# 2020-2021 : DS - Analysis

## Problem Set - 1: 06 - 01 -2021

In Problems 1 - 8, determine if the given sequence $\{a_n\}$ converges or diverges. Give reason in the case of a divergent sequence, and find the limit in the case of a convergent sequence.

In Problems 14 - 16, determine the points of discontinuity, if any; if $x$ is a discontinuity of $f$, find if $f(x+),\ f(x-)$ exist, and the values if they exist. Also draw a graph.

1. $$a_n = \frac{n^2}{n+1} - \frac{n^2+3}{n}.$$

2. $$a_n = \frac{(-1)^n}{n} + \frac{1+(-1)^n}{10}.$$

3. $$a_n = \sin(\frac{n\pi}{2}).$$

4. $$a_n = 2^{\frac{1}{n}}.$$

5. $$a_n = \frac{n\ \sin(n!)}{(n^2+5)}.$$

6. $$a_n = \frac{3^n + (-2)^n}{3^{n+1} + (-2)^{n+1}}.$$

7. $$a_n = \sqrt{n+7} - \sqrt{n+4}.$$

8. $$a_n = n\ a^n,$$

   where $0 < a < 1$. ( Assume $\frac{\log n}{n} \to 0,\ n \to \infty$.)

9. Find

$$\lim_{x\to 1} \frac{2x^2 - 3x + 1}{x - 1}.$$

10. Let $a \neq 0$ be a constant. Find

$$\lim_{x\to a} \frac{x^2 - a^2}{x^2 + 2ax + a^2}.$$

11. Using L'Hospital's rule show that

$$\lim_{x\to 0} \frac{\sin x}{x} = 1.$$

(If you are not familiar with L'Hospital's rule, skip this problem.)

12. Using Problem 11, (or L'Hospital's rule) show the following:

$$\begin{aligned} \lim_{x\to 0} \frac{\sin 5x}{\sin x} &= 5; \\ \lim_{x\to 0} \frac{\tan 3x}{\sin x} &= 3; \\ \lim_{x\to 0} \frac{(1 - \cos x)}{x^2} &= \frac{1}{2}. \end{aligned}$$

13. Let $f(x) = x \sin(1/x),\ x \neq 0$. Can you define $f(0)$ appropriately so that $f$ becomes a continuous function?

14. Let $[x] =$ the largest integer $\leq x$, in

$$f(x) = x - [x],\ x \in \mathbb{R}.$$

15.

$$\begin{aligned} f(x) &= x^2,\ x > 0, \\ &= x,\ x \leq 0. \end{aligned}$$

16.

$$\begin{aligned} f(x) &= \frac{\tan x}{x},\ x \in [-\frac{\pi}{4}, \frac{\pi}{4}],\ x \neq 0, \\ &= 1,\ x = 0. \end{aligned}$$