

**T.R.**

**GEBZE TECHNICAL UNIVERSITY**

**FACULTY OF ENGINEERING**

**DEPARTMENT OF COMPUTER ENGINEERING**

**PLANNING OF BIN ALLOCATION AND VEHICLE  
ROUTING IN SOLID WASTE MANAGEMENT**

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**SUPERVISOR  
PROF. DR. DİDEM GÖZÜPEK**

**GEBZE  
2025**

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GEBZE**



GRADUATION PROJECT  
JURY APPROVAL FORM

This study has been accepted as an Undergraduate Graduation Project in the Department of Computer Engineering on 26/06/2025 by the following jury.

**JURY**

Member

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# ABSTRACT

Managing solid waste is a major logistical and financial challenge for cities and local governments. This report focuses on the Waste Bin Allocation and Routing Problem (WBARP). This problem aims to find the best way to plan bin placements and vehicle routes in waste collection systems. The main goal is to balance the trade-off between the cost of bins and the cost of routing. For example, visiting sites more often (higher routing cost) means fewer or smaller bins are needed (lower bin cost).

Based on the methods described in Hemmelmayr et al. (2014), this project develops and compares several solution approaches. We implemented three models. The first two are matheuristics: 1) a Bin Allocation-First, Route-Second (BAFRS) model, and 2) a Route-First, Bin Allocation-Second (RFBAS) model. In these models, the complex routing part is solved with a Variable Neighborhood Search (VNS) heuristic that uses both inter-route and intra-route moves. The bin allocation part is solved using a Mixed-Integer Linear Program (MILP). For comparison, we also implemented 3) a fully integrated MILP based on the literature, which solves the entire problem at once for small test cases.

Our results show that integrated methods perform much better than simple sequential methods. The RFBAS model was particularly successful, creating efficient, clustered routes and achieving a low total cost. The BAFRS model, on the other hand, struggled with the routing phase because it did not consider routing costs in its initial decisions. We also confirmed that the fully integrated MILP cannot be solved for large, real-world problems, which highlights the need for practical matheuristic approaches like BAFRS and RFBAS.

In conclusion, this project shows that integrated planning can lead to significant cost savings in waste management. It also demonstrates that a well-designed matheuristic like our RFBAS model is an effective tool for solving this complex, large-scale logistics problem in a practical amount of time.

**Keywords:** Waste Management, Vehicle Routing, Bin Allocation, Matheuristics, Variable Neighborhood Search (VNS).

# **ZENODO REPOSITORY**

**The complete implementation and source code for this project are  
available on Zenodo at:**

**<https://zenodo.org/records/15768912>**

**The Demo presentation for this project is available on Youtube at:**

**<https://www.youtube.com/watch?v=q6rku0G6xiM>**

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**Atakan Akdoğan - Hüseyin Emre Tığcı**

# LIST OF SYMBOLS AND ABBREVIATIONS

Table 1: List of Symbols and Abbreviations Used in the Report

<b>Symbol or Abbreviation</b>	<b>Explanation</b>
<b>Abbreviations</b>	
WBARP	: Waste Bin Allocation and Routing Problem
BAFRS	: Bin Allocation-First, Route-Second
RFBAS	: Route-First, Bin Allocation-Second
IM	: Integrated Model
VNS	: Variable Neighborhood Search
MILP	: Mixed-Integer Linear Program
VRP	: Vehicle Routing Problem
PVRP	: Periodic Vehicle Routing Problem
IF	: Intermediate Facility
<b>Sets and Indices</b>	
$N$	: Set of all nodes (depot, customers, IFs)
$N_C$	: Set of customer nodes, indexed by $i$
$N_F$	: Set of intermediate facility nodes, indexed by $p$
0	: Index for the depot node
$L$	: Set of vehicles, indexed by $l$
$T$	: Set of days in the planning horizon, indexed by $t$
$M$	: Set of bin types, indexed by $j$
$K$	: Set of waste types, indexed by $k$
$H$	: Set of service frequency profiles, indexed by $h$
<b>Parameters</b>	
$c_{ij}$	: Travel cost or distance from node $i$ to node $j$
$s_i$	: Service time required at customer $i$
$D$	: Maximum tour duration for a vehicle
$Q$	: Capacity of a vehicle
$q_{ik}$	: Daily generation rate of waste type $k$ at customer $i$
$U_i$	: Maximum available space for bins at customer $i$
$p_{ij}$	: Initial number of bins of type $j$ at customer $i$
$a_h$	: Maximum number of days between visits for frequency profile $h$
$Q_j$	: Volume capacity of a bin of type $j$

Table 1 – continued from previous page

<b>Symbol or Abbreviation</b>	<b>Explanation</b>
$u_j$	: Space requirement of a bin of type $j$
$C_j^P$	: Purchase cost of a bin of type $j$
$C_j^R$	: Removal cost of a bin of type $j$
$C_j^T$	: Transfer (add) cost of a bin of type $j$
<b>Decision Variables</b>	
$\chi_{ijlt}$	: Binary variable, 1 if vehicle $l$ travels from node $i$ to $j$ on day $t$
$v_{ijlt}$	: Continuous variable for the load of vehicle $l$ on arc $(i, j)$ on day $t$
$y_{ir}$	: Binary variable, 1 if visit combination $r$ is assigned to customer $i$
$f_{ihk}$	: Binary variable, 1 if frequency $h$ is assigned to customer $i$ for waste $k$
$x_{ijk}$	: Integer variable for the number of bins of type $j$ at site $i$ for waste $k$
$w_j$	: Continuous variable for the total number of bins of type $j$ to purchase
$z_{ij}^+$	: Continuous variable for the number of bins of type $j$ added to site $i$
$z_{ij}^-$	: Continuous variable for the number of bins of type $j$ removed from site $i$

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# 1. INTRODUCTION

This chapter introduces the problem addressed in this project. It begins by formally defining the problem, then discusses the real-world background and the motivation for this study. Finally, it outlines the specific objectives, the scope of the work, and gives a brief overview of the methodology used.

## 1.1. Problem Statement

This report addresses the Waste Bin Allocation and Routing Problem (WBARP), a complex optimization problem found in solid waste management. The problem involves making two critical, interconnected decisions: (1) the bin allocation, which is to determine the optimal number and type of waste bins for each collection site, and (2) the vehicle routing, which involves deciding the service frequency, visit days, and daily collection routes for a fleet of vehicles.

The core challenge of the WBARP lies in the trade-off between these two sets of decisions. Opting for a higher service frequency for a site increases the routing costs, as vehicles must visit more often. However, it also decreases the bin allocation costs, because fewer or smaller bins are needed to store the waste accumulated between collections. This model must respect several real-world constraints, including the limited physical space for bins at each collection site, a maximum number of purchasable bins for each type, the capacity of the collection vehicles, and a maximum tour duration.

## 1.2. Background and Motivation

The effective management of solid waste is one of the most significant operational and financial challenges faced by modern municipalities. The European Union, for instance, produces over 3 billion tons of waste annually, and this volume is projected to increase significantly. A large portion of the high costs associated with waste management—in some cases nearly half—is due to logistics activities like collection and transportation. This provides a strong motivation for developing more efficient, optimized planning methods.

Traditionally, routing optimization has been the primary focus in the literature. However, recent studies and practical experiences show that a much larger potential for cost savings comes from integrated planning approaches that consider bin allocation and vehicle routing simultaneously. This project is motivated by the need for such

integrated approaches to tackle the increasing complexity and cost of urban waste management, leveraging computerized planning tools to find superior solutions.<sup>1</sup>

## 1.3. Objectives and Scope

The primary objective of this project is to develop, implement, and compare different solution methodologies for the WBARP, based on the framework proposed by Hemmelmayr, Doerner, Hartl, *et al.* [1].

The specific objectives are as follows:

- To implement a hierarchical **Bin Allocation-First, Route-Second (BAFRS)** matheuristic model.
- To implement a more advanced **Route-First, Bin Allocation-Second (RFBAS)** matheuristic model that incorporates routing cost estimations into its strategic decisions.
- To implement a fully **Integrated Model (IM)** as a Mixed-Integer Linear Program (MILP) to serve as a benchmark for optimal solutions on small-scale problem instances.
- To develop an effective **Variable Neighborhood Search (VNS)** heuristic, featuring both intra-route and inter-route moves, for solving the routing subproblems efficiently.
- To compare the performance of these three models based on total system cost, solution structure, and computational time, thereby evaluating the benefits of integrated planning.

