



Learn to Build Automated Software Analysis Tools with Graph Paradigm and Interactive Visual Framework

ASE Tutorial, September 4, 2016

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Module IV: Accuracy and Scalability Barriers for Analyzing Large Software

Acknowledgement: Team members at Iowa State University and EnSoft, DARPA contracts FA8750-12-2-0126 & FA8750-15-2-0080

Module Outline

- *Exponentiality of points to sets*
- *Exponentiality of control flow paths*
- *Exponentiality of path feasibility*
- A tug-of-war between *scalability* and *accuracy*
- Solutions to counter-act exponentiality:
 - Binary Decision Diagram (BDD)
 - Projected Control Graph

Terminology

A **control block** is the sequence of statements forming the body of a control statement.

A **branch node** corresponds to a conditional statement from which mutually exclusive branches emanate. A conditional statement could be a conditional loop.

Structured Program: Every control block has exactly one entry. Could have multiple exits.

Unstructured Program: A control block can have multiple entries.

Control Flow Graph (CFG): The graph with one nodes for each statement and each edge depicting the control flow from one statement to another.

A **CFG path (execution path)** follows branches with one branch at each branch node it hits. Each such branch is called a path segment.

A **path is infeasible** if it includes mutually exclusive path segments

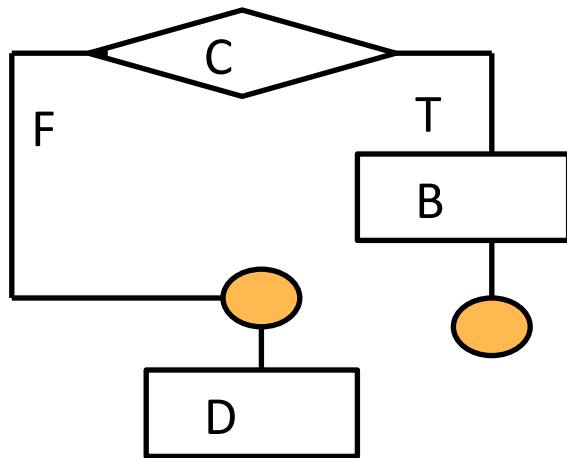
An **event** is execution result of a statement relevant to validating a safety property

Conservative analysis is reasoning based on aggregate of events along all branches at a branch node

Path-sensitive analysis reasoning based on only the events along individual paths

Multiple exits in control block

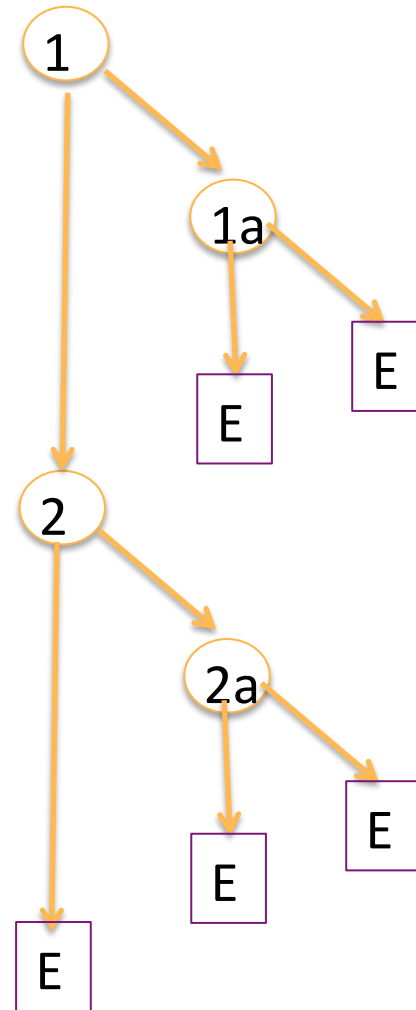
```
if ( (q=dsptr->dreqlst) == DRNULL ) {  
    dsptr->dreqlst = drptr;  
    drptr->drnext = DRNULL;  
    dskstrt(dsptr);  
    return(DONQ);  
}
```



Two execution paths

Execution Path Analysis Examples - I

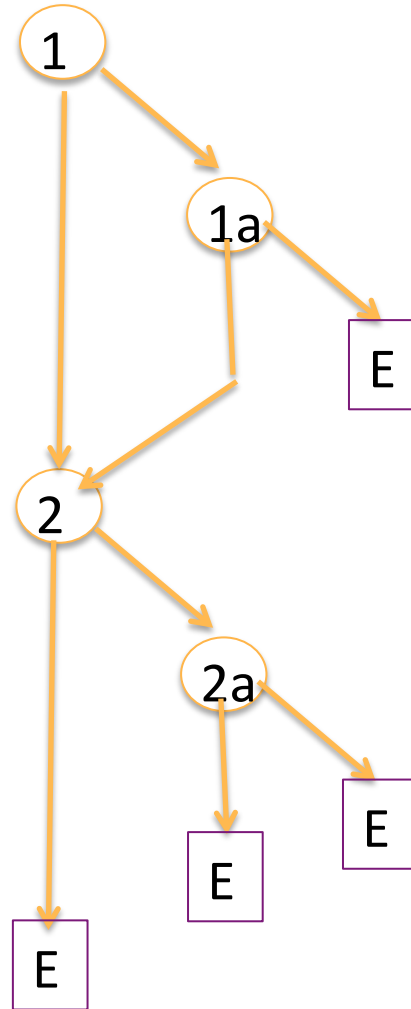
```
if ( C1 ) {  
    if ( C2 ) {  
        do something ;  
        return();  
    }  
    do something;  
    return();  
}  
  
if ( C3 ) {  
    if ( C4 ) {  
        do something ;  
        return();  
    }  
    do something;  
    return();  
}
```



Five execution paths

Execution Path Analysis Examples - II

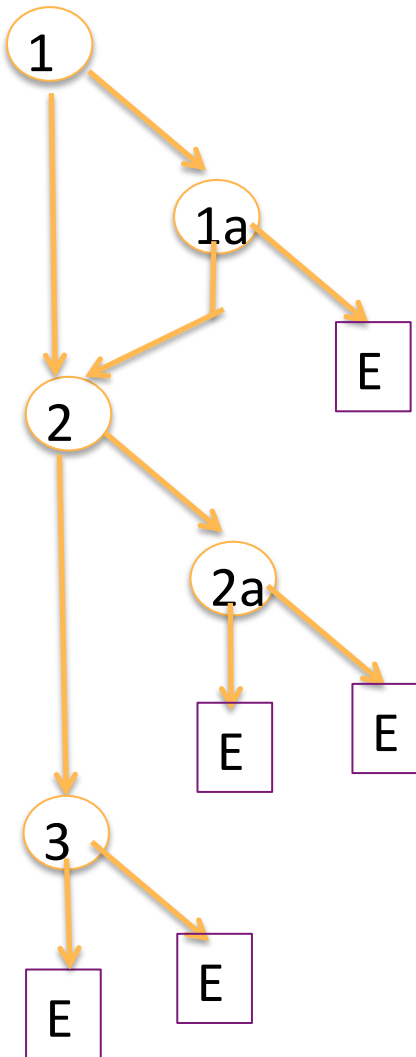
```
if ( C1 ) {  
    if ( C2 ) {  
        do something ;  
        return();  
    }  
    do something;  
}  
  
if ( C3 ) {  
    if ( C4 ) {  
        do something ;  
        return();  
    }  
    do something;  
    return();  
}
```



Seven execution paths

Execution Path Analysis Examples - II

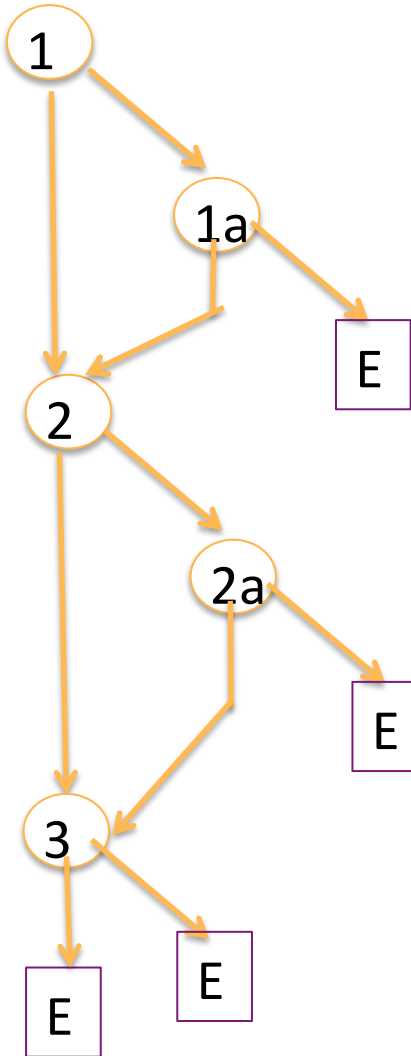
```
if ( C1 ) {  
    if ( C2 ) {  
        do something ;  
        return();  
    }  
    do something;  
}  
if ( C3 ) {  
    if ( C4 ) {  
        do something ;  
        return();  
    }  
    do something;  
    return();  
}  
if ( C5 ){  
    do something ;  
    return();  
}
```



