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21801985
EEE321-001
Lab report 2

PLOTS

PART 1

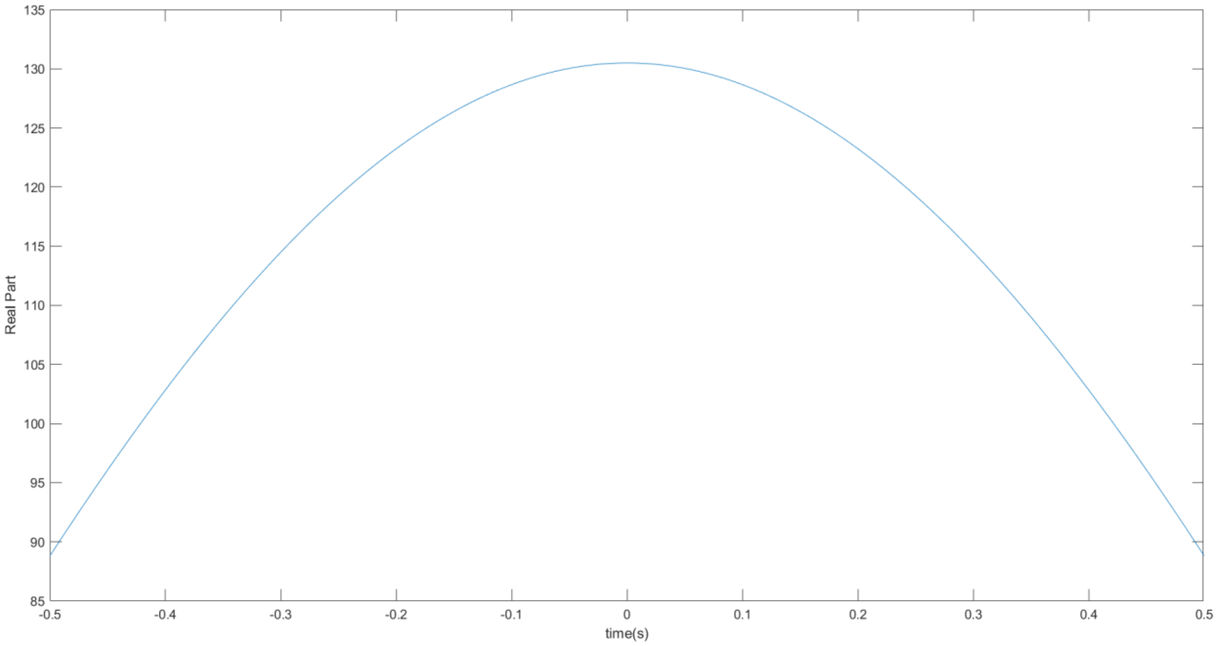

Figure 1: Plot of Real part of the function in Part 1

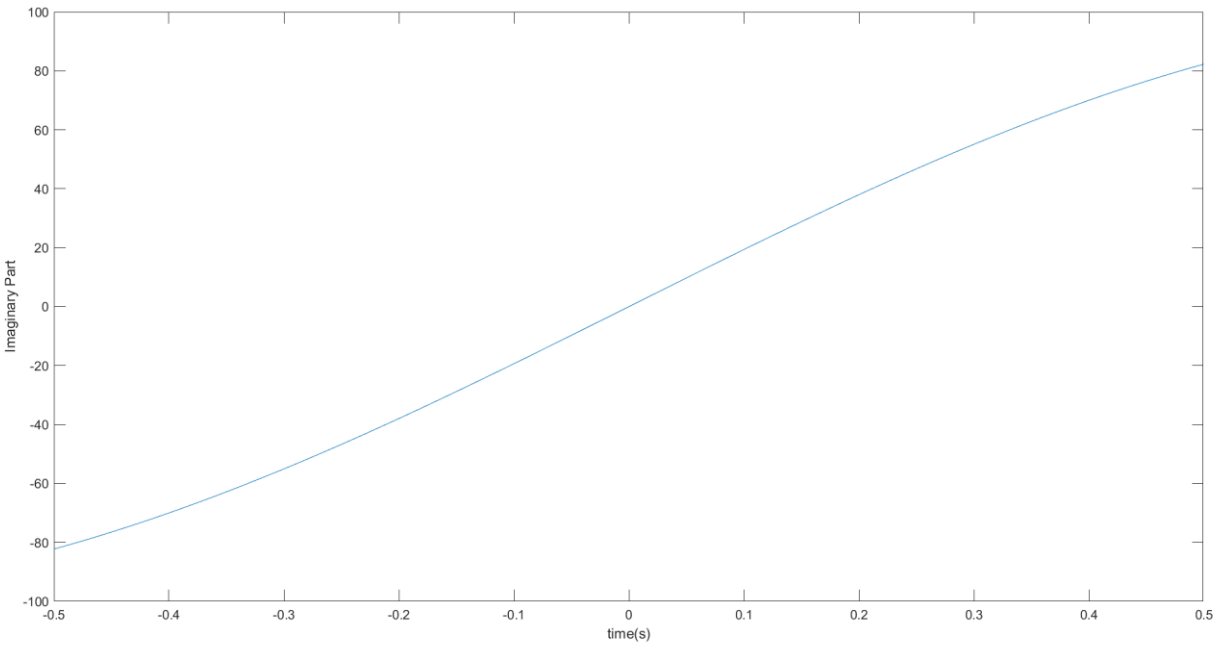