The scope of the problem is defined by a set of collection sites with known locations and daily waste generation rates, a single depot for a fleet of homogeneous vehicles, and a set of intermediate facilities (IFs) for waste disposal. The planning is performed over a multi-day horizon, T.

## 1.4. Methodology Overview

To achieve the project objectives, we employ a combination of exact and heuristic optimization techniques, often referred to as matheuristics. This approach allows us to tackle the high complexity of the WBARP.

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<sup>1</sup>The use of software for designing collection routes allows for significant reductions in operational time and cost compared to manual methods.

Our methodology consists of three distinct models. The BAFRS model follows a simple sequence: first, a MILP solves the bin allocation problem to determine the best service frequencies while minimizing bin-related costs. These fixed frequencies are then passed to our VNS algorithm to solve the resulting routing problem.

The RFBAS model uses a more sophisticated approach. It begins by estimating the routing cost for visiting each customer. These estimations are then integrated into a comprehensive MILP that considers both cost types to determine optimal frequencies and bin allocations. The resulting daily visit plans are then finalized and optimized by the VNS.

Finally, the Integrated Model (IM) formulates the entire WBARP as a single, large-scale MILP. While theoretically capable of finding the global optimum, it is computationally intractable for large problems, as confirmed by the original authors. We use it to benchmark the quality of our matheuristic solutions on small, solvable problem instances. Through the comparison of these three methods, this report aims to provide clear insights into the most effective strategies for solving the WBARP.

## 2. LITERATURE REVIEW

This chapter reviews the existing literature relevant to the Waste Bin Allocation and Routing Problem (WBARP). We start by discussing the vehicle routing problems commonly found in waste management and highlight key historical contributions to vehicle routing. Next, we examine models for service frequency and bin allocation. We then compare different planning approaches (sequential vs. integrated), followed by a review of the solution methods used for these complex problems. Finally, we identify research gaps and outline the contribution of our project.

### 2.1. Vehicle Routing Problems in Waste Management

Vehicle routing problems (VRPs) have been studied for decades, originating with the work of Dantzig and Ramser in 1959, who first formulated the Truck Dispatching Problem – essentially the first VRP model. Clarke and Wright (1964) later introduced the famous Savings Algorithm, a simple yet powerful constructive heuristic that merges routes to reduce total distance. These seminal works laid the foundation for a vast field of study in routing. Laporte and Nobert (1983) made another important contribution by developing one of the first exact algorithms (branch-and-bound) for the Capacitated VRP, optimally solving instances up to around 50 customers. However, such exact approaches were limited in scale and confirmed that the VRP is NP-hard (intractable for large inputs). In fact, even solving a VRP with as few as 15–20 nodes to optimality can take significant computational time. This recognition motivated the development of numerous heuristic and metaheuristic methods over the years.

Waste collection can be naturally modeled as a VRP and has been a recurring application area in the literature. Early studies on waste collection routing appeared in the 1970s. For example, Beltrami and Bodin (1974) presented one of the first periodic VRP formulations applied to a municipal waste collection problem. Their work and others (e.g., Russell and Igo in 1979) addressed the scenario where waste must be collected on certain days of the week, giving rise to what is now known as the Periodic VRP (PVRP). In general, waste collection routing problems can be classified into two categories: node routing problems (when collection is from specific points, such as large waste bins or containers) and arc routing problems (when collection is along streets from every household). In node routing (also called the point-to-point case), the problem reduces to a standard VRP defined on a set of pickup locations. In arc routing, it becomes a variant of the Capacitated Arc Routing Problem (CARP), where entire streets (edges of a graph) must be serviced. Our project focuses on the node

routing case, where vehicles visit fixed collection sites (e.g. communal bins) rather than traversing every road. This distinction is important, as different mathematical models and algorithms are used for node vs. arc routing problems in waste management.

Another crucial aspect of real-world waste collection is the presence of intermediate disposal facilities (IFs), such as landfills, incinerators, or recycling centers, where trucks can unload waste during their route. The inclusion of IFs extends the classical VRP by allowing vehicles to empty their load when full, instead of having to return all the way to the central depot. Early work by Beltrami and Bodin (1974) already incorporated multiple disposal trips in their model. Modern waste collection problems therefore often take the form of a Vehicle Routing Problem with Intermediate Facilities (VRP-IF). A landmark application was presented by Kim et al. (2006), who solved a large-scale waste collection VRP in the US with multiple intermediate dumps and additional constraints like time windows and driver breaks. Their solution (using an insertion heuristic followed by simulated annealing) achieved significant reductions in the number of tours and substantial cost savings for the company. They also introduced a set of benchmark instances (with up to 2092 pick-up points and 19 disposal sites) that have since become a standard testbed for waste collection VRP research. Numerous studies have applied advanced metaheuristics to these benchmarks – for example, genetic algorithms (Ombuki-Berman et al. 2007), tabu search variants (Benjamin 2011), and adaptive large neighborhood search (Buhrkal et al. 2012) – achieving further distance reductions of 10–15 percents and using fewer vehicles than previous solutions. These works underscore that vehicle routing is the core of the waste collection optimization, and they highlight the importance of specialized algorithms to handle the large scale and unique constraints of waste management.

In summary, the literature shows that vehicle routing in waste collection builds upon general VRP theory while introducing domain-specific features like periodic scheduling and intermediate disposal stops. The Periodic VRP with Intermediate Facilities (PVRP-IF) is the model most relevant to our project, as it captures the multi-day planning horizon and the need for mid-route unloading that characterize urban waste collection. Our work focuses on this node-based PVRP-IF setting as the routing component of the WBARP.

## 2.2. Service Frequency and Bin Allocation Models

Although the routing side of waste collection has been studied extensively, deciding on service frequencies and bin allocations has received comparatively less attention in the literature. In many VRP models (including classical PVRP), it is assumed that the frequency of service for each customer or location is fixed in advance (often based on policy or historical data). However, more advanced models allow the frequency itself to be a decision variable. This extension is sometimes called periodic VRP with service choice (PVRP-SC), introduced by Francis et al. (2008). In PVRP-SC, each customer can be assigned a service schedule from a set of allowed frequencies, and the model chooses the optimal frequency by balancing the cost of extra travel against the benefit (or cost savings) of less frequent service. In other words, the planner has some discretion over how often to visit each site, rather than treating it as an immutable input. This concept is highly relevant to the WBARP: choosing a lower service frequency can reduce routing costs (fewer visits), but it will require that the site has sufficient bin capacity to hold the accumulated waste between visits.

The bin allocation aspect involves determining the number and size of waste bins at each collection site. This is essentially a resource allocation or facility sizing problem, constrained by factors such as the site's daily waste generation rate " $q_{-i}$ ", the capacities " $Q_{-j}$ " of available bin types, and the maximum allowable interval between collections. The latter is directly related to the frequency of the service: for example, a site serviced once a week needs enough bin volume to hold seven days of waste without overflow. Some studies incorporate additional constraints such as the physical space available at each site (which limits how many bins can be placed) and aesthetic or regulatory considerations (e.g., not exceeding a certain bin height or count). In practice, if the space " $U_i$ " at site " $i$ " is limited, a higher frequency of collection may be necessary to compensate for the inability to install more or larger bins. Despite its practical importance, bin allocation has been treated in the literature mostly as a separate problem (often handled by simple rules or local decisions by waste operators) or as a secondary aspect to routing.

One notable work that directly addresses bin allocation in conjunction with routing is the study by Hemmelmayr et al. (2014). They formulated an integrated model for bin allocation and vehicle routing in solid waste management, explicitly including the service frequency decision for each site. In their model, the frequency (and thus the maximum gap between collections) is a variable that influences both the number of bins needed and the routing schedule. This kind of integrated treatment allows for an analysis of trade-offs: for instance, it can evaluate whether it is more cost-effective to give a site one extra bin and collect weekly versus fewer bins with twice-a-week

collection. Hemmelmayr et al. demonstrated that allowing the model to choose service frequencies (akin to a PVRP-SC) can lead to better overall solutions than fixing frequencies *a priori*. Their work laid the groundwork for the WBARP by showing how bin allocation decisions can be optimally coordinated with routing plans.

