UUM526E - Optimization Techniques in Engineering Project: Trajectory Optimization for Spacecraft Randezvouz

Emine Elif Yiğit (514211005) * and Ahmet Talha Çetin (511211106)†
Istanbul Technical University, Maslak, Istanbul, 34469

I. Nomenclature

x	=	Downrange separation distance
y	=	Radial separation distance
z	=	Out-of-plane separation distance
v_x	=	Cownrange separation velocity
v_y	=	Radial separation velocity
v_z	=	Out-of-plane separation velocity
ω	=	Angular Velocity that comes from Coriolis acceleration
w	=	Fuel cost
T	=	Thrust
σ	=	Thrust per mass
θ	=	Angle formed by thrust vector and xy-plane
ϕ	=	Angle which the projection of the thrust vector on the xy-plane forms with the x-axis
ζ	=	Perturbation factor
$pdad$	=	primal-dual average distance
niq	=	accounts for the inequality constraints

II. Introduction

Spacecraft Rendezvous consists of maneuvers required for the target spacecraft, usually the space station, to rendezvous with a chaser spacecraft.The first successful meeting of the two spacecraft was in 1965. Although many years have passed, human intervention was needed in the rendezvous of a chaser spacecraft with the target spacecraft.Today, technologies that enable a chaser spacecraft to rendezvous with a target spacecraft autonomously without human intervention are intended to be made more autonomously.However, a rendezvous whose operations are all autonomous has yet to be proven.When the target spacecraft comes into view of the chaser spacecraft, the crew takes manual control and flies away with glideslope aproach that the chaser moves along a quasistraight line toward the target spacecraft.The chaser spacecraft can approach the target spacecraft in a straight line, easily monitoring abnormal conditions.

In this project, efforts will be made to efficiently carry out rendezvous maneuvers of spacecraft in the light of the information given.In the past, a lot of work has been done for rendezvous optimizations of space vehicles.These works can be generally divided into two groups as those using impulsive maneuvers and finite-duration maneuvers methods.The impulse thrust method uses primary vector theory involving instantaneous velocity changes and has been used for a long time as it is suitable for the crew controlling the chasing spacecraft.However, this method is incomplete in collision avoidance maneuvers and causes high fuel costs.For this reason, it was decided to solve the finite-thrust rendezvous problem, considering that the finite thrust model, which ensures the continuity of speed changes, includes the spacecraft optimal rendezvous models

In this study, an optimal control distribution will be calculated as a function of time allowing finite time maneuvers with the Interior Point Methods. With the Interior Point methods, it is aimed to solve the optimal meeting problem seamlessly by looking at the minimum absolute time appointment, the absolute minimum fuel appointment and all the situations in between.In this way, it is aimed to create optimum trajectories with the guidance formulation that can be applied autonomously in a spacecraft.

*MSc. Student, Defence Technologies, ITU Maslak Campus, 34469, yigite21@itu.edu.tr
†MSc. Student, Aeronautics and Astronautics Engineering, ITU Maslak Campus, 34469, cetinah16@itu.edu.tr

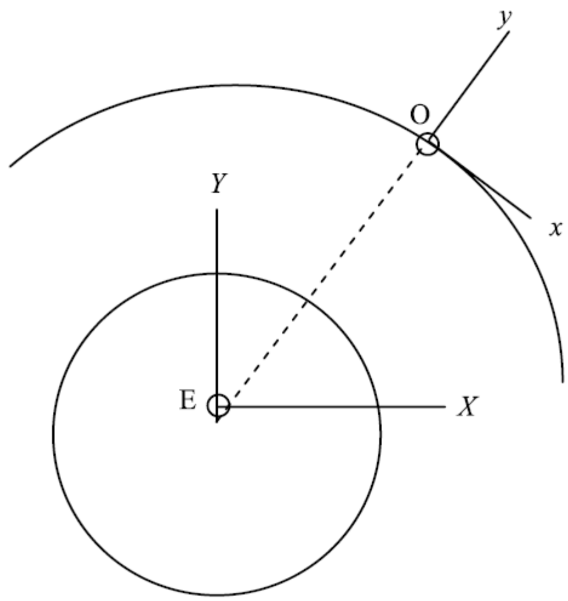


Fig. 1 LVLH Coordinate Frame

Going to space has always been an intriguing subject for humans to study and explore other planets and stars.It can be said that the aim of this project is to contribute to space exploration by solving the appointment problems of the vehicles sent to space and docking with the space station for space explorations and optimizing their trajectories.

Creating the optimum trajectory for Spacecraft rendezvous is very important for the fuel to be used and the rendezvous time. In addition, while optimizing the trajectory, it is necessary to approach the target at the desired amount and avoid collision. Since large amounts of budget are spent for space projects and also weight constraints, it is very important to avoid both high fuel costs and any damage to the spacecraft. As a result of these reasons, the problem of creating an optimum trajectory for Spacecraft rendezvous becomes important in many ways.

With the solution of the problem of creating the optimum orbit for the Spacecraft rendezvous, the most suitable appointment in terms of time and fuel will be optimized and the most suitable trajectory will be created for this optimization. As a result, time and fuel, therefore economy, efficiency will be achieved.

III. Optimization Problem

For the spacecraft randezvouz problem coordinate system Oxyz is of the LVLH type as seen at the Figure 1 (local vertical, local horizontal) and is centered at the center of mass of the target spacecraft: the x-axis is aligned with the target velocity vector, positive opposite to the direction of motion; the y-axis is aligned with the orbital radius vector, positive toward zenith; the z-axis is orthogonal to the xy-plane Dynamical equations of the system given at the Equation 1 [1]

$$\begin{aligned} &\ddot{x} - 2\omega\dot{y} = T_x/m = \sigma_x, \\ &\ddot{y} + 2\omega\dot{x} - 3\omega^2 y = T_y/m = \sigma_y, \\ &\ddot{z} + \omega^2 z = T_z/m = \sigma_z, \end{aligned} \tag{1}$$

With thrust per mass for each axis σ_x, σ_y and σ_z.

