prisoners in a circle

Avi Taicher

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The prisoners are now exactly as many as there are colors (N, instead of 100) and standing in a circle (so that everyone can see everyone else's hat color, but not his own). This time, the prisoners don't call out their color, but write it down secretly on a piece of paper (so no one knows the other's answers). The prisoners win if at least one of them guesses his own hat color right. Come up with a strategy that the prisoners can agree upon beforehand and that will guarantee them to win in any case.

Solution: Number each prisoner P_i and each color C_i from 0 to $N-1$. Let,

$$T = \sum_{j=0}^{N-1} C_j,$$

and suppose without loss of generality that $T\%N = k$. The strategy for at least one prisoner to guess his color is as follows: each prisoner P_i sees the sum,

$$T_i = \sum_{j=0, j\neq i}^{N-1} C_j,$$

and guesses his color $\hat{C}_i$ such that $(T_i + \hat{C}_i)\%N = i$. For this strategy, prisoner P_k will guess his color $\hat{C}_k$ such that $T_k + \hat{C}_k = T$ and hence $\hat{C}_k = C_k$. To see this assume that $T_k + \hat{C}_k \neq T$. This implies that $T_k + \hat{C}_k = T + mN$, $m \in \mathbb{N}$, $m \neq 0$. But this is a contradiction since

$$T - (N-1) \leq T_k + \hat{C}_k \leq T + (N-1)$$