are, **but there is also a probabilty that it is false.**

Inductive Probability

The probabilty of a conclusion, given a set of premises, is called **inductive probability**.

What is the Inductive Probabilty of a Deductive Argument::

The inductive probabilty of an **deductive argument** is **1**

What is the Inductive Probabilty of a Inductive Argument::

The inductive probabilty of an **inductive argument** is typically **less than 1**

Valid Deductive Arguments:

Arguments such that their conclusion cannot be false as long as their basic premises are true

Invalid Deductive Arguments::

They are arguments which purport to be deductive but in fact are not.

Deductiveness and inductiveness are independent of the **actual truth or falsity** of the premises and conclusion.

Sound Argument:

A deductive argument all of whose **basic premises are true** is said to be **sound.** A sound argument establishes **with certainty** that its conclusion is true.

In Deductive Arguments **impossibleness** for the conclusion when all premises are true, is related to **"Logically Impossible"** not **"Impossible in practice"**

Strongly Inductive

When the inductive probability of an inductive argument is **high**, we say that the reasoning of the argument is **Strongly Inductive**

Weakly Inductive

When the inductive probability of an inductive argument is **low**, we say that the reasoning of the argument is **Weakly Inductive**

With regard to complex nonconvergent arguments, if one or more of the steps is weak, then usually the inductive probability of the argument as a whole is low

if all the steps of a complex nonconvergent argument are strongly inductive or deductive, then (if there are not too many of them) the inductive probability of the whole is usually farily high.

The inductive probability of a convergent argument is usually at least as high as the inductive probability of its strongest branch.

Only way to **ensure** an accurate judgment of inductive probability is to examine directly the proabability of the **conclusion** given the **basic premises,** ignoring the intermediate steps.

If all the steps of a complex argument is deductive, then so is the argument as a whole.

A logically necessary statement is a satement whose very conception or meaning requires its truth; its falsehood, in other words, is logically impossible.

In logically necessary statements if one occurs as the conclusion of an argument, then the argument is automatically deductive, regardless of the nature of the premises.

Logically Necessary statements cannot be false under any conditions.

A set of statements is **inconsistent** if it is logically impossible for all of them to be true simultaneously

Any argument with **inconsistent premises** is deductive, regardless of what the conclusion says. Hence, *any conclusion follows deductively from inconsistent premises.*

In classical logic, in which inductive probability and revelance are considered as separate factors in argument evaluation.

Regardless of the nature of the premises, a deductive argument remains deductive if new premises are added.

An inductive argument, can be either strengthened or weakened by the addition of new premises.

Fallacy of Suppressed Evidence::

> if an evidence ommitted tells stongly against the conclusion it commits the Fallacy of Suppressed Evidence

Total Evidence Condition:

> if an argument is inductive, its premises must contain all known evidence that is relevant to the conclusions.

Differences between, Implicit Premises & Suppressed Evidence

Implicit premises are assumptions that the author of an argument intends the audience to take for granted.

Suppressed evidence, by contrast, is information that the author has deliberately concealed or unintentionally ommiteed.

(Implicit Assumptions are part of the author's argument). Suppressed evidence is not.

Propositional Logic

The letters ‘P’ and ‘Q’ fuction as placeholders for **declarative sentences**, aka **sentence letters.**

Modus ponendo ponens

Mode that affirms by affirming

If P, then Q.

P

∴ Q

Modus Tollendo Tollens

Mode that denies by denying

If P, then Q

it is not the case that Q

∴ It is not the case that P

Valid vs Invalid Forms

- The argument froms modus ponens and modus tollens are **valid**. By contrast, **affirming the consequent** (AC), is **invalid:**

- (AC) If P, then Q. Q Hence: P
- Some intances of this from are valid arguments, others are not.

Affirming the Consequent (AC):

- Any form that has even one invalid instance is invalid, the validity proves the invalidity of affirming the consequent (AC)
- Though AC also has valid instance these are not valid as a result of being instances of AC.

If P, then Q.

Q

$\therefore P$

It is INVALID

Logical Operators | Connectives

Domain of argument forms consisting of sentence letters combined with one or more or of the following five expressions:

- ‘it is not the case that’
- ‘and’
- ‘either … or’
- ‘if … then’
- ‘if and only if’

Very many different forms are constructedible from these simple expressions; some of them are among the most widely used <u>patterns of reasoning</u>

- Operators are unary vs binary

1. It is not the case that … **Negation** (unary)
2. ‘… and … ‘, **Conjunction** (binary)
3. ‘either … or …’ **Disjuntion** (binary)
4. ‘If …, then …’, **Conditional** (binary)

5. '… if and only if …' **Bi-conditional** (binary)

Antecedent & Consequent

"x follows y" means the same as "y precedes x"

- 'P only if Q' == 'If P, then Q'

$$P \Rightarrow Q$$

P: Antecedent , Q: Consequence

NL (Natural Language): 'Fire only if oxygen'

LF (Logical Form): 'If P [fire], then Q [oxygen]'

Bi-Conditional

$$\Longleftrightarrow$$

- '… if and only if …'
- 'P if and only if Q'
- '(If P, then Q) and (if Q then P)'
- Logically, a definition of a (technical) term "comes out as" a bi- conditional

Logical Operator	*Symbol*	*Logical Operator*	*Alternative Symbol(s)*
It is not the case that	$\sim$	It is not the case that	$-$ or $\neg$
And	&	And	$\cdot$ or $\wedge$
Either . . . or	$\vee$	Either . . . or	none
If . . . then	$\rightarrow$	If . . . then	$\supset$
If and only if	$\leftrightarrow$	If and only if	$\equiv$

