

2020-2021 Semester - I : Mathematical Methods-Analysis

20-03-2021 End-semester examination Duration:  $2\frac{1}{2}$  hours

**Note:** (i) Submission should be in the form of PDF file.

(ii) Terminology is as used in class.

(iii) Each question carries 12 marks. (Maximum marks: 60)

1. Let  $f(x) = x^2 + ax + b$ ,  $x \in \mathbb{R}$ , where  $a, b$  are constants. Find the values of  $a$  and  $b$  such that the line  $y = 7x + 3$  is tangent to the graph of  $f$  at the point  $(3, 24)$ .
2. Find the area under the curve  $y = |x^3 - 6x^2 + 8x|$  between  $x = 0$  and  $x = 4$ .
3. Let  $b > 0$ . Show that the improper integral

$$\int_{\mathbb{R}} x^2 e^{-b|x|} dx$$

converges, and evaluate it.

4. Let

$$f(x, y) = 5x^2 + 5y^2 - xy - 11x + 11y + 11, (x, y) \in \mathbb{R}^2.$$

Find the global minimum of  $f$ .

5. Find the point in the set  $E = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + 3z = 27\}$  which is closest to the point  $(1, 1, 1)$ .