Figure 2: Plot of Imaginary part of the function in Part 1

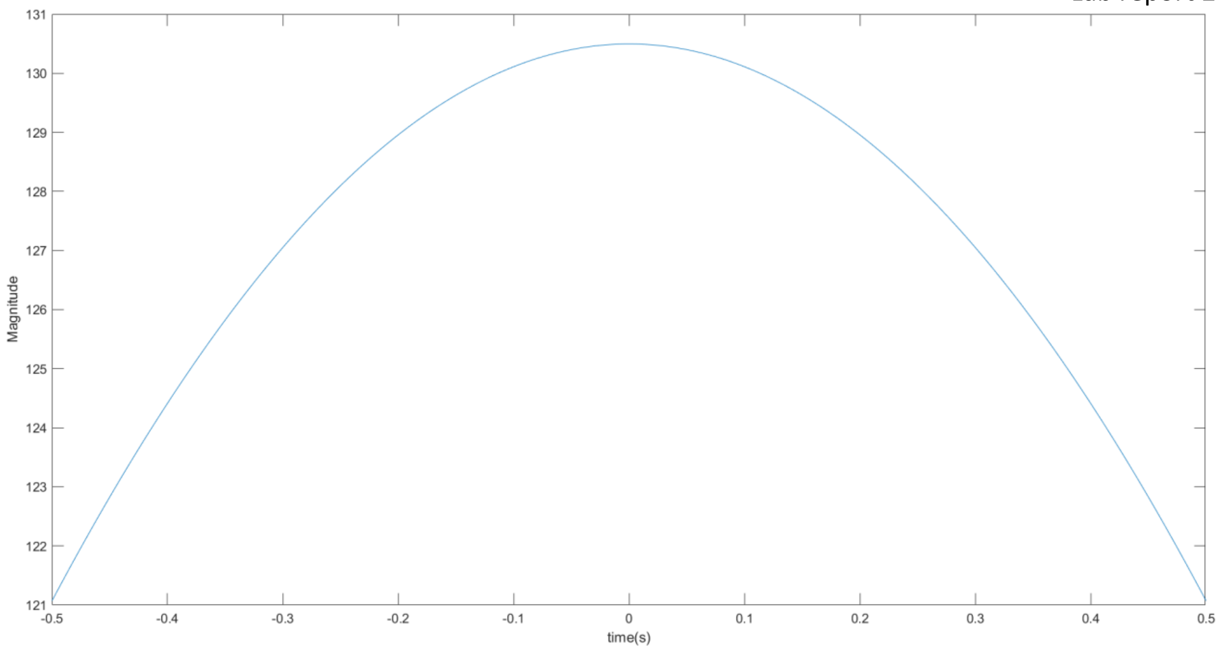

Figure 3: Plot of Absolute value of the function in Part 1

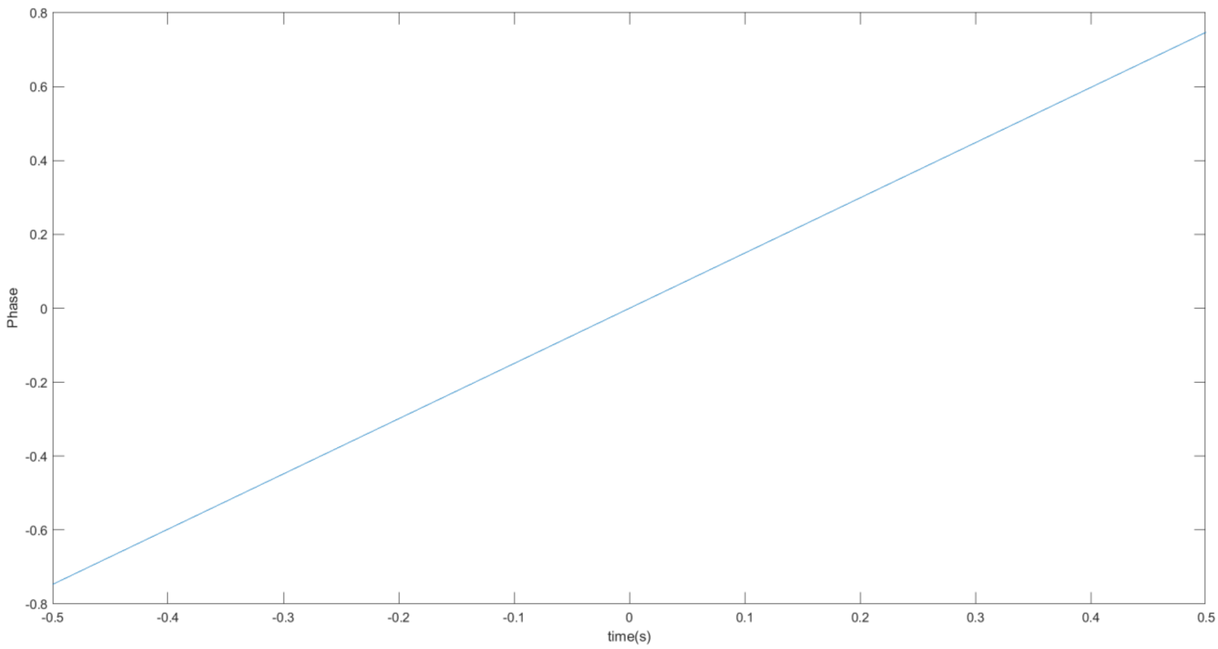

Figure 4: Plot of Phase of the function in Part 1

PART 2 (IS IN THE LAST PAGES ON THE REPORT)