TERM	SYMBOL	LATEX
1. there exists at least one	$\exists$	\exists
2. there exists one and only one	$\exists!$	\exists!
3. there is no	$\nexists$	\nexists
4. for all	$\forall$	\forall
5. not (logical not)	$\neg$	\neg
6. or (logical or)	$\lor$	\lor
7. division	$\div$	\div
8. and (logical and)	$\land$	\land
9. implies	$\implies$	\implies
10. right implication	$\Rightarrow$	\Rightarrow
11. is implied by (only if)	$\Longleftarrow$	\Longleftarrow
12. left implication	$\Leftarrow$	\Leftarrow
13. if and only if, iff	$\iff$	\iff
14. equivalence	$\Leftrightarrow$	\Leftrightarrow

Sign	LaTeX Command	Name of Sign
$\neg$	`$\neg$`	unary not, negation
$\wedge$	`$\wedge$`	caret
$\lor$	`$\lor$`	the vee or wedge
$\leftarrow$	`$\leftarrow$`	left arrow
$\rightarrow$	`$\rightarrow$`	right arrow
$\leftrightarrow$	`$\leftrightarrow$`	double arrow
$\vdash$	`$\vdash$`	right turnstile ”therefore”
$\dashv$	`$\dashv$`	left turnstile
$\models$	`$\models$`	models or semantic entailment

If you are using a different system, here are several other operators:

Sign	LaTeX Command	Name of Sign
$\sim$	`$\mathord{\sim}$`	unary not (the 'tilde')
&	`\& (in tables)`	ampersand
$\supset$	`$\supset$`	horseshoe
$\equiv$	`$\equiv$`	tribar
$\otimes$	`$\otimes$`	circled X

In this horizontal format, we use the symbol ‘ $\vdash$ ’ instead of the triple dots. This symbol is called a **turnstile** or an **assertion sign**.

$$\begin{array}{ll} & P \vee Q \\ & {\sim}Q \\ \therefore & P \end{array}$$

$$P \vee Q, \sim Q \vdash P$$

Formulas

Formulas are all **constructed** from the following three sets of sumbols:

- Sentence Letters: Any capital letter; may have numerical subscripts ‘S_1’, ‘S_2’, ‘S_3’
- Logical Operators: \`~\` , &, v, ->, <->

- Brackets: (,)
- These three sets of symbols constitute the **vocabulary** of the language of **propositional logic**

Vocabulary Contains Logical vs non-logical symbols

- Logical Symbols
  - Logical Operators: ~, &, v, $\rightarrow$, $\leftrightarrow$
  - Brackets: (,)
- Non-logical Symbols
  - Sentence letters: 'S_1', 'S_2', 'S_3'
- A **formula** of thel anguage of propositional logic is **any sequence of elements** of the vocavulary, including **non-sense sequences**.

Well-formed formula (wff)

- Formation rules
  1. Any sentence letter is a wff
  2. If $\phi$is a wff, then so is $\sim \phi$
  3. if ϕ and ψ are wffs, then so are $(\phi \& \psi)$, $(\phi \vee \psi)$,$(\phi \Rightarrow \psi)$, $(\phi \iff \psi)$
- Read the greek letters as phi and psi
- Anything not asserted to be wff by three rules is not a wff.

Atomic/molecular/compound

- Sentence letters are called atomic wffs; all other wffs are said to be molecular or compound

A subwff is a part of a wff which is itself a wff. Thus 'P' is a subwff of '(P & Q)' and 'R' is a subwwf of 'R'

Scope

A particular occurence of an operator in a wff, together with the part of the wff to which that occurence of the operator applies, is called the scope of that occuraence of the operator.

The scope of an occurence of an operator in a wff is the smallest subwff that contains that occurence

Main operator

Each wff has exactly one operator whose scope is the entire wff. This is called the **main operator** of that wff.

Bivalence: two truth values (T, F)

- Vague of spurious statements, statements about the future, statements about infinite processes, or paradoxical statements may have truth values other than True and False, or no truth value at all, and hence are not bivalent. Some have also argued that certain statements may have both truth values. There are logics for these, too!

Truth Table: Negation

ϕ	$\sim \phi$
T	F
F	T

Truth Table: Conjuction

ϕ	ψ	$\phi \& \psi$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table: Disjunction

ϕ	ψ	$\phi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

Truth table : Exclusive Disjunction

ϕ	ψ	$\phi \otimes \psi$
T	T	F
T	F	T
F	T	T
F	F	F

Truth Table: Conditional

ϕ	ψ	$\phi \Rightarrow \psi$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table: Bi-Conditional

ϕ	ψ	$\phi \iff \psi$
T	T	T
T	F	F
F	T	F
F	F	T

P	Q	$(P$	$\Rightarrow$	$Q)$	$\&$	$(Q$	$\Rightarrow$	$P)$
T	T	T	T	T	T	T	T	T
T	F	T	F	F	F	F	T	T
F	T	F	T	T	F	T	F	F
F	F	F	T	F	T	F	T	F

Functional Completeness

- It can be shown that as long as we have **negation** together with either **conjunction, disjunction,** or **material conditional,** we can **express every other** logical operator, at least every logical operator whose meaning can be represented by a **truth table.**

- Truth-functional accout of meaning: The distribution of truth-values under the main operator is treated as the meaning of the statement.

Construction Principle for truth tables for wffs

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Tautology

True in every possible situation

Contradiction (inconsistent)

False in every possible situation

Contingent Formulas

Formulas/Statements that are true and false at some points in the truth talble are said to be **truth-functionally contingent.**

‘Neither … nor’

$$(\sim P \& \sim Q) = \sim (P \vee Q)$$

Truth table is in effect an exhaustive list of possible situations.

Regutation Tree

An open path is any path of a tree which has not been ended with an ‘X ’. Paths which have been ended with an ‘X ’ are said to be closed.

Table 3-1 Refutation Tree Rules

Negation ($\sim$): If an open path contains both a formula and its negation, place an ‘X’ at the bottom of the path.