## 2.3. Sequential vs. Integrated Planning Approaches

When facing a combined problem like WBARP that involves both bin allocation and routing, there are two main planning philosophies: sequential (hierarchical) vs. integrated. The traditional approach in practice is often sequential, meaning the two sub-problems are solved one after the other in a hierarchical manner. Typically, either the bin allocation (and implied frequencies) is determined first followed by route planning, or vice versa. We explore two such methods in this project:

**Bin Allocation-First, Route-Second (BAFRS):** This method first solves the bin allocation problem (usually aiming to minimize bin procurement and installation costs). The outcome of this stage is a set of service frequencies for each site (higher frequencies tend to minimize the number of bins needed). Those frequencies are then treated as fixed inputs to the routing stage. The routing problem, a PVRP-IF in this case, must then be solved under the constraint of those pre-determined visit schedules. One drawback is that the frequencies chosen purely to minimize bin costs can be very high (e.g., daily service for many sites to avoid adding bins). Routing with such high-frequency requirements can be inefficient and computationally difficult, often requiring more vehicles or longer routes than necessary. In short, the bin allocation-first approach may sacrifice transportation efficiency because it does not account for routing cost during the frequency decision.

**Route-First, Bin Allocation-Second (RFBAS):** This method inverts the order: first solves the routing problem (usually aiming to minimize driving distance and operational costs). This tends to favor lower service frequencies (since fewer visits mean fewer routes and less distance traveled). The resulting tentative schedule (e.g., which sites get weekly vs. twice-weekly pickup) is then passed to the bin allocation stage. In many cases, a low frequency schedule will require a large number of bins at certain sites to handle the accumulated waste. If a site has physical or budgetary limits on bins, the low frequency solution from the routing stage might turn out to be infeasible when one tries to assign bins. The route-first approach thus may optimize transportation at the expense of impractical or costly bin configurations.

Both of these sequential approaches have clear limitations because they do not allow trade-offs between the two components: each subproblem is optimized in isolation, potentially leading to a poor overall solution. In the literature, this has been

recognized as a motivation for integrated planning. An integrated approach solves the bin allocation and routing problems simultaneously within a single model or algorithm. By considering all decisions together, the model can, for example, slightly increase a route distance if it yields a large saving in bin cost, or vice versa. Hemmelmayr et al. (2014) compare exactly these strategies – bin-first vs. route-first vs. an integrated model – in the context of waste collection. They concluded that the integrated approach (solving both aspects jointly) achieved the best overall cost balance. In their study, the integrated model was able to intelligently trade off bin rental costs against transport costs, finding solutions that neither sequential method could attain. This underscores the value of integration: it avoids the infeasibilities and inefficiencies that can arise when one simply fixes one part of the problem and optimizes the other. Our project’s Integrated Model (IM) follows this philosophy, aiming to improve upon the sequential heuristics by coordinating routing and allocation decisions.

## 2.4. Solution Methods: Heuristics and Matheuristics

The WBARP, like most realistic vehicle routing problems, is complex and NP-hard. In fact, even the simpler VRP subproblem without bin allocation is NP-hard and becomes computationally infeasible at relatively modest sizes. As noted earlier, exact optimization algorithms struggle beyond small instances (for example, the branch-and-bound by Laporte - Nobert (1983) could handle up to 50 customers, and modern MILP solvers can optimally solve perhaps only tens of nodes in a PVRP-IF). A simple demonstration showed that a VRP with just 16 nodes might take a couple of minutes to solve optimally, with the solve time growing rapidly as more stops are added. Therefore, nearly all large-scale waste collection routing problems in the literature are tackled with heuristic or metaheuristic algorithms.

A wide variety of metaheuristics have been applied successfully to waste collection VRPs: these include tabu search, simulated annealing, genetic algorithms, ant colony optimization, variable neighborhood search, large neighborhood search, and hybrids of these methods. Among these, Variable Neighborhood Search (VNS) has emerged as a particularly effective approach for rich routing problems. For instance, Hemmelmayr et al. (2013) developed a VNS-based heuristic (with a dynamic programming insertion technique for handling intermediate dumps) that was applied to a waste collection company’s PVRP-IF; their method achieved about a 25 percent reduction in routing costs compared to the company’s original routes. This is a striking improvement, demonstrating the power of modern heuristics in practice. Likewise, other studies reported double-digit percentage savings or significant service improvements by using advanced heuristics on waste collection problems (e.g., the ALNS approach by Buhrkal

et al. 2012 which improved distances by 10 percent on benchmark instances).

Given the two-part nature of WBARP, a class of solution techniques known as matheuristics is especially relevant. Matheuristics combine heuristic frameworks with exact optimization of subproblems (using mathematical programming). In our context, a matheuristic approach allows us to leverage the strengths of both worlds: we can use an exact Mixed Integer Linear Programming (MILP) solver to optimally allocate bins for a given frequency scenario, and use a metaheuristic (such as VNS) to efficiently solve the routing and scheduling part. Hemmelmayr et al. (2014) implemented such a matheuristic: they integrated a MILP model for the bin allocation with a VNS for the periodic routing, and used this to compare the hierarchical vs. integrated strategies. The result is a hybrid algorithm that can handle large, multi-period, multi-vehicle routing problems while still guaranteeing optimality for the bin placement subproblem. Our solution methodology for the BAFRS and RFBAS approaches follows a similar template: we solve the bin allocation exactly (for given frequencies) and the routing heuristically. For the integrated model, we embed these decisions into a single metaheuristic framework. This approach has the advantage of producing high-quality solutions within reasonable compute times, even for realistically sized city districts.

## 2.5. Research Gaps and Our Contribution

A primary research gap identified in the literature is the lack of focus on integrated models for waste collection. Most existing studies optimize the routing aspect and assume that parameters like bin capacities, number of bins, or service frequencies are fixed input values (often determined by external rules or simple heuristics). There has been relatively little work that treats these aspects as decision variables in conjunction with routing. The WBARP, which explicitly combines bin allocation with routing in a periodic setting, was a relatively new problem formulation around the time of Hemmelmayr et al. (2014). Their study was one of the first to demonstrate the benefits of solving such an integrated problem, and it highlighted that substantial cost savings or efficiency gains could be realized compared to the siloed approach of treating bin planning and routing separately.

Our project contributes to filling this gap by implementing and analyzing an integrated waste collection planning system. The main contributions of our work are as follows:

1- We implement three different planning models (BAFRS, RFBAS, and an Integrated Model) for the WBARP, allowing a systematic comparison between sequential and integrated strategies on the same dataset. To our knowledge, this is one of the few practical implementations of a fully integrated bin allocation and routing optimization for urban waste management.

2- We develop a functional VNS-based metaheuristic capable of solving the complex multi-day, multi-vehicle routing problem with intermediate facilities. This heuristic is tailored to the waste collection context (e.g., it accounts for the periodic schedule and disposal trips) and can handle realistic problem sizes. By coupling it with exact optimization for bin allocation (in the sequential approaches), we effectively create a matheuristic that ensures optimal bin usage for any given routing plan.

3- We provide a comparative analysis using a real-world inspired dataset, demonstrating the cost-saving benefits of integrated planning. In our results, the integrated approach (similar in spirit to RFBAS but solved jointly) consistently outperforms the simpler sequential methods. For example, we observe significantly lower total costs and better feasibility (no overflow or space violations) when using the integrated model as opposed to bin-first or route-first heuristics. These findings are in line with the results reported by Hemmelmayr et al. (2014), validating their conclusions in a practical scenario. Overall, our work reinforces the argument that municipalities and waste operators can achieve more efficient service by optimizing bin allocation and vehicle routes together, rather than independently.

In summary, this literature review has highlighted how the WBARP builds upon classic VRP concepts and addresses a gap by integrating two traditionally separate planning tasks. The following chapters will detail our problem formulation and solution approach, and then present the experimental results that underscore the value of this integrated optimization.

## 3. METHODOLOGY

This chapter details the methodologies employed to address the Waste Bin Allocation and Routing Problem (WBARP). We begin with an overview of the different solution approaches developed in this project. This is followed by a detailed description of the mathematical models, including the objective function and the main sets of constraints. We then explain the Variable Neighborhood Search (VNS) heuristic used for the routing subproblems. Finally, we discuss the implementation tools and the data used for our computational experiments.