A. Objective

To search for optimal trajectory creation for Spacecraft rendezvous, 4 key optimization problems arise

Problem1 minimum time rendezvous, fuel free,

Problem2 minimum fuel rendezvous, time free,

Problem3 minimum time rendezvous, fuel given,

Problem4 minimum fuel rendezvous, time given.

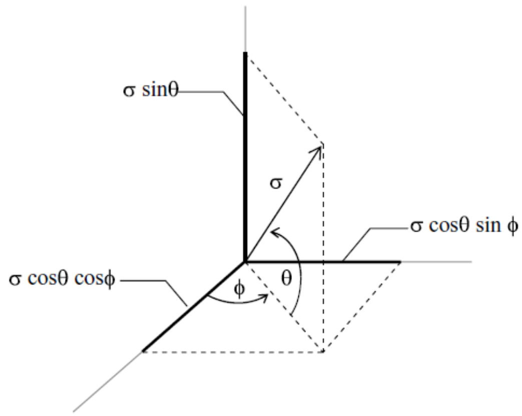


Fig. 2 Components of the thrust acceleration

For this research, it was decided to investigate all problems for optimization.In **Problem1**, the minimum time will be optimized and the fuel restriction will be removed.In **Problem2**, the minimum fuel will be optimized and the time restriction will be removed.Most appropriate timing for the given fuel will be solved in **Problem3**.Most appropriate fuel for the given time will be solved in **Problem4**.There are 4 objectives in total.

B. Constraints

Spacecraft needs to follow dynamical equations. This dynamical equation is added to the optimization problem as equality constraints given as Equation 2.

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) \tag{2}$$

Where **x** and **u** are states and control inputs respectively. Detailed dynamical equations are given at the mathematical formulation section.

Let T be the total thrust acting on the spacecraft and σ thrust per unit mass of T/m. In that case ;

$$T = \sqrt{\left(T_x^2 + T_y^2 + T_z^2\right)}, \tag{3}$$

$$\sigma = \sqrt{\left(\sigma_x^2 + \sigma_y^2 + \sigma_z^2\right)} \tag{4}$$

Thrust and thrust per unit mass are limited to the following;

$$\begin{aligned} 0 \leq T \leq T_{\max} \\ 0 \leq \sigma \leq \sigma_{\max} \end{aligned} \tag{5}$$

States at the start and end are formulated as equality constraints. By formulating as equality constraints solution ensures that start and end of the trajectory will be as specified. The rendezvous scenario between the target spacecraft and the chasing spacecraft will be considered as in [1].

The orbit of the target spacecraft is circular and is located at Space Station altitude. The trajectory of the chaser spacecraft is elliptical.

The initial conditions for the chaser spacecraft are as follows;

$$\begin{aligned}
&x(0) = 30,000 \text{ m} && (\text{ downrange separation distance}) \\
&y(0) = -15,000 \text{ m} && (\text{radial separation distance}) \\
&z(0) = 7,500 \text{ m} && (\text{out-of-plane separation distance}), \\
&v_x(0) = -60 \text{ m/s} && (\text{downrange separation velocity}), \\
&v_y(0) = -15 \text{ m/s} && (\text{radial separation velocity}), \\
&v_z(0) = -6 \text{ m/s} && (\text{out-of-plane separation velocity}), \\
&w(0) = 0.00 \text{ m/s} && (\Delta V \text{ expended}).
\end{aligned} \tag{6}$$

Table 1 Target spacecraft and chaser spacecraft information

	Target Spacecraft	Chaser Spacecraft
Orbit	Circular	Eliptical
Altitude	390 km	390 km
Orbit Radius	6768 km	min = 373 km, max = 500 km
Eccentricity	0,00°	0,0093°
Orbital Inclination	0,00°	0,0789°

Since the Chaser spacecraft is intended to dock with the target spacecraft, the desired final conditions(at time T) for the chaser spacecraft are;

$$\begin{aligned} &x(T) = 0 \text{ m} \\ &y(T) = 0 \text{ m} \\ &z(T) = 0 \text{ m} \\ &v_x(T) = 0 \text{ m/s}, \\ &v_y(T) = 0 \text{ m/s} \\ &v_z(T) = 0 \text{ m/s}, \\ &w(T) = \text{ free or given (m/s), depending on the} \\ &\quad \text{particular problem being solved.} \end{aligned} \tag{7}$$

C. Optimization Variables

Optimization variables for this optimization problem control inputs

$$U = (u_0, u_1, ..., u_N)$$

and states

$$X = (x_0, x_1, ..., x_N)$$

along trajectory. These variables will be changed to optimize the trajectory.

IV. Mathematical Formulation of the Optimization Problem

By transforming equations from second order to first order system follows dynamics at the Equation 8

$$\begin{aligned} \dot{x} &= v_x \\ \dot{y} &= v_y \\ \dot{z} &= v_z \\ \dot{v}_x &= 2\omega v_y + \sigma_x \\ \dot{v}_y &= -2\omega v_x + 3\omega^2 y + \sigma_y \\ \dot{v}_z &= -\omega^2 z + \sigma_z \end{aligned} \tag{8}$$

σ is collective thrust per mass which constrained by

$$0 \leq \sigma \leq \sigma_{\max} \tag{9}$$

σ_x, σ_y and σ_z written at Equation 10 while θ is angle formed by the thrust vector and the xy-plane and ϕ angle which the projection of the thrust vector on the xy-plane forms with the x-axis.

$$\begin{aligned} \sigma_x &= \sigma \cos\theta \cos\phi \\ \sigma_y &= \sigma \cos\theta \sin\phi \\ \sigma_z &= \sigma \sin\theta \end{aligned} \tag{10}$$

Control inputs for this problem is

$$\mathbf{u} = (\sigma, \theta, \phi)^T \tag{11}$$

In order to add fuel quantity to the problem, time integral of the thrust acceleration along trajectory at the Equation 12is used for fuel criterion. This criterion added to system dynamics equations as state and used for minimum fuel problems.