Negated Negation ($\sim\sim$): If an open path contains an unchecked wff of the form $\sim\sim\phi$, check it and write ϕ at the bottom of every open path that contains this newly checked wff.

Conjunction (&): If an open path contains an unchecked wff of the form $\phi \& \psi$, check it and write ϕ and ψ at the bottom of every open path that contains this newly checked wff.

Negated Conjunction ($\sim$&): If an open path contains an unchecked wff of the form $\sim(\phi \& \psi)$, check it and split the bottom of each open path containing this newly checked wff into two branches, at the end of the first of which write $\sim\phi$ and at the end of the second of which write $\sim\psi$.

Disjunction ($\vee$): If an open path contains an unchecked wff of the form $\phi \vee \psi$, check it and split the bottom of each open path containing this newly checked wff into two branches, at the end of the first of which write ϕ and at the end of the second of which write ψ.

Negated Disjunction ($\sim\vee$): If an open path contains an unchecked wff of the form $\sim(\phi \vee \psi)$, check it and write both $\sim\phi$ and $\sim\psi$ at the bottom of every open path that contains this newly checked wff.

Conditional ($\rightarrow$): If an open path contains an unchecked wff of the form $\phi \rightarrow \psi$, check it and split the bottom of each open path containing this newly checked wff into two branches, at the end of the first of which write $\sim\phi$ and at the end of the second of which write ψ.

Negated Conditional ($\sim\rightarrow$): If an open path contains an unchecked wff of the form $\sim(\phi \rightarrow \psi)$, check it and write both ϕ and $\sim\psi$ at the bottom of every open path that contains this newly checked wff.

Biconditional ($\leftrightarrow$): If an open path contains an unchecked wff of the form $\phi \leftrightarrow \psi$, check it and split the bottom of each open path containing this newly checked wff into two branches, at the end of the first of which write both ϕ and ψ, and at the end of the second of which write both $\sim\phi$ and $\sim\psi$.

Negated Biconditional ($\sim\leftrightarrow$): If an open path contains an unchecked wff of the form $\sim(\phi \leftrightarrow \psi)$, check it and split the bottom of each open path containing this newly checked wff into two branches, at the end of the first of which write both ϕ and $\sim\psi$, and at the end of the second of which write both $\sim\phi$ and ψ.

The Propositional Calculus

Proof strategies:

If the conclusion is a(n):	*Then do this:*
Atomic formula	If no other strategy is immediately apparent, hypothesize the negation of the conclusion for ~I. If this is successful, then the conclusion can be obtained after the ~I by ~E.
Negated formula	Hypothesize the conclusion without its negation sign for ~I. If a contradiction follows, the conclusion can be obtained by ~I.
Conjunction	Prove each of the conjuncts separately and then conjoin them with &I.
Disjunction	Sometimes (though not often) a disjunctive conclusion can be proved directly simply by proving one of its disjuncts and applying ∨I. Otherwise, hypothesize the negation of the conclusion and try ~I.
Conditional	Hypothesize its antecedent and derive its consequent by →I.
Biconditional	Use →I twice to prove the two conditionals needed to obtain the conclusion by ↔I.

Equivalances

Equivalence	*Name*
$\sim(P \,\&\, Q) \leftrightarrow (\sim P \vee \sim Q)$	De Morgan's law (DM)
$\sim(P \vee Q) \leftrightarrow (\sim P \,\&\, \sim Q)$	De Morgan's law (DM)
$(P \vee Q) \leftrightarrow (Q \vee P)$	Commutation (COM)
$(P \,\&\, Q) \leftrightarrow (Q \,\&\, P)$	Commutation (COM)
$(P \vee (Q \vee R)) \leftrightarrow ((P \vee Q) \vee R)$	Association (ASSOC)
$(P \,\&\, (Q \,\&\, R)) \leftrightarrow ((P \,\&\, Q) \,\&\, R)$	Association (ASSOC)
$(P \,\&\, (Q \vee R)) \leftrightarrow ((P \,\&\, Q) \vee (P \,\&\, R))$	Distribution (DIST)
$(P \vee (Q \,\&\, R)) \leftrightarrow ((P \vee Q) \,\&\, (P \vee R))$	Distribution (DIST)
$P \leftrightarrow \sim\sim P$	Double negation (DN)
$(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)$	Transposition (TRANS)
$(P \rightarrow Q) \leftrightarrow (\sim P \vee Q)$	Material implication (MI)
$((P \,\&\, Q) \rightarrow R) \leftrightarrow (P \rightarrow (Q \rightarrow R))$	Exportation (EXP)
$P \leftrightarrow (P \,\&\, P)$	Tautology (TAUT)
$P \leftrightarrow (P \vee P)$	Tautology (TAUT)

Ten Basic Inference Rules

Table 4-3 The Ten Basic Rules

Negation introduction (~I)	Given a derivation of an absurdity from a hypothesis ϕ, discharge the hypothesis and infer $\sim\phi$.
Negation elimination (~E)	From a wff of the form $\sim\sim\phi$, infer ϕ.
Conditional introduction ($\rightarrow$I)	Given a derivation of a wff ψ from a hypothesis ϕ, discharge the hypothesis and infer $\phi \rightarrow \psi$.
Conditional elimination ($\rightarrow$E)	From a conditional and its antecedent, infer its consequent.
Conjunction introduction (&I)	From any wffs ϕ and ψ, infer the conjunction $\phi \,\&\, \psi$.
Conjunction elimination (&E)	From a conjunction, infer either of its conjuncts.
Disjunction introduction ($\vee$I)	From a wff ϕ, infer the disjunction of ϕ with any wff.
Disjunction elimination ($\vee$E)	From wffs of the forms $\phi \vee \psi$, $\phi \rightarrow \chi$, and $\psi \rightarrow \chi$, infer χ.
Biconditional introduction ($\leftrightarrow$I)	From wffs of the forms $\phi \rightarrow \psi$ and $\psi \rightarrow \phi$, infer $\phi \leftrightarrow \psi$.
Biconditional elimination ($\leftrightarrow$E)	From a wff of the form $\phi \leftrightarrow \psi$, infer either $\phi \rightarrow \psi$ or $\psi \rightarrow \phi$.

