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# Antiferromagnetic and bond-order-wave phases in the half-filled 2D optical Su-Schrieffer-Heeger- Hubbard model

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**APS Global Physics Summit**  
**18 March 2025**



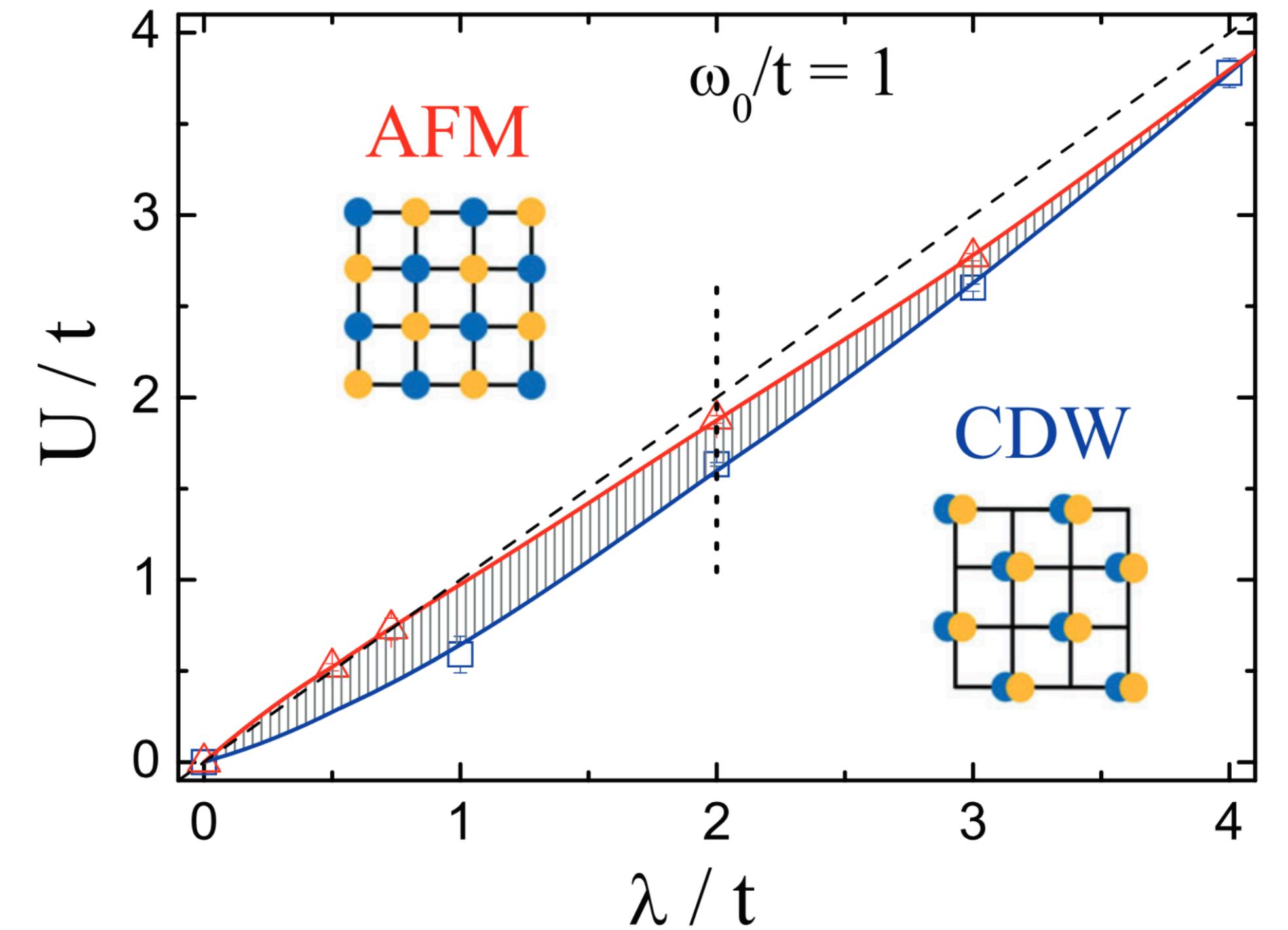
This work was primarily  
supported by the National  
Science Foundation under  
Grant No.~DMR-2401388.



Additional support provided by U.S. Department of Energy, Office of Science, Office of Workforce Development for Teachers and Scientists, Office of Science Graduate Student Research (SCGSR) program. The SCGSR program is administered by the Oak Ridge Institute for Science and Education (ORISE) for the DOE. ORISE is managed by ORAU under contract number DE-SC0014664.

# Electron-phonon models

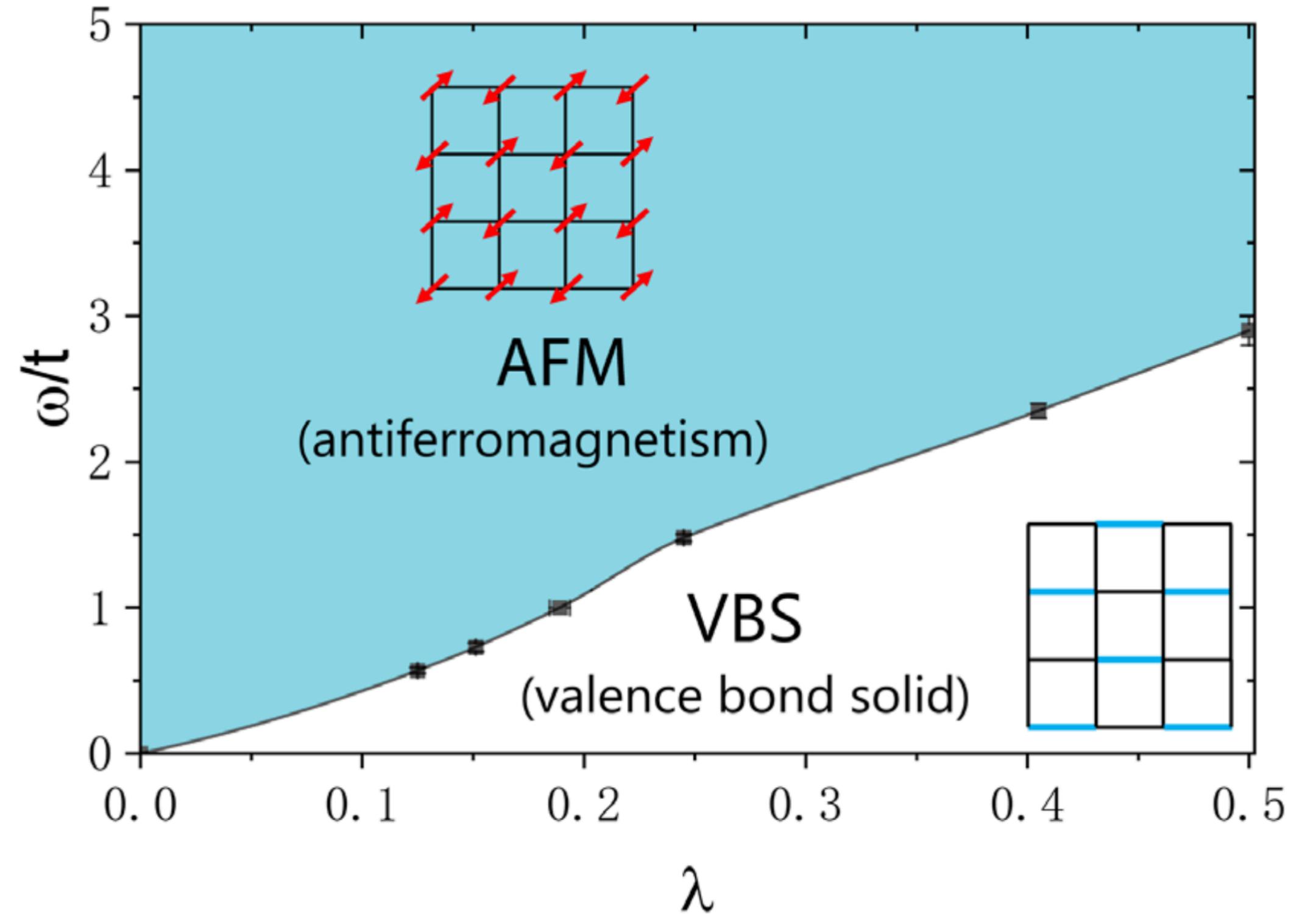
## Hubbard-Holstein model



N.C. Costa et al. *Commun. Phys.* **3**, 80 (2020)