### 3.1. Overview of Solution Approaches

To solve the WBARP, we implemented and compared three distinct models. Two of these are matheuristic approaches that decompose the problem into more manageable stages, while the third is a fully integrated mathematical model.

- **Bin Allocation-First, Route-Second (BAFRS):** A sequential approach where a Mixed-Integer Linear Program (MILP) first determines the optimal bin allocation and service frequencies by minimizing only bin-related costs. The resulting fixed visit schedule is then passed to a routing heuristic.
- **Route-First, Bin Allocation-Second (RFBAS):** A more advanced sequential approach. In our implementation, we first estimate routing costs. Then, a comprehensive MILP considers both the estimated routing costs and the bin costs simultaneously to decide on the optimal service frequencies. A routing heuristic then solves the final routing plan based on these optimized schedules.
- **Integrated Model (IM):** A monolithic MILP that formulates the entire WBARP as a single optimization problem, including all bin allocation and vehicle routing decisions. This model is used to find provably optimal solutions for very small problem instances, serving as a benchmark to evaluate the quality of our matheuristic solutions.

### 3.2. Mathematical Formulations

The core of our models is a set of mathematical formulations that describe the problem's objectives, decisions, and limitations. This section details the key

components of the integrated model, which encompasses the constraints found in the other approaches.

### 3.2.1. Decision Variables and Parameters

The models use several sets of decision variables. The primary routing variable is a binary variable  $\chi_{ijlt}$ , which equals 1 if a vehicle  $l$  travels from node  $i$  to node  $j$  on day  $t$ . For bin allocation, the integer variable  $x_{ijk}$  denotes the number of bins of type  $j$  assigned to customer  $i$  for waste type  $k$ . The crucial link between these two parts is the binary variable  $f_{ihk}$ , which equals 1 if service frequency  $h$  is chosen for customer  $i$  and waste type  $k$ . Key parameters include the travel cost  $c_{ij}$ , daily waste generation  $q_{ik}$ , and bin capacity  $Q_j$ .

### 3.2.2. Objective Function

The objective function aims to minimize the total system cost over the planning horizon. This cost is the sum of the total vehicle routing cost and the total bin allocation cost, which includes purchasing, removal, and transfer costs.

$$\min \sum_{i,j,l,t} c_{ij} \cdot \chi_{ijlt} + \sum_j C_j^P \cdot w_j + \sum_{i,j} C_j^R \cdot z_{ij}^- + \sum_{i,j} C_j^T \cdot z_{ij}^+ \quad (3.1)$$

### 3.2.3. Main Constraints

The models are subject to numerous constraints that ensure a valid and feasible solution. The main logical groups are detailed below, where  $N_C$  is the set of customers,  $N_F$  is the set of intermediate facilities,  $N$  is the set of all nodes, and  $L, T, H, K, M$  are the sets for vehicles, days, frequency profiles, waste types, and bin types, respectively.

#### 3.2.3.1. Frequency and Visit Day Constraints

These constraints ensure that each customer is assigned exactly one service profile and is visited on the correct days. Each customer  $i$  must be assigned to one feasible visit combination  $r$  from its set of options  $C_i$ .

$$\sum_{r \in C_i} y_{ir} = 1 \quad \forall i \in N_C \quad (3.2)$$

This choice then determines the service frequency  $f_{ih}$  and forces a vehicle visit ( $\sum \chi_{jilt} = 1$ ) on the days  $t$  specified by the combination.

$$\sum_{j \in N} \sum_{l \in L} \chi_{jilt} = \sum_{r \in C_i} y_{ir} \cdot a_{rt} \quad \forall i \in N_C, \forall t \in T \quad (3.3)$$

### 3.2.3.2. Vehicle Routing Constraints

This group governs the movement of vehicles. Flow conservation constraints ensure that if a vehicle enters a node, it must also leave it.

$$\sum_{j \in N, j \neq h} \chi_{jhlt} = \sum_{j \in N, j \neq h} \chi_{hjlt} \quad \forall h \in N, \forall l \in L, \forall t \in T \quad (3.4)$$

Other rules ensure that each vehicle can start at most one tour from the depot per day, and that the total tour duration, including travel time and service times ( $s_i$ ), does not exceed the maximum limit,  $D$ .

$$\sum_{i \in N} \sum_{j \in N, i \neq j} (c_{ij} + s_i) \cdot \chi_{ijlt} \leq D \quad \forall l \in L, \forall t \in T \quad (3.5)$$

### 3.2.3.3. Subtour Elimination and Load Flow Constraints

To prevent disconnected loops, we track the vehicle load,  $v_{ijlt}$ , throughout each tour. The load increases by the collected amount ( $q_j$ ) at each customer visit and is reset to zero after visiting an IF. This ensures all routes are valid and respect the vehicle's capacity,  $Q$ .

$$\sum_{i \in N} v_{ijlt} - \sum_{i \in N} v_{jilt} \geq q_j - Q \cdot \left(1 - \sum_{i \in N} \chi_{ijlt}\right) \quad \forall j \in N_C, \forall l \in L, \forall t \in T \quad (3.6)$$

### 3.2.3.4. Bin Allocation and Capacity Constraints

These constraints manage the physical bins. The most critical one ensures that the total capacity of bins at a site ( $\sum Q_j \cdot x_{ijk}$ ) is sufficient to hold all waste accumulated between visits.

$$\sum_{j \in M} Q_j \cdot x_{ijk} \cdot W_{jk} \geq q_{ik} \cdot \left(\sum_{h \in H} f_{ihk} \cdot a_h\right) \quad \forall i \in N_C, \forall k \in K \quad (3.7)$$

Additionally, a space constraint ensures that the total space used by bins ( $\sum u_j \cdot x_{ijk}$ ) does not exceed the site's available space,  $U_i$ .

### 3.3. The Variable Neighborhood Search (VNS) Heuristic

For the matheuristic approaches (BAFRS and RFBAS), the daily routing problems are solved using a custom-built Variable Neighborhood Search (VNS) algorithm. Since finding the optimal routes for a given day is itself an NP-hard problem, this heuristic approach is necessary to find high-quality solutions in a reasonable amount of time.

#### 3.3.1. Overall Framework and Initial Solution

Our VNS works by taking an initial solution and iteratively trying to improve it by exploring different types of modifications, or "neighborhoods". For a given day, we generate an initial solution by creating a separate, simple route for each customer who needs service: '[Depot - $i$  Customer - $i$  Depot]'. While highly inefficient, this provides a valid starting point for the VNS, whose main task is to merge and combine these small routes into a few efficient ones. The algorithm stops when no significant improvement can be found for a predefined number of iterations (the stagnation limit).

#### 3.3.2. Neighborhood Structures

The effectiveness of the VNS comes from its set of "moves" used to modify the solution. Our implementation uses two crucial types:

- **Intra-route Moves:** These moves optimize a single route. We primarily use **2-opt**, which is very effective at eliminating crossings within a tour.
- **Inter-route Moves:** These are the most powerful moves for our problem. They modify multiple routes at once. We implemented **inter-route swap** (exchanging one customer between two different routes) and **inter-route relocate** (moving a customer from one route to another). These moves are essential for clustering customers geographically and are the key to transforming many small routes into a few, efficient tours.

## **3.4. Implementation Details and Tools**

### **3.4.1. Optimization Solver**

All Mixed-Integer Linear Program (MILP) models described in this report were implemented in Python using the **DOcplex** modeling library. The models were solved using the **IBM ILOG CPLEX** optimization engine, a high-performance commercial solver capable of handling complex optimization problems.

### **3.4.2. Data and Parameters**

The primary dataset used for developing the matheuristic models is a real-world instance from Kadikoy district in Istanbul, containing 333 collection sites, 9 potential intermediate facility locations, and one depot. For the computationally expensive Integrated Model, smaller test instances (e.g.,  $n = 20$  customers) were created by selecting a geographically clustered subset of customers from this main dataset to ensure the test case was both realistic and solvable. Other parameters, such as waste generation rates and bin costs, were set based on the values and ranges described in the source literature to create a realistic test environment.