Nine execution paths

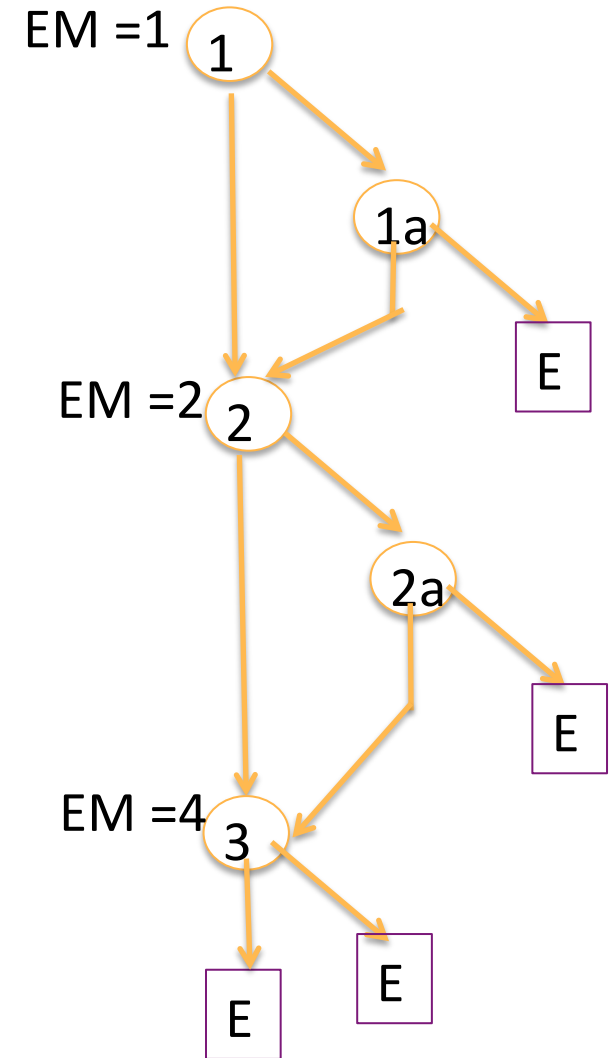
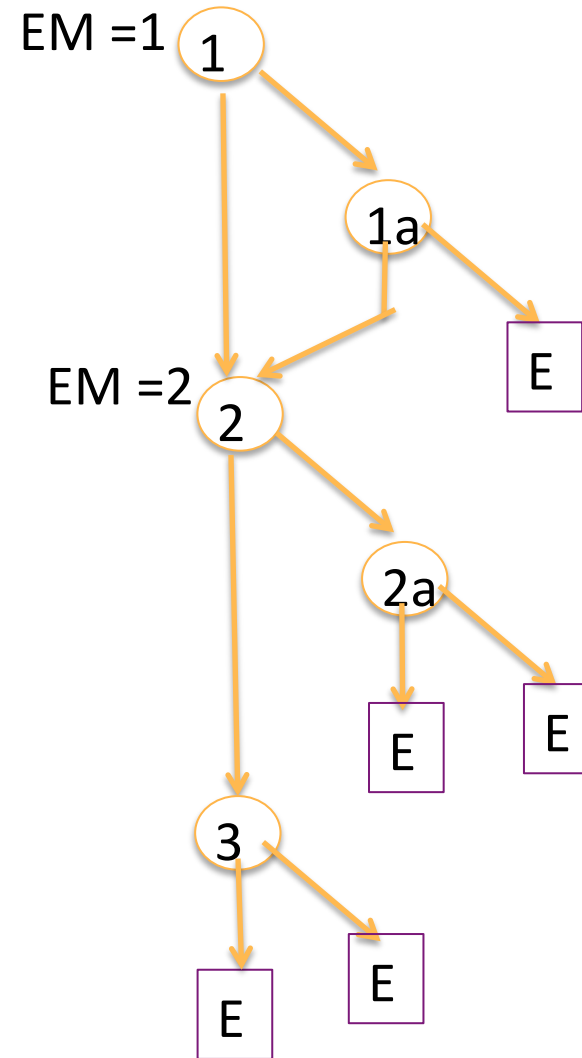
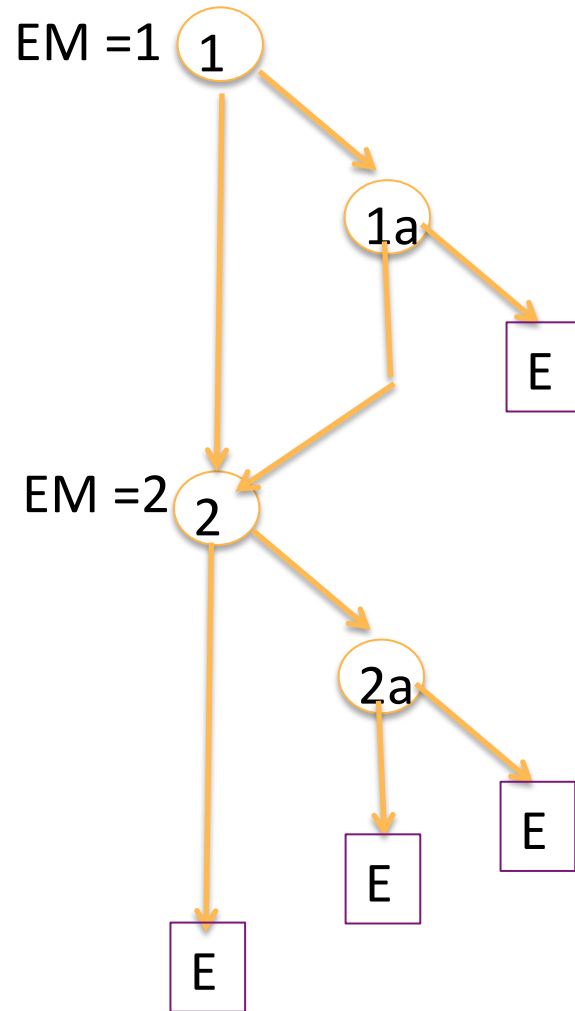
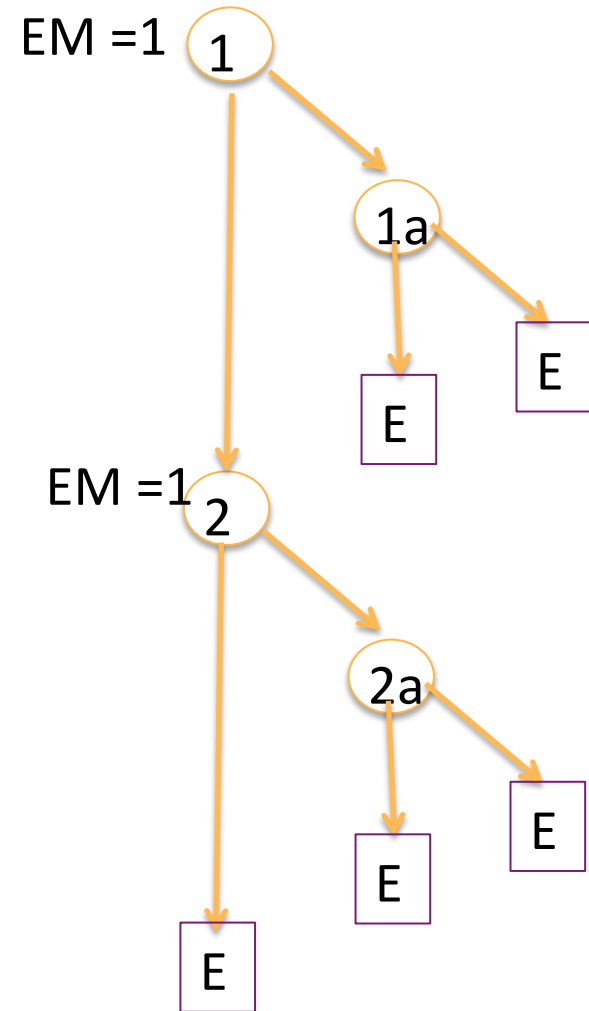
Execution Path Analysis Examples - III

```
if ( C1 ) {  
    if ( C2 ) {  
        do something ;  
        return();  
    }  
    do something;  
}  
if ( C3 ) {  
    if ( C4 ) {  
        do something ;  
        return();  
    }  
    do something;  
}  
if ( C5 ) {  
    do something ;  
    return();  
}  
}
```

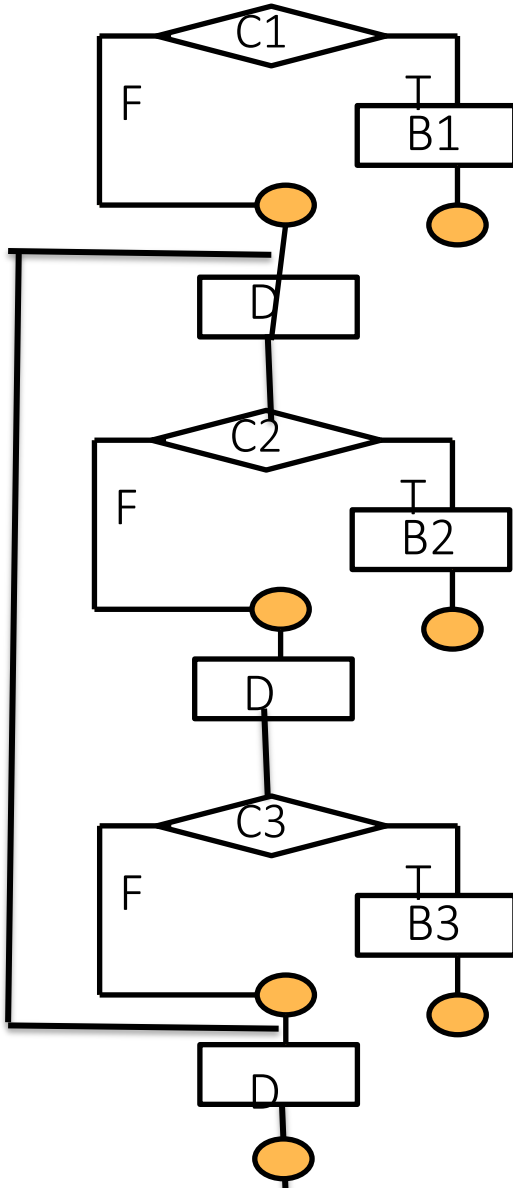


Eleven execution paths

Entry point multiplicity (EM)



A Loop Example



Path 1: C1 = F, C2 = F, C3 = F, Loop executed

Path 2: C1 = F, Loop not executed

Path 3: C1 = F, C2 = T, BREAK B2

Path 4: C1 = F, C2 = F, C3 = T, BREAK B3

Path 5: C1 = T, BREAK B1

Loops without BREAKs

- Scenario 1: Loop contains A(X) (allocation) followed by D(X) (deallocation) on all feasible execution paths within a loop - safe, no matter how many times the loop iterates
- Scenario 2: Loop contains D(X) (deallocation) followed by A(X) (allocation) on all feasible execution paths within a loop – safe if the loop is preceded A(X) and succeeded by D(X).
- OK to treat multiple iterations of the Loop as a single execution path.

Loops with BREAK

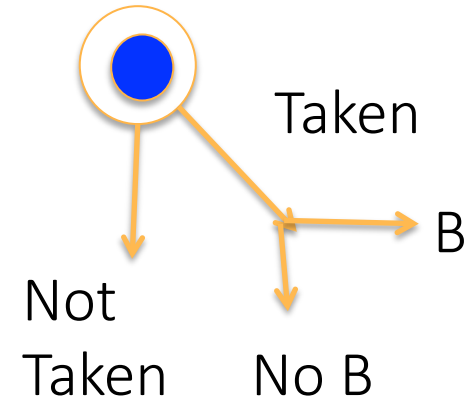
- Scenario 3: Loop contains A(X) (allocation) followed a followed by D(X) (deallocation) on all feasible execution paths within a loop but with a BREAK in between on some paths.
- OK to treat multiple iterations of the Loop as two equivalent execution paths:
 - One path without a BREAK – safe.
 - Another path with a BREAK – not safe without D(X) on the path following BREAK.

Loops with BREAK

- Scenario 4: Loop contains D(X) (deallocation) followed a followed by A(X) (allocation) on all feasible execution paths within a loop but with a BREAK in between on some paths.
- OK to treat multiple iterations of the Loop as two equivalent execution paths:
 - One path without a BREAK – safe if loop is followed by D(X).
 - Another path with a BREAK – safe without D(X) on the path following BREAK.

A Flow diagram example for a loop

```
for(i=1;i<=n;++i){  
    read num;  
    if(num<0)  
        break;  
    sum= sum+ num;  
}
```

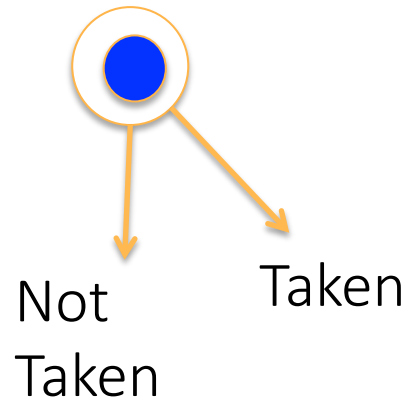


Three execution paths: (i) FOR skipped, (ii) FOR exited with BREAK, (iii) FOR exited without BREAK

