BILKENT UNIVERSITY ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

EEE - 321: Signals and Systems
Lab Assignment 2

Please carefully study this assignment before coming to the laboratory. You may begin working on it or even complete it if you wish, but you do not have to. There will be short quizzes both at the beginning and end of the lab session; these may contain conceptual, analytical, and Matlab-based questions. Within one week, complete the assignment in the form of a report and turn it in to the assistant. Some of the exercises will be performed by hand and others by using Matlab. What you should include in your report is indicated within the exercises.

Part 1

Consider a signal defined as

$$x_s(t) = \sum_{i=1}^{M} A_i e^{j\omega_i t}. \tag{1}$$

As you see, $x_s(t)$ is obtained as the superposition of M complex exponentials, where A_i and ω_i respectively denote the complex amplitude and angular frequency of the ith complex exponential.

In this part, you will write a Matlab program (in the form of a Matlab function) that computes the values of $x_s(t)$. Your function should look like

function [xs] = SUMCS(t,A,omega)

where

- **t**: $1 \times N$ vector that contains the time instants over which $x_s(t)$ is computed.
- **A**: $1 \times M$ complex-valued vector. ith element is A_i.
- **omega**: $1 \times M$ vector. ith element is ω_i.

While writing your function, make use of the **exp** function of Matlab. Use a single for loop over the index i of Eq. 1. Do not use any for loop over the time points. Doing so will result in significant loss of grade. Your program does not need to show any error message if a wrong value is given for any of the inputs (for instance, if **A** is of size 1×5 but **omega** is of size 1×7). Simply assume that you will always run your function with correct inputs. Also, throughout all your programs, in order to avoid programming bugs, reserve the letter **j** for $\sqrt{-1}$, i.e., do not use it as the counter of any for loop or so. Include your code to your report.

Hint: Given **A** (or **omega**), you can use the **length** command of Matlab to determine its length. Type **help length** in Matlab to learn how it works.

After writing your function, take **t=[-0.5:0.001:0.5]** and n = mod(your ID number, 37). Then, create a random $1 \times n$ complex amplitude array for **A**. The real and imaginary parts of the complex amplitude should be between 1 and 5. Also create another $1 \times n$ real frequency array for **omega**. The created frequency array should be between 0 and π. You can create these random arrays using **rand** function of Matlab. Type **help rand** to learn how to use **rand** function. (Note that Matlab's random number generator, and the other random number generators used in other computer programs and languages, are psuedo-random generators. That means, they does not produce completely random numbers. You can make a web search to learn more about this topic.)

Using your function, compute **xs** (which is complex valued). Then, extract the real and imaginary parts of **xs**. Plot the real and imaginary parts (versus **t**) separately. Also, extract the magnitude and phase of **xs** and plot them separately. In all your plots, put the labels and titles properly. Include your code and plots to your report.

Part 2

Consider a signal $x(t)$ which is periodic with T such that $x(t) = x(t+T)$ for all t. In the lectures, you learned that we can write

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\frac{2\pi k t}{T}} \quad (2)$$

where

$$X_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-j\frac{2\pi kt}{T}}\mathrm{d}t \tag{3}$$

Above, Eq.2 is called the Fourier series expansion of $x(t)$, and the expansion coefficients X_k can be computed according to Eq.3. As you see, in its Fourier series expansion, we write a signal $x(t)$ as the weighted superposition of complex sinusoidal signals $e^{j\frac{2\pi kt}{T}}$ where the weights are given by X_k.

Note that Eq.2 contains an infinite number of complex sinusoids. In practical situations, if we wish to use Eq.2 to compute $x(t)$, we can not add all the sinusoids, but instead we can only add a finite number of them. In this respect, consider the signal $\tilde{x}(t)$ which is defined as

$$\tilde{x}(t) = \sum_{k=-K}^{K} X_k e^{j\frac{2\pi kt}{T}} \tag{4}$$

where K is a positive integer. Note that if $K = \infty$, we would have $\tilde{x}(t) = x(t)$. When K is finite, $\tilde{x}(t)$ is only an approximation to $x(t)$. Now, suppose, within $t \in \left[-\frac{T}{2}, \frac{T}{2}\right)$, $x(t)$ is given as

$$x(t) = \begin{cases} 1-3t^2 & \text{if } -\frac{W}{2} < t < \frac{W}{2}, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

where

$$W < T \tag{6}$$

(Recall that in the beginning we assumed that $x(t)$ is periodic with T. The above equation describes $x(t)$ in only ONE period. It does not specify $x(t)$ FOR ALL t. Do not get confused on this point.)

Take $T = 3$, $W = 1.5$ and sketch (with your hand) $x(t)$ over $-1T < t < 1T$. Include this sketch to your report. Clearly indicate the signal amplitudes on the sketch.