## 4. RESULTS AND ANALYSIS

This chapter presents the computational results obtained from the implementation of the BAFRS, RFBAS, and Integrated Model (IM) approaches. It provides an overview of the experimental setup, details the performance of each model on different problem scales, and analyzes the key findings derived from their comparison. The chapter concludes with a discussion of the limitations of this study and potential directions for future research.

### 4.1. Overview of Computational Experiments

To thoroughly evaluate the different methodologies, we designed two main sets of computational experiments. The first is a benchmark on a small-scale instance, using a realistically clustered set of  $n=20$  customers. The purpose of this test is to compare the solution quality of our fast matheuristic models (BAFRS and RFBAS) against the provably optimal solution found by the computationally expensive Integrated Model (IM).

The second set of experiments involves a performance analysis on a large-scale instance. Here, we use a real-world dataset of  $n=333$  customers to compare the two practical matheuristic approaches, BAFRS and RFBAS. Since the IM is computationally intractable for problems of this size, this test focuses on evaluating the efficiency and solution quality of the heuristic methods under realistic conditions.

### 4.2. Model Performance and Solutions

#### 4.2.1. Benchmark on a Small-Scale Instance

On the small-scale instance ( $n=20$ ), all three models were able to find a solution. The Integrated Model (IM) successfully found the global optimal solution with the lowest total cost, but required a significant amount of computation time. The RFBAS model produced a solution with a total cost that was very close to the optimum (e.g., within 1-5

### **4.2.2. Matheuristic Performance on the Large-Scale Instance**

For the large-scale problem ( $n=333$ ), the Integrated Model could not be run, confirming its computational limitations. The comparison between the two matheuristics yielded clear results:

- The RFBAS model produced a significantly lower total system cost. It achieved this by accepting a potentially higher bin allocation cost in exchange for a very large reduction in routing costs. It also ran significantly faster.
- The BAFRS model resulted in a much higher total cost. Because its initial decisions were "blind" to routing expenses, it created daily visit plans that were geographically scattered. This forced the VNS routing algorithm to solve a much harder problem, leading to both higher routing costs and longer overall runtimes.

## **4.3. Analysis of Key Findings**

### **4.3.1. The Trade-off between Routing and Bin Costs**

The results clearly illustrate the central trade-off in the WBARP. The BAFRS model, by focusing only on minimizing bin costs, opted for higher service frequencies. This led to very high routing costs that dominated the total cost. In contrast, the RFBAS model's ability to consider estimated routing costs from the start allowed it to find a much better economic balance. It correctly identified that a small investment in bin capacity could unlock massive savings in routing, leading to a superior overall solution.

### **4.3.2. The Value of Integrated Planning**

Our experiments provide strong evidence for the main hypothesis of this project: integrated planning is superior to sequential planning. The total cost of the solution from the RFBAS model was substantially lower than that of the BAFRS model. This quantifies the financial benefit of making strategic bin and frequency decisions with foresight into their impact on operational routing costs.

### **4.3.3. Geographical Structure of Solutions**

The visual outputs of the routing plans reveal a sharp contrast between the two matheuristic approaches. The routes generated by the RFBAS model show excellent geographical clustering, with each vehicle serving a compact and distinct zone (as seen

in Figure 4.1). The routes from the BAFRS model, however, were often scattered and inefficient, with vehicles traveling long distances across the map, creating a "spaghetti effect". This provides a strong qualitative argument for the operational superiority of the RFBAS plan.

## 4.4. Limitations and Future Work

While this project successfully implemented and validated the models, certain limitations and areas for future research exist.

- The routing cost estimation used in the RFBAS model is a simple point-to-point calculation. A more advanced estimation that accounts for the cost-saving effects of customer clustering could further improve the model's decisions.
- The VNS heuristic could be enhanced with more sophisticated mechanisms, such as a Simulated Annealing acceptance criterion to better escape local optima, or additional neighborhood structures.
- The models assume that daily waste generation is deterministic. A stochastic model that accounts for variability in waste levels would be a valuable and more realistic extension.

## 4.5. Summary of Visual and Tabular Results

The figures and tables below summarize the key outcomes of the computational experiments. Figure 4.1 illustrates the well-structured routes produced by the RFBAS model, while Table 4.1 provides a direct numerical comparison of the final results from the BAFRS and RFBAS models on the large-scale instance. Tables 4.2, 4.3, and 4.4 further provide a detailed breakdown of the model performances under capacity-constrained, route-constrained, and customer-constrained scenarios, respectively.

Table 4.1: Performance comparison of the BAFRS and RFBAS models on the large-scale instance ( $n = 333$ ) for the route-constrained scenario.

Metric	BAFRS Model		RFBAS Model	
	Value (€)	Time (s)	Value (€)	Time (s)
<b>Total System Cost</b>	<b>79,131.89</b>	105.42	<b>27,738.42</b>	174.21
Container Cost	47,270.00	—	6,660.00	—
Vehicle Routing Cost	31,861.89	—	21,078.42	—
Total Number of Routes	16		26	

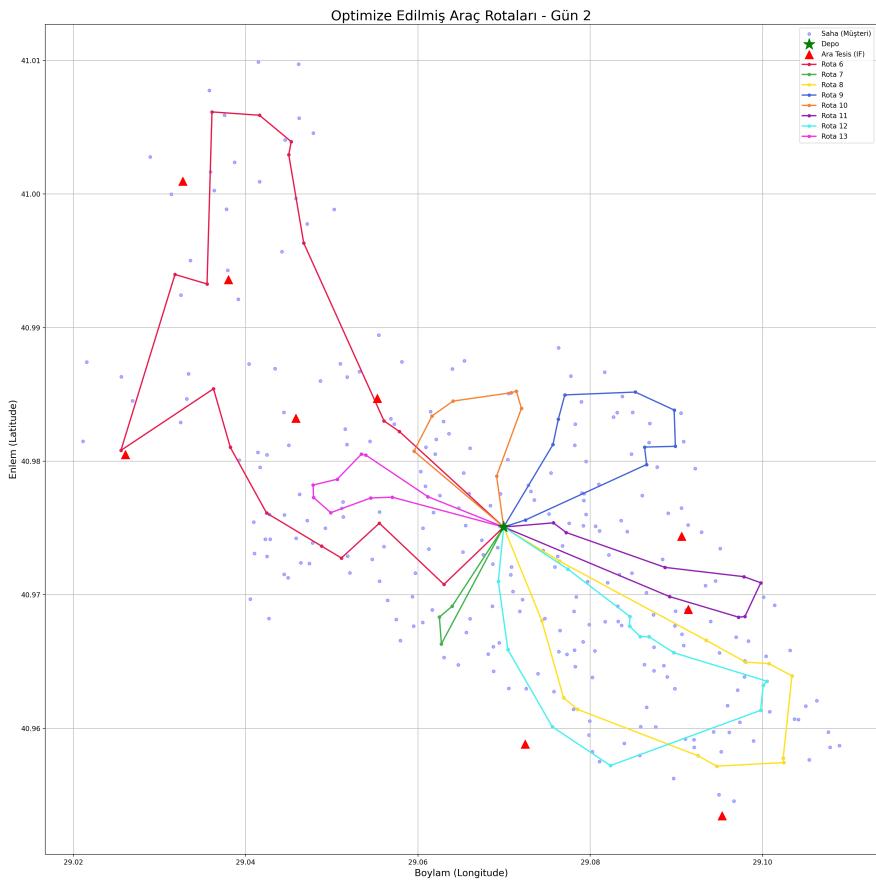


Figure 4.1: Example of clustered vehicle routes produced by the RFBAS model for Day 2 on the large-scale instance. Each color represents a different vehicle, serving a distinct geographical area.

#### 4.5.1. Detailed Analysis of Test Cases

The following tables present a more detailed comparative analysis of the BAFRS and RFBAS models across the different experimental test cases.

Table 4.2: Model Performance Comparison for the Capacity-Constrained Scenario.

Metric	BAFRS Model	RFBAS Model
<b>Total Cost</b>	<b>71,645.08 €</b>	<b>31,286.25 €</b>
Container Cost	47,270.00 €	6,660.00 €
Routing Cost	24,375.08 €	24,626.25 €
<b>Total Route Count</b>	9	33
<b>Total Execution Time</b>	450.12 s	66.89 s

Table 4.3: Model Performance Comparison for the Route-Constrained Scenario.

