Problem set 3

Ds Analysis Jan

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Question 1

$$f(x) = \sqrt{e^{2x} + 3}$$

The function is defined everywhere since $e^{2x}+3$ is defined everywhere and square root of positive real numbers is also defined everywhere on real line.

As for the derivative, it exists everywhere too.
Differentiating, we get :

$$\frac{df(x)}{dx} = \frac{e^{2x}}{\sqrt{e^{2x} + 3}}$$

Question 2

Hint:

$$f(x) = \frac{(2x^2 + x - 1)^{\frac{5}{2}}}{(3x + 2)^9}$$

As you can see $3x+2 = 0$ has a real root $-\frac{2}{3}$. So,$f(x)$ is not defined at $x = -\frac{2}{3}$

$f'(x)$ exists everywhere else. for

$$f(x) = \frac{g(x)}{h(x)}$$

$$f'(x) = \frac{g'(x)h(x) - h'(x)g(x)}{(h(x))^2}$$

Question 3

$$f(x) = \sin[\log(2x + 1)]$$

The function is not defined at $x \leq -1/2$ but is defined everywhere else. Similarly, the derivative exists for $x > -1/2$.

$$\frac{df(x)}{dx} = \frac{2\cos[\log(2x + 1)]}{2x + 1}$$

Question 4

Hint:

$$g(x) = \frac{\sin(5x+2)}{\cos(x^2-1)}$$

$g(x)$ is not defined for

$$\cos(x^2-1) = 0$$

$$\Rightarrow x^2 - 1 = (2n+1)\pi/2$$

$$\Rightarrow x = \pm\sqrt{(2n+1)\pi/2+1}$$

$g(x)$ is well defined and differentiable everywhere else.

Question 5

$$f(x) = e^{sin(x^3+1)}$$

Since x^3+1 , sin and exponential are differentiable(and exists) everywhere, thus the composition is differentiable(and exists) everywhere too.

$$\frac{df(x)}{dx} = e^{\sin(x^3+1)}\cos(x^3+1)3x^2$$

Question 6

Hint:

$$g(x) = \frac{\log(x^2+2)}{e^{-x}} = e^x \log(x^2+2)$$

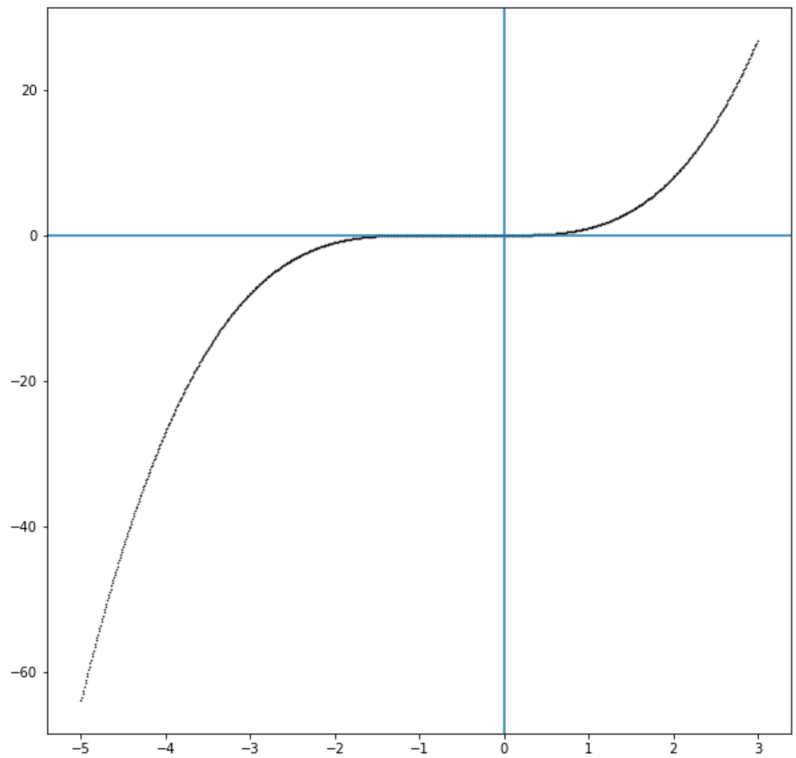
$x^2+2 > 0$ for all $x \in \mathbb{R} \Rightarrow g(x)$ well defined and differentiable everywhere.
for