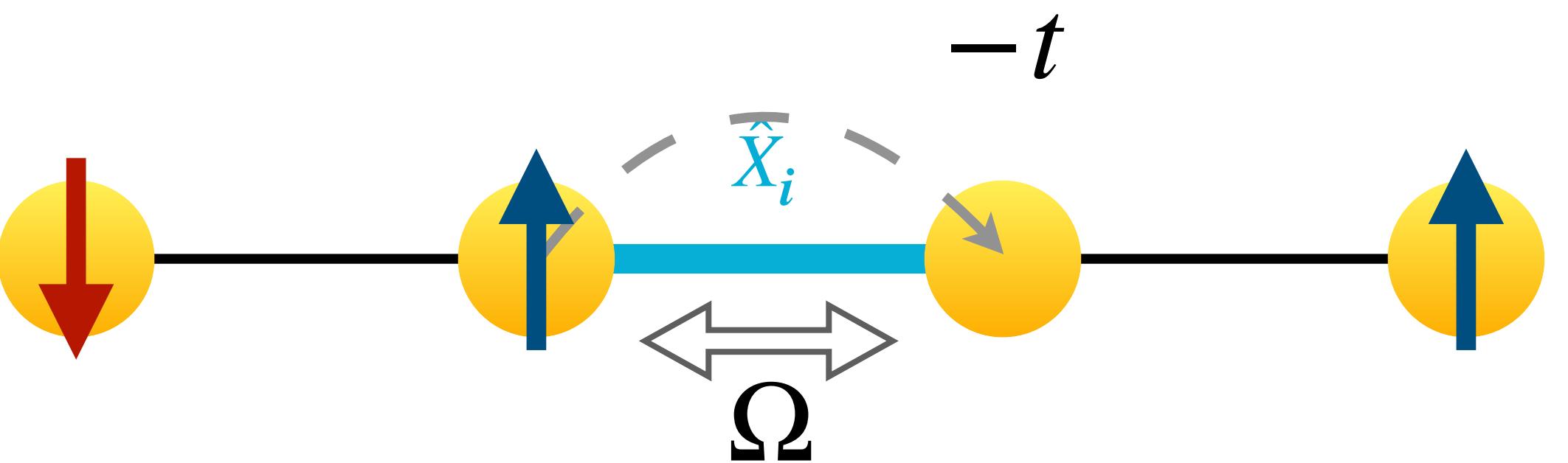
# Electron-phonon models

## Bond Su-Schrieffer-Heeger model



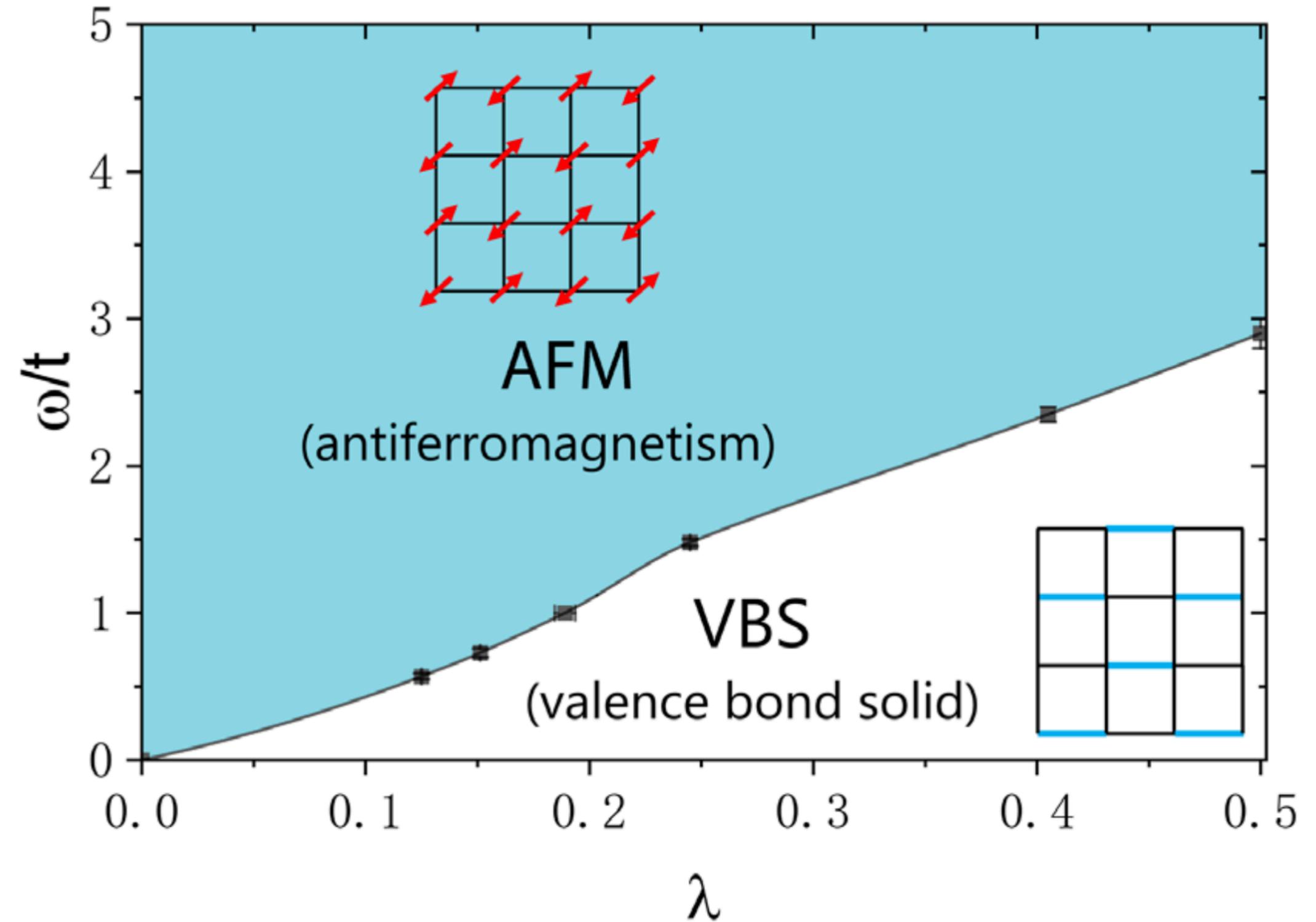
X.Cai et al. Phys. Rev. Lett. **127**, 247203 (2021)

$$\hat{H}_{e-\text{ph}} = \alpha \sum_{i,\nu,\sigma} \hat{X}_{i,\nu} (\hat{c}_{i+a_\nu, \sigma}^\dagger \hat{c}_{i, \sigma} + \text{H.c.})$$