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PART 3

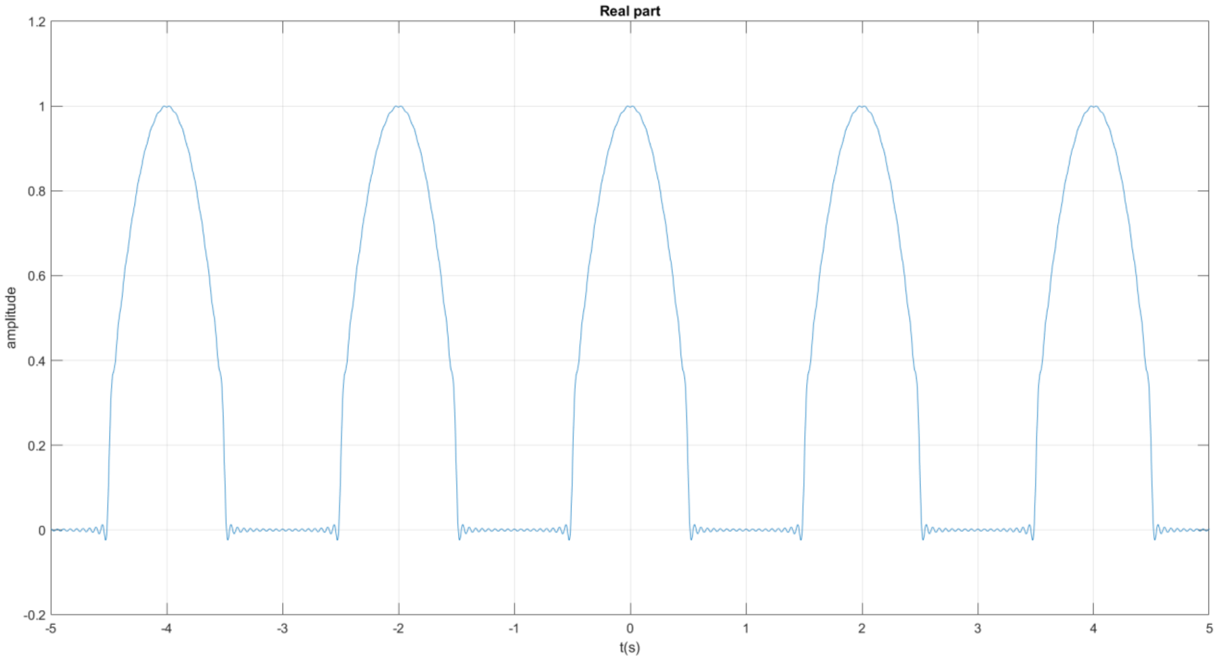

Figure 5: Plot of Real part of the function in Part 3

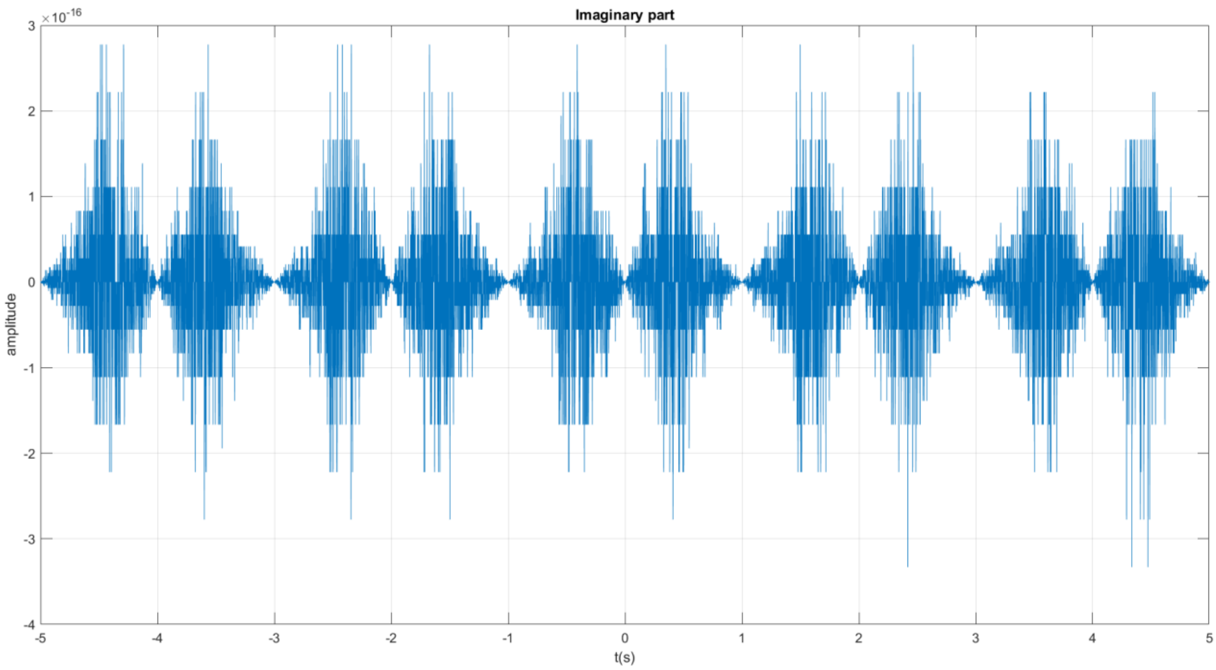

Figure 6: Plot of Imaginary part of the function in Part 3

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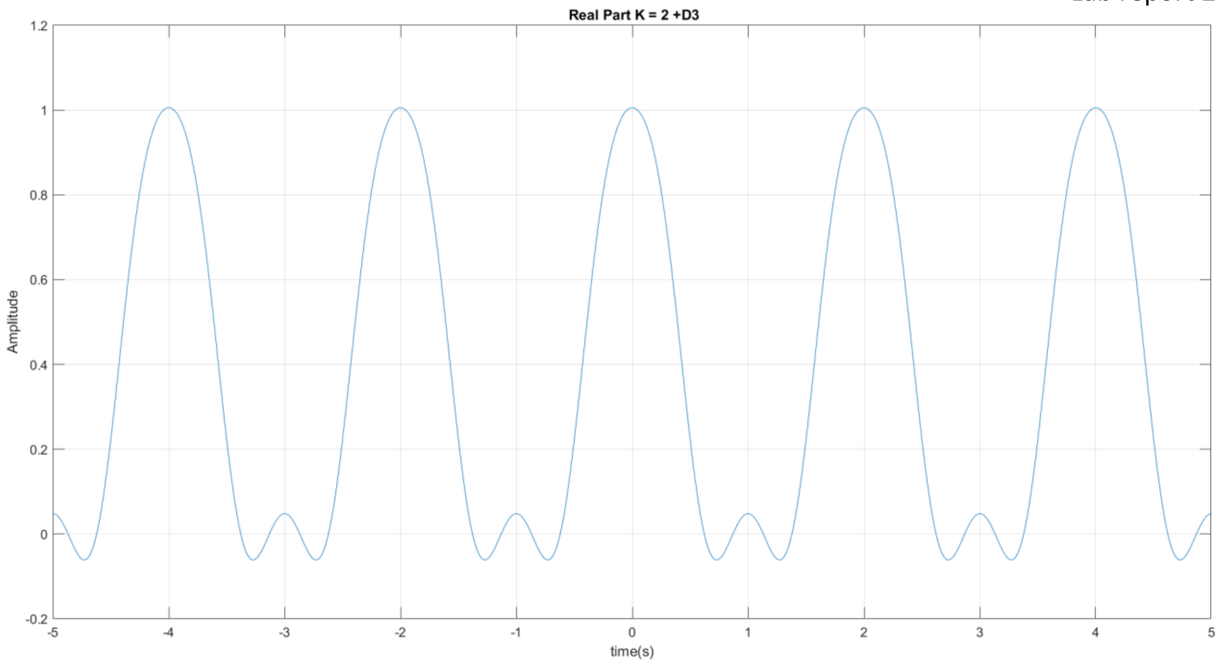