Important Derived Rules

Table 4-4 Important Derived Rules

Modus tollens (MT)	From wffs of the forms $\phi \rightarrow \psi$ and $\sim\psi$, infer $\sim\phi$.
Hypothetical syllogism (HS)	From wffs of the forms $\phi \rightarrow \psi$ and $\psi \rightarrow \chi$, infer $\phi \rightarrow \chi$.
Absorption (ABS)	From a wff of the form $\phi \rightarrow \psi$, infer $\phi \rightarrow (\phi \,\&\, \psi)$.
Constructive dilemma (CD)	From wffs of the forms $\phi \vee \psi$, $\phi \rightarrow \chi$, and $\psi \rightarrow \omega$, infer $\chi \vee \omega$.
Repeat (RE)	From any wff ϕ, infer ϕ.
Contradiction (CON)	From wffs of the forms ϕ and $\sim\phi$, infer any wff.
Disjunctive syllogism (DS)	From wffs of the forms $\phi \vee \psi$ and $\sim\phi$, infer ψ.
Theorem introduction (TI)	Any substitution instance of a theorem may be introduced at any line of a proof.
Equivalence introduction (Abbreviation used depends on equivalence used; see Table 4-2, the table of equivalences.)	If ϕ and ψ are interderivable and ϕ is a subwff of some wff χ, from χ infer the result of replacing one or more occurrences of ϕ in χ by ψ.

Propositional Calculus is a different method for establishing deductive validity without explicit reference to truth values

Guidelines on **Hypothetical Reasoning**

Several important guidelines are to be observed when we use hypothetical reasoning:

(1) *Each hypothesis introduced into a proof begins a new vertical line.* This line continues downward until the hypothesis is discharged by application either of $\rightarrow$I or of $\sim$I (the last rule we shall introduce).

(2) *No occurrence of a formula to the right of a vertical line may be cited in any rule applied after that vertical line has ended.* This is to ensure that a formula derived under a hypothesis is not surreptitiously used after that hypothesis has been discharged. Thus, for example, though 'P' appears at line 3 in Problem 4.22, we must derive it anew at line 9 to show that it follows from the second hypothesis, '$P \& R$'. We could not use it (or, indeed, any of lines 2 to 6) in proving '$(P \& R) \rightarrow (P \& (Q \vee R))$', since these lines represent a "logical fantasy" based on the hypothesis of line 2, which is no longer in effect after line 6.

(3) *If two or more hypotheses are in effect simultaneously, then the order in which they are discharged must be the reverse of the order in which they are introduced.* Thus in Problem 4.20, the hypothesis of line 3 must be discharged before the hypothesis of line 2 is.

(4) *A proof is not complete until all hypotheses have been discharged.* Hypotheses which are not discharged are in effect additional assumptions, introduced illicitly into the proof.

Theorems

Some wffs are provable without making any nonhypothetical assumptions. These are the *theorems* or *laws* of the propositional calculus. (Semantically, they are just the tautologies, i.e., those wffs all of whose instances are logically necessary.) To indicate that a wff is a theorem, we write the symbol '$\vdash$' in front of it. As before, this symbol asserts that the formula on its right is provable using only the formulas on its left as assumptions; hence when there are no formulas on its left, it asserts that the formula on its right is a theorem. The proof of a theorem typically begins with one or more hypotheses, which are later discharged by $\rightarrow$I or $\sim$I.

Equivalences

We shall call a biconditional which is a theorem an *equivalence*. If $\phi \leftrightarrow \psi$ is an equivalence, then ϕ and ψ validly imply one another and are said to be *interderivable*. Thus 'P' and '$\sim\sim P$' are interderivable in view of the equivalence proved in Problem 4.43. To prove an equivalence, we follow the usual strategy for proving biconditionals: prove the two conditionals needed for $\leftrightarrow$I by two separate conditional proofs.

Categorical Logic

Class terms (or: predicates)

Class terms (also called predicates) denotes **classes (sets) of objects**. They are not sentences like in propositional logic they are **placeholders.**

Let S be **subect term** and P be **predicate term.**

1. A and O; E and I are **contradictictories**

Designation	Form
A	**All** *S* are *P*.
E	**No** *S* are *P*.
I	**Some** *S* are *P*.
O	**Some** *S* are **not** *P*.

THE LETTERS (AEIO) DESIGNATE THE FORM

		QUALITY	
		Affirmative	Negative
QUANTITY	Universal	*A*	*E*
	Particular	*I*	*O*

code	quantifier	subject	copula	predicate	type	example
A	All	S	are	P	**universal affirmative**	All humans are mortal.
E	No	S	are	P	**universal negative**	No humans are perfect.
I	Some	S	are	P	**particular affirmative**	Some humans are healthy.
O	Some	S	are not	P	**particular negative**	Some humans are ***not*** clever.