$$w(t) = w(0) + \int_0^t \sigma dt, \quad w(0) = 0 \tag{12}$$

Fuel dynamics can be written as

$$\dot{w} = \sigma \tag{13}$$

System dynamics with state vector $\mathbf{x} = \left(x, y, z, v_x, v_y, v_z\right)^T$ for this problem can be written as

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} v_x \\ v_y \\ v_z \\ 2\omega v_y + \sigma_x \\ -2\omega v_x + 3\omega^2 y + \sigma_y \\ -\omega^2 z + \sigma_z \end{bmatrix} \tag{14}$$

Optimization problem will be formulated using those dynamics as equality constraint. Four different optimization problem will be formulated which are:
- Minimum time rendezvous, fuel free
- Minimum fuel rendezvous, time free
- Minimum time rendezvous, fuel given
- Minimum fuel rendezvous, time given

A. Minimum Time Rendezvous, Fuel Free

For minimum time condition without fuel constraints optimization problem is

$$\begin{aligned} \underset{x(\cdot),u(\cdot)}{\text{minimize}} \quad & T \\ \text{subject to} \quad & \dot{x}(t) = f(x(t), u(t)) \quad t \in [0, T] \\ & x(0) = x_0 \\ & x(T) = x_T \\ & 0 \leq \sigma \leq \sigma_{max} \end{aligned} \tag{15}$$

where x_0 is initial condition of the states and x_T is final condition of the states.

B. Minimum Fuel Rendezvous, Time Free

For minimum fuel condition without time constraints optimization problem is formulated as

$$\begin{aligned} \underset{x(\cdot),u(\cdot)}{\text{minimize}} \quad & \int_0^T \sigma dt, \\ \text{subject to} \quad & \dot{x}(t) = f(x(t), u(t)) \quad t \in [0, T] \\ & x(0) = x_0 \\ & x(T) = x_T \\ & 0 \leq \sigma \leq \sigma_{max} \end{aligned} \tag{16}$$

C. Minimum Time Rendezvous, Fuel Given

For minimum time optimization with given fuel problem is defined as

$$
\begin{aligned}
\underset{x(\cdot),u(\cdot)}{\text{minimize}} \quad & T \\
\text{subject to} \quad & \dot{x}(t) = f(x(t), u(t)) \quad t \in [0, T] \\
& x(0) = x_0 \\
& x(T) = x_T \\
& w(T) = w_T \\
& 0 \leq \sigma \leq \sigma_{max}
\end{aligned}
\tag{17}
$$

D. Minimum Fuel Rendezvous, Time Given

For minimum fuel optimization problem with given time is formulated as

$$
\begin{aligned}
\underset{x(\cdot),u(\cdot)}{\text{minimize}} \quad & \int_0^T \sigma dt, \\
\text{subject to} \quad & \dot{x}(t) = f(x(t), u(t)) \quad t \in [0, T] \\
& T = T_{specified} \\
& x(0) = x_0 \\
& x(T) = x_T \\
& 0 \leq \sigma \leq \sigma_{max}
\end{aligned}
\tag{18}
$$

V. Methodology

Continuous time optimization problem will be discretized for using numerical methods. Discrete system dynamics will be integrated with Runge-Kutta 4th-order integration and next state enforced to follow the dynamical constraints. Initial and final conditions for states are given to system. Additional constraints that came from time and fuel will be added according to structure of the optimization problems. Then structured optimization problem will be solved by multiple shooting method.

VI. Optimization Algorithm

Multiple shooting method will be used for structuring this nonlinear optimization problem. Method divides the interval, solves an initial value problem in each of the smaller intervals, and imposes additional matching conditions to form a solution on the whole interval. Multiple shooting methods can overcome nonlinearity. This method also has more advantage in sense of numerical stability for multiple interval problem.

When optimization problem is structured, resulting problem will be solved by Nonlinear Interior Point Methods. Since the problem is nonlinear, solver for nonlinear optimization problems are investigated. Interior point methods can solve nonlinear optimization problems with reasonable amount of time. Other methods such as Sequential Quadratic Programming (SQP) is also investigated but in order to use SQP methods, it is needed that objective function and the constraints are twice continuously differentiable which is not the case for this problem.[2] Casadi [3] software package in Python is used for formulating and solving trajectory optimization problem.

A. Nonlinear Interior Point Algorithm

To ensure that the optimization problem remain in the feasible region, a perturbation factor, μ, is added to "penalize" close approaches to the boundaries. This approach is analogous to the use of an invisible fence to keep lambs in an unfenced yard. As the lambs moves closer to the boundaries, the more shock he will feel. In the case of the IP method, the amount of shock is determined by μ. A large value of μ gives the analytic center of the feasible region. As μ decreases and approaches zero , the optimal value is calculated by tracing out a central path. With small incremental decreases in μ during each iteration, a smooth curve is generated for the central path. This method is accurate, but time consuming and computationally intense. Instead, Newton's method is often used to approximate the central path for non-linear programming. Using one Newton step to estimate each decrease in μ for each iteration, a polynomial ordered time complexity is achieved, resulting in a small zig-zag central path and convergence to the optimal solution.

The logarithmic barrier function is based on the logarithmic interior function:

$$B(x,\mu) = f(x) - \mu \log(x) = f(x) - \mu \sum_{i=1}^{m} \ln(x_i) \tag{19}$$

The IP method for NLP have been commonly used.To solve problems, the perturbation factor is used in addition to the typical Karush-Kuhn-Tucker (KKT) methods.