Figure 7: Plot of Real part of the function in Part 3 when K= 2 +D3

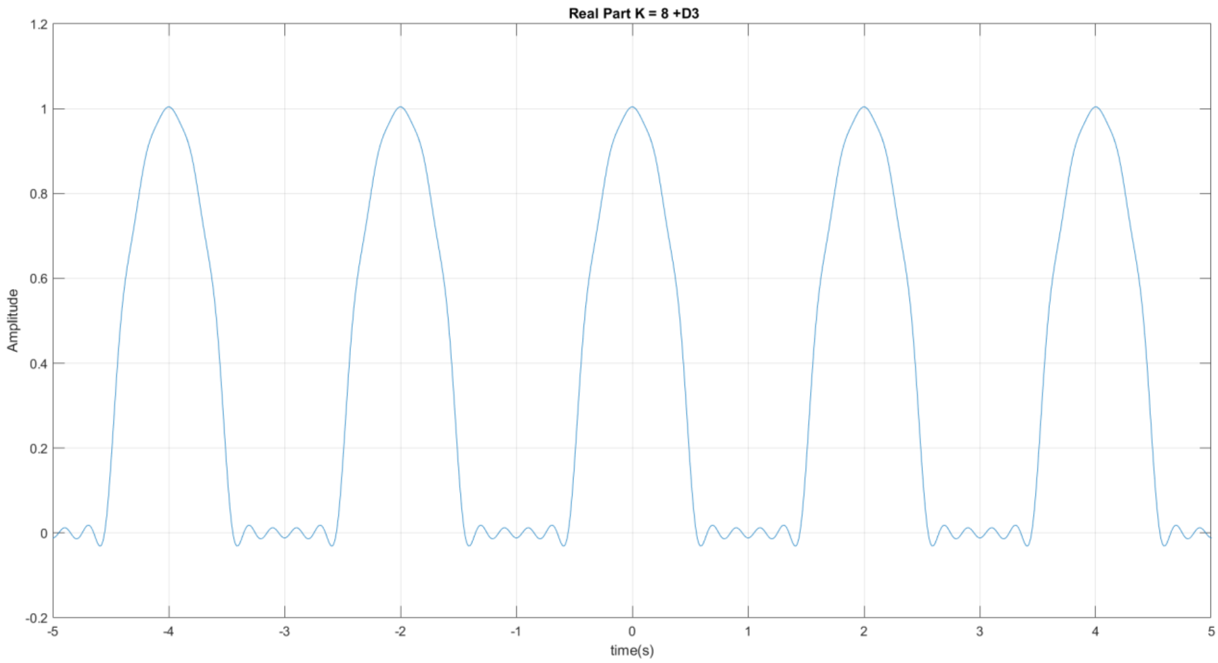

Figure 8: Plot of Real part of the function in Part 3 when K= 8 +D3

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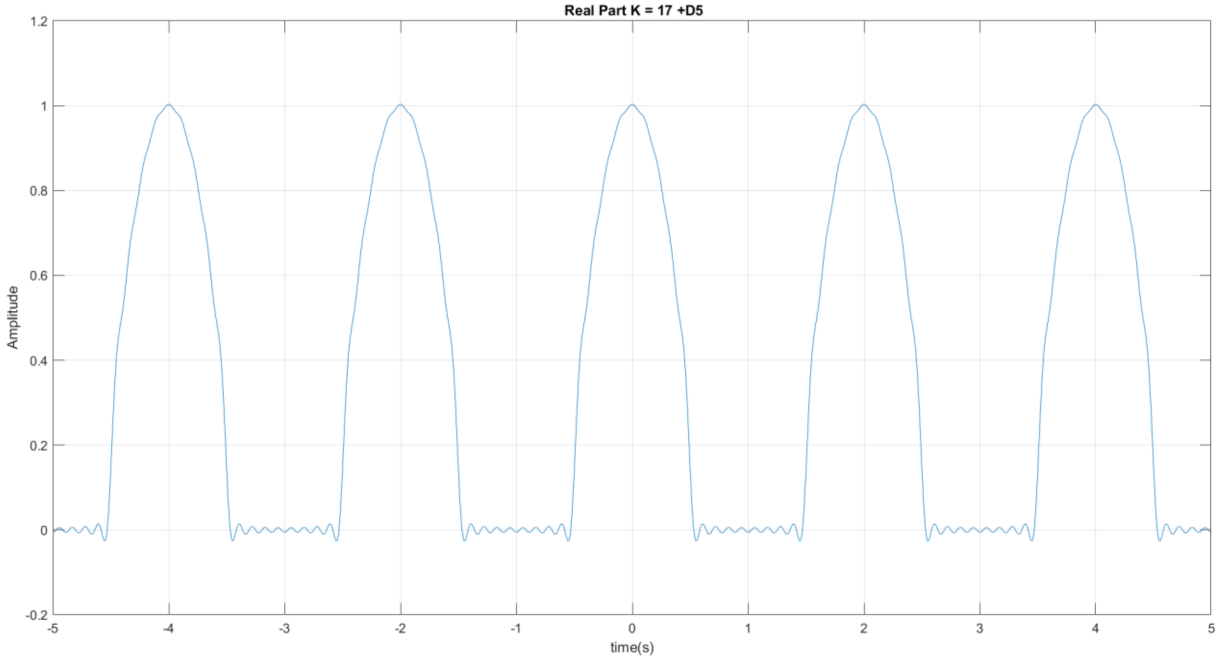