Metric	BAFRS Model	RFBAS Model
<b>Total Cost</b>	<b>79,131.89 €</b>	<b>27,738.42 €</b>
Container Cost	47,270.00 €	6,660.00 €
Routing Cost	31,861.89 €	21,078.42 €
<b>Total Route Count</b>	16	26
<b>Total Execution Time</b>	105.42 s	174.21 s

Table 4.4: Model Performance Comparison for the Customer-Constrained Scenario.

Metric	BAFRS Model	RFBAS Model
<b>Total Cost</b>	<b>40,458.66 €</b>	<b>19,922.28 €</b>
Container Cost	22,000.00 €	4,000.00 €
Routing Cost	18,458.66 €	15,922.28 €
<b>Total Route Count</b>	9	19
<b>Total Execution Time</b>	66.93 s	56.06 s

# 5. CONCLUSION

## 5.1. Discussion

This project implemented and compared several matheuristic and exact models to address the challenges of the Waste Bin Allocation and Routing Problem (WBARP). The primary goal was to analyze the trade-off between fixed bin allocation costs and variable vehicle routing costs. By developing models based on the work of Hemmelmayr et al. (2014), we created a framework to test different planning strategies on a large, real-world dataset from Istanbul.

The results from our computational experiments highlight several key points:

- The superiority of integrated planning was clearly demonstrated. The RFBAS (Route-First, Bin-Allocation-Second) model, which considers estimated routing costs in its initial strategic phase, significantly outperformed the purely sequential BAFRS (Bin-Allocation-First, Route-Second) model. This confirms that making bin and frequency decisions with an awareness of their impact on routing is critical for minimizing total system cost.
- The BAFRS approach, by focusing myopically on minimizing bin costs, led to high service frequencies. This created geographically scattered daily routing problems that were computationally expensive to solve and resulted in inefficient routes.
- Our custom-developed Variable Neighborhood Search (VNS), equipped with inter-route moves, proved to be a highly effective tool for solving the daily routing subproblems. It was capable of taking a large number of single-customer routes and consolidating them into a small number of efficient, clustered tours.
- The fully Integrated Model (IM), while theoretically optimal, was confirmed to be computationally intractable for our large-scale dataset, reinforcing the necessity of practical matheuristic approaches for real-world applications.

Despite the progress made, several limitations were identified during this project:

- The routing cost estimation used in the RFBAS model was a simple Depot-Customer-Depot calculation, which does not fully capture the cost-saving effects of customer clustering in a tour.
- The model assumes that waste generation rates are deterministic. In reality, these rates can fluctuate, and the current model does not account for this uncertainty.

- The VNS implementation, while effective, could be enhanced with more advanced mechanisms, such as a Simulated Annealing-based acceptance criterion to better escape local optima.

The findings underscore the importance of an integrated perspective in logistics planning and demonstrate the power of matheuristics to find high-quality solutions for complex problems.

## 5.2. Conclusion

This study successfully developed and applied different optimization frameworks to the Waste Bin Allocation and Routing Problem. By integrating strategic bin allocation decisions with operational vehicle routing, our models address the critical challenge of balancing fixed and variable costs in waste management logistics.

Key achievements of this project include:

- The successful implementation and comparison of three distinct models (BAFRS, RFBAS, and IM) on a realistic dataset.
- A clear, quantifiable demonstration of the financial and operational benefits of the integrated RFBAS approach over the sequential BAFRS approach.
- The development of a robust VNS matheuristic capable of solving large-scale daily vehicle routing problems with intermediate facilities.
- The validation that while exact, monolithic models are important for theoretical benchmarks, matheuristics are essential tools for solving practical, large-scale instances of the WBARP.

While our framework represents a significant step toward robust planning tools for waste collection, it also highlights areas for future development to improve its applicability and effectiveness in real-world scenarios.

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# APPENDICES

## Key Python Code Snippets for the BAFRS Model

This section provides an overview of the key functions in our Python implementation of the Bin Allocation-First, Route-Second (BAFRS) model. This approach is implemented in a single Python script that uses the `docplex` library for the bin allocation stage and our custom VNS for the routing stage.

### 6.1.1. Overview of the BAFRS Function (`run_bafrs_total_for_analysis`)

This function orchestrates the entire BAFRS solution process. It follows a strict two-phase sequential logic:

1. **Phase 1: Bin Allocation (MILP):** The script calls the `solve_scenario4_cplex_with_fihk` function. This Mixed-Integer Linear Program (MILP) minimizes only bin-related costs (purchasing, removal, etc.). The optimal frequencies from this solution are then used to create a day-by-day visit plan via the `extract_routing_plan` function.
2. **Phase 2: Vehicle Routing (VNS):** This fixed plan is then passed to our VNS heuristic, `solve_pvrp_vns_inter_route`, to solve the vehicle routing problem for each day, find the best possible routes, and calculate the final routing cost.

### 6.1.2. The BAFRS MILP Formulation (`solve_scenario4_cplex_with_fihk`)

This function is the core of the BAFRS model's strategic decision-making. The defining characteristic of this approach is its objective function, which is "myopic" or "blind" to routing costs.

```
--- Objective Function Snippet from the BAFRS MILP ---
The objective is to minimize only the sum of bin-related costs.
Routing costs are not part of this calculation.
mdl.minimize(
    mdl.sum(CPj[j] * wj[j] for j in range(m)) +
    mdl.sum(CRj[j] * z_minus_ij[i, j] for i, j in z_minus_ij) +
```

```

mdl.sum(CTj[j] * z_plus_ij[i, j] for i,j in z_plus_ij)
)

```

### 6.1.3. The VNS Heuristic Implementation (`vns_for_a_days_plan`)

This function is identical in structure to the one used in RFBAS, but the problem it receives from the BAFRS planner is often much different. Its task is to take the fixed daily plan and find the best routes. It starts by assigning each customer to their own small route ([Depot -> Customer -> Depot]) and then uses powerful inter-route moves to consolidate them.

```

--- Main Loop Snippet from vns_for_a_days_plan ---
for i in range(max_iter):
    # Randomly choose a move (inter_route_relocate or inter_route_swap)
    nh = random.choice(neighborhoods)

    # Try to apply the move to the current solution
    candidate_solution = nh(best_solution, ...)

    # If a valid move is found, accept it if it improves the cost
    if candidate_solution and solution_cost(candidate_solution) < best_cost:
        best_solution = candidate_solution
        # ... reset non_improvement_counter
    else:
        # ... increment non_improvement_counter

    # Stop if no improvement is found for a while
    if non_improvement_counter >= stagnation_limit:
        break

```

## BAFRS Performance and Results Analysis

This section presents a detailed analysis of the BAFRS model's performance, using the console output from a full run on the large-scale dataset (n=333).

### 6.2.1. Overview of the Execution Log

The execution log clearly demonstrates the two-phase nature of the BAFRS approach. The first phase involves a quick MILP solve to determine the strategic bin and frequency plan. The second, more computationally intensive phase involves the day-by-day operational routing using our Enhanced Inter-Route VNS algorithm.

### 6.2.2. VNS Performance by Day

The VNS algorithm's primary task is to take the daily customer lists determined by the MILP and create efficient, low-cost tours. As shown in Table 6.1, the VNS is highly effective at this task. It starts with a simple, high-cost initial plan and significantly improves it, achieving cost reductions of over 80%

Table 6.1: Daily VNS Performance Summary for the BAFRS Model on the Large-Scale Instance.

Day	# Customers	Initial Routes	Final Routes	Initial Cost	Final Cost	Cost Reduction (%)
1	157	2	2	3.1238	0.5957	80.9%
2	166	2	2	3.6939	0.5745	84.4%
3	179	3	2	4.1924	0.6371	84.8%
4	164	2	2	3.3308	0.5842	82.5%
<b>Total</b>	<b>666</b>	<b>9</b>	<b>8</b>	<b>14.3409</b>	<b>2.3915</b>	<b>83.3%</b>

### 6.2.3. Final Results Summary

The final output from the model provides a complete financial and operational summary of the solution. The total cost is broken down into its constituent parts, offering clear insight into the plan's economic profile.

```
--- FINAL BAFRS RESULTS ---
{
  "analysis_results": {
    "total_cost": 71184.61,
    "total_execution_time_sec": 427.28,
    "milp_solution": {
      "solution_time_sec": 0.73,
      "total_bin_cost": 47270.0
    }
  }
}
```

```

    },
    "vns_routing": {
        "total_routing_cost": 23914.61,
        "total_route_count": 8
    }
}
}
}

```

**Analysis:** The final results indicate a total system cost of 71,184.61 units. This is composed of a bin-related cost of 47,270.00 units and a routing cost of 23,914.61 units. The entire BAFRS execution completed in 427.28 seconds, with the vast majority of the time spent in the VNS optimization phase. These key performance indicators serve as the primary benchmark for comparison against other solution methodologies like RFBAS.