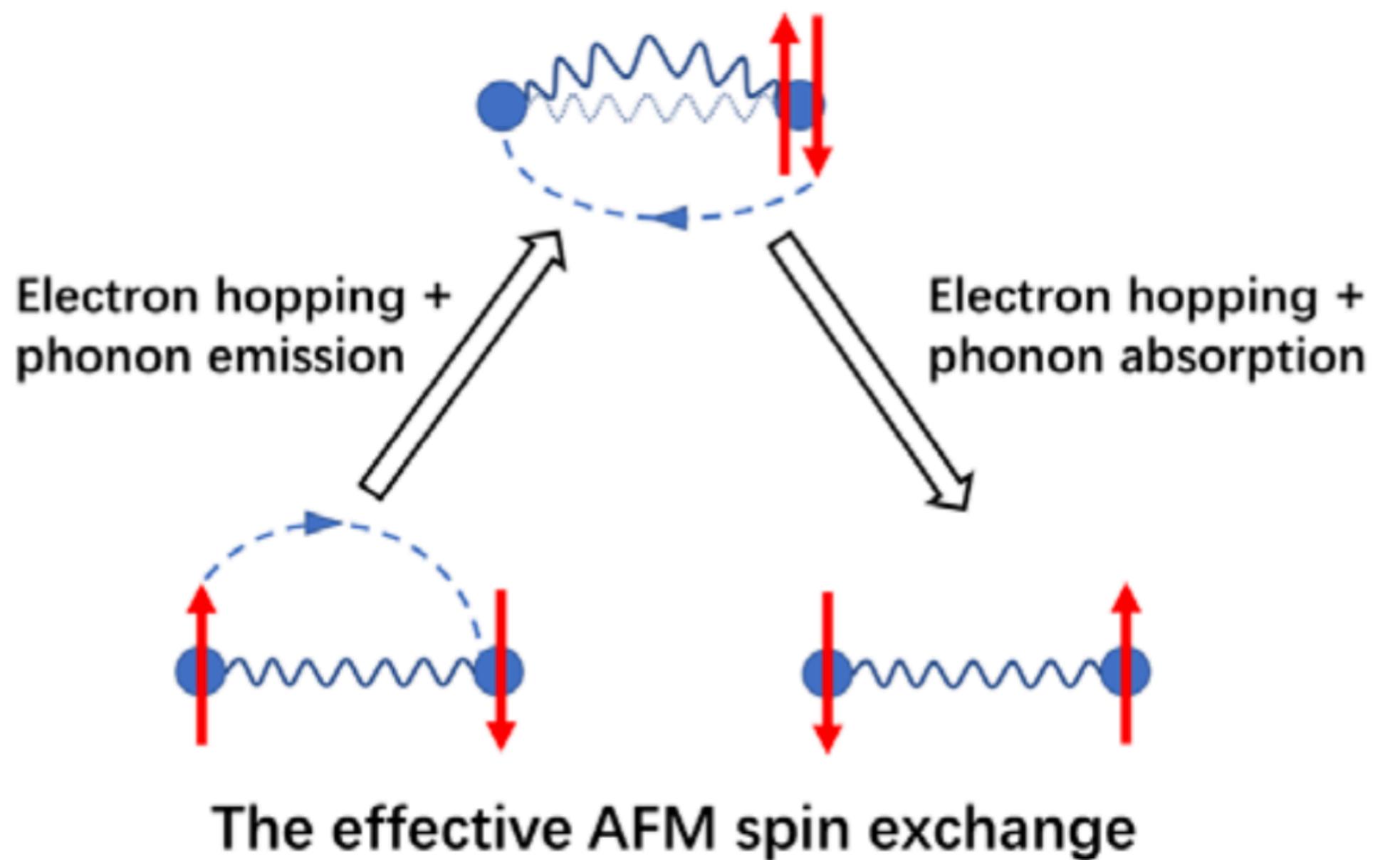


# Electron-phonon models

## Bond Su-Schrieffer-Heeger model

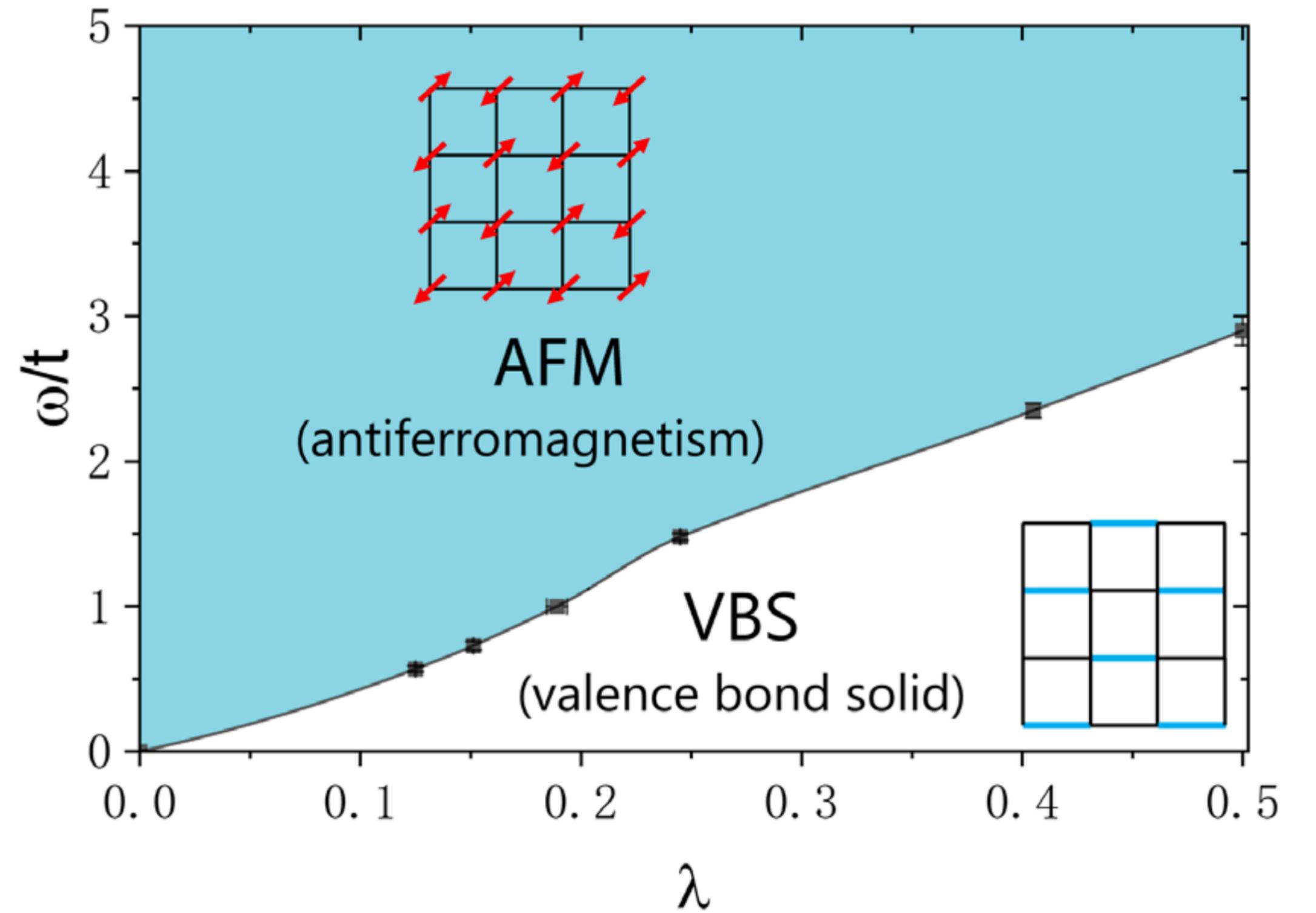


X.Cai et al. *Phys. Rev. Lett.* **127**, 247203 (2021)