Figure 9: Plot of Real part of the function in Part 3 when K= 17 +D5

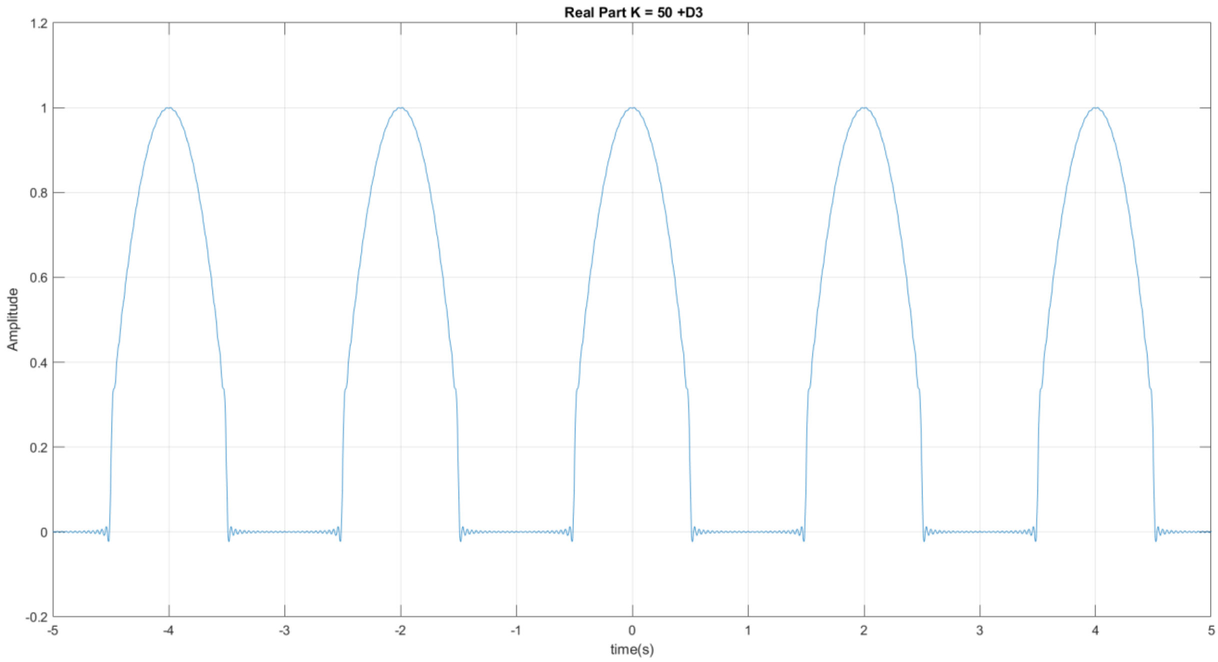

Figure 10: Plot of Real part of the function in Part 3 when K= 50 +D3

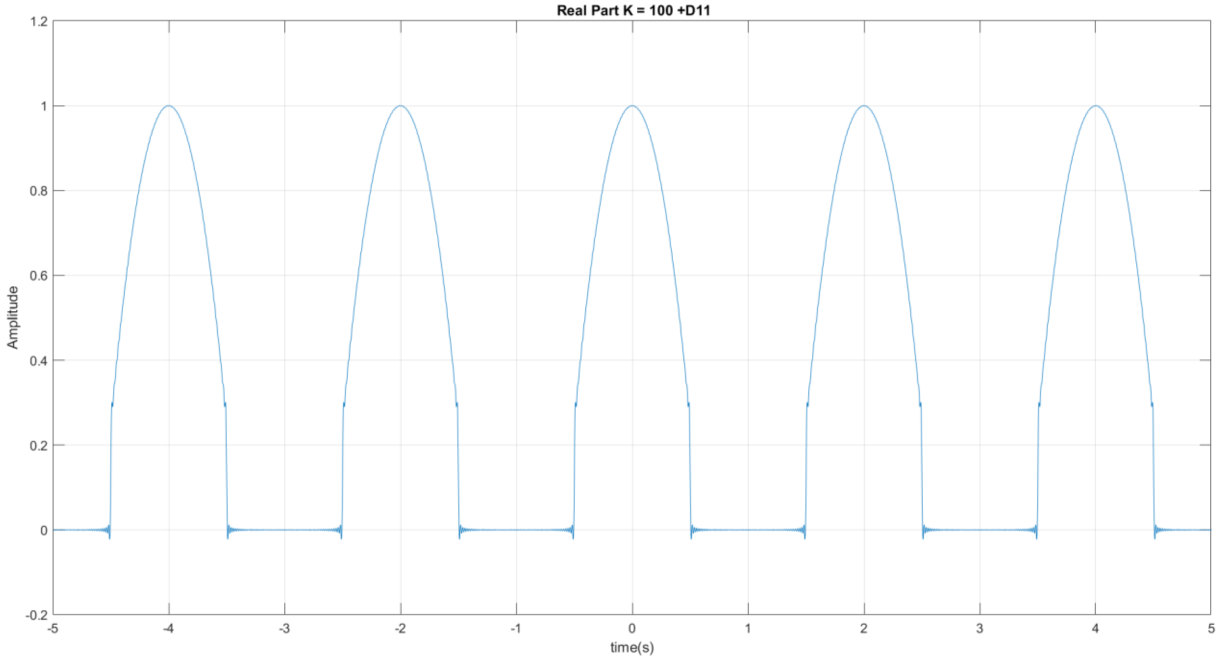

Figure 11: Plot of Real part of the function in Part 3 when K= 100 +D11

PART 4

a)

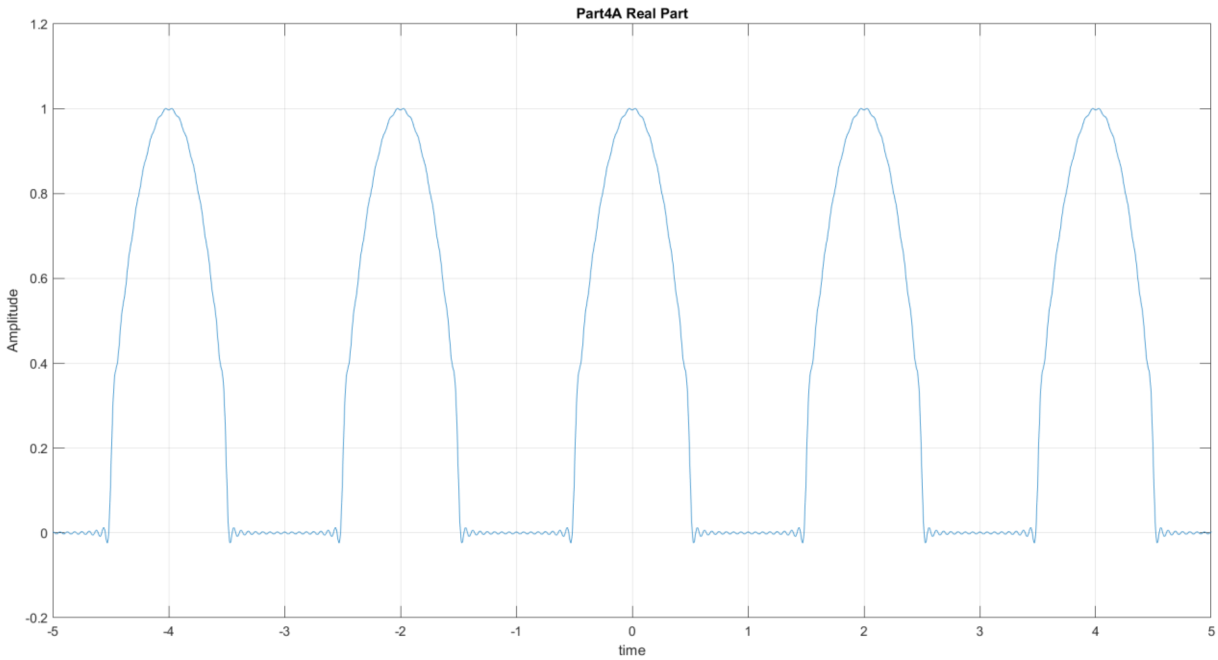

Figure 12: Plot of The Real Part of Time Reversed Function x(-t) from part 4A

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Since the function is an even function, the effect of this operation is only a time rehearsal, which makes function time reversed. Only thing that changed is the order of FSE coefficients.

