Time 2 hours
25/09/2018

CHENNAI MATHEMATICAL INSTITUTE

Max : 30

Subject : Optimization Techniques

I (a) Maximize $5x_1 + 12x_2 + 4x_3$, subject to

$$x_1 + 2x_2 + x_3 \le 5$$
$$2x_1 - x_2 + 3x_3 = 2$$ and $x_i \geq 0$ for $i = 1, 2, 3$

(b) Is your solution to (a), a basic feasible solution ? [3 + 2]

II Let $S = \{(x_1, x_2) : x_1 + x_2 \le 1\}$

(a) Find the extreme points and directions of S

(b) Can you represent any point in S as a convex combination of extreme points plus a nonnegative linear combination of extreme directions ? [3+2]

III Let $f(x) = \text{minimum}\{f_1(x), f_2(x)\}$ where $f_1(x) = 4 - |x|$ and $f_2(x) = 4 - (x-2)^2$ for $x \in \mathbb{R}$.

(a) Examine whether f is concave in $\mathbb{R}$?

(b) Find a subgradient vector for f at $x = 4$.

Is it unique ? [3+2]

PLEASE TURN TO OTHER SIDE FOR MORE QUESTIONS.

IV Examine whether the following statements are true or false. Give reasons in either case.

(i) Consider minimizing: $\sum_{i=1}^{n} \xi_i$ subject to $\xi \in X$ where X is a nonempty, closed subset of nonnegative orthant in R^n. This problem has an optimal solution.

(ii) Origin in R^3 is a local minimum for $f(x_1, x_2, x_3) = x_1 x_2 + 2x_1^2 + x_2^2 + 2x_3^2 - 6x_1 x_3 + 1$

(iii) Let $f : R^n \to R$ be a convex and differentiable function. Let $\nabla f(x)$ stand for the gradient vector at $x \in R^n$. Then

$$(\nabla f(x') - \nabla f(x''))^t (x' - x'') \geq 0 \quad \text{for}$$

every $x', x'' \in R^n$.

(Hint: Note subgradient vector at x coincides with the gradient vector in the given problem).

[4+4+4]

Brevity + Neatness . . . . [3]