$$f(x) = g(x)h(x)$$

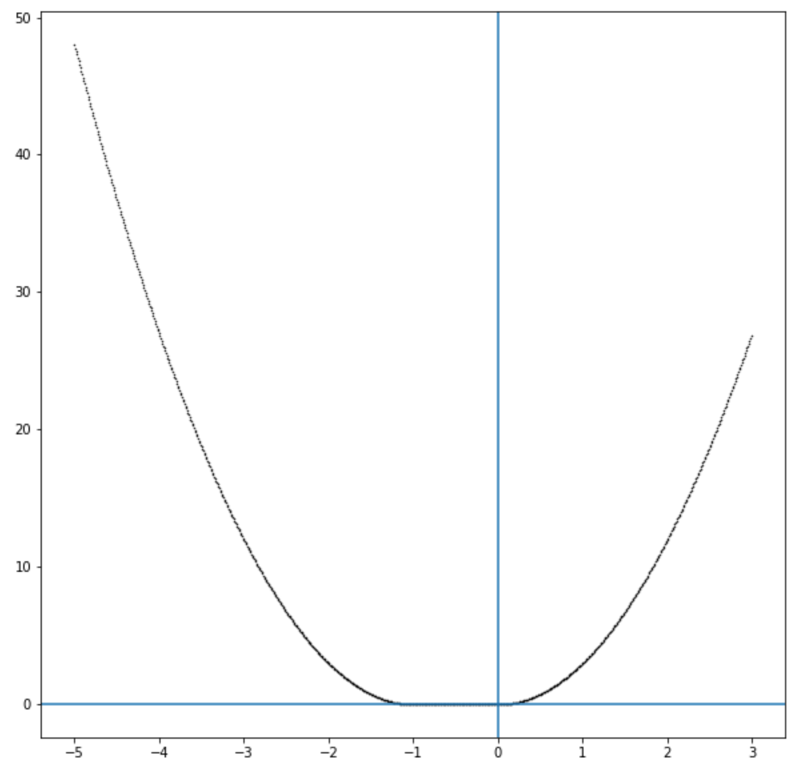
$$f'(x) = g'(x)h(x) + g(x)h'(x)$$

Question 7

The graph of the function between [-5,3] is



Now, the function is differentiable everywhere. (The RHD = LHD at x = -1 and 0. Since the function is a polynomial function at all other points, its differentiable everywhere).
Between x = [-1,0], for all values of x, $\frac{df(x)}{dx} = 0$



Question 8

Hint:

$$g(t) = \frac{t}{t+5}$$

$g(t)$ is well defined and differentiable except for $t+5=0 \Rightarrow t=-5$.

$$g'(t) = \frac{(t+5)-t}{(t+5)^2} = \frac{5}{(t+5)^2}$$

$$g'(2) = \frac{5}{49}$$

Let,$y = mx + b$ be the equation of the tangent line then $m = g'(2)$ (Since, $y' = m$ and both g' and y' have to be equal at $t = 2$ for y to be the the tangent line of g at $t = 2$) $y = mx + b$ must pass through (2,g(2))=(2,$\frac{2}{7}$) $\Rightarrow b = \frac{4}{49}$

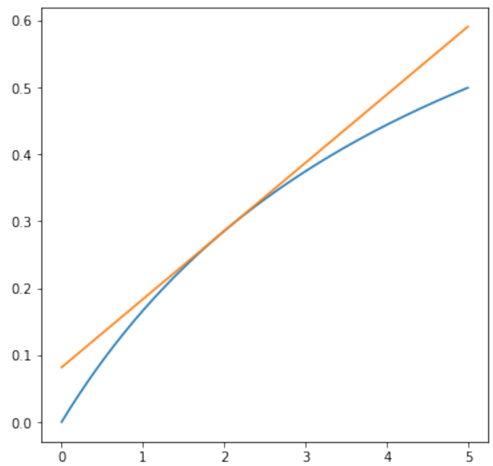
$\Rightarrow$ equation of the tangent line is $y = \frac{5}{49}x + \frac{4}{49}$
Just for fun you can check in python whether the equation is right or not :

In [1]:
```python
import numpy as np
import matplotlib.pyplot as plt
```

In [2]:
```python
def f(x):
    return (x/(x+5))

x=np.arange(0,5,0.01)
y=[f(t) for t in x]
z=[((5/49)*t+(4/49)) for t in x]
```

In [3]:
```python
plt.figure(figsize=(6,6))
plt.plot(x,y)
plt.plot(x,z)
```



Question 9

$$f(x) = x^{\frac{1}{4}} + 2x^{\frac{3}{4}}$$

At x = 16, f(x) = 18

$$f'(x) = \frac{1}{4x^{\frac{3}{4}}} + \frac{3}{2x^{\frac{1}{4}}}$$

At x = 16, f'(x) = $\frac{25}{32}$
Thus the line will be

$$y = \frac{25x}{32} + \frac{11}{2}$$

Question 10

The volume of the sphere :

$$V = \frac{4}{3}\pi r^3$$

Rate of change of volume w.r.t time:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

When radius is 5 cm , the rate of change in volume :

$$50 = \frac{dV}{dt}_{r=5} = 100\pi \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{50}{100\pi}$$

Question 11

The volume of the cone :

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (\frac{3h}{2})^2 h = \frac{3}{4}\pi h^3$$

(Since, $Hight = \frac{1}{3} diameter\ of\ the\ base$ always.)

Rate of change of volume w.r.t time:

$$3 = \frac{dV}{dt} = \frac{9}{4}\pi h^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt}_{h=4} = \frac{4}{3\pi 4^2}$$