In <u>Venn Diagrams:</u>

1. A shaded are is "empty"
2. An "x" denotes there exist at least one element which is a set-member.
3. non-S is complement of S

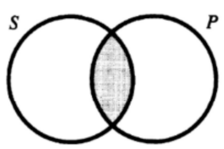


Fig. 5-1

No *S* are *P*

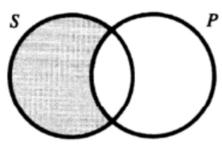


Fig. 5-2

All *S* are *P*

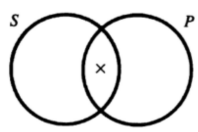


Fig. 5-3

Some *S* are *P*

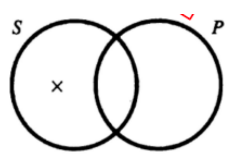


Fig. 5-4

Some *S* are *not P*

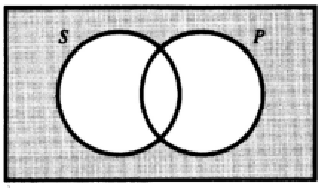


Fig. 5-5

All *non-S* are *P*

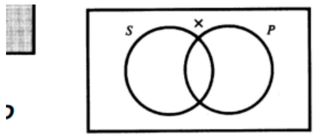


Fig. 5-6

Some *non-S* are *not P*

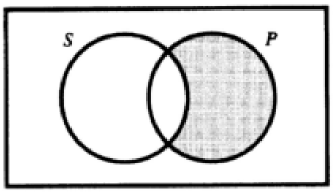


Fig. 5-7

No non-S is P

Immediate:: “nothing in the middle” or “there is nothing that mediates” the inference from one categorical statement to another such statement.

Types of immediate inferences
☐ 1. Negation
☐ 2. Conversion
☐ 3. Contraposition
☐ 4. Obversion

To test the validity of an inference form using Venn diagrams, **diagram the premise**. If, in doing so, you have also **diagramed its conclusion**, then the form is **valid.** ☐ **If not,** it is **invalid.**

In Immediate Inferences, when two categorical statement forms are converses if one results from the other by exchanging subject and predicate terms.

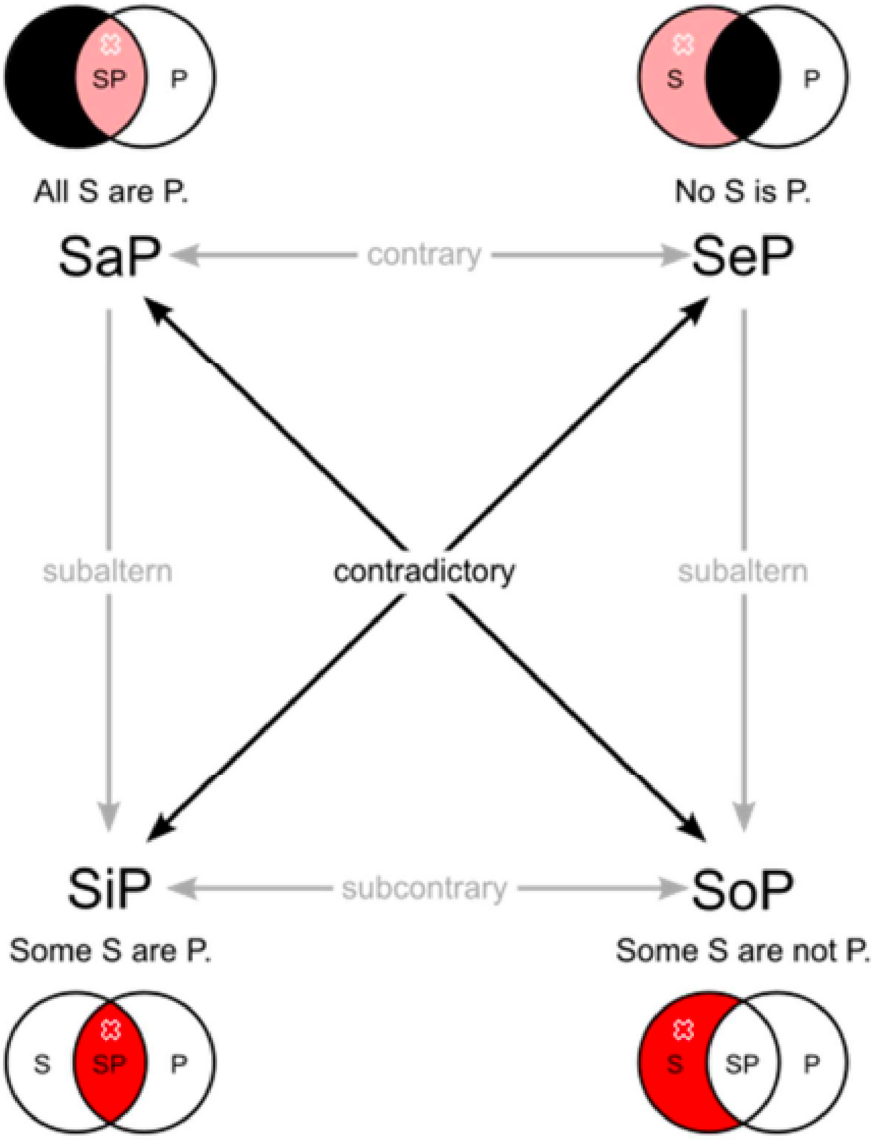
Conversion is a valid immediate inference for forms E and I, but invalid for forms A and O.

Two categorical statements are **contrapositives** if one results from the other when we replace its **subject term** with the **complement** of its **predicate term** and its **predicate term** with the **complement** of its **subject term**.

Contraposition is valid for forms A and O, but invalid for forms E and I.

Obversion: **Changing the quality of a categorical statement** (while **keeping** its **quantity the same**) and **replacing the predicate term by its complement**.

Obverse categorical statements are always logically equivalent



Contraries cannot be true at the same time, but they can be false at the same time.

Categorical syllogisms are **two-premise arguments** consisting entirely of categorical statements.

To diagram a **Categorical syllogistic form**, we draw **three overlapping circles** to represent the **three terms in the premises**, labeled.