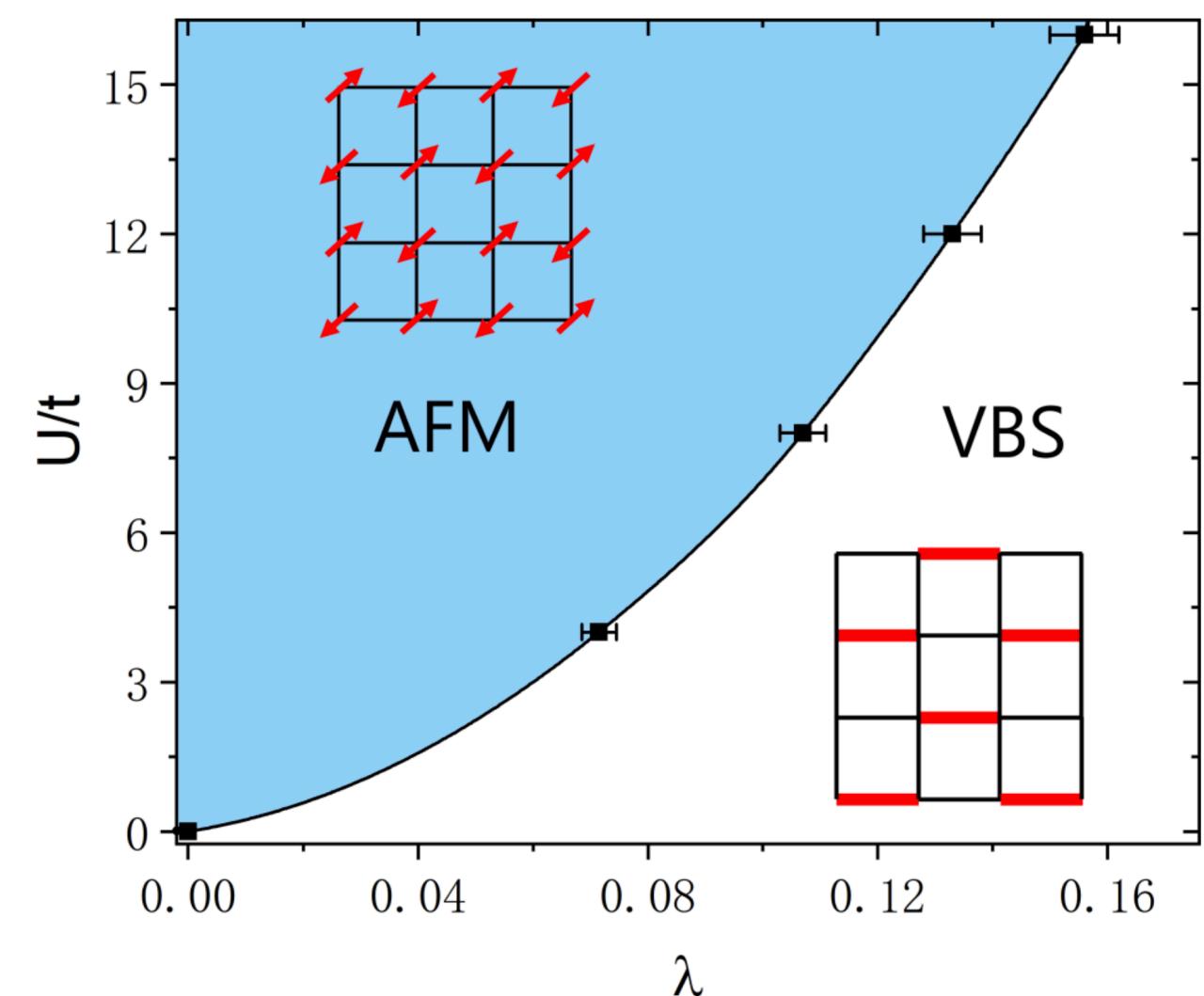
# Electron-phonon models

## Bond Su-Schrieffer-Heeger model

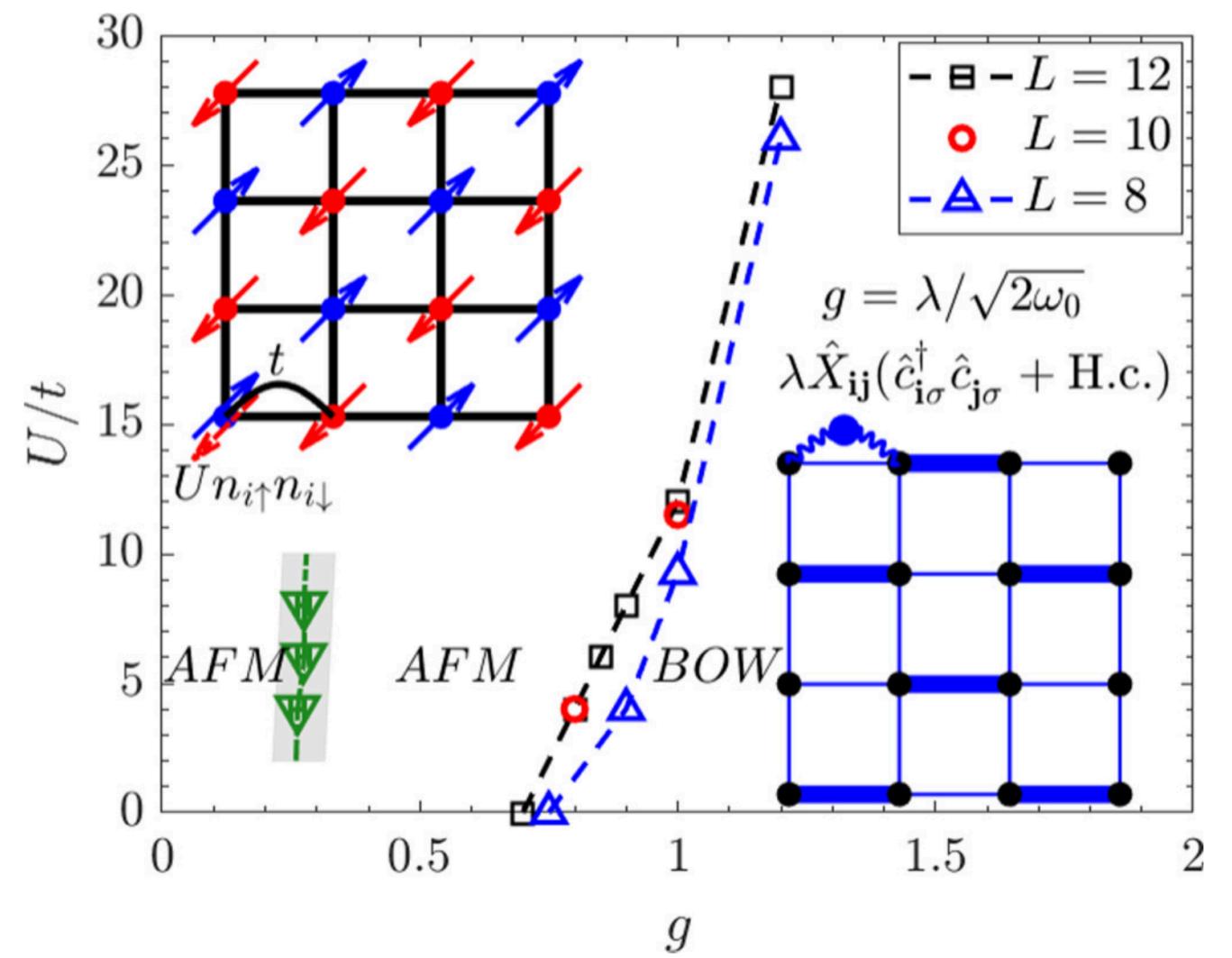


X.Cai et al. *Phys. Rev. Lett.* **127**, 247203 (2021)

+ Hubbard  
→



X.Cai et al. *Phys. Rev. B* **106**, L081115 (2022)