Three paths

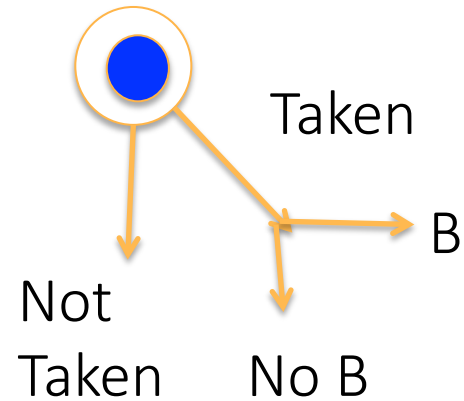
Flow diagrams for loops

Loop



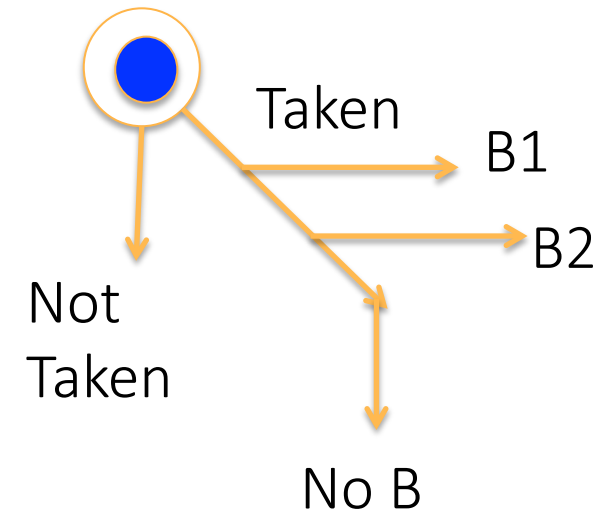
Two paths

Loop with one
BREAK



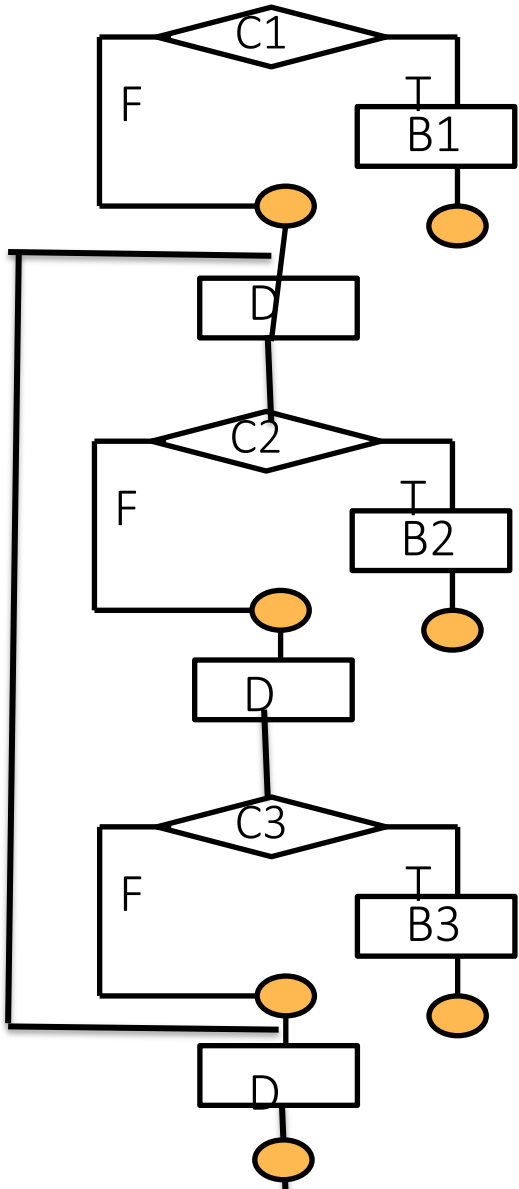
Three paths

Loop with two
BREAKs

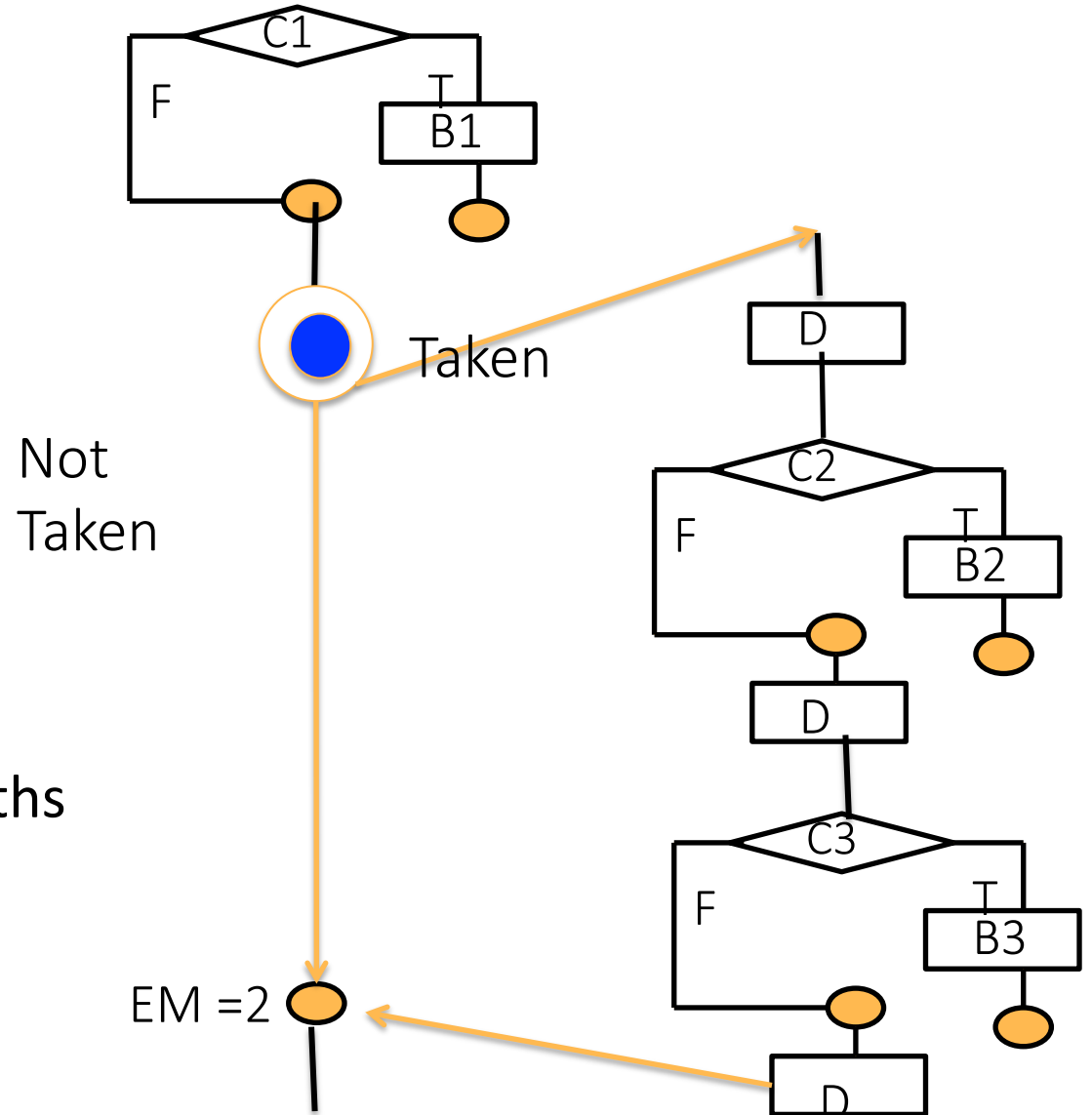


Four paths

A Loop Example

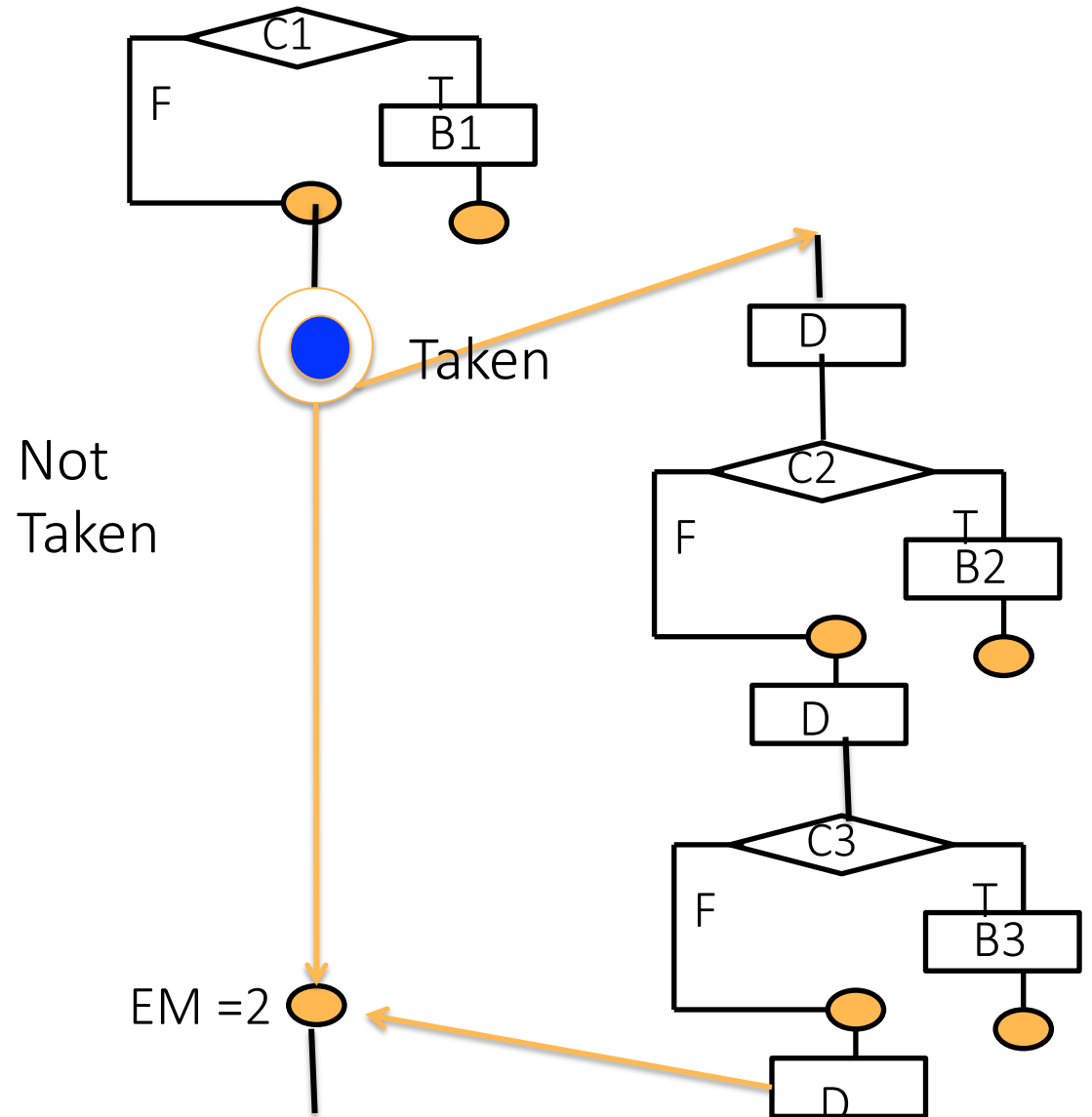


Five Execution Paths

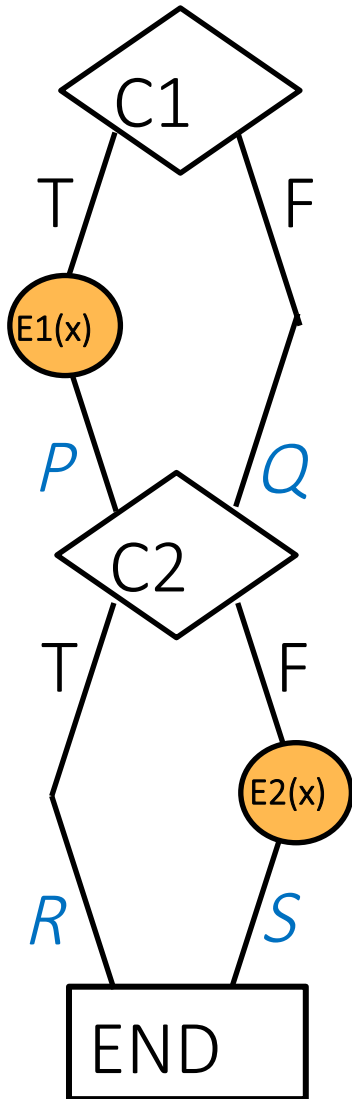


A Loop Example

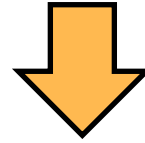
```
if (C2) {  
    dosomething;  
    return();  
}  
for ( ) {  
    if (C2) {  
        dosomething;  
        return();  
    }  
    if ( C3) {  
        dosomething;  
        return();  
    }  
}  
dosomething;
```



Failed Verification of Infeasible Path



When C1 is TRUE C2 *cannot* be TRUE



PR is an infeasible path

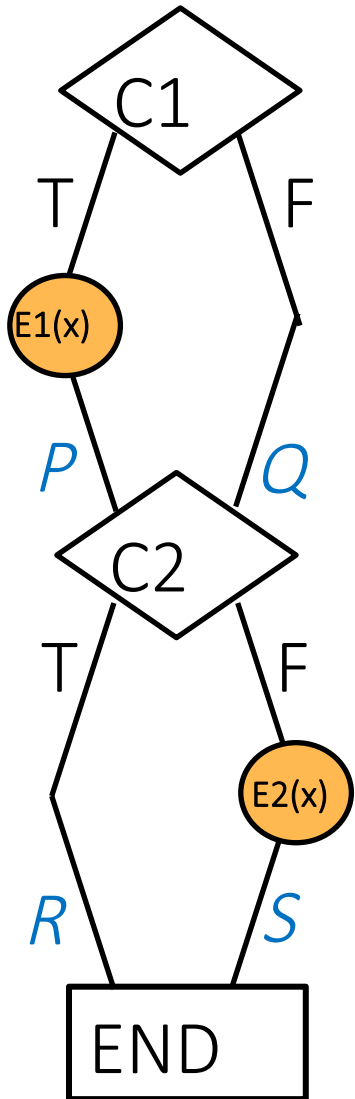
For MP, a FP if the path *PR* is infeasible.