Predicate Logic

- **A-Form** (*universal affirmative*)
  - All *S* are *P*.
  - For all *x*, **if** *x* is *S* **then** *x* is *P*.
  - ∀*x*(*Sx*→*Px*) **[∀=universal quantifier]**

NOTICE THE CONDITIONAL

- **E-Form** (*universal negative*)
  - No *S* are *P*.
  - For all *x*, **if** *x* is *S* **then** *it is not the case that x* is *P*.
  - ∀*x*(*Sx*→~*Px*)

- **I-Form** (*particular affirmative*)
  - Some *S* are *P*.
  - For some *x*, *x* is *S* **and** *x* is *P*.
  - There exists an *x* such that *x* is *S* **and** *x* is *P*.
  - ∃*x*(*Sx* & *Px*) **[∃=existential quantifier]**

NOTICE THE CONJUNCTION

- **O-Form** (*particular negative*)
  - Some *S* are not *P*.
  - For some *x*, *x* is *S* **and** *it is not the case that x* is *P*.
  - There exists an *x* such that *x* is *S* **and** *x* is not *P*.
  - ∃*x*(*Sx* & ~*Px*)

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Simple subject-predicate statements which attribute a property to an individual person or thing.

Predicate Calculus

Table 7-2 Basic Rules of Inference for Predicate Logic with Identity*

Universal Elimination ($\forall E$): From a universally quantified wff $\forall\beta\phi$ infer any wff of the form $\phi^{\alpha}/_{\beta}$ which results from replacing each occurrence of the variable β in ϕ by some name letter α.

Universal Introduction ($\forall I$): From a wff ϕ containing a name letter α not occurring in any assumption or in any hypothesis in effect at the line on which ϕ occurs, infer any wff of the form $\forall\beta\phi^{\beta}/_{\alpha}$, where $\phi^{\beta}/_{\alpha}$ is the result of replacing all occurrences of α in ϕ by some variable β not already in ϕ.

Existential Introduction ($\exists I$): Given a wff ϕ containing some name letter α, infer any wff of the form $\exists\beta\phi^{\beta}/_{\alpha}$, where $\phi^{\beta}/_{\alpha}$ is the result of replacing one or more occurrences of α in ϕ by some variable β not already in ϕ.

Existential Elimination ($\exists E$): Given an existentially quantified wff $\exists\beta\phi$ and a derivation of some conclusion ψ from a hypothesis of the form $\phi^{\alpha}/_{\beta}$ (the result of replacing each occurrence of the variable β in ϕ by some name letter α not already in ϕ), discharge $\phi^{\alpha}/_{\beta}$ and reassert ψ. *Restriction:* The name letter α may not occur in ψ, nor in any assumption, nor in any hypothesis that is in effect at the line at which $\exists E$ is applied.

Identity Introduction ($=I$): For any name letter α, we may assert $\alpha = \alpha$ at any line of a proof.

Identity Elimination ($=E$): If ϕ is a wff containing a name letter α, then from ϕ and either $\alpha = \beta$ or $\beta = \alpha$ we may infer $\phi^{\beta}/_{\alpha}$, the result of replacing one or more occurrences of α in ϕ by β.

Fallacies

Fallacies (in the broadest sense) are simply mistakes that occur in arguments and affect their cogency. In Latin, the verb fallere means ‘to deceive’.

6 classes of fallacies.

Fallacies of relevance occur when the premises of an argument have no bearing upon its conclusion. In addition, such fallacies often involve a distractive element which diverts attention away from this very problem.

Circular reasoning is the fallacy of assuming what we are trying to prove.

Semantic fallacies result when the language employed to construct arguments has multiple meanings or is excessively vague in a way that interferes with assessment of the argument.

Inductive fallacies occur when the probability of an argument’s conclusion, given its premises—i.e., its inductive probability—is low, or at least less than the arguer supposes.

Formal fallacies occur when we misapply a valid rule of inference or else follow a rule which is demonstrably invalid.

And finally, there is a class of mistakes traditionally classified as fallacies which consist in an argument’s having *false premises*.

Fallacies of Relevance

Fallacies of relevance occur when the premises of an argument have no bearing upon its conclusion. Such arguments are often called **non sequiturs** (from the Latin phrase 'non sequitur', meaning ''it does not follow")

Ad hominem arguments try to discredit a claim or proposal by attacking its proponents instead of providing a reasoned examination of the proposal itself. 'Ad hominem' means ''against the person.'' Ad hominem arguments come in at least five varieties.

1. **Ad hominem abusive arguments** attack a person's age, character, family, gender, ethnicity, social or economic status, personality, appearance, dress, behavior, or professional, political, or religious affiliations. The implication is that there is no reason to take the person's views seriously.
2. **The fallacy of guilt by association** is the attempt to repudiate a claim by attacking not the claim's proponent, but the company he or she keeps, or by questioning the reputations of those with whom he or she agrees. This is also known as **poisoning the well.**
3. **Tu quoque ("you too") arguments** attempt to refute a claim by attacking its proponent on the grounds that he or she is a hypocrite, upholds a double standard of conduct, or is selective and therefore inconsistent in enforcing a principle. The implication is that the arguer is unqualified to make the claim, and hence that there is no reason to take the claim seriously.
4. **Vested interest arguments** attempt to refute a claim by arguing that its proponents are motivated by the desire to gain something (or avoid losing something). The implication is that were it not for this vested interest, the claim's proponents would hold a different view, and hence that we should discount their argument.
5. **Circumstantial ad hominem fallacies** are sometimes grouped in a single category with vested interest fallacies, but there is a distinction between them. The circumstantial version of the ad hominem fallacy is the attempt to refute a claim by arguing that its proponents endorse two or more conflicting propositions. The implication is that we may therefore safely disregard one or all of those propositions.

Straw Man Argument attempt to refute a claim by confusing it with a less plausible claim (the straw man) and then attacking that less plausible claim instead of addressing the original issue.

Ad baculum arguments (also called appeals to force, appeals to the stick) are attempts to establish a conclusion by threat or intimidation.