- Step 1 : Start with a general optimization problem:

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & h(x) = 0 \\ & g(x) \leq 0 \end{array} \tag{20}$$

- Step 2 : Modify the KKT conditions by adding convergence properties with slack variables and the perturbation factor:

$$\begin{aligned} & \nabla_x L\left(x, \lambda_h, \lambda_g\right) = 0 h(x) = 0 \\ & g(s) + s = 0 \\ & \left[\lambda_g\right] s - \mu e = 0 \\ & \left(s, \lambda_g, \mu\right) \geq= 0 \end{aligned} \tag{21}$$

- Step 3 : Solve the nonlinear equations iteratively by Newton's methods. First determine Δx and $\Delta \lambda_h$ with reduced linear equations.
- Step 4 :Calculate slack variables and corresponding multipliers with:

$$\begin{aligned} & \Delta s = -g(x) - s - \nabla g(x) \Delta x \\ & \Delta \lambda_g = -\lambda_g + \left[s^{-1}\right] * \mu e - \left[\lambda_g\right] \Delta s \end{aligned} \tag{22}$$

- Step 5 :To calculate the perturbation factor, μ, use primal-dual distances:

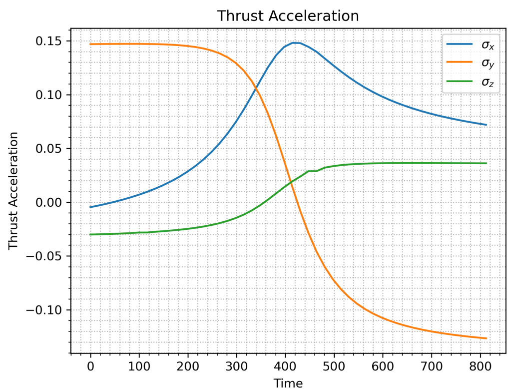
$$\mu = \zeta * p\text{dad} = \zeta * \frac{\lambda_g^t s}{niq} \tag{23}$$

  where ζ defines the trajectory of the optimal solution, pdad is the primal-dual average distance, and niq accounts for the inequality constraints. ζ ranges between 0 and 1 .For the extreme conditions:
  $\zeta = 0$ corresponds to affine-scaling direction where the optimal point is obtained through non-perturbed solution of KKT.
  $\zeta = 1$ corresponds to centralization direction where the non-optimal solution is found with a primal-dual distance equal to the initial value of μ.

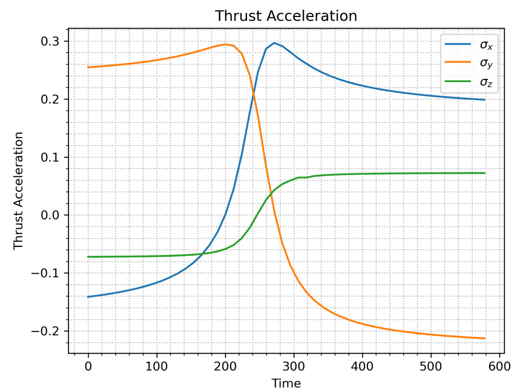
VII. Minimum Time Rendezvous, Fuel Free

Problem P1 was solved via IP for the three values of the max-thrust acceleration (0.15, 0.30, 0.60 m/s^2). Problem discretized with $N = 50$ intervals. The results as can be seen in figure 3 and 4. The minimum time to rendezvous for a max-thrust acceleration of 0.15 m/s^2 is 811.9s, the minimum time to rendezvous for a max-thrust acceleration of 0.30 m/s^2 is 578.1s, and the minimum time to rendezvous for a max-thrust acceleration of 0.60 $m/s^2 is$ 421.18s.As can be see in figure 4b 4d and 4f velocities are increases first then decrease to zero but the largest velocity changes were seen at V_x and V_y.

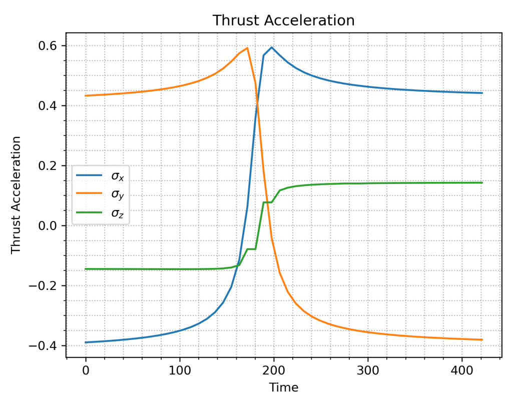
Problem P1 and reference paper[1] compared as in figure 5, the minimum time to rendezvous values were higher in this study, even though the minimum time value found and the reference value approached as the max-thrust acceleration increased.Besides, separation distance separation velocity and thrust acceleration values are compatible with reference.



(a) Problem P1. Time-optimal rendezvous. Thrust acceleration components vs. time, $\sigma_{max} = 0.15 m/s^2$

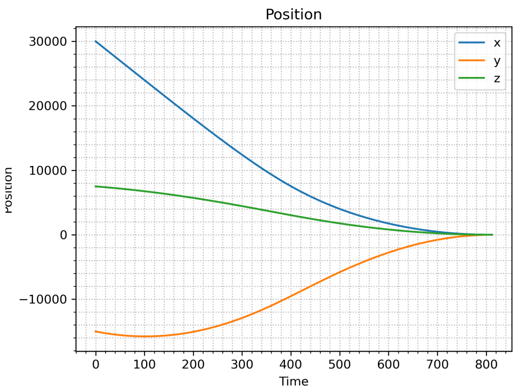


(b) Problem P1. Time-optimal rendezvous. Thrust acceleration components vs. time, $\sigma_{max} = 0.3 m/s^2$

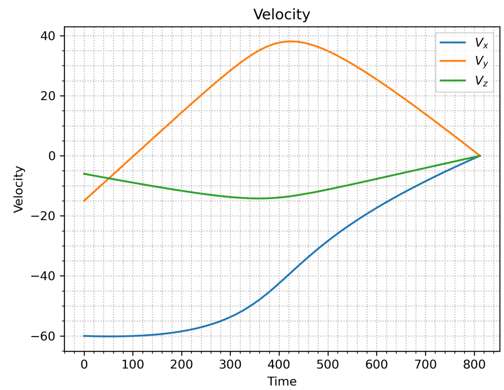


(c) Problem P1. Time-optimal rendezvous. Thrust acceleration components vs. time, $\sigma_{max} = 0.6 m/s^2$