C. Feng et al. *Phys. Rev. B* **106**, L081114 (2022)

# Optical Su-Schrieffer-Heeger-Hubbard model

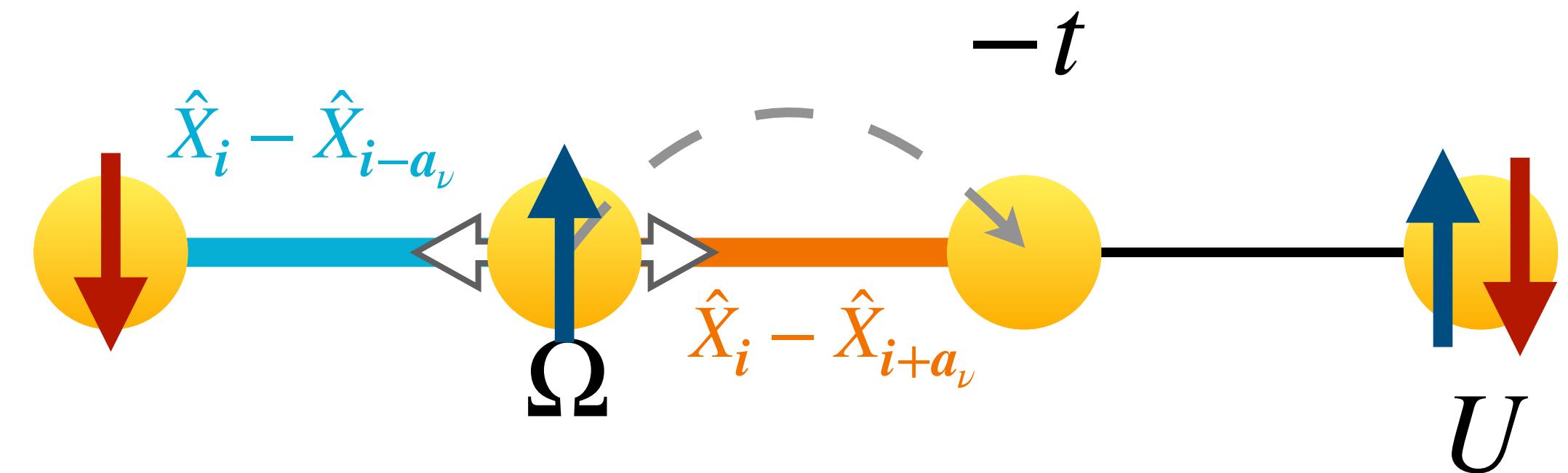
## Hubbard model

$$\hat{H}_e = -t \sum_{i,\nu,\sigma} (\hat{c}_{i+a_\nu,\sigma}^\dagger \hat{c}_{i,\sigma} + \text{H. c.}) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

## Optical SSH model

$$\hat{H}_{e-\text{ph}} = \alpha \sum_{i,\nu,\sigma} (\hat{X}_{i+a_\nu,\nu} - \hat{X}_{i,\nu})(\hat{c}_{i+a_\nu,\sigma}^\dagger \hat{c}_{i,\sigma} + \text{H. c.})$$

$$\hat{H}_{\text{ph}} = \sum_{i,\nu} \left( \frac{1}{2M} \hat{P}_{i,\nu}^2 + \frac{M\Omega^2}{2} \hat{X}_{i,\nu}^2 \right)$$



M. Capone, W. Stephen, M Grilli. *Phys. Rev. B.* **56**, 4484 (1997)

**Comparative studies:** [ATL et al. Phys. Rev. B 108, 184501 \(2023\)](#)

S. Malkaruge Costa et al. *Phys. Rev. B* **108**, 165138 (2023)

# Methods

## Dimensionless coupling constant

$$\lambda = \frac{2}{WN^2} \sum_{\mathbf{k}, \mathbf{q}, \nu} \frac{|g_\nu(\mathbf{k}, \mathbf{q})|^2}{\Omega(\mathbf{q})} = \boxed{\frac{8\alpha^2}{M\Omega^2 W}}$$

See [ATL](#) et al. *Phys. Rev. B* **108**, 184501 (2023) and S. Malkaruge Costa et al. *Phys. Rev. B* **108**, 165138 (2023)

## Determinant quantum Monte Carlo

- with hybrid Monte Carlo updates
- Finite temperature
- Square clusters ( $L = 8 - 14$ )
- Phonon energy:  $\Omega/t = 0.5$
- Half-filling:  $\langle n \rangle = 1$
- **SmoQyDQMC** package



