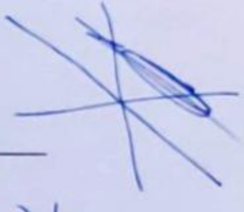
$\frac{\text{no. of lic}}{r} = \frac{\text{no. of lir}}{r}$

$\dim(\text{rowspace}) = n - r.$

$f(x_0) < f(y_0)$ y_0

$f(\lambda x_0 + (1-\lambda) y_0) = \lambda f(x_0) + (1-\lambda) f(y_0)$
$> \lambda f(x_0) + (1-\lambda) f(x_0)$

$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$



LPCO **Quiz 1 (15 marks)**

Note: Each question carries 3 marks

1. Show that a basis for a subspace can be extended to a basis for the whole vector space.
2. Prove that Gaussian elimination preserves the number of linearly independent columns.
3. Show that on a convex set, a local optimum for a linear function is also a global optimum.
4. In an LP with m constraints and 2 variables x_1, x_2, show that the optimum of an objective function $c_1x_1 + c_2x_2$ can be attained at atmost 2 extreme points (note that this is independent of m).
5. Does a result similar to Question 4 hold in general for $n \geq 3$ dimensions? In other words, is it true that in an LP with n variables $x_1, \ldots, x_n$, the optimum of $c^T\overline{x}$ is attained at atmost n extreme points? Justify.

1. Solve the following LP using the simplex tableau method.

$$
\begin{array}{llcll}
\text{maximize} & x_1 & + & x_2 & \\
\text{subject to} & -x_1 & + & x_2 & \leq 2 \\
 & & & x_2 & \leq 4 \\
 & x_1 & + & x_2 & \leq 9 \\
 & x_1 & & & \leq 6 \\
 & x_1 & - & x_2 & \leq 5 \\
 & x_1 & , & x_2 & \geq 0
\end{array}
$$

$X_7 = 2 - X_5 + 2X_6$

$-X_7 = -2 + X_5 - 2X_6$

Farkas: $Ax \leq b, x \geq 0$ has a sol^n

$\exists y \geq 0 \ni y^T A = 0$ or $y^T b < 0$.

$X_2 = 1 + X_7 - X_6$

$-X_2 = -1 - X_7 + X_6$

$z = 5 - X_7 + 2 + 2X_7 - 2X_6$

$Ax \leq b$, $x \geq 0$ has a sol^n

$\exists y \in \mathbb{R}^m \ni y^T A \geq 0, y^T b < 0$.

2. Consider a primal-dual pair of LPs: 1) maximize $c^T x$ subject to $Ax \leq b, x \geq 0$ (primal) and 2) minimize $b^T y$ subject to $A^T y \geq c, y \geq 0$ (dual).

   Prove the following statement (of the strong duality theorem) using Farkas' lemma: if the primal is feasible and bounded, the dual is feasible and bounded, and moreover the optima of the primal and dual coincide.

3. Suppose in an instance of LP, we have n variables that are unconstrained in sign. Show how they can be replaced by $n + 1$ variables that are constrained to be non-negative.

4. Prove or dispove: if A and B are totally unimodular matrices, then the composed matrix $(A \mid B)$ is totally unimodular.

$x_1 + x_2 \geq 0$

$x_1 - x_2 \geq 3$

x_1^+, x_2^+, x^-

$x_1 = x_1^+ - x^-$

$x_2 = x_2^+ - x^-$