2020-2021 : DS - Analysis

Problem Set - 5: 30 - 01 - 2021

In Problems 1 - 7, find the indicated definite / indefinite integrals.

1. $$\int_0^{\pi} (\cos(x) + 5x^4) dx.$$

2. $$\int_0^{4\pi} |\sin(x)| dx.$$

3. $$\int \frac{x}{x+1} dx.$$

4. $$\int \frac{e^x}{(e^x+1)} dx.$$

5. $$\int x^2 e^x dx.$$

6. $$\int x^3 \cos(x^2) dx.$$

   (Hint: First take $u = x^2$; so $du = 2xdx$.)

7. $$\int_2^3 \frac{x+1}{\sqrt{x^2+2x+3}}.$$

8. Find $\int_0^{\pi} f(x)dx$, where f is defined by

$$\begin{aligned} f(x) &= \sin(x),\ 0 \le x < (\pi/2), \\ &= \cos(x),\ (\pi/2) \le x \le \pi. \end{aligned}$$

9. Find the area between the curves $y = x$ and $y = x^2$ from 0 to their first point of intersection for $x > 0$.

10. Find the area under the curve $y = x\sin(x^2)$ between $x = 0$ and $x = \sqrt{\pi}$.

11. Using integration by parts, first show that

$$\int \sin^2(x)dx = -\sin(x)\cos(x) + \int \cos^2(x)dx,$$

and then prove that

$$\int \sin^2(x)dx = \frac{1}{2}x - \frac{1}{4}\sin(2x).$$

12. Let f be a continuous function on $[a, b]$. Define

$$\begin{aligned} f^+(x) &= \max\{0, f(x)\} &&= f(x), \text{ if } f(x) \geq 0, \\ & &&= 0, \text{ if } f(x) < 0, \\ f^-(x) &= \max\{0, (-f(x))\} &&= 0, \text{ if } f(x) \geq 0, \\ & &&= (-f(x)), \text{ if } f(x) < 0. \end{aligned}$$

Show that

(i) f^+ and f^- are non-negative continuous functions on $[a, b]$.

(ii)

$$\int_a^b f(x)dx = \int_a^b f^+(x)dx - \int_a^b f^-(x)dx.$$