For AMP, a FP if the path *PS* is infeasible.

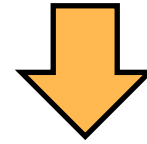
Definitions:

1. Matching Pair (MP) Property: $E1(x)$ is followed by $E2(x)$ on all feasible execution paths.
2. Anti-Matching Pair (AMP) Property: $E1(x)$ is *not* followed by $E2(x)$ on all feasible execution paths

Verification Misses a Path



Both **C1** and **C2** *can* be TRUE



PR is a feasible path

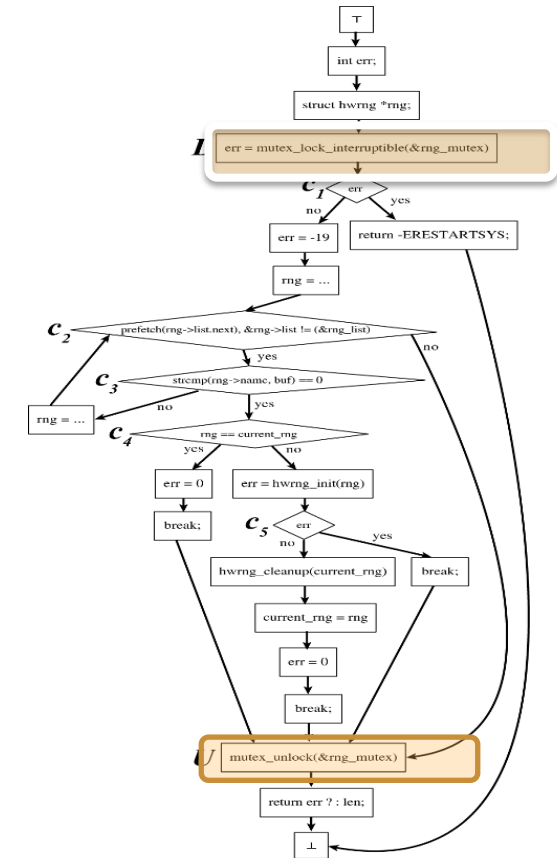
For MP, a FN if the feasible path *PR* is missed.

For AMP, a FN if the feasible path *PS* is missed.

An example from Linux kernel

```
1 static ssize_t hwrng_attr_current_store(...) {  
2     int err;  
3     struct hwrng *rng;  
4     err = mutex_lock_interruptible(&rng_mutex);  
5     if (err)  
6         return -ERESTARTSYS;  
7     err = -ENODEV;  
8     list_for_each_entry(rng, &rng_list, list) {  
9         if (strcmp(rng->name, buf) == 0) {  
10             if (rng == current_rng) {  
11                 err = 0;  
12                 break;  
13             }  
14             err = hwrng_init(rng);  
15             if (err)  
16                 break;  
17             hwrng_cleanup(current_rng);  
18             current_rng = rng;  
19             err = 0;  
20             break;  
21         }  
22     }  
23     mutex_unlock(&rng_mutex);  
24     return err ? : len;  
25 }
```

Linux kernel (v 2.6.31)

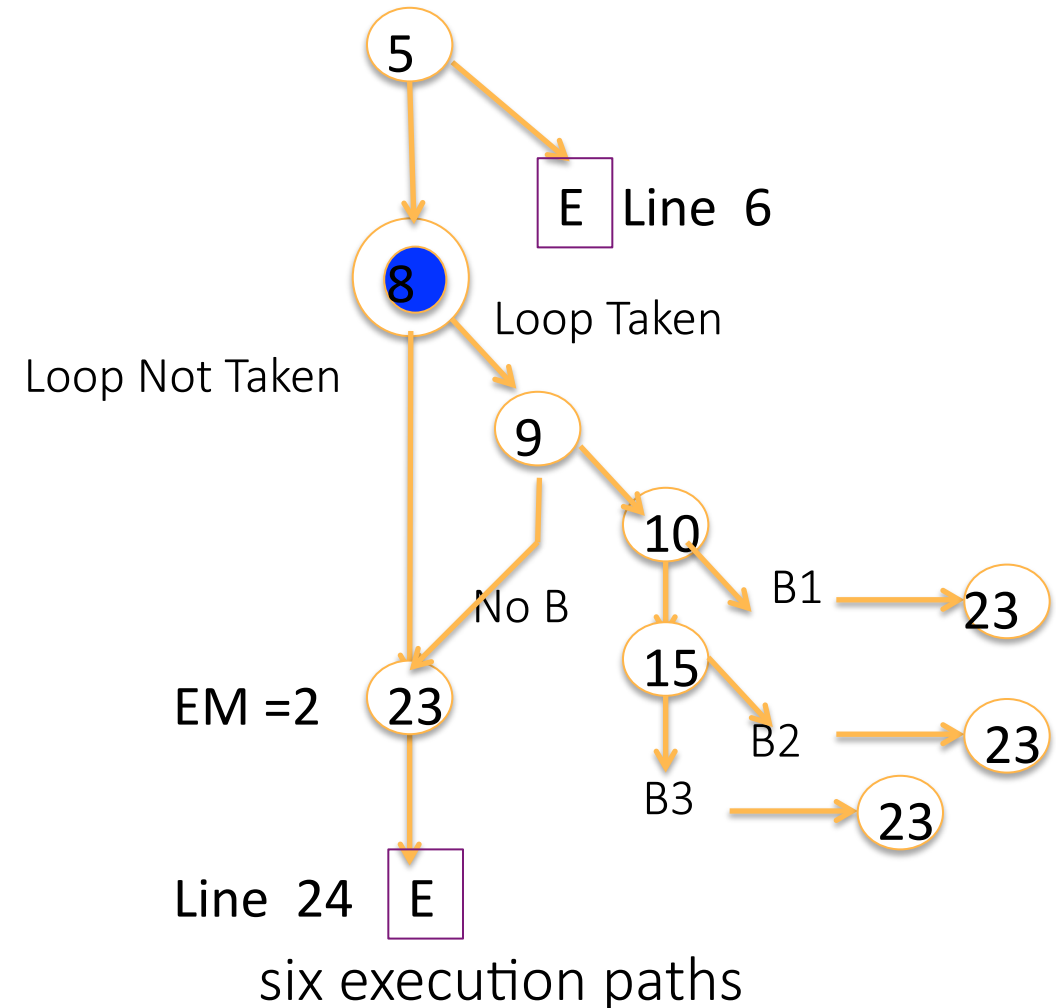


How many execution paths?

An example from Linux kernel

```
1 static ssize_t hwrng_attr_current_store(...){
2     int err;
3     struct hwrng *rng;
4     err = mutex_lock_interruptible(&rng_mutex);
5     if (err)
6         return -ERESTARTSYS;
7     err = -ENODEV;
8     list_for_each_entry(rng, &rng_list, list) {
9         if (strcmp(rng->name, buf) == 0) {
10             if (rng == current_rng) {
11                 err = 0;
12                 break;
13             }
14             err = hwrng_init(rng);
15             if (err)
16                 break;
17             hwrng_cleanup(current_rng);
18             current_rng = rng;
19             err = 0;
20             break;
21         }
22     }
23     mutex_unlock(&rng_mutex);
24     return err ? : len;
25 }
```

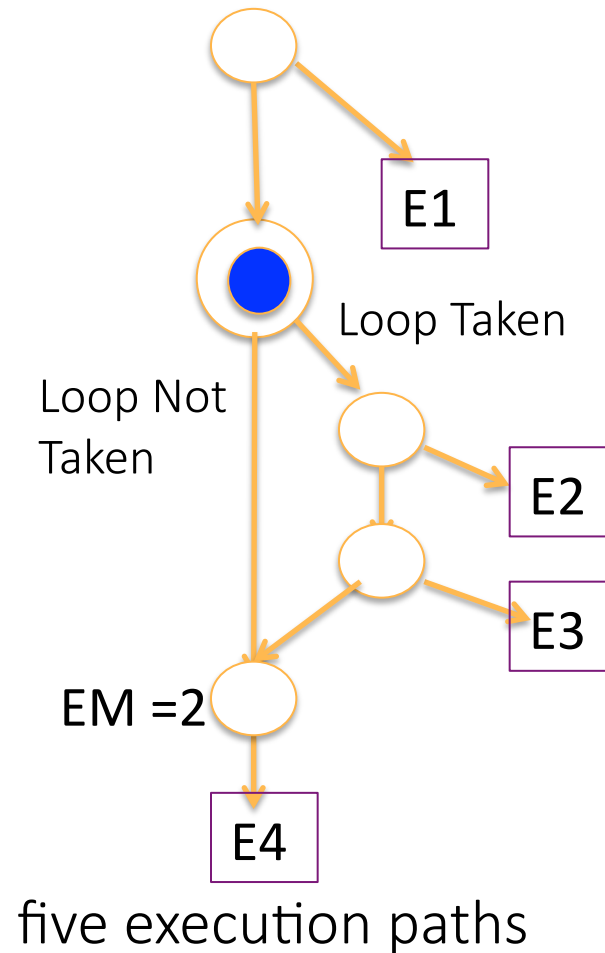
Linux kernel (v 2.6.31)



Execution path analysis of a call

dskeng(drprr, dsprr)

```
{  
  if () {  
    return(DONQ);  
  }  
  for () {  
    if () {  
      return(st);  
    }  
    if () {  
      return(DONQ);  
    }  
  }  
  return(DONQ);  
}
```



five execution paths

Analysis of dskenq

```
dskenq(drprr, dsprr)
{
    if () {
        return(DONQ);          EXIT1, EM=1
    }
    for () {
        if () {
            return(st);        EXIT2, EM=1
        }
        if () {
            return(DONQ);      EXIT3, EM=1
        }
    }
    return(DONQ); EXIT4, EM=2
}
```

5 execution paths

Analysis of dskqopt

```
dskqopt(p, q, drpctr)

    if ( ) {
        return(SYSERR);
    }
    if (drpctr->drop == DSEEK) {
        return(OK);
    }
    if (p->drop == DSEEK) {
        return(OK);
    }
    if (p->drop==DWRITE && drpctr->drop==DWRITE) {
        return(OK);
    }
    if (drpctr->drop==DREAD && p->drop==DWRITE) {
        for ( );
        return(OK);
    }
    if (drpctr->drop==DWRITE && p->drop==DREAD) {
        for ( ) {
            for ( );
        }
        return(OK);
    }
    return(SYSERR);
}
```

EXIT1, EM=1

EXIT2, EM=1

EXIT3, EM=1

EXIT4, EM=1

EXIT5, EM=2

EXIT6, EM=3

EXIT7, EM=1

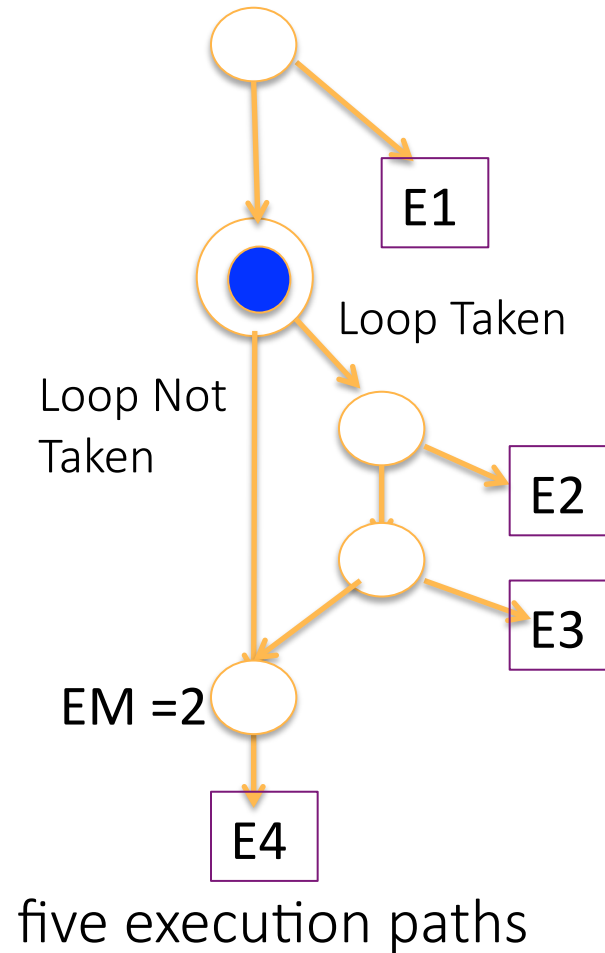
10 execution paths

Total # paths in dskqopt(), which is its total exit multiplicity (EM), is the sum of all exit multiplicities = 10.

