Exercises for the course
Machine Learning for Data Science
Winter Semester 2024/25

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Exercise Sheet 4 (theory part)

Exercise 1: Convex Optimization (10 + 10 + 20 + 20 + 10 P)

The SVDD method for anomaly detection is formulated as a convex optimization problem. Here, we consider a simplified variant of SVDD, the hard-margin SVDD, consisting of finding a maximum enclosing hypersphere of the data:

$$\min_{S,\boldsymbol{c}} \quad S$$

$$\text{s.t.} \quad \forall_{i=1}^{N} : \|\boldsymbol{c} - \boldsymbol{x}_i\|^2 \leq S$$

where S denotes the squared radius of the hypersphere.

(a) *Verify* that there is a point $(S, \boldsymbol{c})$ such that all constraints above are satisfied.

(b) *Write* the Lagrange function associated to this optimization problem.

(c) *State* the KKT conditions for this problem and simplify them.

(d) *Show* that the dual of SVDD is given by:

$$\max_{\boldsymbol{\alpha}} \quad \sum_{i=1}^{N} \alpha_i \|\boldsymbol{x}_i\|^2 - \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j \boldsymbol{x}_i^\top \boldsymbol{x}_j$$

subject to:

$$\sum_{i=1}^{N} \alpha_i = 1,$$

$$\forall_{i=1}^{N} : 0 \leq \alpha_i$$

(e) *Express* the primal variables $S, \boldsymbol{c}$ as a function of dual variables $\alpha_1, \ldots, \alpha_N$.