Note that: $25+D_{11}=32$ from 25 + mod(21801985,11)

b)

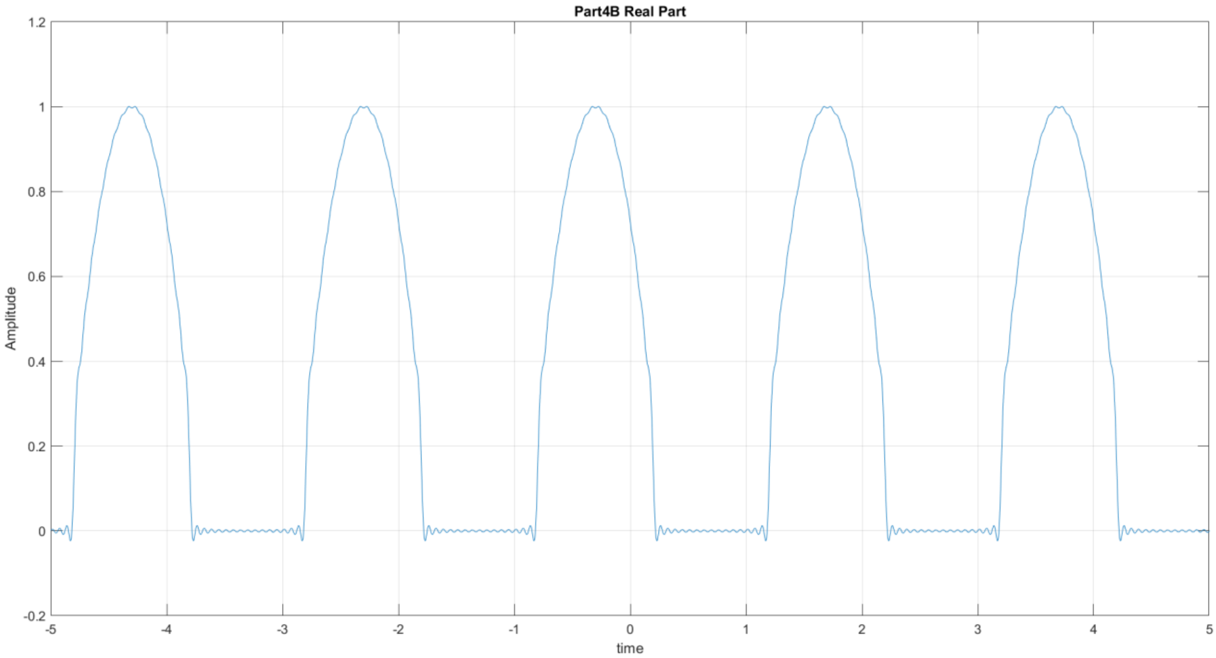

Figure 13: Plot of The Real Part of The Time Shifted Function x(t - t0) = x(t - 0.7) from part 4B

In the part4b, we used time shifting on the function. Therefore, FSE coefficients multiplied by $e^{-j\frac{2\pi k t_0}{T}}$. Then, we expected a 0.7 shift to the right in the plot.

c)

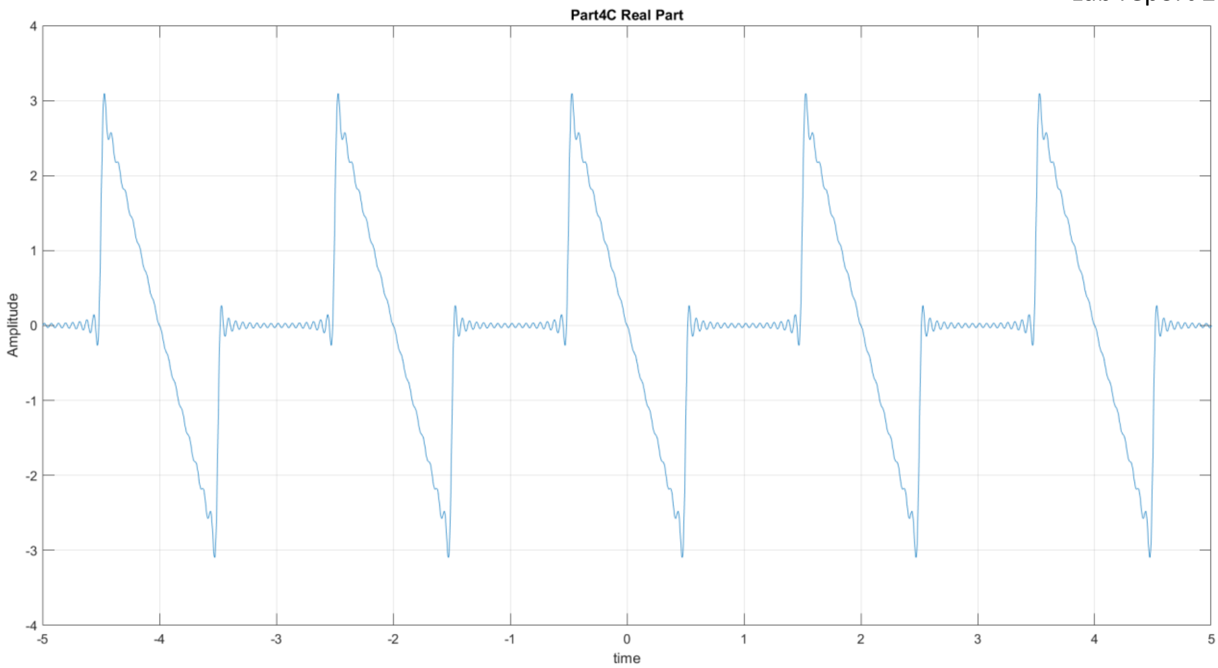

Figure 14: Plot of The Real Part of The Differentiated Function from part 4C

In this part, all FSE coefficients are multiplied by . Therefore, the function is differentiated.

d)