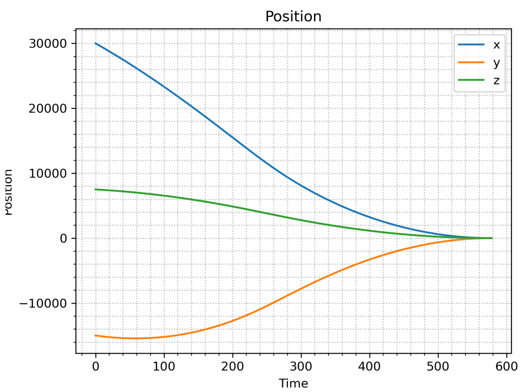
Fig. 3 Minimum Time Rendezvous, Fuel Free: Thrust Profiles



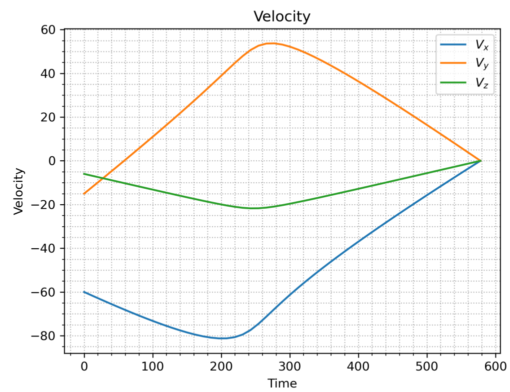
(a) Problem P1. Time-optimal rendezvous. Separation distances vs. time, $\sigma_{max} = 0.15 m/s^2$



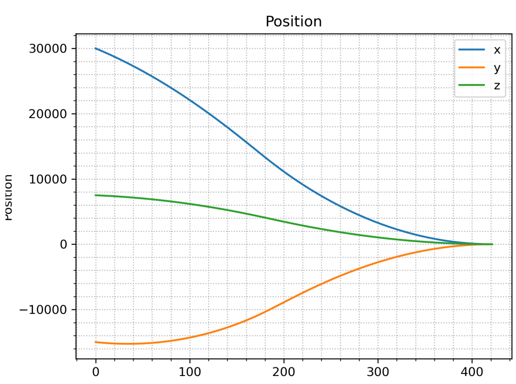
(b) Problem P1. Time-optimal rendezvous. Separation velocities vs. time, $\sigma_{max} = 0.15 m/s^2$



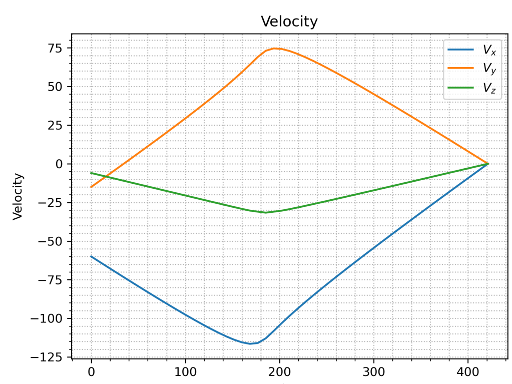
(c) Problem P1. Time-optimal rendezvous. Separation distances vs. time, $\sigma_{max} = 0.30 m/s^2$



(d) Problem P1. Time-optimal rendezvous. Separation velocities vs. time, $\sigma_{max} = 0.30 m/s^2$

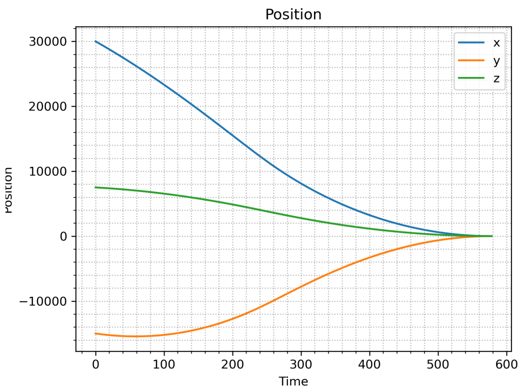


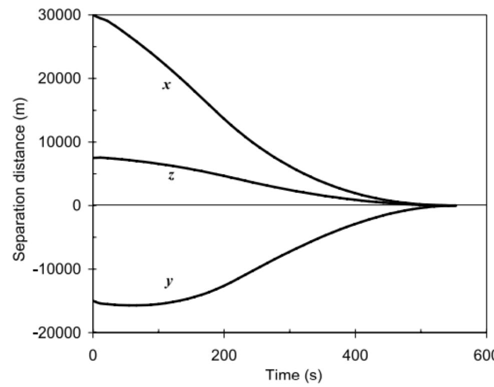
(e) Problem P1. Time-optimal rendezvous. Separation distances vs. time, $\sigma_{max} = 0.6 m/s^2$



(f) Problem P1. Time-optimal rendezvous. Separation velocities vs. time, $\sigma_{max} = 0.6 m/s^2$

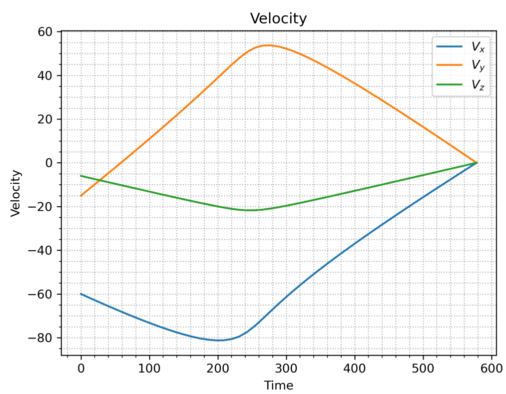
Fig. 4 Minimum Time Rendezvous, Fuel Free: State Trajectories