**SmoQy Suite:** <https://github.com/SmoQySuite>

B. Cohen-Stead et al. *SciPost Physics Codebases*. **24** (2024)

# Correlation Ratios

$$R_\gamma(L) = 1 - \frac{S_\gamma(Q - \delta q_L, 0)}{S_\gamma(Q, 0)}$$

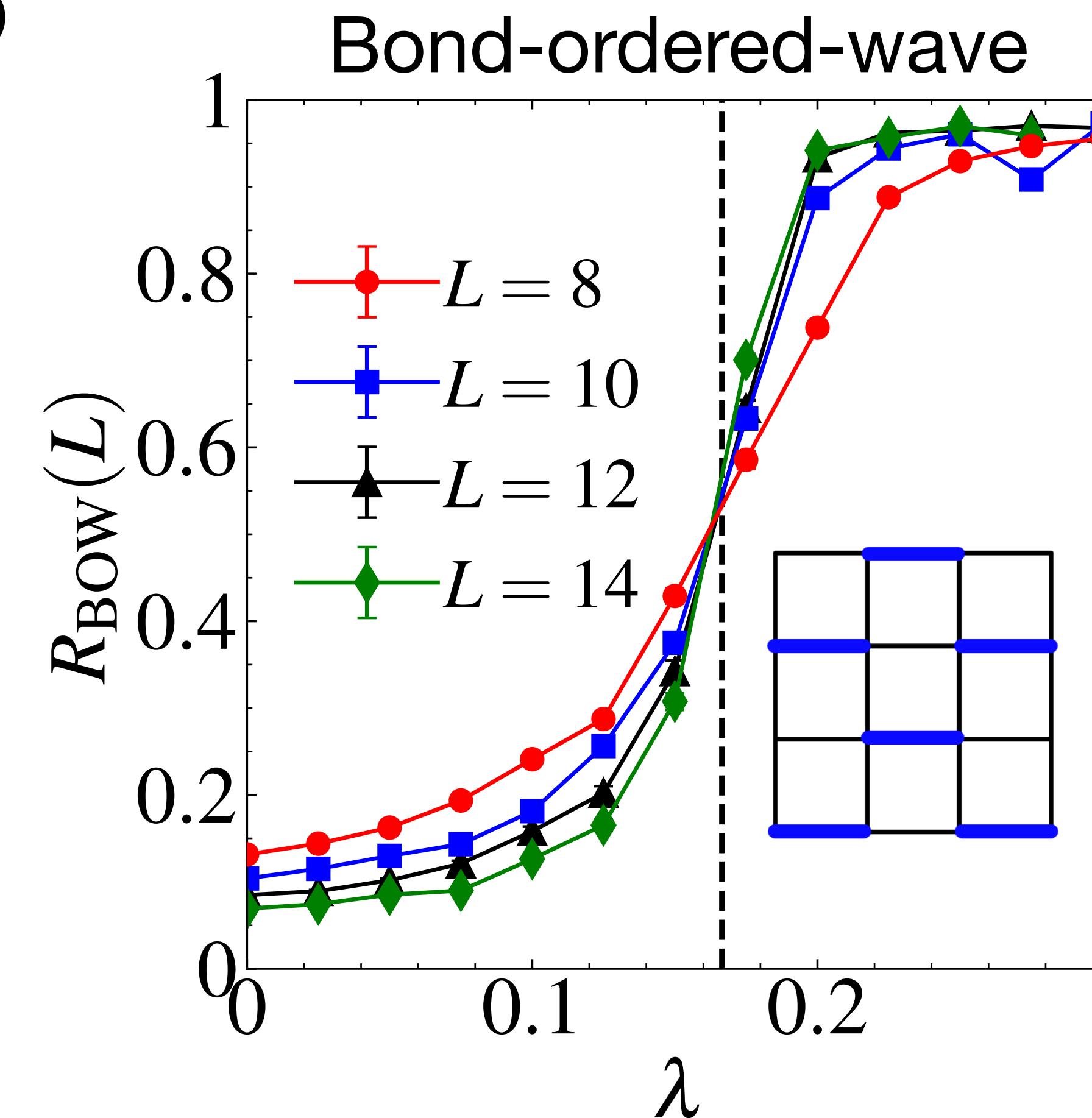
$$|\delta q_L| = \frac{2\pi}{La}$$

$$Q = (\pi/a, \pi/a)$$

Finite size scaling

$$\beta t = L$$

$$U/t = 3$$
$$\Omega/t = 0.5$$



# Correlation Ratios

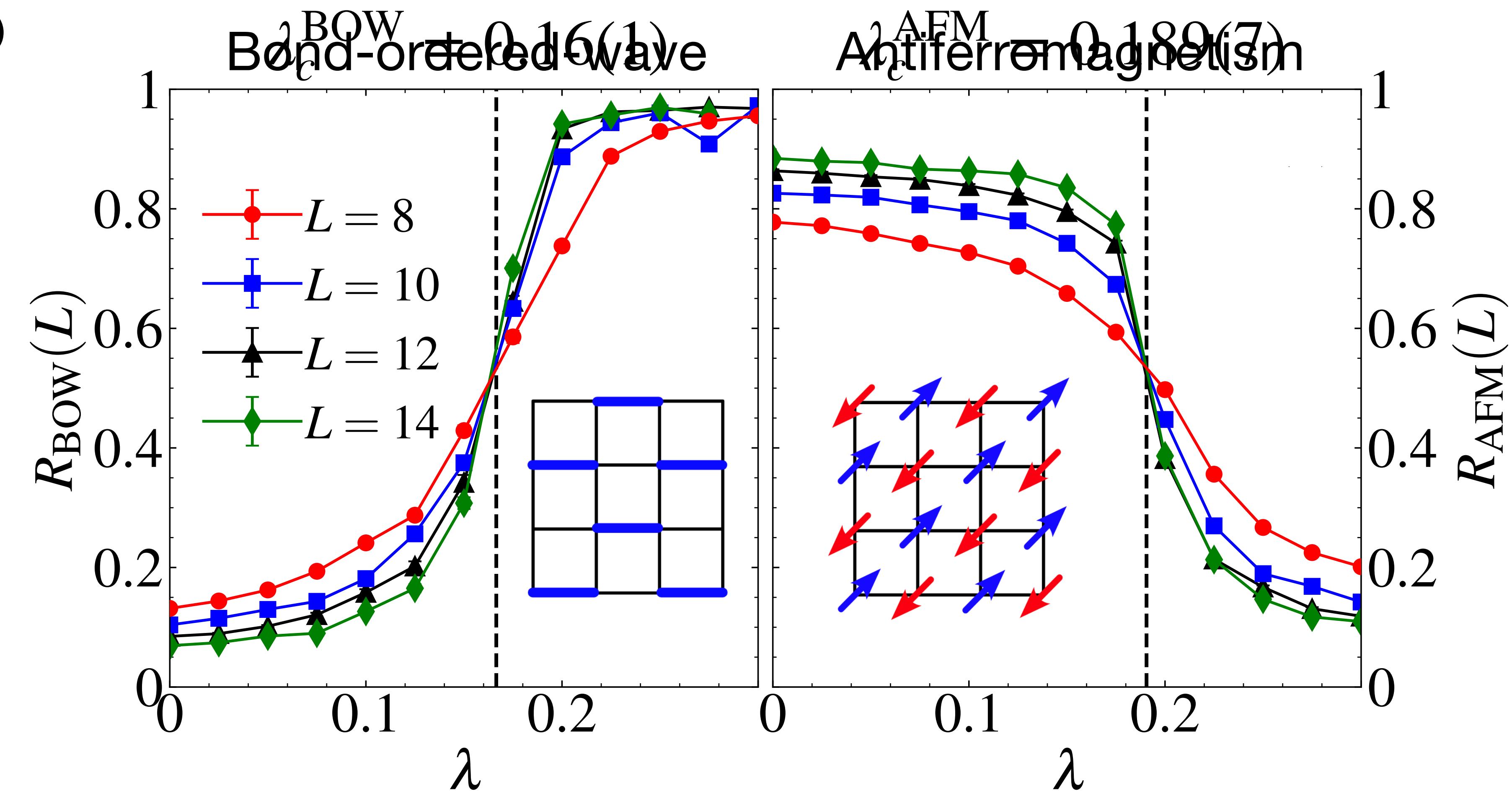
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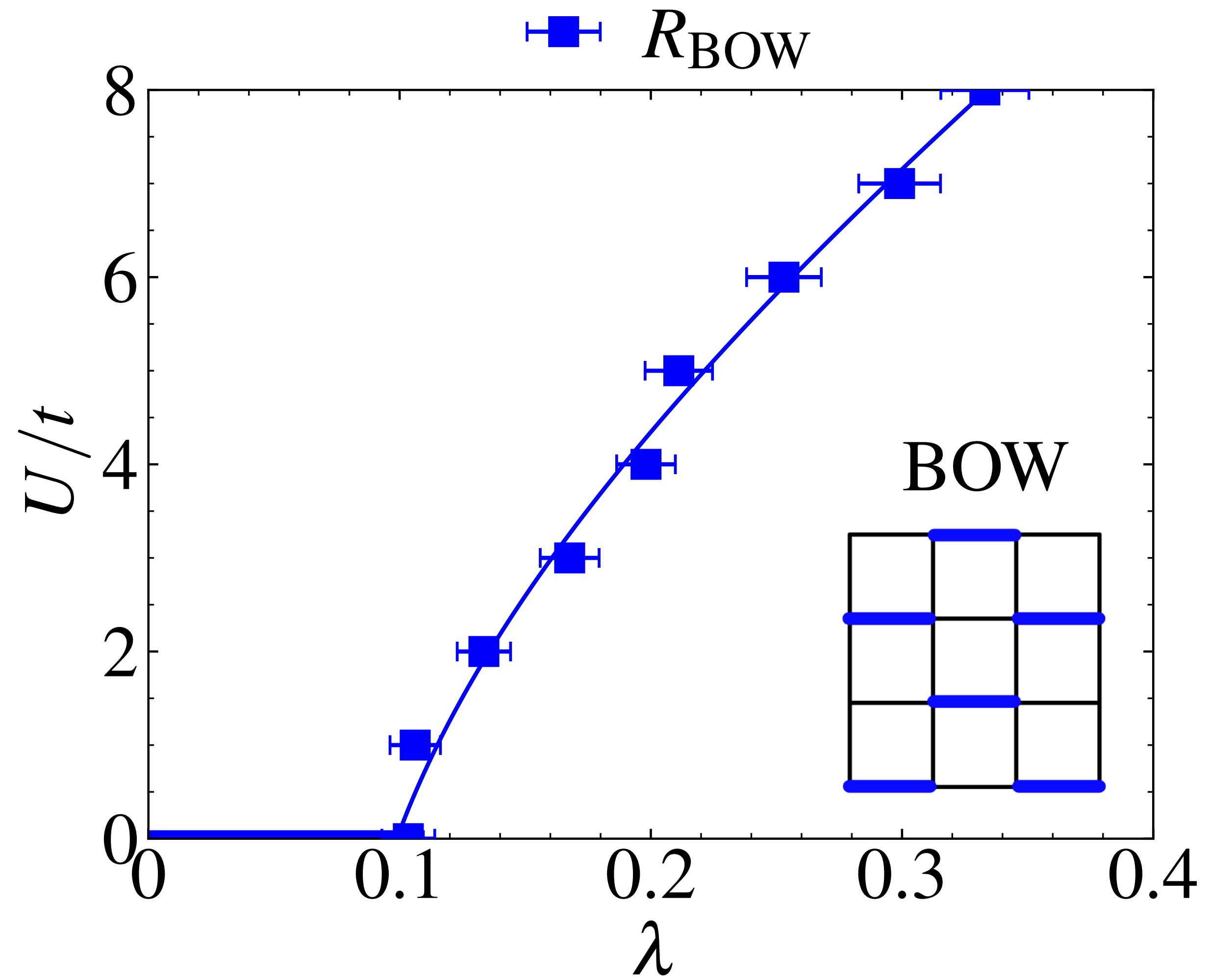
$$Q = (\pi/a, \pi/a)$$