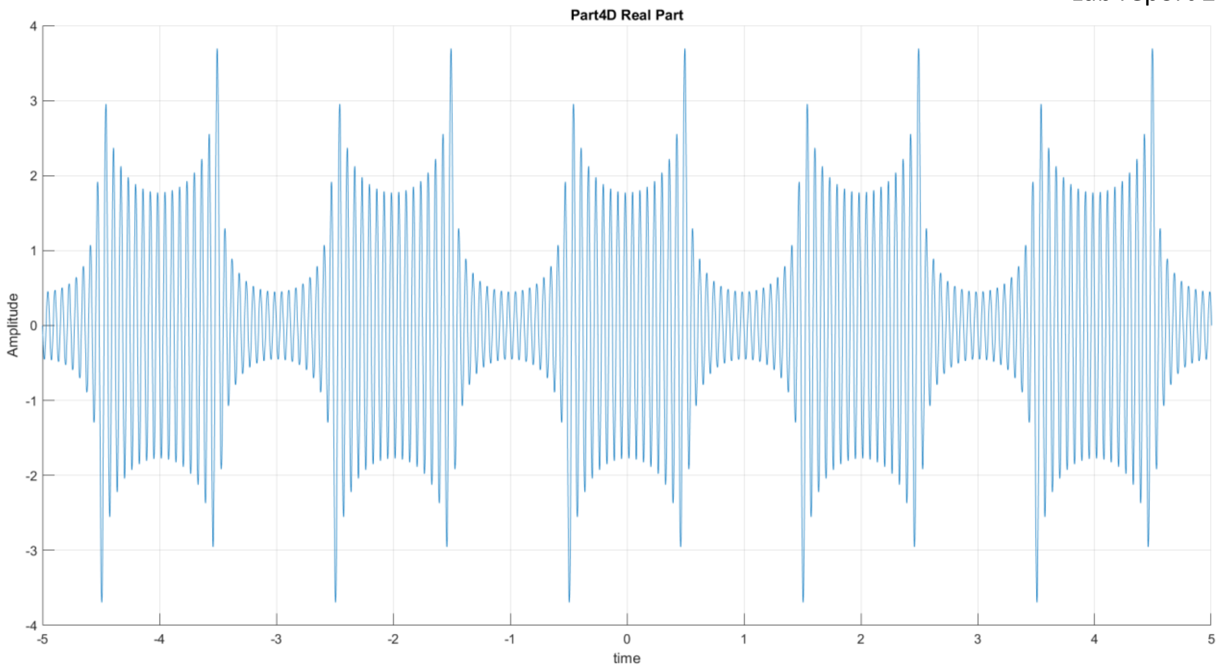

Figure 15: Plot of The Real Part of The Customized Function from part4D

In the plot of part4D, we changed the positions of the coefficients. When k=0, they remains same. When k>0, their order is reversed. When k<0, their order is reversed as well.

MATLAB CODES

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```
%% PART1
t = [-0.5:0.001:0.5];
n = mod(21801985,37); %n=31
A = 1 + (5 - 1) .* rand([1, n]);
B = (1i) * (1 + (5 - 1) .* rand([1, n]));
C = abs(A + B);
omega = 0 + (pi - 0) .* rand([1, n]);

signalOne = SUMCS(t, C, omega);

figure; plot(t, real(signalOne)); ylabel('Real Part'); xlabel('time(s)'); axis
auto;
figure; plot(t, imag(signalOne)); ylabel('Imaginary Part'); xlabel('time(s)');
axis auto;
figure; plot(t, abs(signalOne)); ylabel('Magnitude'); xlabel('time(s)'); axis
auto;
figure; plot(t, angle(signalOne)); ylabel('Phase'); xlabel('time(s)'); axis auto;


%% PART 3
T = 2;
W = 1;
K = 25 + mod(21801985,13);
t=[-5:0.001:5];

xt = FSWave(t,K,T,W);
xt_r = real(xt); %real
figure; plot(t,xt_r); title("Real part"); xlabel("t(s)"); ylabel("amplitude");
grid on;

xt_i = imag(xt);  %imaginary
figure; plot(t,xt_i); title("Imaginary part"); xlabel("t(s)");
ylabel("amplitude"); grid on;

t= [-5: 0.001: 5];
K_1 = 2 + mod(21801985,3);
K_2 = 8 + mod(21801985,3);
K_3 = 17 + mod(21801985,5);
K_4 = 50 + mod(21801985,3);
K_5 = 100 + mod(21801985,11);

xtk_1 = FSWave(t,K_1,T,W);
xtk_2 = FSWave(t,K_2,T,W);
xtk_3 = FSWave(t,K_3,T,W);
xtk_4 = FSWave(t,K_4,T,W);
xtk_5 = FSWave(t,K_5,T,W);

figure; plot(t, real(xtk_1)); title(" Real Part K = 2 +D3"); xlabel("time(s)") ;
ylabel("Amplitude"); grid on;
figure; plot(t, real(xtk_2)); title(" Real Part K = 8 +D3"); xlabel("time(s)") ;
ylabel("Amplitude"); grid on;
figure; plot(t, real(xtk_3)); title(" Real Part K = 17 +D5"); xlabel("time(s)") ;
ylabel("Amplitude"); grid on;
figure; plot(t, real(xtk_4)); title(" Real Part K = 50 +D3"); xlabel("time(s)") ;
ylabel("Amplitude"); grid on;
figure; plot(t, real(xtk_5)); title(" Real Part K = 100 +D11"); xlabel("time(s)")
; ylabel("Amplitude"); grid on;
```

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```
%% PART 4
% PART A
T = 2; W = 1; K = 25 + mod(21801985,11); t = [-5:0.001:5];
xt_p4 = FSWave(t, K, T, W);
figure; plot(t, real(xt_p4)); title("Part4A Real Part"); xlabel("time");
ylabel("Amplitude"); grid on;
% PART B
xt_p4_2 = FSWave2(t, K, T, W);
figure; plot(t, real(xt_p4_2)); title("Part4B Real Part"); xlabel("time");
ylabel("Amplitude"); grid on;
% PART C
xt_p4_3 = FSWave3(t, K, T, W);
figure; plot(t, real(xt_p4_3)); title("Part4C Real Part"); xlabel("time");
ylabel("Amplitude"); grid on;
% PART D
xt_p4_4 = FSWave4(t, K, T, W);
figure; plot(t, real(xt_p4_4)); title("Part4D Real Part"); xlabel("time");
ylabel("Amplitude"); box off; grid on;
```

```
%% Functions
function x_s = SUMCS(t, A, omega)
    x_s = 0;
    for j = 1 : length(A)
       xs = A(j) * exp(1i * omega(j) * t) ;
       x_s = x_s + xs;
    end
end

function x_t = FSWave(t, K, T, W)
    omega2 = [];
    t2 = -W / 2 : 0.001 : W / 2;
    X_K = [];
    ind = 1;

    for j = -K : 1 : K
        y = (1 - 3 * t2 .^ 2) .* exp((-1i) * (2 * pi * j / T) * t2);
        X_K(ind) = (1 / T) * trapz(t2, y);
        omega2(ind) = (2 * pi * j / T);
        ind = ind + 1;
    end
    x_t1 = SUMCS(t, X_K(1 : K), omega2(1 : K));
    x_t2 = SUMCS(t, X_K(K + 2 : 2*K + 1), omega2(K + 2 : 2*K + 1));
    x_t3 = X_K(K + 1);
    x_t = x_t1 + x_t2 + x_t3;
end

function x_t = FSWave2(t, K, T, W)
    omega3 = [];
    t2 = -W / 2 + 0.7 : 0.001 : W / 2 + 0.7;
    X_K = [];
    ind = 1;
    for j = -K : 1 : K
        y = (1 - 3 * (t2 - 0.7) .^ 2) .* exp((-1i) * (2 * pi * j / T) * t2);
        X_K(ind) = (1 / T) * trapz(t2, y) .* exp((-1i) * (2 * pi * j / T));
        omega3(ind) = (2 * pi * j / T);
        ind = ind + 1;
    end
    x_t1 = SUMCS(t, X_K(1 : K), omega3(1 : K));
    x_t2 = SUMCS(t, X_K(K + 2 : 2*K + 1), omega3(K + 2 : 2*K + 1));
    x_t3 = X_K(K + 1);
    x_t = x_t1 + x_t2 + x_t3;
end
```