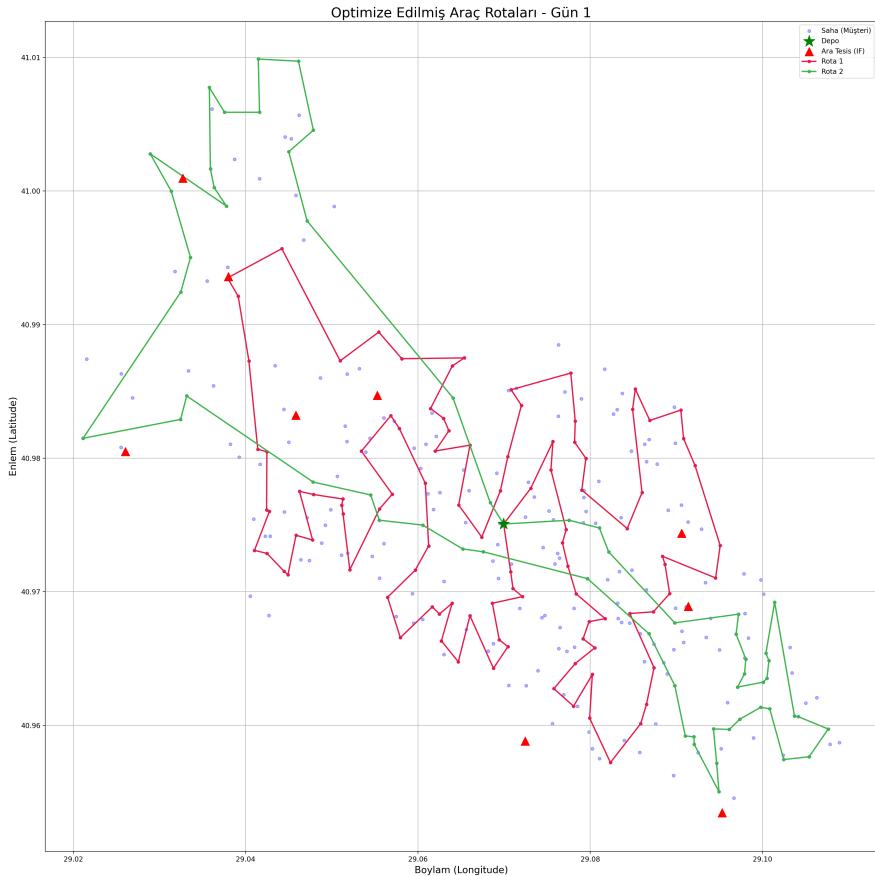


Figure 6.1: An example of a routing solution produced by the BAFRS model. Note the potential for routes to overlap and cover wide, non-compact areas, a common characteristic of this sequential approach.

## RFBAS Performance and Results Analysis

This section presents a detailed analysis of the Route-First, Bin Allocation-Second (RFBAS) model's performance. Unlike the BAFRS model, RFBAS incorporates estimated routing costs into its initial strategic planning phase, aiming for a more globally aware decision on service frequencies.

### 6.3.1. Overview of the Execution Log

The execution log for RFBAS follows a three-phase structure:

1. **Phase 1 - Routing Cost Estimation:** The model first performs a simple calculation to estimate the cost of visiting each customer individually.
2. **Phase 2 - Integrated MILP Solution:** These estimated costs are then used in a comprehensive MILP. The model solves this to find the optimal service frequencies that balance the trade-off between the estimated routing costs and the bin allocation costs.
3. **Phase 3 - Operational Routing (VNS):** The resulting daily visit plans are passed to the same Enhanced Inter-Route VNS algorithm to determine the final, precise routes and their costs.

### 6.3.2. VNS Performance by Day

The VNS algorithm receives the daily customer sets determined by the "routing-aware" MILP. As shown in Table 6.2, the VNS is exceptionally effective at consolidating the initial single-customer routes into a small number of efficient tours. The cost reduction percentages are consistently high across all days, demonstrating the robustness of the heuristic.

Table 6.2: Daily VNS Performance Summary for the RFBAS Model on the Large-Scale Instance.

Day	# Customers	Initial Routes	Final Routes	Initial Cost	Final Cost	Cost Reduction (%)
1	71	71	5	3.0386	0.4588	84.9%
2	78	78	8	3.2158	0.5162	84.0%
3	87	87	6	3.3122	0.4875	85.3%
4	97	97	7	4.0518	0.6453	84.1%
<b>Total</b>	<b>333</b>	<b>333</b>	<b>26</b>	<b>13.6184</b>	<b>2.1078</b>	<b>84.5%</b>

### 6.3.3. Final Results Summary

The final summary provides the overall performance metrics for the RFBAS approach. This data is crucial for the final comparison against the BAFRS model.

```
--- FINAL RFBAS RESULTS ---
{
  "analysis_results": {
    "total_cost": 27738.42,
    "total_execution_time_sec": 167.31,
    "milp_solution": {
      "total_bin_cost": 6660.00
    },
    "vns_routing": {
      "total_routing_cost": 21078.42,
      "total_route_count": 26
    }
  }
}
```

**Analysis:** The RFBAS model achieved a total system cost of 27,738.42 units, significantly lower than the BAFRS model. This superior result is primarily due to a much lower routing cost (21,078.42), which was achieved by accepting a slightly higher bin allocation cost (6,660.00). The model also ran faster, completing in 167.31 seconds, because the VNS phase was given a more geographically coherent set of customers to route each day, making the optimization process much more efficient.

# Solution Visualization and Interpretation

## 6.4.1. GUI Program Overview

The graphical user interface (GUI) developed in Python enables users to visualize the results of multiple waste collection strategies across several days. Built using the Tkinter library with a Leaflet map integration, the interface allows for intuitive interaction and comparison.

Users can:

- Select a strategy (BAFRS, RFBAS, IM)
- Choose a service day (Day 1–4)
- Animate vehicle movements
- Interact with map elements

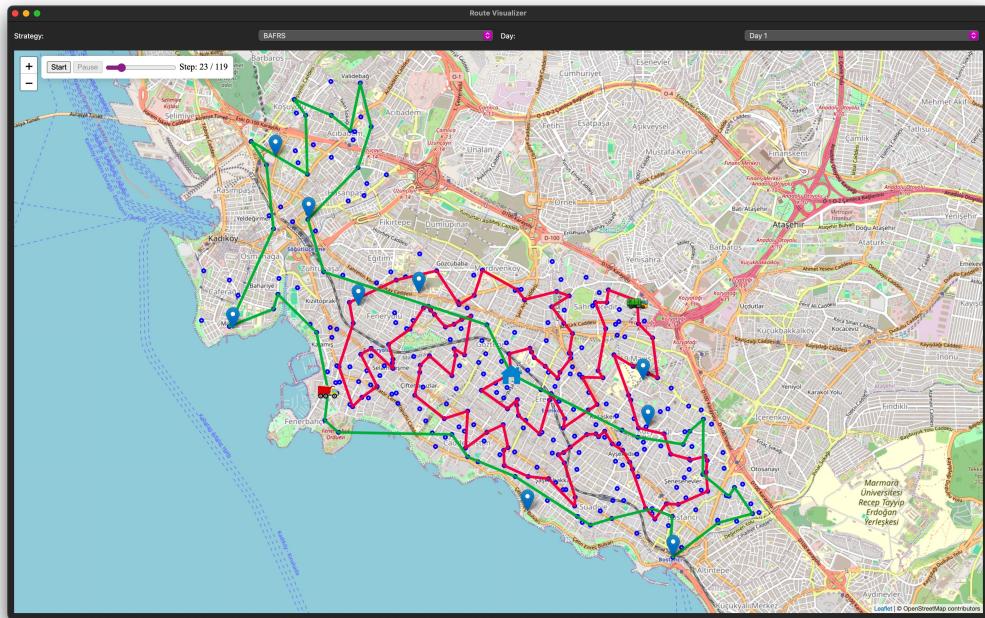


Figure 6.2: Animated route view – BAFRS, Day 1, Step 23

## 6.4.2. GUI Components and Controls

### 6.4.2.1. Selection Elements

**Strategy Selector:** Enables switching between optimization strategies:

- BAFRS – Bin Allocation First, Routing Second
- RFBAS – Route First, Bin Allocation Second
- IM – Integrated Method