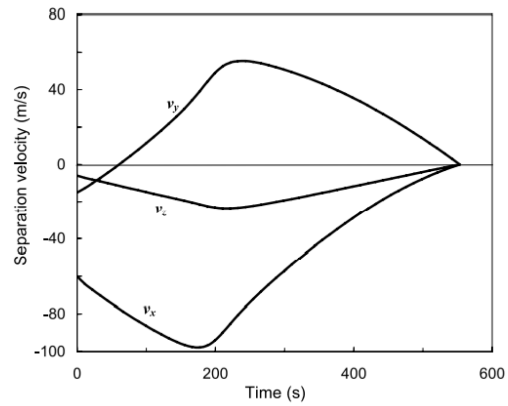




(a) Problem P1. Time-optimal rendezvous. Separation distances vs. time, $\sigma_{max} = 0.30 m/s^2$

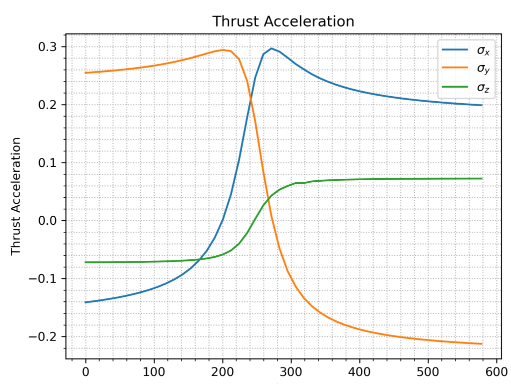
(b) Problem P1. Time-optimal rendezvous. Separation distances vs. time, $\sigma_{max} = 0.30 m/s^2$

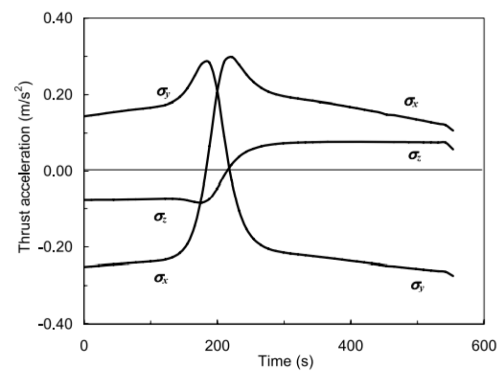




(c) Problem P1. Time-optimal rendezvous. Separation velocities vs. time, $\sigma_{max} = 0.30 m/s^2$

(d) Problem P1. Time-optimal rendezvous. Separation distances vs. time, $\sigma_{max} = 0.30 m/s^2$





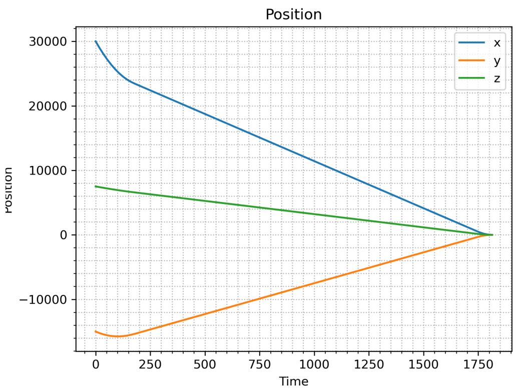
(e) Problem P1. Time-optimal rendezvous. Thrust acceleration components vs. time, $\sigma_{max} = 0.3 m/s^2$

(f) Problem P1. Time-optimal rendezvous. Thrust acceleration components vs. time, $\sigma_{max} = 0.30 m/s^2$

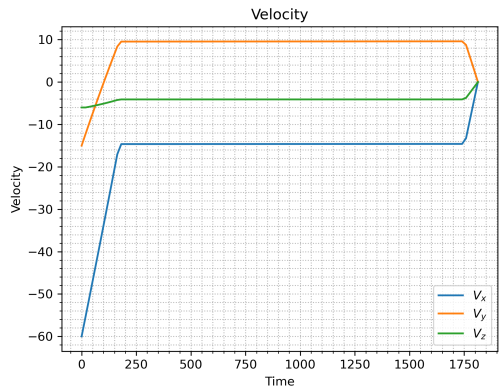
Fig. 5 Minimum Time Rendezvous, Fuel Free: Comparison With Paper

VIII. Minimum Fuel Rendezvous, Time Free

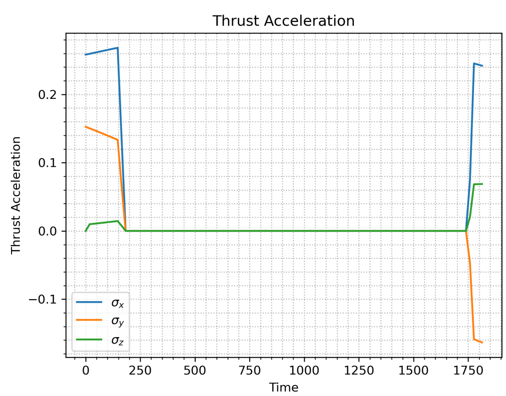
Problem P2 solved for the same three maximum total thrust acceleration as in the reference paper[1]. It is seen that for the all maximum thrusts, ΔV takes same value and ΔV is nearly independent from the maximum thrust. In our implementation solution for the time for fuel optimal solution takes bigger values than reference paper. This is because of the difference in the solution method. To take faster solution maximum value of the time is limited.Problem discretized with $N = 100$ intervals. Optimal ΔV found $\Delta V = 71.5$ for $\sigma_{max} = 0.15$, $\Delta V = 69.6$ for $\sigma_{max} = 0.30$, $\Delta V = 68.8$ for $\sigma_{max} = 0.60$. Fuel-optimal position, velocity and thrust trajectories for minimum fuel problem can be seen at the Figure 6a, 6b and 6c respectively. It is seen that thrust is only applied in the beginning and at the end of the trajectory. During mid part of the trajectory velocity stays constant. The braking thrust action at the end of trajectory seems to establish the chaser and target spacecraft on a collision course, while a small out-of plane component is applied to begin removing the out-of-plane separation distance.



