

$$n^2, n^3, n^2 \log n, \sqrt{n}, \log n, 10^n, 2^n, 8^{\log_2 n}$$

Of course;

$$\sqrt{n} < n^2 < n^3$$

$$\lim_{n \rightarrow \infty} \frac{\log n}{\sqrt{n}} \Rightarrow \text{Derivative } \frac{\frac{1}{n}}{\frac{1}{2} \cdot \frac{1}{n}} \Rightarrow n = \infty \log n < \sqrt{n}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 \log n}{n^3} \log n = \infty \Rightarrow n^2 < n^2 \log n$$

$$\lim_{n \rightarrow \infty} \frac{n^2 \log n}{n^3} \Rightarrow n^2 \log n < n^3$$

$$\lim_{n \rightarrow \infty} \frac{2^n}{n^2} \Rightarrow f(n) = \infty \quad n^3 < 2^n < 10^n$$

$$\log n < \sqrt{n} < n^2 < n^2 \log n < n^3 < 8^{\log_2 n} < 2^n < 10^n$$


```

Q-) int p-1(int my_array[]) {
    for(int i=2; i<=n; i++)
        if(i%2==0) {
            count++;
        }
        else {
            i=(i-1);
        }
}

```

$\theta(1)$ $\theta(n)$

→ Time complexity: $\theta(n)$

→ Space complexity: $\theta(1)$

```

b-) int p-3(int my_array[]) {
    first_element = my_array[0]; →  $O(1)$ 
    second_element = my_array[1]; →  $O(1)$ 
    for(int i=0; i<size of Array; i++) {
        if(my_array[i] < first_element) {
            second_element = first_element;
            first_element = my_array[i];
        }
        else if(my_array[i] < second_element) {
            if(my_array[i] != second_element) {
                second_element = my_array[i];
            }
        }
    }
}

```

$O(n)$ $O(1)$

→ Time complexity: $O(n)$

→ Space complexity: $O(1)$

```

c-) int p-3(int array[]) {
    return array[0] * array[2]; →  $O(1)$ 
}

```

→ Time complexity: $\theta(1)$

→ Space complexity: $\theta(1)$


```

d-) int p-4(int[] array, int n) {
    int sum = 0;  $\rightarrow \Theta(1)$ 
    for (int i = 0; i < n; i = i + 5)
        sum += array[i] * array[i];  $\rightarrow \Theta(1)$   $\rightarrow \Theta(n)$ 
    return sum;  $\rightarrow \Theta(1)$ 
}

```

\rightarrow Time complexity: $\Theta(1)$

\rightarrow Space complexity: $\Theta(1)$

```

e-) int p-5(int[] array, int n) {
    for (int i = 0; i < n; i++)
        for (int j = 1; j < i; j = j * 2)
            printf("%d", array[i] * array[j]);  $\rightarrow \Theta(1)$ 
    }
}

```

$\left. \begin{array}{l} \text{ } \end{array} \right\} \Theta(n \log_2 n)$
 $\left. \begin{array}{l} \text{ } \end{array} \right\} \Theta(\log_2 n)$

\rightarrow Time complexity: $\Theta(n \log_2 n)$

\rightarrow Space complexity: $\Theta(1)$

```

f-) int p-6(int array[], int n) {
    if (p-4(array, n) > 1000)  $\rightarrow \Theta(n)$ 
        p-5(array, n)  $\rightarrow \Theta(n \log_2 n)$ 
    else
        printf("%d", p-3(array) * p-4(array, n))  $\rightarrow \Theta(1, n) = \Theta(n)$ 
    }
}

```

$\rightarrow T_{\text{worst}}(n) = \Theta(n) + \Theta(n \log_2 n) = \Theta(n \log_2 n)$

$\rightarrow T_{\text{best}}(n) = \Theta(n) + \Theta(n) = \Theta(n)$

\rightarrow Space complexity: $\Theta(1)$

(A-1)

$$I \Rightarrow 2^{n+1} = \Theta(2^n) \Leftrightarrow c_1 \cdot 2^n \geq 2^{n+1} \geq c_2 \cdot 2^n, c_1, c_2 > 0 \text{ for all } n \geq n_0$$

$$c_1 \cdot 2^n \geq 2^{n+1} \quad 2^{n+1} \geq c_2 \cdot 2^n$$

for $n=1$ $c_1 \cdot 2 \geq 4$ for $n=1$ $4 \geq c_2 \cdot 2$ $2 \geq c_2$
 $c_1 \geq 2$

$$n \geq n_0 = 1, \quad \underline{c_1 = c_2} \quad \checkmark$$

$$II \Rightarrow 2^{2^n} = \Theta(2^n) \Leftrightarrow c_1 \cdot 2^n \geq 2^{2^n} \geq c_2 \cdot 2^n, c_1, c_2 > 0 \text{ for all } n \geq n_0$$

$$c_1 \cdot 2^n \geq 2^{2^n}$$

$$c_1 \geq 2^n, \text{ false, } c_1 \text{ is constant.}$$


```

public void pairs(int[] array, int value) {
    for (int i = 0; i < array.length - 1; i++) {
        for (int j = 0; j < array.length; j++) {
            if (array[i] + array[j] == value) {
                System.out.println(array[i] + " and " + array[j]
                    + " is pairs");
            }
        }
    }
}

```

```

public static void rec_pairs(int[] array, int index, int iter, int value) {
    if (array[index] + array[iter] == value) {
        System.out.println(array[index] + array[iter] + " is pairs");
    }
    if (index == array.length - 1) {
        return;
    }
    if (iter == array.length - 1) {
        rec_pairs(array, index + 1, 0, value);
    }
    else {
        rec_pairs(array, index, iter + 1, value);
    }
}

```