Our initial wave fucntion is

$$\psi(x,0) = C \exp\left(-\frac{(x-x_0)^2}{4s^2} + \mathrm{i}k_0 x\right) \tag{1}$$

where $C$ is some normalization constant, and $k_0$ and $x_0$ are real numbers.

The eigenvalues in the interior region of infinite well are

$$\begin{aligned}
\varphi_n(x,t) &= A_n \cos(k_n x) e^{-\mathrm{i}\frac{k_n^2}{2}t} + B_n \sin(k_n x) e^{-\mathrm{i}\frac{k_n^2}{2}t} \quad \text{where} \quad k_n = n\frac{\pi}{2L}, \quad n \in \mathbb{Z} && (2)\\
&= \left[A_n \frac{e^{\mathrm{i}k_n x} - e^{-\mathrm{i}k_n x}}{2\mathrm{i}} + B_n \frac{e^{\mathrm{i}k_n x} - e^{-\mathrm{i}k_n x}}{2\mathrm{i}}\right] e^{-\mathrm{i}\frac{k_n^2}{2}t} && (3)\\
&= \left[A_n \frac{e^{\mathrm{i}k_n x} + e^{-\mathrm{i}k_n x}}{2} + B_n \frac{e^{\mathrm{i}k_n x} - e^{-\mathrm{i}k_n x}}{2\mathrm{i}}\right] e^{-\mathrm{i}\frac{k_n^2}{2}t} && (4)\\
&= \left[\underbrace{\left(\frac{A_n}{2} + \frac{B_n}{2\mathrm{i}}\right)}_{\tilde{A}_n} \underbrace{e^{\mathrm{i}k_n x}}_{\varphi_n^{(1)}(x)} + \underbrace{\left(\frac{A_n}{2} - \frac{B_n}{2\mathrm{i}}\right)}_{\tilde{B}_n} \underbrace{e^{-\mathrm{i}k_n x}}_{\varphi_n^{(2)}(x)}\right] e^{-\mathrm{i}\frac{k_n^2}{2}t} && (5)\\
&= \left[\tilde{A}_n \varphi_n^{(1)}(x) + \tilde{B}_n \varphi_n^{(2)}(x)\right] \exp\left(-\mathrm{i}\frac{k_n^2 t}{2}\right) && (6)
\end{aligned}$$

By applying boundary conditions, we can conclude that whenever $n$ is even, we must take into account only the sine part of the solution. On the other hand, if it is odd, we should take cosine part.

$$\begin{aligned}
\sin(k_n L) = \sin(-k_n L) = 0 &\implies k_n := n\frac{\pi}{2L} = \frac{m\pi}{L} \quad \text{where} \quad n, m \in \mathbb{Z} && (7)\\
\cos(k_n L) = \cos(-k_n L) = 0 &\implies k_n := n\frac{\pi}{2L} = (2m+1)\frac{\pi}{2L} \quad \text{where} \quad n, m \in \mathbb{Z} && (8)
\end{aligned}$$

Therefore, we can obtain our solution with

$$\begin{aligned}
\psi(x,t) &= \sum_{n=0}^{\infty} A_{2n+1} \cos(k_{2n+1}x) e^{-\mathrm{i}\frac{k_{2n+1}^2}{2}} + B_{2n} \sin(k_{2n}x) e^{-\mathrm{i}\frac{k_{2n}^2}{2}} && (9)\\
&= \sum_{n=0}^{\infty} \left(\tilde{A}_{2n+1} + \tilde{B}_{2n+1}\right) \cos(k_{2n+1}x) e^{-\mathrm{i}\frac{k_{2n+1}^2}{2}} + \mathrm{i}\left(\tilde{A}_{2n} - \tilde{B}_{2n}\right) \sin(k_{2n}x) e^{-\mathrm{i}\frac{k_{2n}^2}{2}} && (10)
\end{aligned}$$

I tired to find projection of our initial wave function to eigenstates by using Gaussian integration

$$
\begin{aligned}
\tilde{A}_n &= \left\langle \varphi_n^{(1)} \middle| \psi \right\rangle = \int\limits_{-L}^{L} \mathrm{d}x \ \varphi_n^{(1)*} \psi(x,0) \ = C \int\limits_{-L}^{L} \mathrm{d}x \ \ \exp\left( -\frac{(x-x_0)^2}{4s^2} + \mathrm{i}(k_0 - k_n)x \right) \\
&= C \int\limits_{-L-x_0}^{L-x_0} \mathrm{d}x' \ \ \exp\left( -\frac{x'^2}{4s^2} + \mathrm{i}(k_0-k_n)(x'+x_0) \right) \\
&= C e^{\mathrm{i}(k_0-k_n)x_0} \int\limits_{-L-x_0-\mathrm{i}2s^2(k_0-k_n)}^{L-x_0-\mathrm{i}2s^2(k_0-k_n)} \mathrm{d}x'' \ \ \exp\left( -\frac{(x''+\mathrm{i}2s^2(k_0-k_n))^2}{4s^2} + \mathrm{i}(k_0-k_n)(x''+\mathrm{i}2s^2(k_0-k_n)) \right) \\
&= C e^{\mathrm{i}(k_0-k_n)x_0} \int\limits_{-L-x_0-\mathrm{i}2s^2(k_0-k_n)}^{L-x_0-\mathrm{i}2s^2(k_0-k_n)} \mathrm{d}x'' \ \ \exp\left( -\frac{x''^2 + \mathrm{i}4s^2(k_0-k_n)x'' - 4s^4(k_0-k_n)^2}{4s^2} \right. \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \left. + \mathrm{i}(k_0-k_n)(x''+\mathrm{i}2s^2(k_0-k_n)) \right) \\
&= C e^{\mathrm{i}(k_0-k_n)x_0} \int\limits_{-L-x_0+\mathrm{i}2s^2(k_0-k_n)}^{L-x_0+\mathrm{i}2s^2(k_0-k_n)} \mathrm{d}x'' \ \ \exp\left( -\frac{x''^2}{4s^2} + s^2(k_0-k_n)^2 - 2s^2(k_0-k_n)^2 \right) \\
&= C e^{-s^2(k_n-k_0)**2-\mathrm{i}(k_n-k_0)x_0} \int\limits_{-L-x_0+\mathrm{i}2s^2(k_0-k_n)}^{L-x_0+\mathrm{i}2s^2(k_0-k_n)} \mathrm{d}x'' \ \ \exp\left( -\frac{x''^2}{4s^2} \right) \\
&\approx C e^{-s^2(k_n-k_0)**2-\mathrm{i}(k_n-k_0)x_0} \int\limits_{-\infty}^{\infty} \mathrm{d}x'' \ \ \exp\left( -\frac{x''^2}{4s^2} \right) \ = C\sqrt{4s^2\pi} e^{-s^2(k_n-k_0)**2-\mathrm{i}(k_n-k_0)x_0}
\end{aligned}
$$

$$
\Longrightarrow \ \tilde{A}_n = C 2s\sqrt{\pi} e^{-s^2(k_n-k_0)**2-\mathrm{i}(k_n-k_0)x_0} \tag{11}
$$

$$
\tilde{B}_n = A_{(-n)} = C 2s\sqrt{\pi} e^{-s^2(k_n+k_0)**2+\mathrm{i}(k_n+k_0)x_0} \tag{12}
$$