(a) Problem P2. Time-optimal rendezvous. Seperation Distances vs. time, $\sigma_{max} = 0.30 m/s2$



(b) Problem P2. Time-optimal rendezvous. Seperation Velocities vs. time, $\sigma_{max} = 0.30 m/s2$



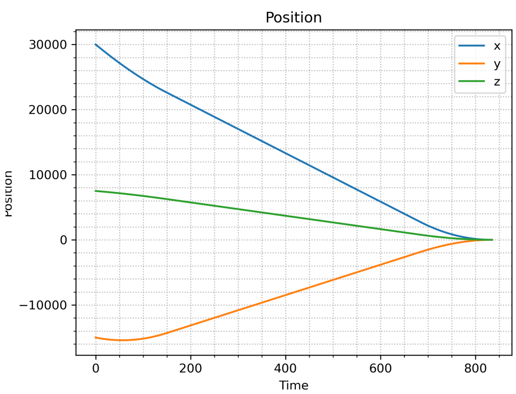
(c) Problem P2. Time-optimal rendezvous. Thrust acceleration components vs. time, $\sigma_{max} = 0.30 m/s2$

Fig. 6 Minimum Fuel Rendezvous, Time Free: Position, Velocities and Thrust Profiles

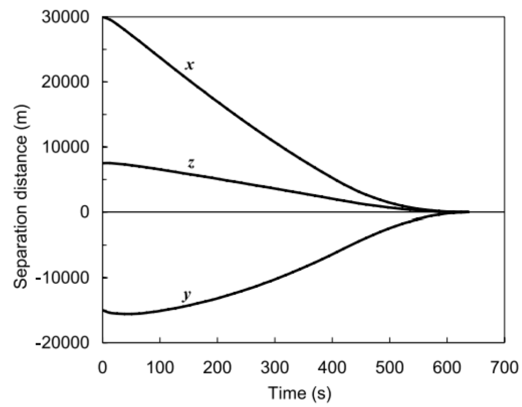
IX. Minimum Time Rendezvous, Fuel Given

Problem P3 was solved via IP for the same three values of the max-thrust accceleration (0.15, 0.30, 0.60 m/s^2). Problem discretized with $N = 100$ intervals. The results for max-thrust accceleration equal to 0.30 as can be seen in figure 7.The minimum time to rendezvous for a max-thrust acceleration of 0.15 m/s^2 is 988.91s, the minimum time to rendezvous for a max-thrust acceleration of 0.30 m/s^2 is 834.7s, and the minimum time to rendezvous for a max-thrust acceleration of 0.60 $m/s^2 is$ 764.4s.Optimal ΔV found $\Delta V = 90$ for $\sigma_{max} = 0.30$.

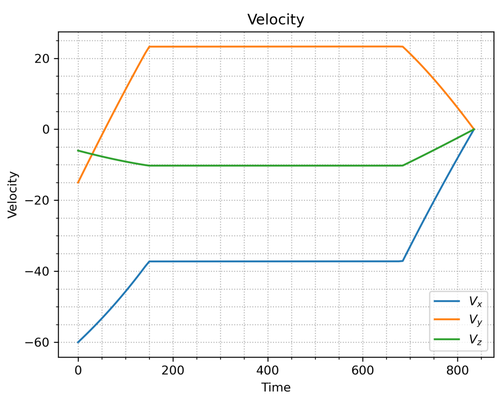
Also comparison of Problem P3 with reference paper [1] is given in figure 7.The optimal time found for the given fuel is still higher than the paper, but the values of the other graphics found are compatible with each other. This may be due to the different optimization solution algorithms used with the paper.



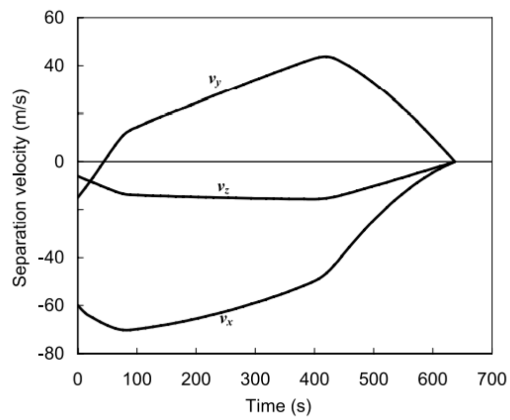
(a) Problem P3. Time-optimal rendezvous, fuel given. Seperation Distances vs. time, $\sigma_{max} = 0.30 m/s^2$ and $\Delta V = 90.0 m/s$



(b) Problem P3. Time-optimal rendezvous, fuel given. Seperation Distances vs. time, $\sigma_{max} = 0.30 m/s^2$ and $\Delta V = 90.0 m/s$

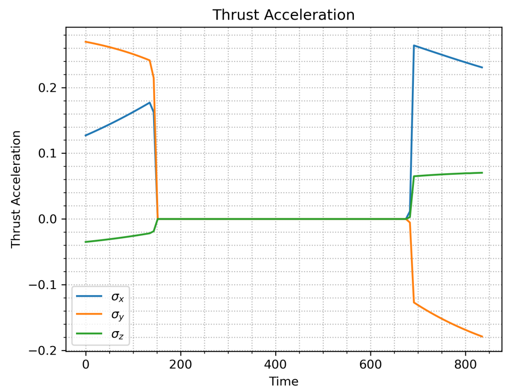


(c) Problem P3. Time-optimal rendezvous, fuel given. Seperation Velocities vs. time, $\sigma_{max} = 0.30 m/s^2$ and $\Delta V = 90.0 m/s$

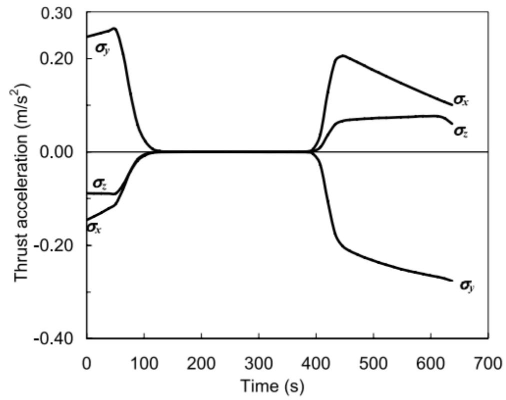


(d) Problem P3. Time-optimal rendezvous, fuel given. Seperation Velocities vs. time, $\sigma_{max} = 0.30 m/s^2$ and $\Delta V = 90.0 m/s$