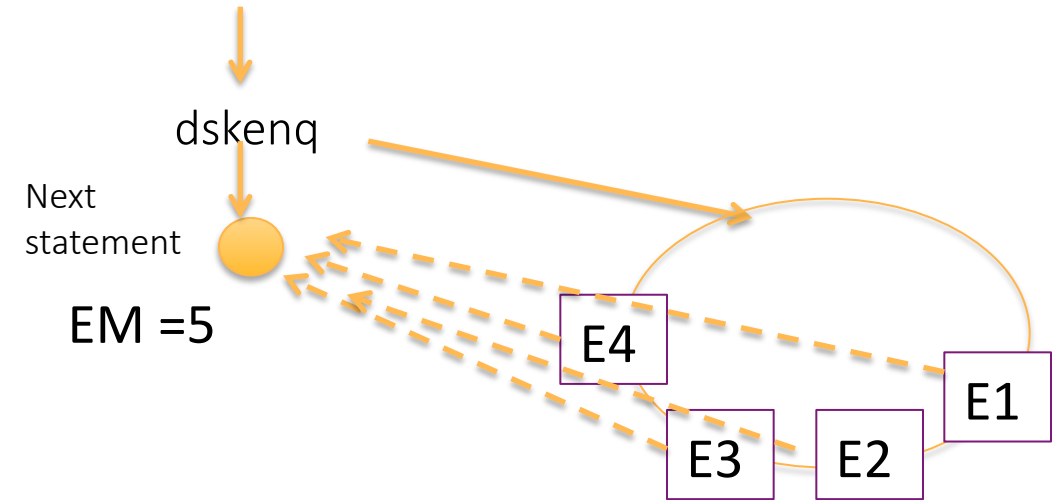
Execution path analysis of a call

dskenq(drptr, dsptr)

```
{  
  if () {  
    return(DONQ);  
  }  
  for () {  
    if () {  
      return(st);  
    }  
    if () {  
      return(DONQ);  
    }  
  }  
  return(DONQ);  
}
```



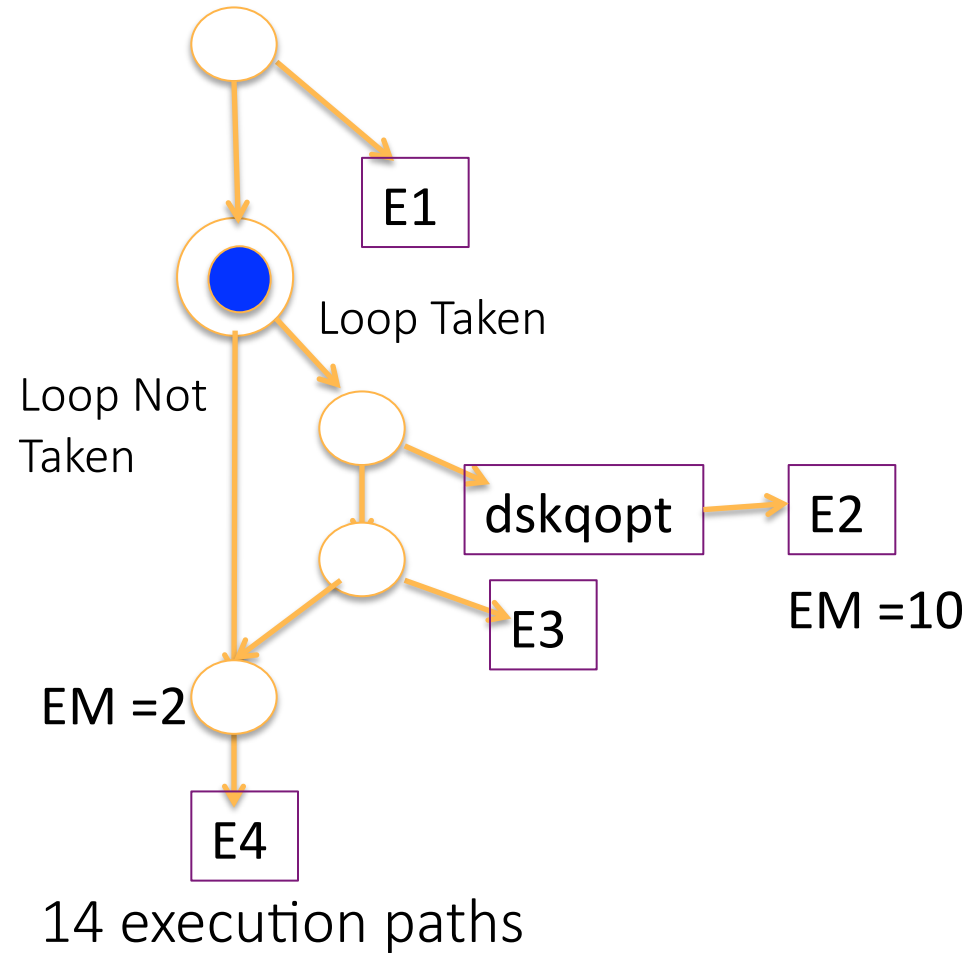
Modeling a call to dskenq



Execution path analysis of with nested calls

```
dskmq(drpqr, dspqr)
```

```
{  
  if () {  
    return(DONQ);  
  }  
  for () {  
    if () {  
      return(st);  
    }  
    if () {  
      return(DONQ);  
    }  
  }  
  return(DONQ);  
}
```



Three accuracy and scalability challenges

- Points-to analysis – exponential due to the power set of objects.
- Path-sensitive analysis – exponential due to the multiplicative growth of the number of paths
- Path feasibility analysis – exponential due to the satisfiability problem.
- We will discuss two approaches to address these problems:
 - Binary Decision Diagram (PCG)
 - Projected Control Graph (PCG)

On Binary Decision Diagram (BDD)

Binary decision diagrams (BDDs) are wonderful, and the more I play with them the more I love them. For fifteen months I've been like a child with a new toy, being able now to solve problems that I never imagined would be tractable.

~Donald Knuth, The Art of Computer Programming, Volume 4, Preface

BDD is a compressed representation of a Boolean Function.

Boolean function representations

- $X1, X2, X3$ – three Boolean variables
- $f(X1, X2, X3)$ – a Boolean function
- $f(0,0,0) = 0, f(0,0,1) = 0, f(0,1,0) = 0, f(0,1,1) = 1, f(1,0,0) = 0, f(1,0,1) = 1, f(1,1,0) = 1, f(1,1,1) = 1$
- Boolean vector representation of f : 00010111
- A Truth Table representation of f :

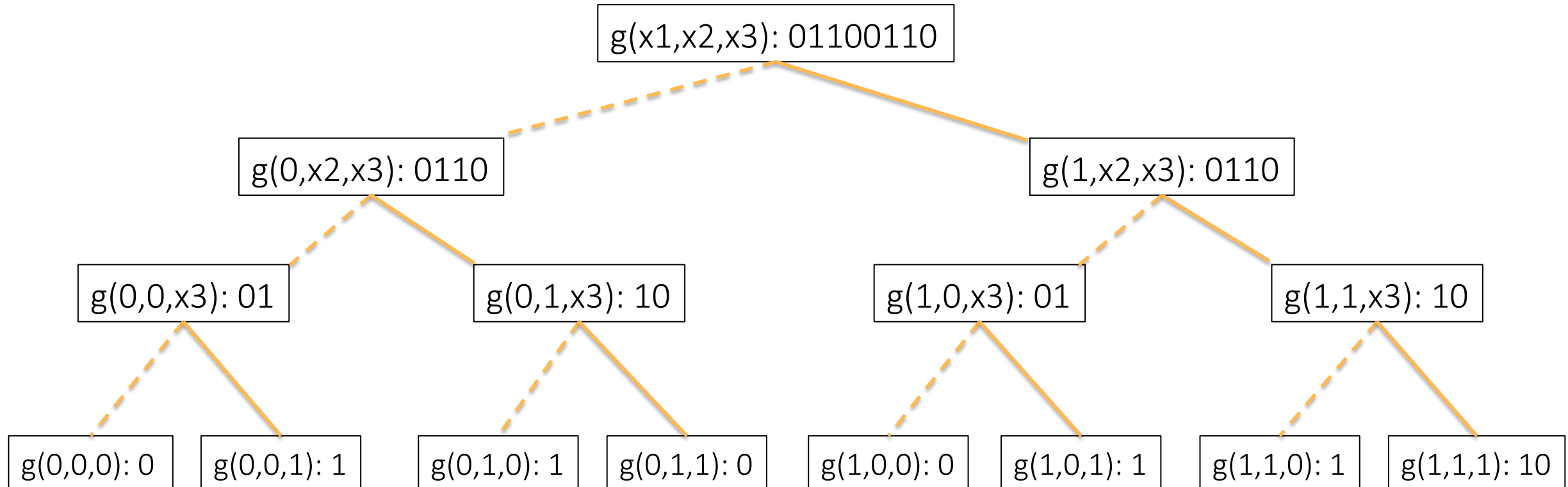
x1	0	0	0	0	1	1	1	1
x2	0	0	1	1	0	0	1	1
x3	0	1	0	1	0	1	0	1
f	0	0	0	1	0	1	1	1

How can the representation be made compact?

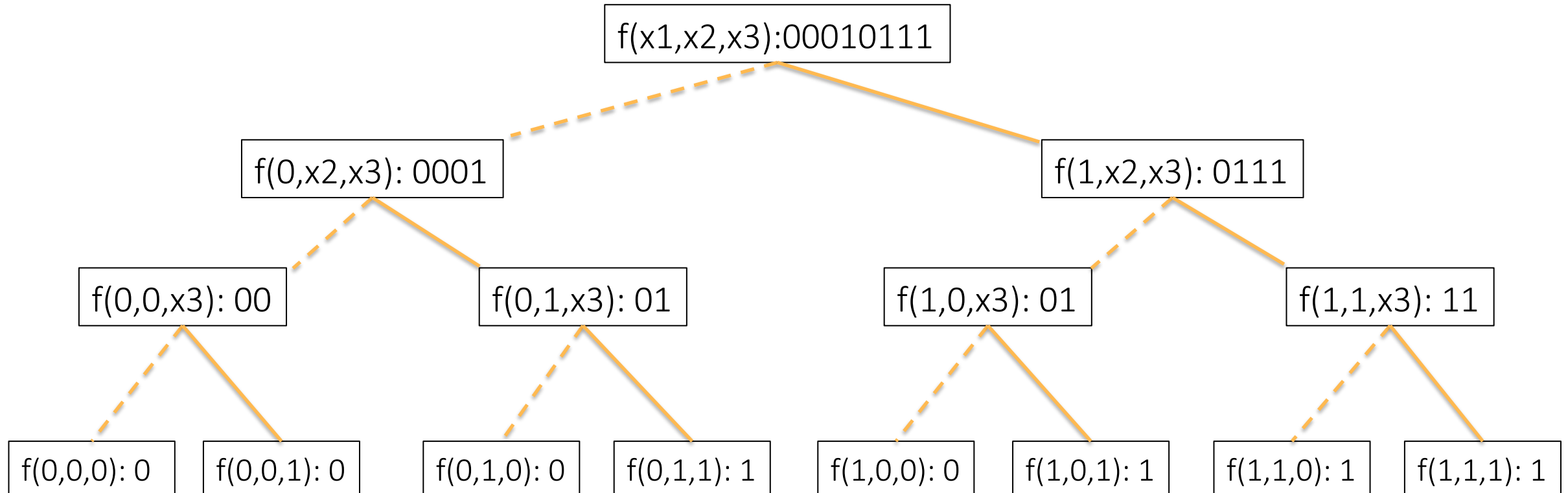
- Consider two Boolean functions:
 - $f(x_1, x_2, x_3)$: 00010111
 - $g(x_1, x_2, x_3)$: 01100110

Which function can be compressed? Why?

