Instructions

- *The duration of the exam is 3 hours. You can refer to the prescribed textbook and lecture notes.*
- *No doubts/clarifications would be entertained during the examination. If needed, make appropriate assumptions and state them in your answers.*

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1. Write the basic feasible solutions of the following LP: **(5 marks)**

$$\begin{array}{ccccccccc} 2x_1 & + & x_2 & + & 4x_3 & & & = & 8 \\ & & x_2 & & & + & x_4 & = & 4 \\ x_1, & & x_2, & & x_3, & & x_4 & \geq & 0 \end{array}$$

2. Let S be the subspace given by the solutions of $x_1 + x_2 + x_3 = 0$. **(5 marks)**

   (a) Exhibit a basis for S. Justify your answer.

   (b) Let T be the affine subspace formed by the solutions of $x_1 + x_2 + x_3 = 1$. Provide $|dim(S)| + 1$ points in T such that every point in T can be written as an affine combination of these $|dim(S)| + 1$ points. Here $dim(S)$ denotes the dimension of S.

3. Write an LP for the following problem: **(10 marks)**

   minimize $x_1 + 2|x_2| + 3|x_3 - 10|$ subject to $|x_1| + |x_2 + x_3| \leq 10$

   Justify your answer: show why the optimum value of the LP you write is equal to the optimum value of the above problem.

4. Let C_{2n} for $n \geq 1$ denote the (undirected) cycle with $2n$ edges. Vertices are $\{v_1, v_2, \ldots, v_{2n}\}$ and edges are the unordered pairs $\{(v_1, v_2), (v_2, v_3), \ldots, (v_{2n-1}, v_{2n}), (v_{2n}, v_1)\}$. **(10 marks)**

   (a) Write the ILP for maximum independent set in C_{2n}.

   (b) Show that the optimum of the ILP coincides with the optimum of its LP relaxation.

5. Consider the following problem. **(10 marks)**

   Input. A universe D consisting of finite number of elements, and a family $S_1, S_2, \ldots, S_m$ of sets with $S_i \subseteq D$ and $|S_i| = 3$ for every $i \in \{1, \ldots, m\}$.

   Goal. Find a minimum size subset $W \subseteq D$ of the universe that intersects with each S_i: that is, $W \cap S_i$ is non-empty for every $i \in \{1, \ldots, m\}$.

   (a) Write an ILP for the above problem.

   (b) Show that the ILP optimum is at most 3 times the optimum of the LP relaxation.