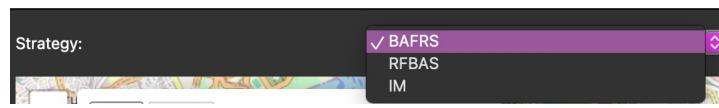


Figure 6.3: Strategy Selection Dropdown Menu

**Day Selector:** Allows day-wise filtering (Day 1–4) to observe daily variations.

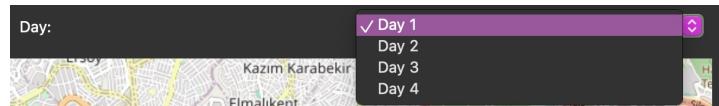


Figure 6.4: Day Selection Dropdown Menu

### **6.4.3. How to Use the Interface (Demo Flow)**

1. Select a strategy and day using the dropdown menus.
2. Press **Start** to begin the animation.
3. Use the slider to explore vehicle movement through time.
4. Compare between BAFRS and RFBAS to evaluate routing differences.

### **6.4.4. Key Visualization Elements**

- **Depot:** Home location, marked with a green base icon.
- **Intermediate Facilities (IFs):** Shown as large map pins.
- **Customer Nodes:** Represented by blue dots.
- **Vehicle Routes:** Colored polylines, one per truck.
- **Step Indicator:** Current animation step (e.g., Step 23 / 119).

#### 6.4.4.1. Map and Playback Features Visual

- **Interactive Map:** Live vehicle paths, depot (home), IFs, and customer nodes are displayed with clear icons.
- **Playback Controls:** Start, pause, and time-step slider to review simulation progress.

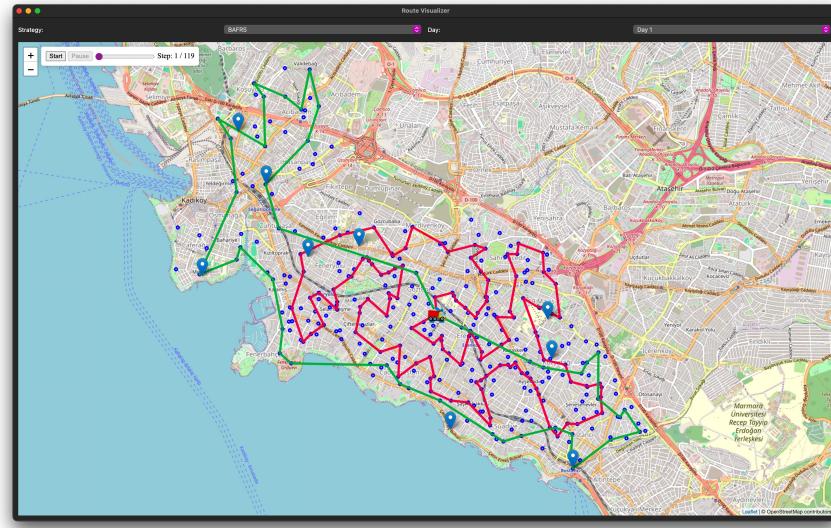


Figure 6.5: BAFRS, Day 1 - Step 1 animation view

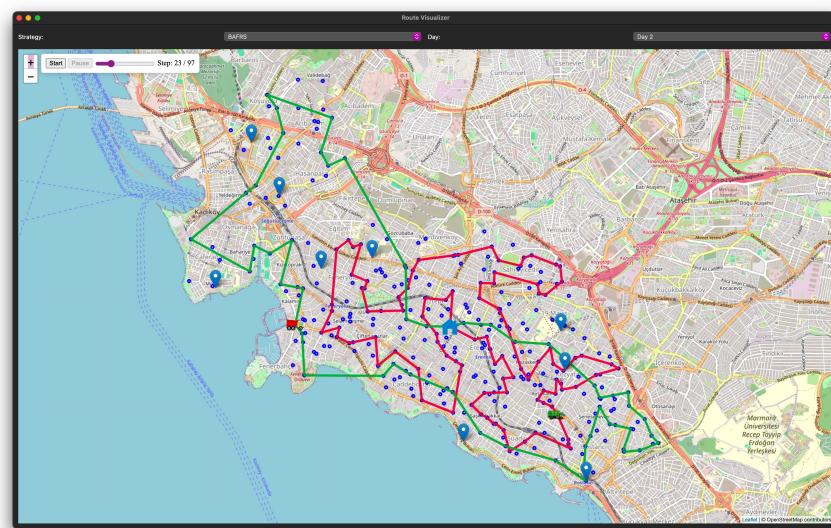


Figure 6.6: BAFRS – Day 2 - Step 23 animation view

#### 6.4.5. Cross-Day and Cross-Strategy Comparison

By switching days and strategies, the user can compare performance, route compactness, and service spread.

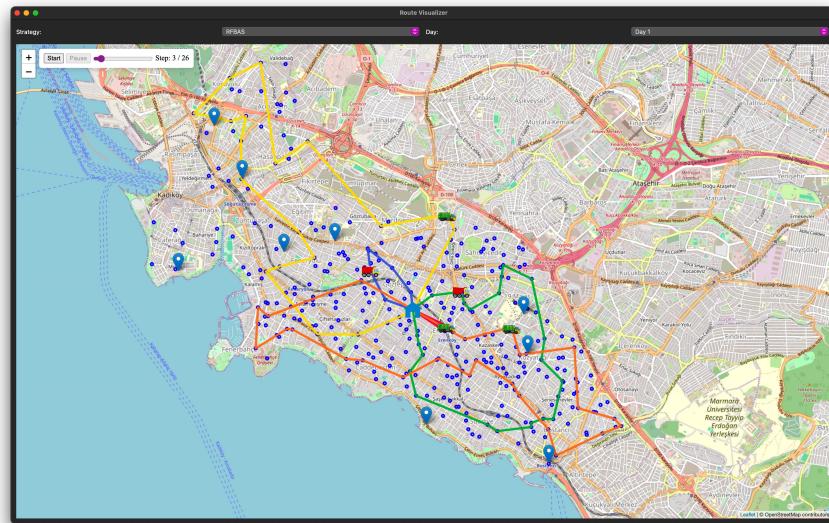


Figure 6.7: RFBAS – Day 1 - Step 3 animation view

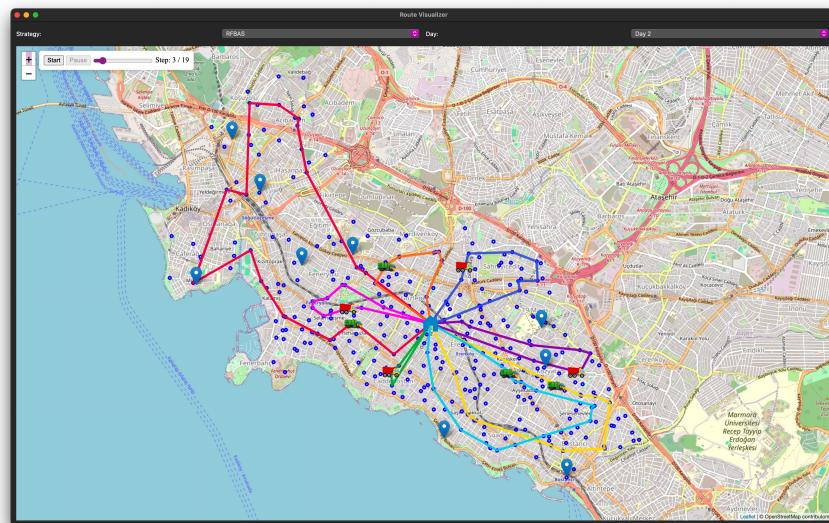


Figure 6.8: RFBAS – Day 2 with higher clustering and fewer routes