Determine the Fourier series expansion coefficients X_k of $x(t)$ in Eq. 5 for generic values of T and W by evaluating the integral in Eq. 3 with your hand. (You may utilize the integration by parts while calculating the integral.) Simplify your result as much as possible. Include your work to your report.

Part 3

In this part, you will write a Matlab function which makes the **Fourier Synthesis** for the signal given in Eq. 5. In other words, your code will compute $\tilde{x}(t)$ from the given parameters K, T, W for a given time interval. Your function should look like

function [xt] = FSWave(t,K,T,W)

where

- **t** denotes the time grid over which $\tilde{x}(t)$ is computed.
- **xt** denotes the values of $\tilde{x}(t)$ computed over **t**.
- **K**, **T** and **W** denote the parameters K, T and W that appear in Eq. 4 and Eq. 5.

While writing your code, use the function **SUMCS** that you developed in Part 2. Use only a single for loop over the index parameter k of Eq. 4. (Note that, within the code you need to compute X_k.) Include the code to your report.

Let D_{11} = mod(your ID number, 11) and D_3 = mod(your ID number, 3)

Take **T=2**, **W=1**, **K=25+**D_{13} and **t=[-6:0.001:6]**. Compute **xt**.

Extract the real and imaginary parts of **xt** and plot them versus **t**. Include the plots to your report. Examine the maximum and minimum values that you see on the plots. What are these values for real and imaginary parts? What can you say when you compare them? Can you ignore the imaginary part when compared to the real part? Actually, since $x(t)$ in Eq. 5 is a real valued signal, we would expect the imaginary part to be zero. But then, why do you think that the imaginary part is not perfectly zero? (Hint: We know that $\sin\left(\frac{\pi}{6}\right)$ is equal to 0.5. Then, $\sin\left(\frac{\pi}{6}\right) - 0.5$ must be equal to 0. Try computing $\sin\left(\frac{\pi}{6}\right) - 0.5$ in Matlab. What result do you get?)

Again take **T=2**, **W=1** and **t=[-5:0.001:5]** and produce five plots taking **K=2+**D_3, **K=8+**D_3, **K=17+**D_5, **K=50+**D_3, **K=100+**D_{11}. Now you know that **xt** is essentially real valued you can display only the real part of **xt**. Include all five plots to your report. What do you observe as K gets larger? In particular, what can you say about the success of $\tilde{x}(t)$ in approximating $x(t)$? Do you observe some irregularities or oscillations within the neighborhood of the discontinuities?

Part 4

In this part, you will make small changes in the function **FSWave** that you wrote in Part 2, and you will observe the resulting changes. Consider the following equation:

$$y(t) = \sum_{k=-K}^{K} Y_k e^{j\frac{2\pi k t}{T}} \tag{7}$$

- **Part a**) Suppose $Y_k = X_{-k}$. What is the required change in your **FSWave** function to compute $y(t)$ from X_ks. Take **T=2**, **W=1**, **K=25+**D_{11} and **t=[-5:0.001:5]**. Run your code, and compute and plot $y(t)$ versus t (only the real part). Include the plot to your report. What is the effect of the operation that we performed? Include the answer to your report.

- **Part b**) Suppose $Y_k = X_k e^{-j\frac{2\pi k t_0}{T}}$ where t_0 is a new parameter. What is the required change in your **FSWave** function to compute $y(t)$ from X_ks. Take **T=2**, **W=1**, **K=25+**D_{11}, **t=[-5:0.001:5]** and $t_0 = 0.7$. Run your code, and compute and plot $y(t)$ versus t (only the real part). Include the plot to your report. What is the effect of the operation that we performed? Include the answer to your report.

- **Part c**) Suppose $Y_k = jk\frac{2\pi}{T}X_k$. What is the required change in your **FSWave** function to compute $y(t)$ from X_ks. Take **T=2**, **W=1**, **K=25+**D_{11} and **t=[-5:0.001:5]**. Run your code, and compute and plot $y(t)$ versus t (only the real part). Include the plot to your report. What is the effect of the operation that we performed? Include the answer to your report.

- **Part d**) Suppose

$$Y_k = \begin{cases} X_{K+1-k} & \text{if } k = 1,2,....,K \\ X_k & \text{if } k = 0 \\ X_{-(K+1+k)} & \text{if } k = -K, -K+1, -K+2, ..., -1 \end{cases} \tag{8}$$

  What is the required change in your **FSWave** function to compute $y(t)$ from X_ks. Take **T=2**, **W=1**, **K=25+**D_{11} and **t=[-5:0.001:5]**. Run your code, and compute and plot $y(t)$ versus t (only the real part). Include the plot to your report. What is the effect of the operation that we performed? Include the answer to your report.