# QUESTION 1 – INDUCTION PROOF

size m = size’ m acc

Base Case

The base case is for a mobile with just an object, ie Obj w.

size Obj w

size Obj w = 1 //From program

size Obj w + acc = 1 + acc

size’ Obj w acc = acc + 1 = 1+0 = 1 //From program

Hence in the base case where m = Obj w,

Inductive Hypothesis

As the inductive hypothesis, let us assume that for 2 arbitrary mobiles m1, m2 , that

size m1 = size’ m1 acc,

and that size m2 = size’ m2 acc.

Since acc is arbitrary, we can also say that the above statements hold when acc = 0, ie that

size m1 = size’ m1 0.

Step Case

Now we can consider a mobile Wire(m1,m2). We need to prove that

(size Wire(m1,m2)) = size’ Wire(m1,m2) acc.

size’: Wire(m1,m2) acc = size’ m1 (size’ m2 (1+ acc)) //From program

size’: Wire(m1,m2) acc = size’ m1 (size’ m2 (1)) //acc=0

size: size m1 + (size m2 + (1)) //From i.h

size = size m1 + (size m2 + (1))

size’= size (Since, the base case is equal they must be the equal)

Thus proving the left hand size of the statement. Since the statement was generalized for an arbitrary acc, it means that the inductive hypotheses could be used even for values such as acc = (size m2 + (1+acc)).