2020-2021 : DS - Analysis

Problem Set - 3: 16 - 01 -2021

In Problems 1- 6, find the derivatives of the given functions, indicating where the functions and the derivatives are well-defined.

1. $$f(x) = \sqrt{e^{2x} + 3}.$$

2. $$f(x) = \frac{(2x^2 + x - 1)^{\frac{5}{2}}}{(3x + 2)^9}.$$

3. $$f(x) = \sin[\log(2x + 1)].$$

4. $$g(x) = \frac{\sin(5x + 2)}{\cos(x^2 - 1)}.$$

5. $$g(x) = e^{\sin(x^3+1)}.$$

6. $$g(x) = \frac{\log(x^2 + 2)}{e^{-x}}.$$

7. Let f be given by

$$\begin{aligned} f(x) &= x^3, \ 0 \leq x, \\ &= 0, \ -1 \leq x \leq 1, \\ &= (x + 1)^3, \ x \leq -1. \end{aligned}$$

   Find $f'(x)$, wherever f is differentiable. Find also the points where $f'(x) = 0$. Draw the graphs of f and f', over the interval $[-5, 3]$.

8. Find the equation of the tangent line to the graph of the function

$$g(t) = \frac{t}{t + 5},$$

   when $t = 2$.

9. Find the equation of the tangent line to the graph of the function

$$f(x) = x^{\frac{1}{4}} + 2x^{\frac{3}{4}},$$

when $x = 16$.

10. Suppose a gas is pumped into a spherical balloon at a constant rate of 50 $\text{cm}^3/$ sec. Assume that the gas pressure remains constant, and that the balloon always has a spherical shape. How fast is the radius increasing when the radius is 5 cm.? (Volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.)

11. Sand is falling on a pile, always having the shape of a cone, at the rate of 3 $\text{cm}^3/$ sec. Assume that the diameter at the base of the pile is always three times the height. At what rate is the height is increasing when the height is 4 cm.? (Volume of a cone of height h, and radius of the base r is $\frac{1}{3}\pi r^2 h$.)