23 April 2021

MSc. Data Science
LAA - Homework 1

1. Let A be an $m \times n$ matrix. Define
$$||A||_F = (\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2)^{\frac{1}{2}}.$$
   (i) Prove that this is indeed a matrix norm.
   (ii) Evaluate $||A||_1$, $||A||_2$, $||A||_\infty$ and $||A||_F$ for
$$A = \begin{pmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{pmatrix}.$$

2. Evaluate $||I_{n\times n}||_F$.

3. Prove that for the induced 2-norm and the Frobenius norm on matrices are invariant under multiplication by unitary matrices.

4. Study Section 1.3 of Strang's book 'Linear Algebra and Learning from Data' (you can find it here). Then solve problems 1, 2, 4 and 6 from problem set 1.3 (page 20).