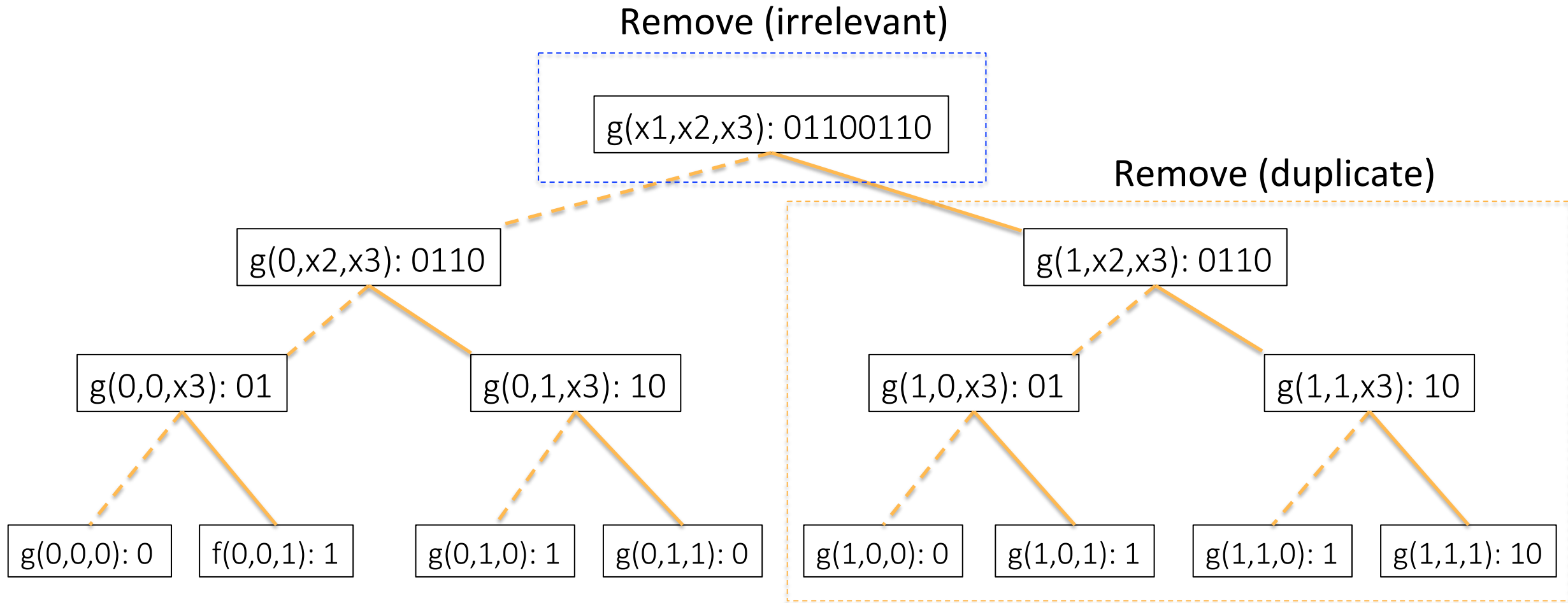
Sub-functions of a function



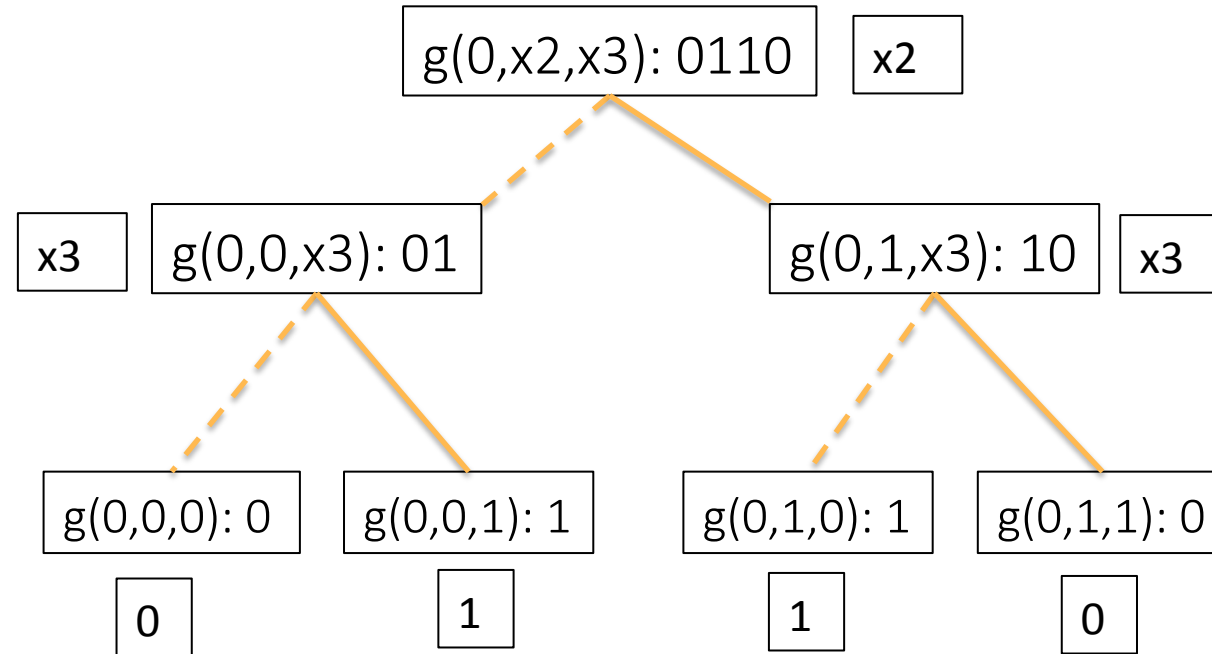
Sub-functions of a function



Retain relevant sub-functions & remove duplicates



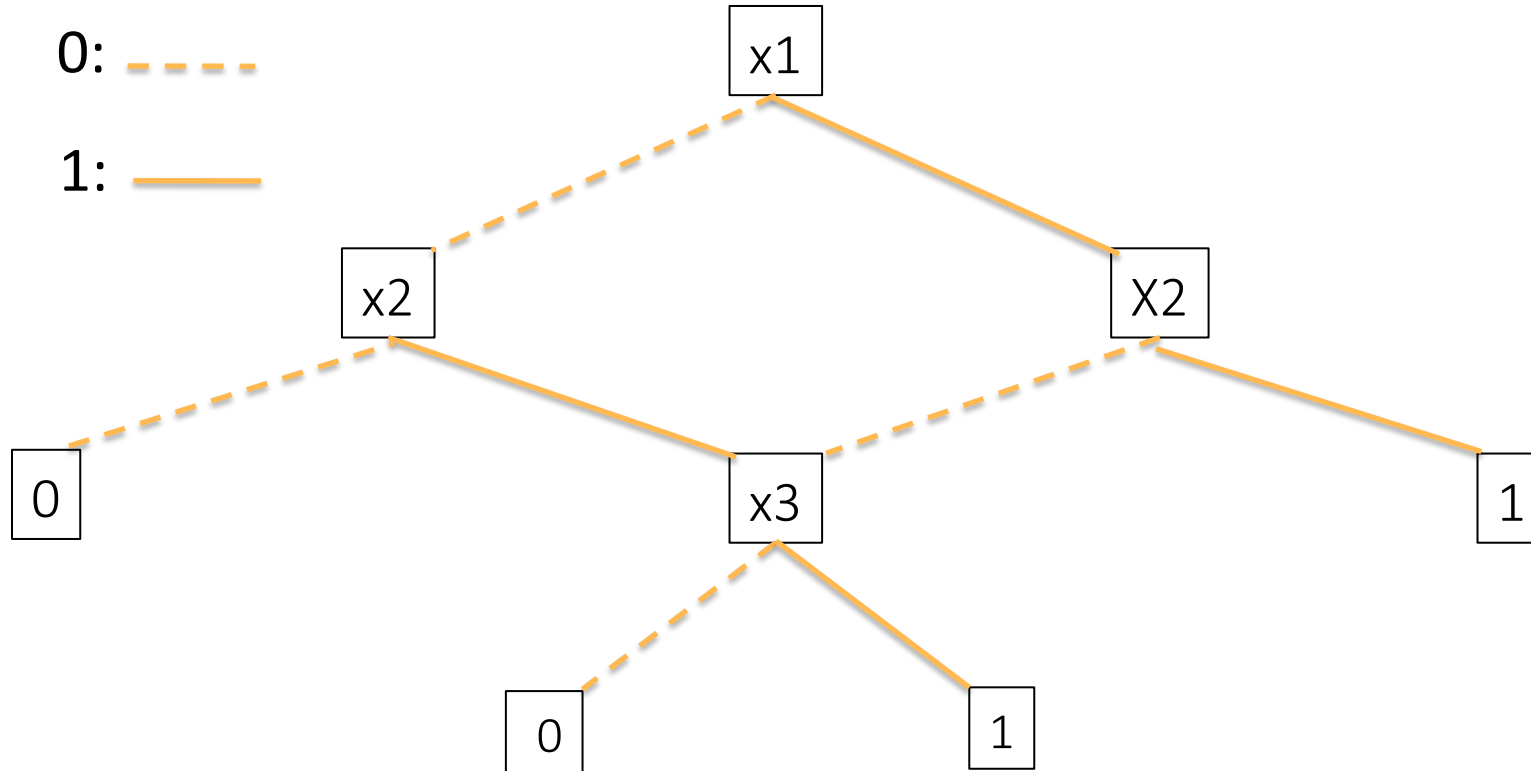
Resulting BDD



The boxes should be relabeled in BDD as shown

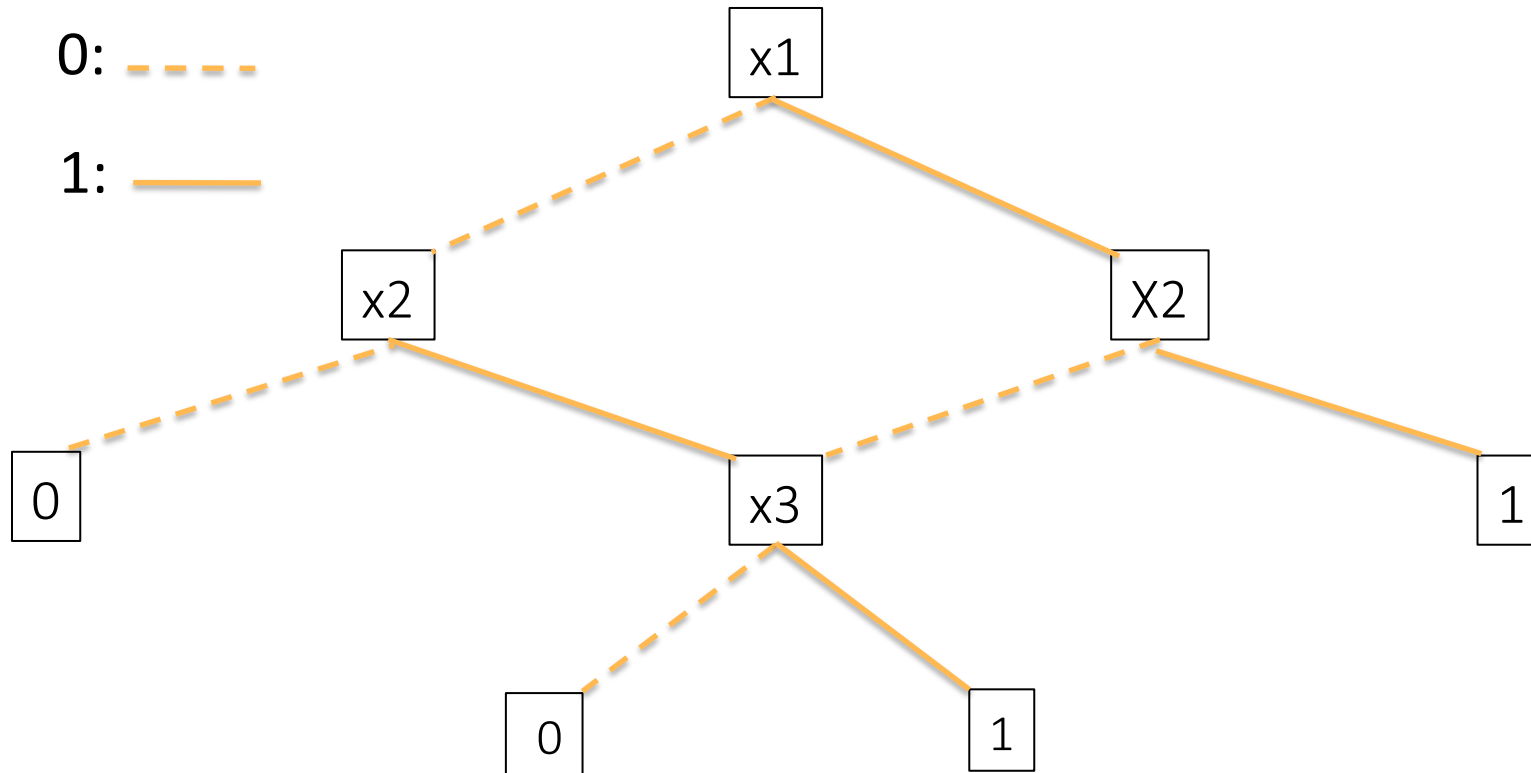
The input values of x_1 are irrelevant for computing the Boolean function.