Finite size scaling  
 $\beta t = L$

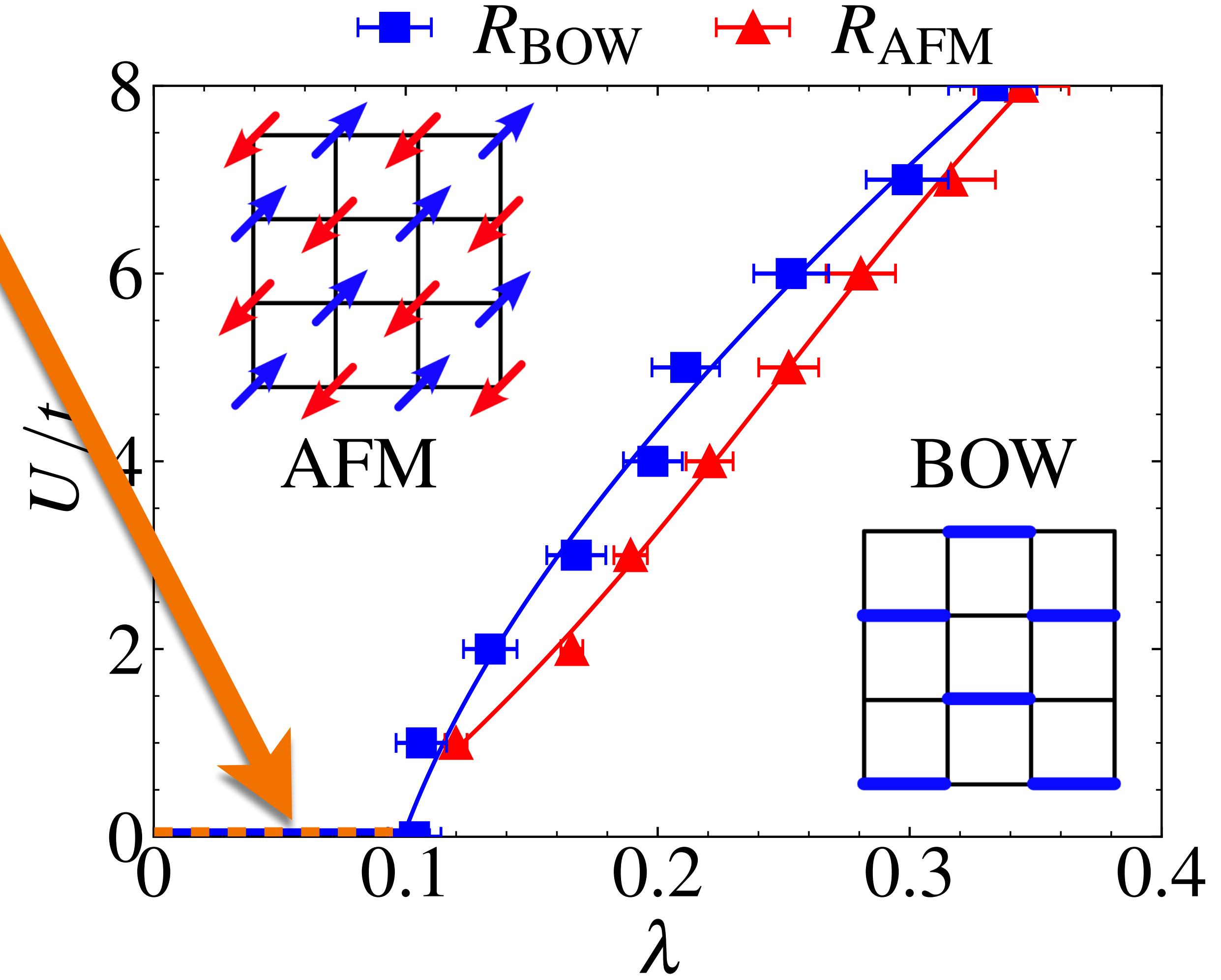
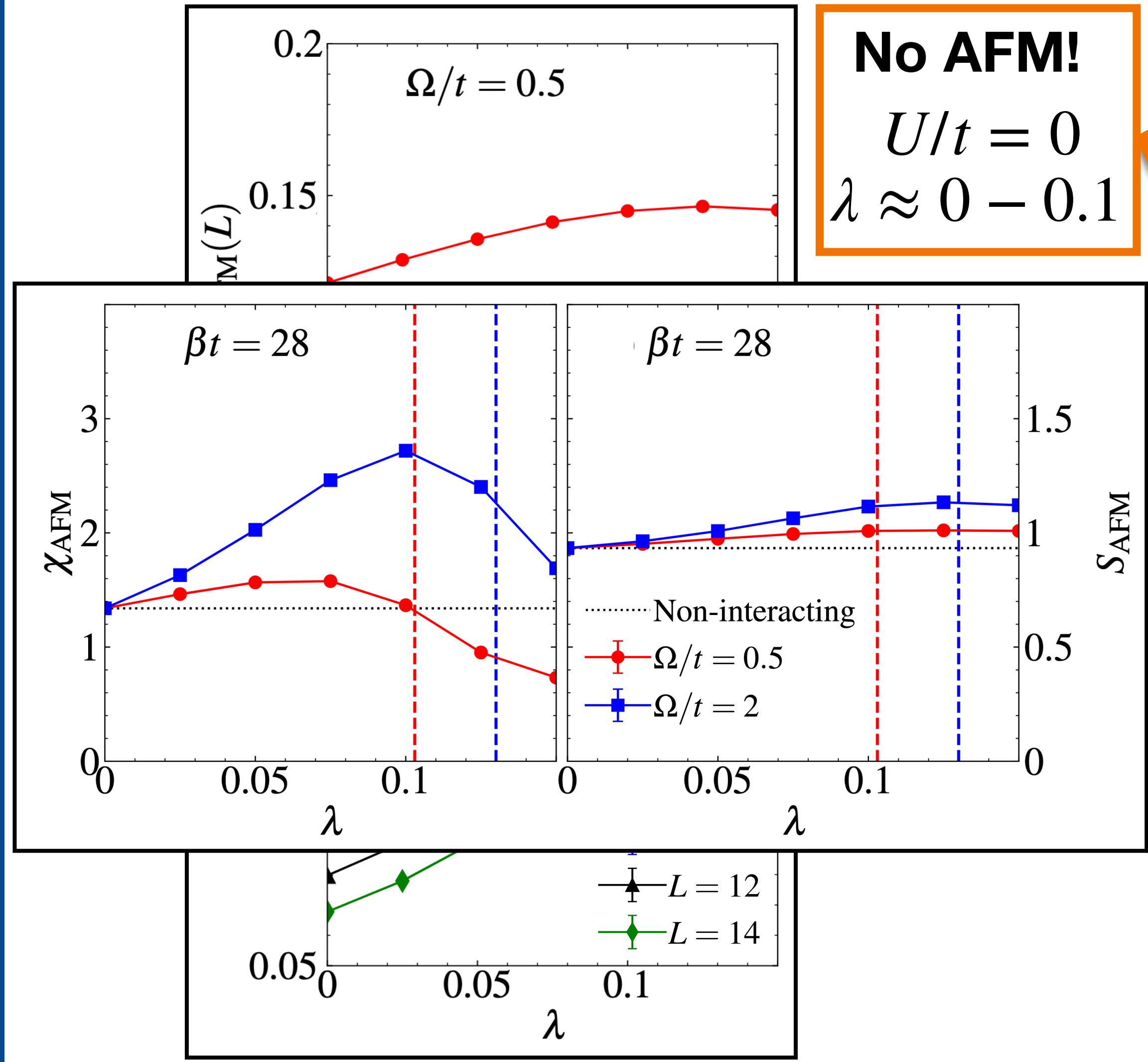
$$\boxed{U/t = 3}$$
$$\boxed{\Omega/t = 0.5}$$



