Q1 Time Complexity

$T(n) = T(n-1) + n$

$T(n-1) = T(n-2) + n-1$

$T(n) = T(n-2) + n + n-1$

$T(n) = T(n-k) + n + n-1 \ldots + n-k+1$

$n-k=0 \quad n=k$

$T(n) = n + n-1 + \ldots + 1$

$T(n) = \frac{n.n-1}{2} \qquad \underline{T(n) = \Theta(n^2)}$

Q2 Time Complexity

$T(n) = T(n-1) + T(n-2) + n-1 + n-2$

$T(n-1) = T(n-2) + T(n-3) + n-2 + n-3$

$T(n-2) = T(n-3) + T(n-4) + n-3 + n-4$

$T(n) = T(n-2) + 2T(n-3) + T(n-4) + n-1 + 2(n-2) + 2(n-3) + n-4$

$n = k-2$

$T(n) = 2T(k-1) + T(k-2) + k+1 + 2k + 2k-2 + k-2$

Q3 Time Complexity

$T(n) = T(n-1) + n$ $\quad T(n-1) = T(n-2) + n-1$

$T(n) = T(n-2) + n + n-1$

$T(n-2) = T(n-3) + n-2$

$T(n) = T(n-3) + n + n-1 + n-2$

$T(n) = T(n-k) + n + n-1 \cdots + n-k+1$

$n-k=0 \quad n=k$

$T(n) = n + n-1 + \ldots + 1$

$T(n) = \frac{n \cdot n-1}{2} \qquad \underline{T(n) = \Theta(n^2)}$

Q4 Time Complexity

$T(n) = 3T(n/3) + cn$

$(n/3^k)(n^k) = n$

$n/3^k = 1$

$n = 3^k$

$\log n = \log 3^k$

$\log n = k \log 3$

$k = \log_3 n$

$\Rightarrow \underline{O(\log_3 n)}$