What is the function represented by the BDD?



x1	x2	x3	f()
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

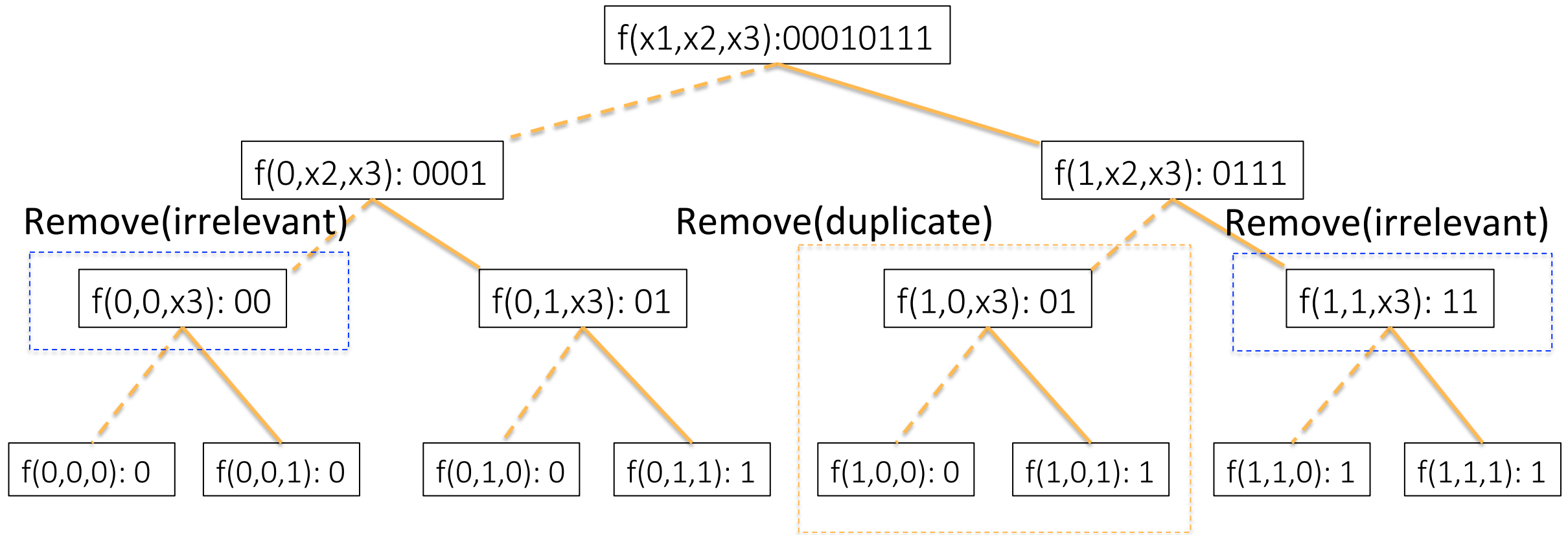
What is the Boolean function represented by the BDD?



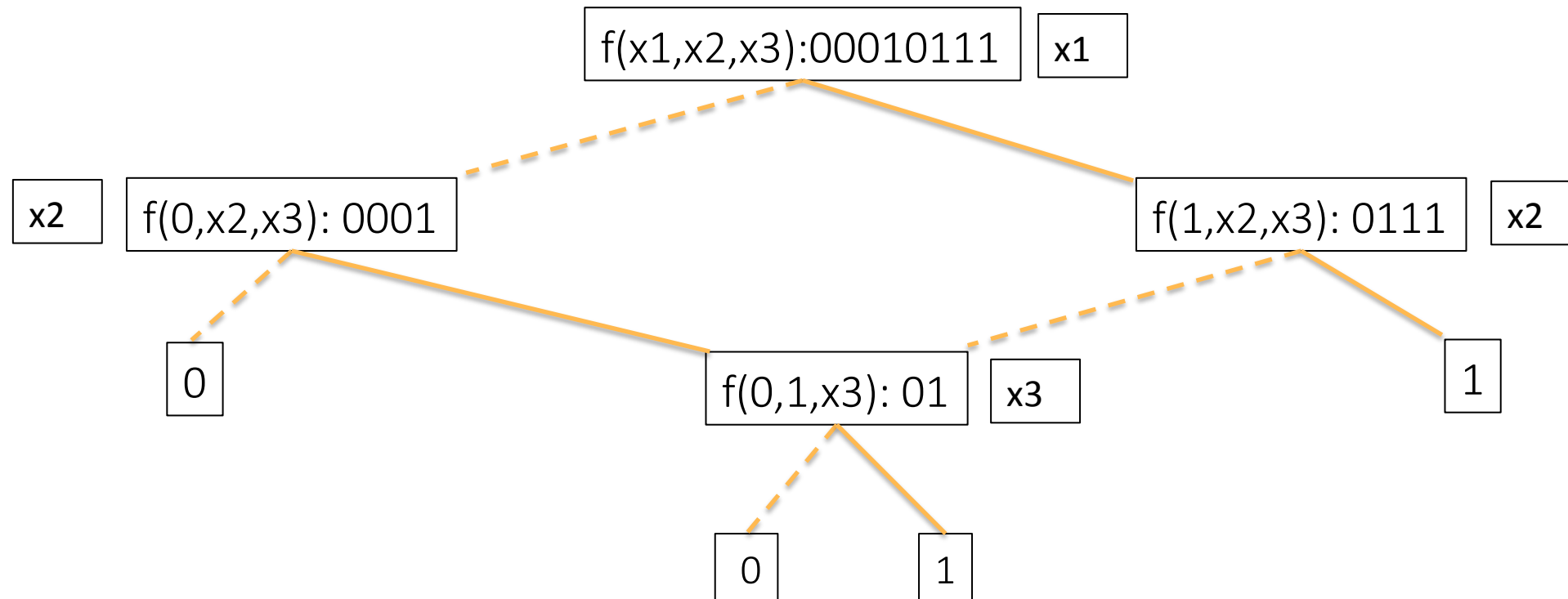
x1	x2	x3	f()
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Binary vector representation of the Boolean function: 00010111

Retain relevant sub-functions and remove duplicate



Resulting BDD



The boxes should be relabeled in BDD as shown

Observations from BDD experiment with two Boolean functions

- Boolean function: 00010111
 - Retained sub-functions in the BDD: 00010111, 0001, 0111, 01
 - Eliminated sub-functions: 00, 11
- Boolean function: 01100110
 - Retained sub-functions in the BDD: 0110, 01, 10
 - Eliminated sub-functions: 01101110
- Definitions to describe the observations succinctly:
 - A sub-function is *non-primitive* if the binary vector is of the form B^2 (BB), otherwise it is *primitive*. The primitive sub-functions are also called the *beads*.
 - Only one copy of each primitive will be retained.

A succinct definition of BDD

- BDD retains only the primitive sub-functions of a Boolean function.
- BDD nodes correspond to an ordered set (sequence) of Boolean variables X_1 , X_2 , and X_n .
- Each node has two edges.
- *Convention*: left edge from node for X_i is dotted and it corresponds to $X_i == 0$.
- BDD is acyclic if there is an edge from X_i node to X_j node then $i < j$.
- The leaves of the BDD are the 0 or 1 values of the Boolean function.
- The paths in the BDD correspond to 2^n binary inputs.
- For the visualization, we may show multiple leaves for 0 or 1 but for the actual storage we need only two leaves.

Ordering of Boolean variables can affect the size of the BDD

a	b	c	f()
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

c	b	a	f()
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

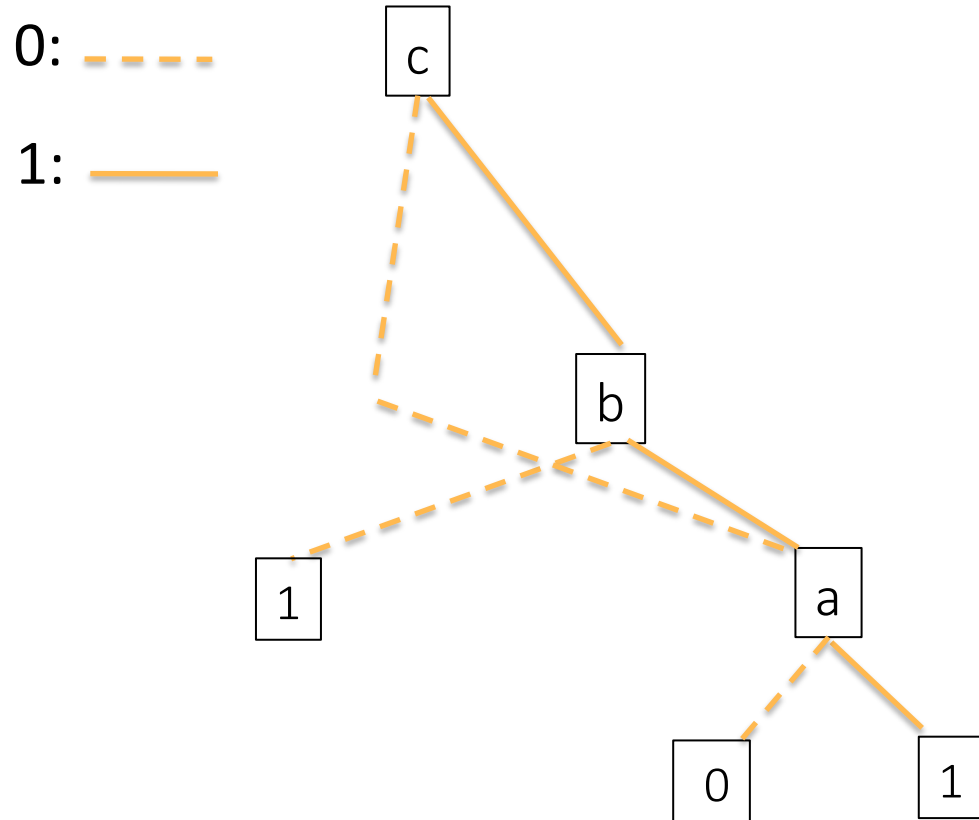
Boolean representation for the ordering a, b, c:
01001111