Ad verecundiam arguments (appeals to authority) occur when we accept (or reject) a claim merely because of the prestige, status, or respect we accord its proponents (or opponents).

<u>Ad populum</u> arguments (*appeals to the people*) occur when we infer a conclusion merely on the grounds that most people accept it. This fallacy has the form:

$$\begin{array}{ll} & X \text{ says that } P. \\ \therefore & P. \end{array}$$

Ad misericordiam arguments (appeals to pity) ask us to excuse or forgive an action on the grounds of extenuating circumstances. They seek clemency for breaches of duty, or sympathy for someone whose poor conduct or noncompliance with a rule is already established. An appeal to pity may be either legitimate or fallacious, depending on whether or not the allegedly extenuating circumstances are genuinely relevant to the case.

<u>Ad ignorantiam</u> arguments (*appeals to ignorance*) have one of the following two forms:

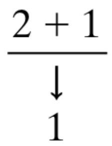
$$\begin{array}{ll} & \text{It has not been proved that } P. \\ \therefore & \sim P. \end{array} \qquad \begin{array}{ll} & \text{It has not been proved that } \sim P. \\ \therefore & P. \end{array}$$

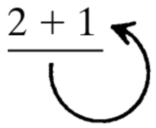
Ignoratio elenchi (missing the point) occurs when the premises of an argument warrant a different conclusion from the n t one the arguer draws. This can be very embarrassing, especially if the conclusion which does follow contradicts or undermines the one actually drawn. The expression ‘‘missing the point’’ is also used as a general catchphrase to describe the fallacies of relevance.

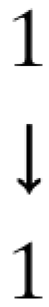
Circular Reasoning

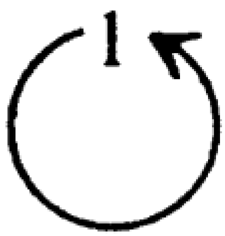
Circular reasoning (also called petitio principii and begging the question) occurs when an argument assumes its own conclusion.

in argument diagrams









Semantic Fallacies

Semantic fallacies occur when the language employed to express an argument has multiple meanings or is excessively vague in ways that interfere with assessment of the argument's cogency.

Ambiguity (or equivocation) is multiplicity of meaning.

Vagueness is indistinctness of meaning, as opposed to multiplicity of meaning

Doublethink is an extreme version of vagueness. In which predecessor and contradicts its successor.

Accent refers to emphases that generate multiple (and often misleading) interpretations. Newspaper headlines, contractual fine print, commercial ‘‘giveaways,’’ and deceptive contest entry forms are frequent sources of fallacies of accent.

Inductive Fallacies

Inductive fallacies occur when the inductive probability of an argument is low, or at least lower than the arguer thinks it is.

Hasty Generalization means fallaciously inferring a conclusion about an entire class of things from inadequate knowledge of some of its members.

Faulty analogy is an inductive fallacy associated with analogical reasoning. In analogical reasoning we assert that object x has certain similarities with object(s) y, and that y has a further property P. We then conclude that x has P. The inductive probability of analogical reasoning depends quite sensitively, however, on the degree and relevance of the similarity. **If the similarity is slight or not particularly relevant, then a fallacy is likely to result.**

The *gambler's fallacy* is an argument of the form:

x has not occurred recently.
$\therefore$ x is likely to happen soon.

‘Falsecause’ is a term covering a variety of logical sins. Most simply, it means confusing a cause with an effect. But it may also mean offering an immediate causal explanation for an event without considering alternatives. Another variant is **post hoc ergo propter hoc** (after this, therefore because of this—often abbreviated to **post hoc**), in which a causal relationship is inferred merely from the temporal proximity of two or more events.

The fallacy of suppressed evidence

This is the fallacy of **ignoring evidence** bearing negatively on an inductively inferred conclusion.

Formal Fallacies

Formal fallacies occur when we misapply a valid rule of inference or else follow a rule which is demonstrably invalid.

Fallacy of denying the antecedent

$$\begin{array}{ll} & R \rightarrow P \\ & \sim R \\ \therefore & \sim P \end{array}$$

Fallacy of affirming the consequent

$$\begin{array}{ll} & P \rightarrow Q \\ & Q \\ \therefore & P \end{array}$$

The fallacy of *composition* occurs when we invalidly impute characteristics of one or more parts of a thing to the whole of which they are parts.[3] This fallacy has the following form:

$$\begin{array}{ll} & p_1, \ldots, p_n \text{ are parts of } w. \\ & p_1, \ldots, p_n \text{ have property } F. \\ \therefore & w \text{ has property } F. \end{array}$$

Fallacy of Division

$$
\begin{array}{ll}
 & w \text{ has property } F. \\
 & p_1, \ldots, p_n \text{ are parts of } w. \\
\therefore & p_1, \ldots, p_n \text{ have property } F.
\end{array}
$$

Fallacies of False Premises

So arguments that commit this fallacy may be valid, but are never sound.

False dichotomy

This fallacy is committed when we make the false assumption that only one of a number of alternatives holds.

A *slippery slope* fallacy occurs when the conclusion of an argument rests upon an alleged chain reaction, suggesting that a single step in the wrong direction will result in a disastrous or otherwise undesirable outcome. We may represent this form of reasoning as follows:

$$
\begin{array}{ll}
 & A_1 \rightarrow A_2 \\
 & A_2 \rightarrow A_3 \\
 & \quad \vdots \\
 & A_n \rightarrow A_{n+1} \\
 & \text{It should not be the case that } A_{n+1}. \\
\therefore & \text{It should not be the case that } A_1.
\end{array}
$$

Induction

his holds in all cases; the negation of a weak statement is strong and the negation of a strong statement is weak.

The strength of a statement is approximately inversely related to what is called its a priori probability, that is, probability prior to or in the absence of evidence.