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```
function x_t = FSWave3(t, K, T, W)
    omega4 = [];
    t2 = -W / 2 : 0.001 : W / 2;
    X_K = [];
    ind = 1;
    for j = -K : 1 : K
        y = (-6 * t2) .* exp((-1i) * (2 * pi * j / T) * t2);
        X_K(ind) = (1 / T) * trapz(t2, y);
        omega4(ind) = (2 * pi * j / T);
        ind = ind + 1;
    end
    x_t1 = SUMCS(t, X_K(1 : K), omega4(1 : K));
    x_t2 = SUMCS(t, X_K(K + 2 : 2*K + 1), omega4(K + 2 : 2*K + 1));
    x_t3 = X_K(K + 1);
    x_t = x_t1 + x_t2 + x_t3;
end

function x_t = FSWave4(t, K, T, W)
    omega5 = [];
    t2 = -W / 2 : 0.001 : W / 2;
    X_K = [];
    ind = 1;
    for j = -K : 1 : K
        y = (-6 * t2) .* exp((-1i) * (2 * pi * j / T) * t2);
        X_K(ind) = (1 / T) * trapz(t2, y);
        omega5(ind) = (2 * pi * j / T);
        ind = ind + 1;
    end
    x_t1 = SUMCS(t, X_K(K : -1 : 1), omega5(1 : K));
    x_t2 = SUMCS(t, X_K(2*K + 1 : -1 : K + 2), omega5(K + 2 : 2*K + 1));
    x_t3 = X_K(K + 1);
    x_t = x_t1 + x_t2 + x_t3;
end
```

PART 2

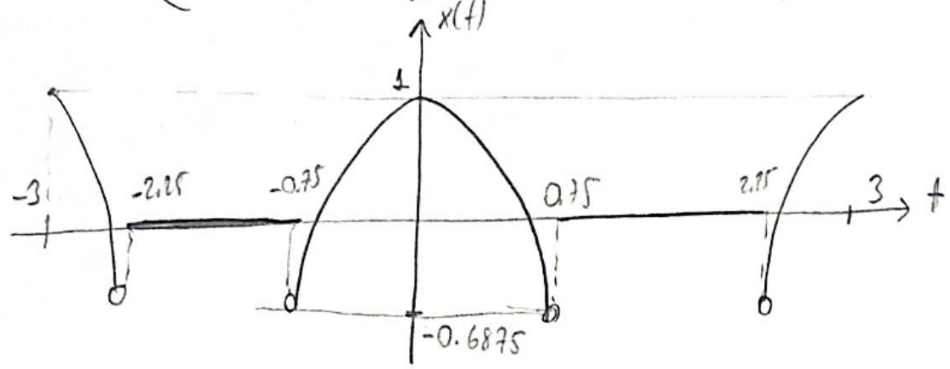
Part 2

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\frac{2\pi k t}{T}} \quad \text{where} \quad X_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-j\frac{2\pi k t}{T}} dt$$

$$\tilde{x}(t) = \sum_{k=-K}^{K} X_k e^{j\frac{2\pi k t}{T}} \quad \text{where } K \text{ is positive integer} \quad t \in \left[-\frac{T}{2}, \frac{T}{2}\right]$$

$$x(t) = \begin{cases} 1-3t^2 & \text{if } -\frac{W}{2} < t < \frac{W}{2} \\ 0 & \text{, o.w} \end{cases}$$

$T=3$, $W=1.5$ sketch over $-3<t<3$



Finding FSE Coefficients

$$X_k = \frac{1}{3}\int_{-1.5}^{1.5}(1-3t^2)\, e^{-j\frac{2\pi t}{3}} dt = \frac{1}{3}\int_{-0.75}^{0.75}(1-3t^2)\, e^{-j\frac{2\pi t}{3}} dt = \frac{1}{3}\int_{-0.75}^{0.75} e^{-j\frac{2\pi t}{3}} - \int_{-0.75}^{0.75} t^2 \cdot e^{-j\frac{2\pi t}{3}} dt$$

This equals to $\frac{1}{3}\left(\frac{-1}{j2\pi k} e^{-j\frac{2\pi k t}{3}}\right)\Big|_{-0.75}^{0.75} - \left(\frac{t^2 e^{-j\frac{2\pi k t}{3}}}{-j2\pi k}\Big|_{-0.75}^{0.75} - \int_{-0.75}^{0.75} \frac{2t\, e^{-j\frac{2\pi k t}{3}}}{-j2\pi k} dt\right.$

$$\int_{-0.75}^{0.75} \frac{2t\, e^{-j\frac{2\pi k t}{3}}}{-j2\pi k} dt = \frac{2}{-j2\pi k}\left(\frac{e^{-j\frac{2\pi k t}{3}}}{-j2\pi k}\Big|_{-0.75}^{0.75} - \int_{-0.75}^{0.75} \frac{e^{-j\frac{2\pi k t}{3}}}{-j2\pi k} dt\right) = \frac{2}{-j2\pi k}\left(\frac{e^{-j\frac{2\pi k t}{3}}}{-j2\pi k}\Big|_{-0.75}^{0.75} + \frac{e^{-j\frac{2\pi k t}{3}}}{-4\pi^2 k^2}\Big|_{-0.75}^{0.75}\right)$$

putting the values, we get

$$X_k = \frac{e^{-j(1.58k)}}{k^3}\left(j(-0.109k^2) + e^{j(3.142k)} \cdot \left(j(0.109k^2) - 0.342k - j(0.218)\right) - 0.342k + j \cdot (0.218)\right)$$

Also equals $-\frac{(11\pi^2 k^2 - 216)\sin\left(\frac{\pi k}{2}\right) + 108\pi k\cos\left(\frac{\pi k}{2}\right)}{16\pi^3 k^3}$, with writing T, W we got this; $-\frac{T\left((13\pi^2 W^2 - 4\pi^2)k^2 - 6T^2\right)\sin\frac{\pi W k}{T} + 6\pi T \cos\left(\frac{\pi W k}{T}\right)}{12\pi^3 k^3}$