Boolean representation for the ordering c, b, a:
01011101

Primitives: 01001111, 0100, 01

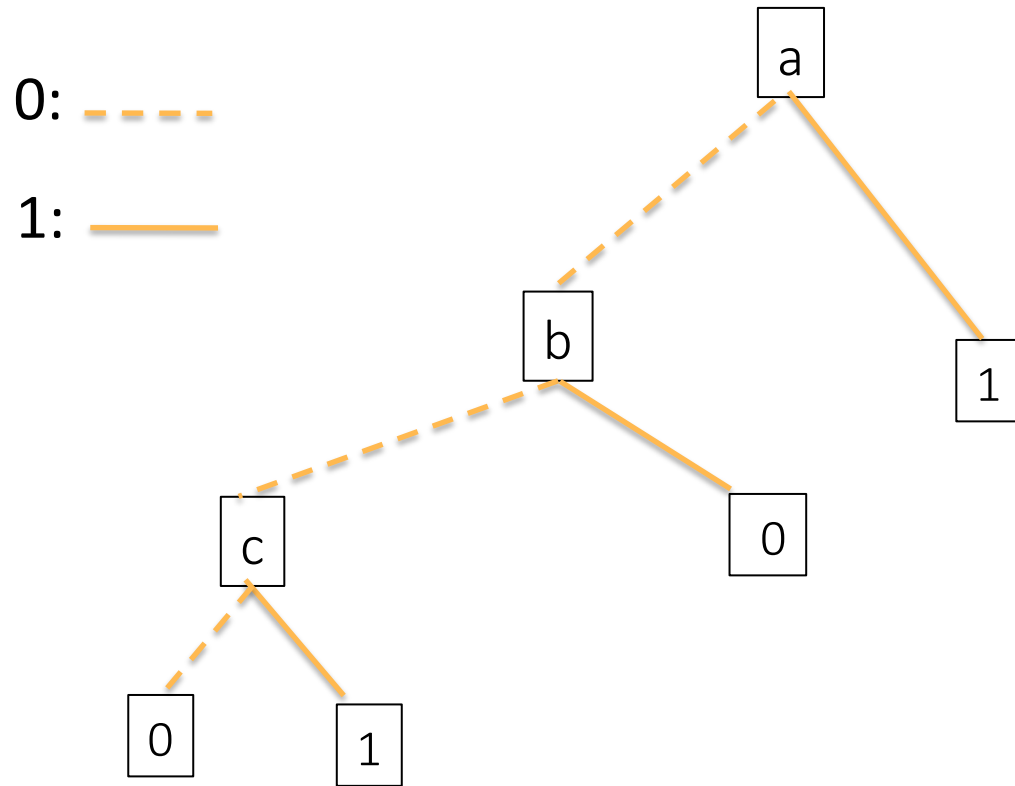
Primitives: 01011101, 1101, 01

What is the function represented by the BDD?



a	b	c	f()
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

What is the function represented by the BDD?



a	b	c	f()
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

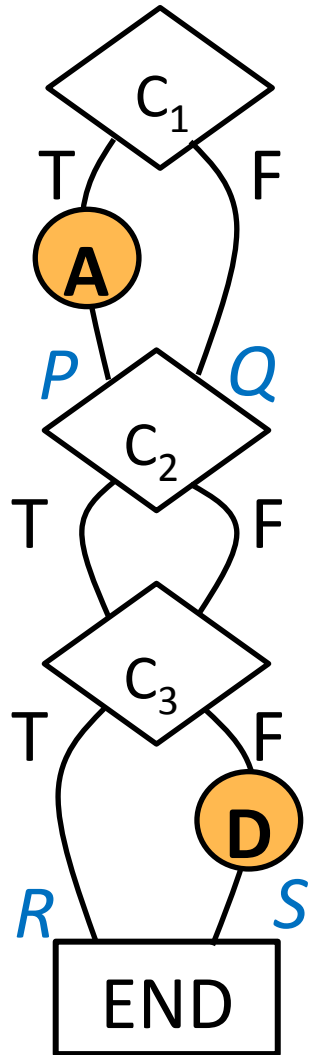
Equivalence between Boolean functions and subsets of $\mathbb{P}(S)$

- $S = \{O_1, O_2, O_3\}$, Powerset: $\mathbb{P}(S) = \{\Phi, \{O_1\}, \{O_2\}, \{O_3\}, \{O_1, O_2\}, \dots\}$
- Each subset corresponds to a Boolean input:
 - $(x_1=0, x_2=0, x_3=0)=\{\Phi\}$, $(x_1=0, x_2=0, x_3=1)=\{O_3\}$, $(x_1=0, x_2=1, x_3=0)=\{O_2\}$, $(x_1=0, x_2=1, x_3=1)=\{O_2, O_3\}$, $(x_1=1, x_2=0, x_3=0)=\{O_1\}$, $(x_1=1, x_2=0, x_3=1)=\{O_1, O_3\}$, $(x_1=1, x_2=1, x_3=0)=\{O_1, O_2\}$, $(x_1=1, x_2=1, x_3=1)=\{O_1, O_2, O_3\}$
- Subset T of $\mathbb{P}(S)$ maps to Boolean function. Given a set S with n elements, the cardinality of $\mathbb{P}(S)$ is $k=2^n$ and the number of Boolean functions is 2^k .
 - $T = \{\{O_1, O_2\}, \{O_3\}, \{O_2, O_3\}, \{O_1, O_2, O_3\}\}$ corresponds to the Boolean function 01010011
- Points-to sets corresponds to subset of T of $\mathbb{P}(S)$
 - e.g. $P(V_1) = \{O_1, O_2\}$, $P(V_2) = \{O_3\}$, $P(V_3) = \{O_2, O_3\}$, $P(V_4) = \{O_1, O_2, O_3\}$, corresponds to $T = \{\{O_1, O_2\}, \{O_3\}, \{O_2, O_3\}, \{O_1, O_2, O_3\}\}$.
 - Every variable V_i maps to one path in the BDD
- Implication: BDDs can serve as compact representations for Boolean functions as well as for points-to sets.

Intuition: Efficient Path-Sensitive Analysis

- A large number of paths could be partitioned into a small number of groups.
- All Paths in a group are equivalent – have the same execution behavior w.r.t. the property to be verified.
- Efficient computation by examining only one path from each group.
- Challenge: How can the groups be formed without examining each path at least once?

Irrelevant Branch Conditions

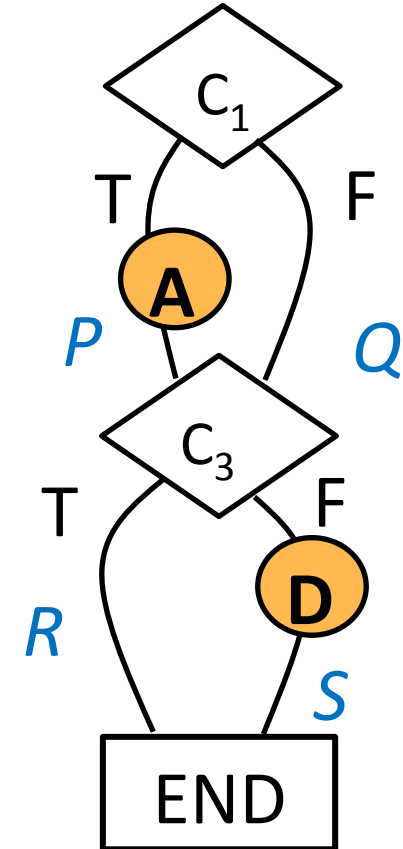


C_2 Irrelevant to
path-sensitive analysis



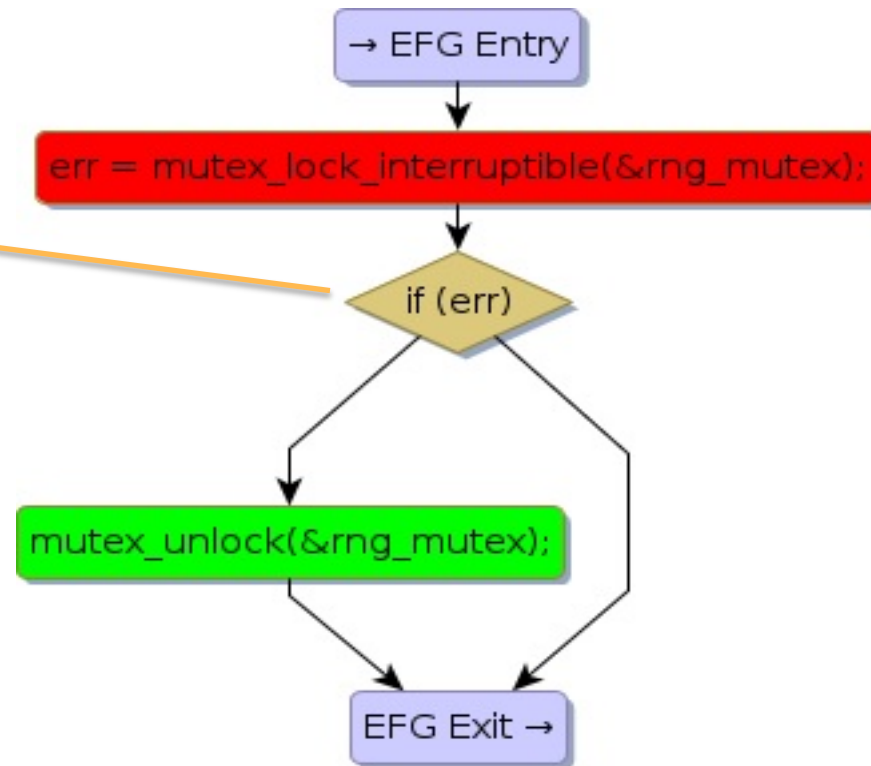
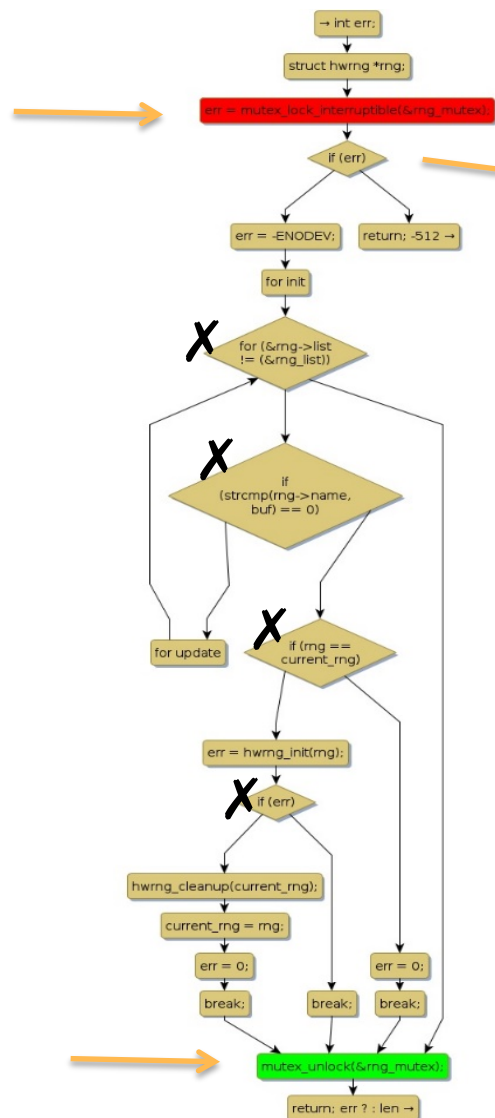
Remove the Irrelevant branch nodes to
avoid unnecessary path explosion & also
simplify the path feasibility check.

paths reduced from 8 to 4



conditions for feasibility check
reduced from 3 to 2

A Linux Example to Illustrate Efficient Analysis



EFG for Efficient Path-Sensitive Analysis

- Event Flow Graph (EFG) is derived from the Control Flow Graph (CFG).
- Relevant events: CFG nodes relevant to the property (e.g. getbuf, freebuf, nodes of type p = drptr (pointer to the allocated memory)).
- Behavior: An event trace along a CFG path
- EFG has one path per group of CFG paths with identical behaviors.
- EFG construction: $O(K)$ where $K = N + E$, N : # nodes in CFG, E : # edges in CFG.

Linux Lock/Unlock Stats by L-SAP

- Versions 3.17-rc1, 3.18-rc1, 3.19-rc1
 - 37 MLOC & 66,609 instances
 - 62,663 Intra-Procedural (MPG size =1), average MPG size 1.3, the maximum MPG size 40 for one MPG
 - 55,251 functions in the union of all MPGs
 - 4 nodes per EFG, the maximum EFG size 63 for one EFG

CFG to EFG Reduction – Top Ten

Linux version 3-19-rc-1

Function Name	Nodes		Edges		Branch Nodes	
	CFG	EFG	CFG	EFG	CFG	EFG
<code>client_common_fill_super</code>	1,101	15	1,179	28	249	13
<code>kiblnd_create_conn</code>	731	18	925	34	197	15
<code>CopyBufferToControlPacket</code>	392	20	559	39	180	18
<code>kiblnd_cm_callback</code>	662	38	831	56	170	15
<code>kiblnd_passive_connect</code>	622	22	784	44	164	20
<code>dst_ca_ioctl</code>	349	2	518	1	163	0
<code>qib_make_ud_req</code>	621	10	821	15	156	5
<code>cfs_cpt_table_al</code>	522	7	672	13	153	6
<code>private_ioctl</code>	569	16	732	24	148	8
<code>vCommandTimer</code>	490	47	623	75	143	28