Fig. 7 Minimum Time Rendezvous, Fuel Given: $\sigma_{max} = 0.30 m/s^2$



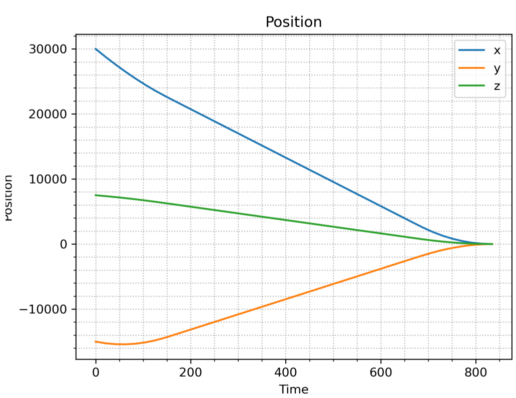
(a) Problem P3. Time-optimal rendezvous, fuel given. Thrust acceleration components vs. time, $\sigma_{max} = 0.30 m/s^2$ and $\Delta V = 90.0 m/s$



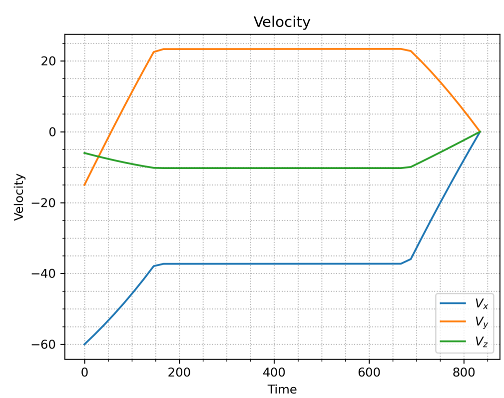
(b) Problem P3. Time-optimal rendezvous, fuel given. Thrust acceleration components vs. time, $\sigma_{max} = 0.30 m/s^2$ and $\Delta V = 90.0 m/s$

X. Minimum Fuel Rendezvous, Time Given

There exists a connection between Problems P3 and P4. Pursuant to in the general property of the calculus of variations, this is the reciprocity of isoperimetric integrals. It is stated as follows: "The solution extremizing a functional J for a given value of the functional K is the same as the solution extremizing the functional K for a given value of the functional J ." However it is decided to solve this problem too for validating the results in Problem P3. Problem discretized with $N = 100$ intervals. With found $T_{final} = 834.0s$ value, solution of the fuel optimal problem found as $\Delta V = 90.08 m/s$ which validates the result of the Problem P3. Optimal position, velocity and thrust trajectories can be seen at the Figure 9a, 9b and 10 respectively.



(a) Problem P4. Fuel-optimal rendezvous, time given. Seperation Distances vs. time, $\sigma_{max} = 0.30 m/s^2$ and $T_{final} = 834.0$



(b) Problem P4. Fuel-optimal rendezvous, time given. Seperation Velocities vs. time, $\sigma_{max} = 0.30 m/s^2$ and $T_{final} = 834.0$

Fig. 9 Minimum Fuel Rendezvous, Time Given: $\sigma_{max} = 0.30 m/s^2$ and $T_{final} = 834.0$

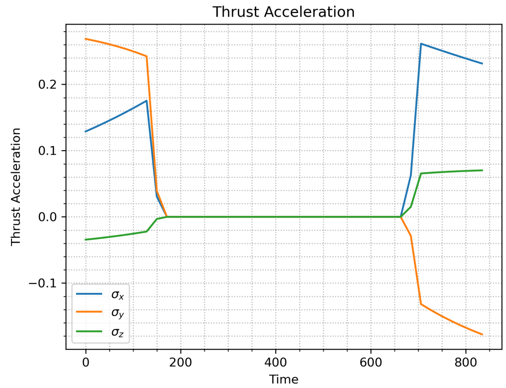


Fig. 10 Problem P4. Fuel-optimal rendezvous, time given. Thrust acceleration components vs. time, $\sigma_{max} = 0.30 m/s^2$ **and** $T_{final} = 834.0$

XI. Conclusion

In this project trajectory optimization for spacecraft randezvouz problem is solved for different objectives and constraints which are:
- Minimum time rendezvous, fuel free
- Minimum fuel rendezvous, time free
- Minimum time rendezvous, fuel given
- Minimum fuel rendezvous, time given

Each problem implemented and solved with Nonlinear Interior-point solver. Even though some of the solutions differ from implemented paper, this difference comes from difference in the solution methods. Reference paper aims to find smooth trajectories and uses SGRA algorithm to ensure that, however we discretized the trajectory and solved this discretized problem. Nevertheless, general behaviour of the trajectories are similar and trajectory optimization problem for spacecraft randezvouz successfully solved.

When the time and fuel restriction are removed, the trajectories that provide the most optimum time and the most optimal fuel is found. It is seen that the rendezvous time had shorten as the maximum thrust value increases. In addition, when expended speed values ΔV decreased, randezvouz time is increased and also the transfer angle decreased as the rendezvous time decreased.

References

[1] Miele, A., Weeks, M., and Ciarcia, M., "Optimal trajectories for spacecraft rendezvous," *Journal of optimization theory and applications*, Vol. 132, No. 3, 2007, pp. 353–376.

[2] Wright, S., Nocedal, J., et al., "Numerical optimization," *Springer Science*, Vol. 35, No. 67-68, 1999, p. 7.

[3] Andersson, J. A. E., Gillis, J., Horn, G., Rawlings, J. B., and Diehl, M., "CasADi – A software framework for nonlinear optimization and optimal control," *Mathematical Programming Computation*, Vol. 11, No. 1, 2019, pp. 1–36. https://doi.org/10.1007/s12532-018-0139-4.