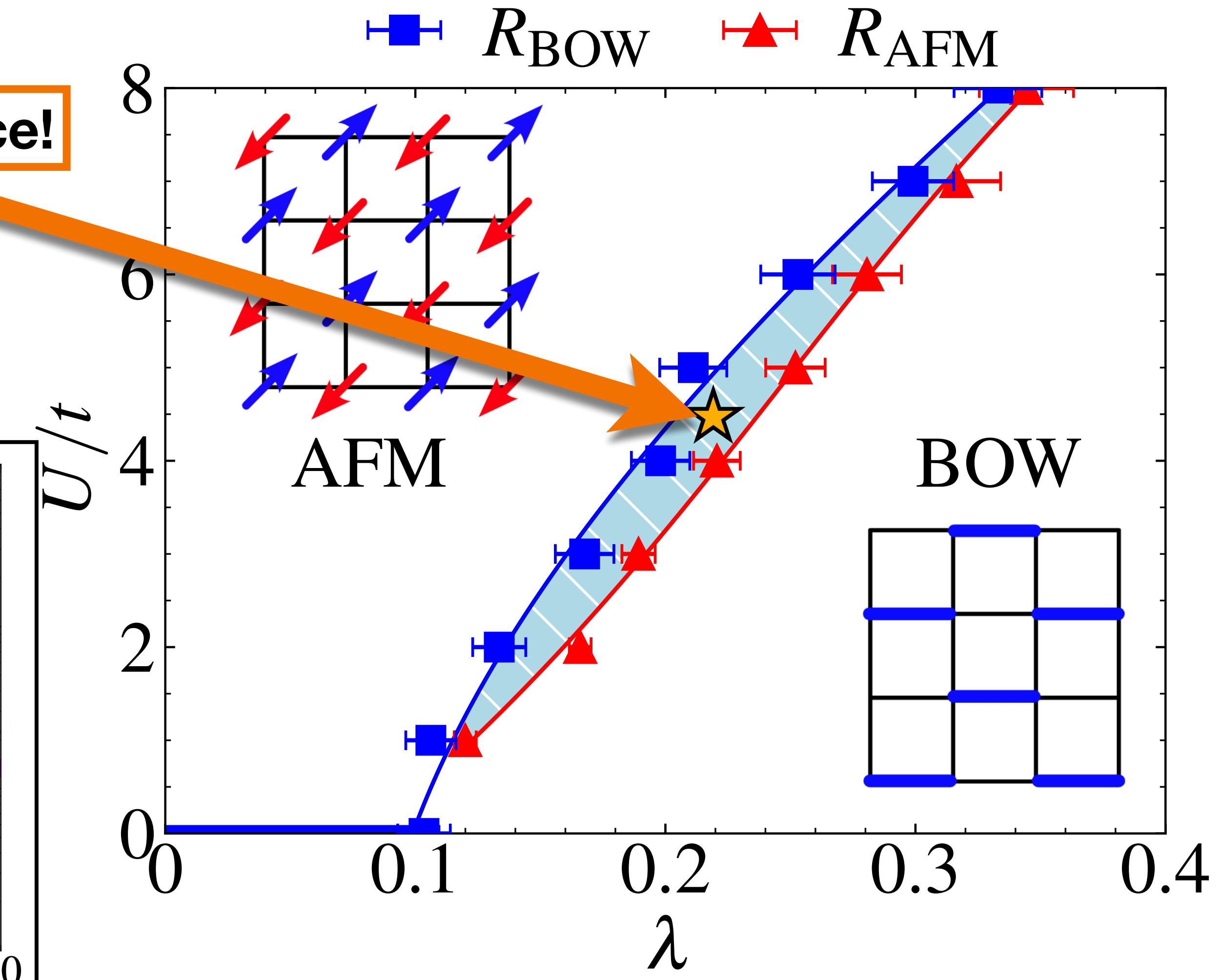
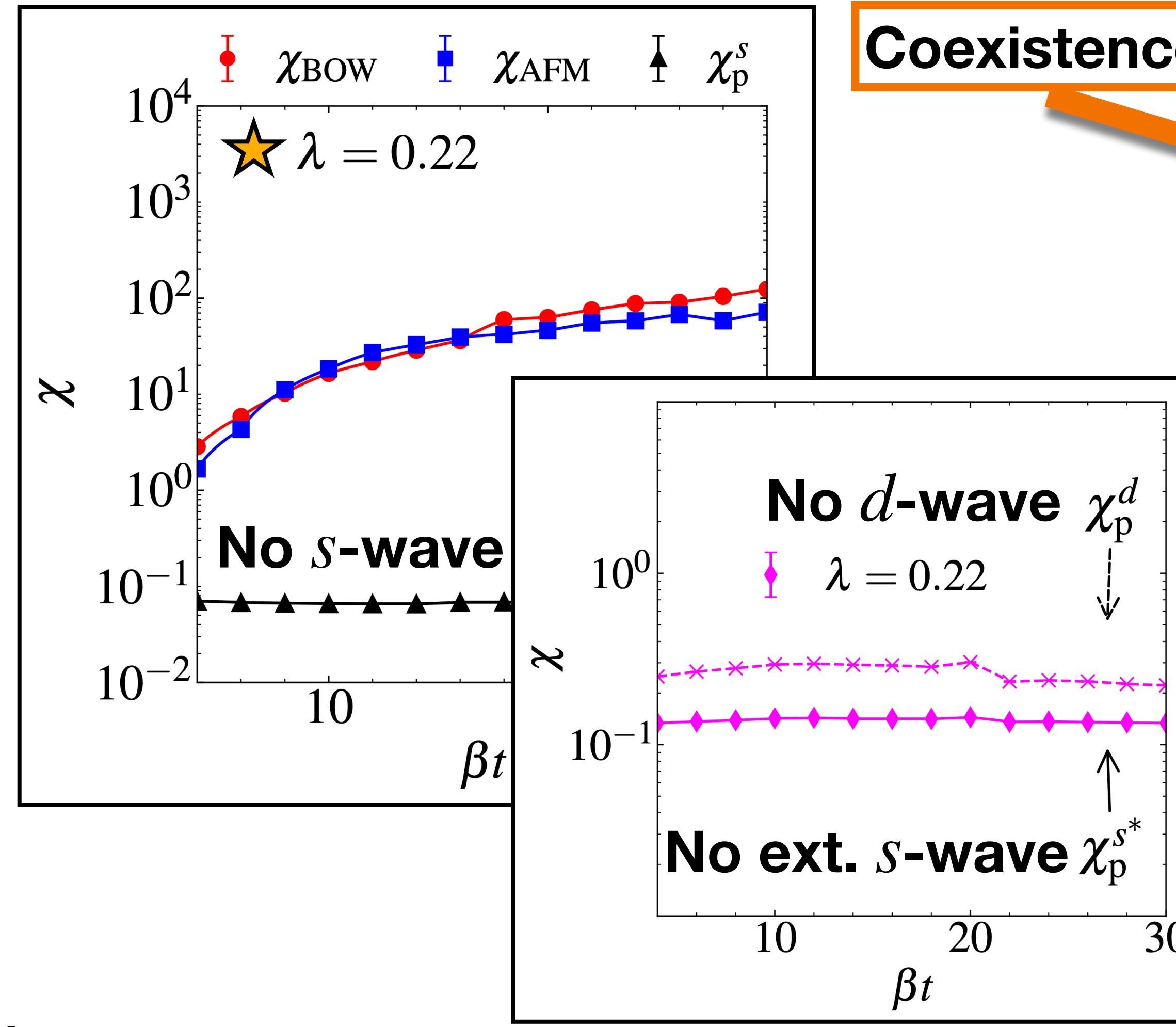
# Phase diagram $\Omega/t = 0.5$



# Phase diagram $\Omega/t = 0.5$



# Phase diagram $\Omega/t = 0.5$



# Summary

- Ground state phase diagram of the optical SSH-Hubbard model.
- No (long-range) antiferromagnetism in the small coupling ( $\lambda = 0 - 0.1$ ) limit at  $U/t = 0$ .
- Narrow region of coexistence between antiferromagnetic and bond-ordered-wave phases.
- Overall, the pure ( $U/t = 0$ ) optical SSH model behaves quite differently from the pure bond SSH model.
- Preprint available at: [arXiv:2502.14196 \(2025\)](https://arxiv.org/abs/2502.14196)