The stronger a statement is, the less
inherently likely it is to be true; the weaker it is, the more inherently probable it is.

Rule 1: If statement A deductively implies statement B but B does not deductively imply A, then A is stronger than B.

Rule 2: If statement A is logically equivalent to statement B (i.e., if A and B deductively imply one another), then A and B are equal in strength.

Statistical Syllogism

Inductive arguments are divisible into two types, according to **whether or not they presuppose that**
the universe or some aspect of it is or is likely to be uniform or lawlike.

1. Those which do not require this presupposition may be called **statistical arguments**; the premises of a statistical argument support its conclusion for **purely statistical or mathematical reasons**.
2. Those which do require it we shall call **Humean arguments**

Statistical Syllogism can be represented as follows:ü

n percent of F are G.
x is F.
$\therefore$ x is G.

For n < 50, it is more natural for the argument to take the form

n percent of F are G.
x is F.
$\therefore$ x is not G.

STATISTICAL GENERALIZATION

Statistical syllogism is an inference from statistics concerning a set of individuals to a (probable) conclusion about some member of that set.

The general form of statistical generalization is as follows:

n percent of s randomly selected F are G.
$\therefore$ About n percent of all F are G.

INDUCTIVE GENERALIZATION AND SIMPLE INDUCTION

The general form of the argument is

n percent of s thus-far-observed F are G.
$\therefore$ About n percent of all F are G.

We shall call this form inductive generalization.

<u>Inductive generalizations are therefore Humean inferences.</u>

It is called **simple induction**, **induction by enumeration**, or the **simple predictive inference:**

n percent of the s thus-far-observed F are G.
$\therefore$ If one more F is observed, it will be G.

INDUCTION BY ANALOGY

argument by analogy is a Humean argument. It has the form of

$$F_1x \ \& \ F_2x \ \& \ldots \& \ F_nx$$
$$F_1y \ \& \ F_2y \ \& \ldots \& \ F_ny$$
$$Gy$$
$$\therefore \ Gx$$

Contrary evidence to analogical arguments often takes the form of a **relevant disanalogy**.

MILL'S METHODS

If C is a necessary cause for E, then E will never occur without C, though perhaps C can occur without E

If C is a sufficient cause for E, then C will never occur without E, though there may be cases in which E occurs without C

That is, the effect never
occurs without the cause nor the cause without the effect.

A variable quantity B is causally dependent on a second variable quantity A if a change in A always produces a corresponding change in B

Now, to reiterate, Mill's methods aim to narrow down a list of suspected causes (of one of the four
kinds just described) in order to find a particular cause for an effect E. Each of the four methods listed
below is appropriate to a different kind of cause:

Mill's Method of:	*Rules Out Conditions Suspected of Being:*
Agreement	Necessary causes of E
Difference	Sufficient causes of E
Agreement and difference	Necessary and sufficient causes of E
Concomitant variation	Quantities on which the magnitude of E is causally dependent

If by using the appropriate method we are able to narrow the list of suspected causes down to one
entry, then this entry is a cause of the kind we are looking for.

The Method of Agreement

deductive procedure for ruling out suspected causally necessary
conditions.

to determine which of a list of suspected causally necessary conditions really is
causally necessary for E, we examine a number of different cases of E. If any of the suspected necessary
conditions fails to occur in any of these cases, then it can certainly be ruled out as not necessary for E.

Case	*Circumstances (suspected causes present in this case)*	*Effect*
Patient 1	V_1, V_3, V_4	E
Patient 2	V_1, V_4, V_5	E
Patient 3	V_1, V_2	E
Patient 4	V_1, V_5	E

The Method of Difference

If we are seeking a sufficient cause the method to use is the method of difference.

If cause C ever occurs without E, then C is not sufficient for E.

Case	*Circumstances (suspected sufficient causes)*	*Effect*
Person 1	F_1, F_2, F_3, F_4, F_5	P
Person 2	F_2, F_3, F_4, F_5	None

The Joint Method of Agreement and Difference

Mill's joint method of agreement and difference is a procedure for eliminating items from a list of
suspected necessary and sufficient causes.

If C is a necessary and sufficient cause of E, then C never occurs without E and E never occurs
without C.

if we find any case in which C occurs but E does not or E occurs but C does not, C can be ruled out as a necessary and sufficient cause for E

The Method of Concomitant Variation

A variable is rejected as not responsible for a particular change if that variable remains constant throughout the change.

If all but one of a list of variables remain constant
while the magnitude of an effect changes, then, presuming that the variable responsible for the change
appears on the list, it must be the one which has not remained constant.

SOLVED PROBLEM

9.27 A houseplant exhibits a sudden spurt of growth. We suspect that the variables relevant to its growth rate are these:

S = sunlight
W = water
F = fertilizer
T = temperature

But we observe that only one of these variables, namely, the amount of water the plant receives, has been altered recently. This observation may be schematized as follows:

Case	*Circumstances (variables on which we suspect G to be relevant)*	*Effect*
1	S, F, T, W	G
2	$S, F, T, W+$	$G+$

Here G is the growth rate and the plus signs stand for increases of magnitude. No plus sign indicates no change. Which, if any, of the variables on our list is causally relevant to the observed change in the growth rate of the plant?

SCIENTIFIC THEORIES

A scientific theory is an account of some natural phenomenon which in conjunction with further known facts or conjectures (called auxiliary hypotheses) enables us to deduce consequences which can be tested by observation.

Nevertheless, it is often held that as more and more of the predictions entailed by a theory are verified, the theory itself becomes more probable. This principle may be formulated more precisely as follows:

(P): If E is some initial body of evidence (including auxiliary hypotheses) and C is the additional verification of some of the theory's predictions, the probability of the theory given E & C is higher than the probability of the theory given E alone.