Graph Theory

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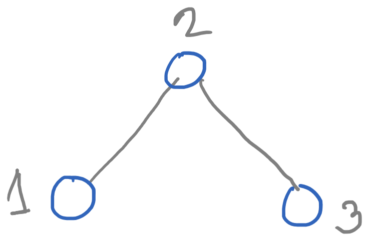
Definitions

What is Graph ?

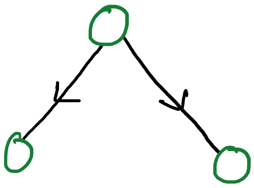
a (simple) graph is a pair of sets (V, E), where V is an arbitrary non-empty finite set, whose elements are called vertices3 or nodes, and E is a set of pairs of elements of V, which we call edges.



undirected graph

Directed and Undirected graph

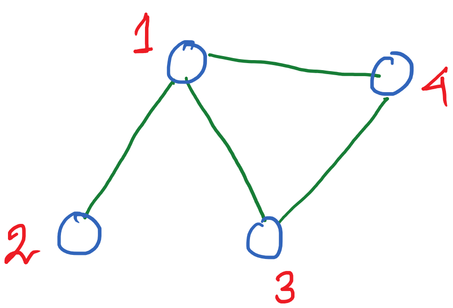
In an undirected graph, the edges are unordered pairs.
In a directed graph, the edges are ordered pairs of vertices; (u,v) denote the directed edge from u to v.



directed graph.

Neighbour

For any edge uv in an undirected graph, we call u a neighbour of v and vice versa and we say that u and v are adjacent.



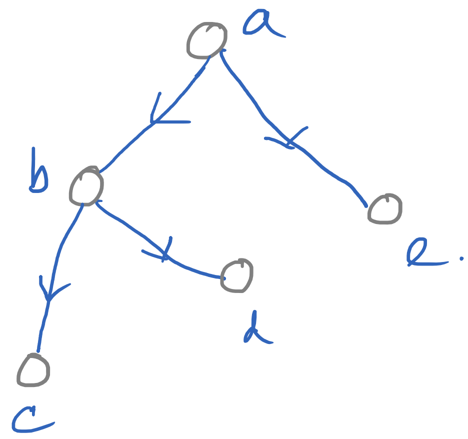
4, 2 are neighbour of 1

1, 4 are neighbours of 3

. . . .

Predecessor and Successor (Directed Graph)

In directed graphs, we distinguish two kinds of neighbours, For any directed edge (u,v) we call u as a predecessor of v and we call v a successor of u.



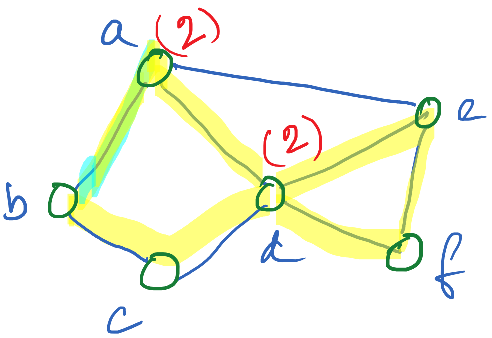
a is predecessor of b, e

b is successor of a.

a is ancestor of c,d

Walk

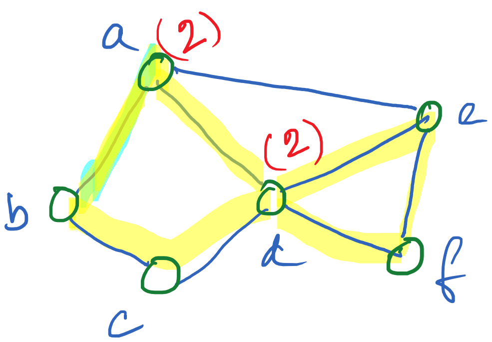
A walk in an undirected graph G is a sequence of vertices, where each adjacent pair of vertices are adjacent in G. We can also think of a walk as a sequence of edges.



a b c d e f d a b

Path

A walk is called a path if it visits each vertex at most once.

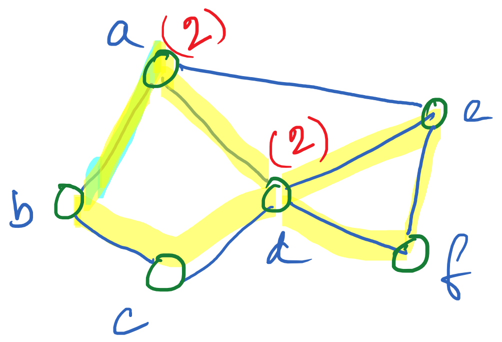


a b c d

a d f e

Reachability

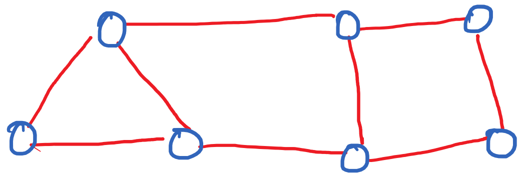
For any two vertices u and v in a graph G, we say that v is reachable from u if G contains a walk (and therefore a path) between u and v.



f is reachable from a.

Connected Graph

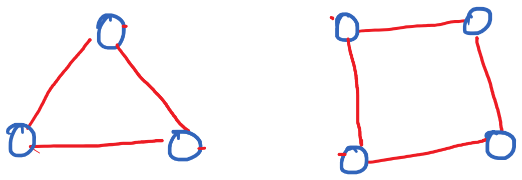
An undirected graph is connected if every vertex is reachable from every other vertex. A graph is said to be connected if it has exactly one connected component.



connected graph.

Connected Component

Every undirected graph consists of one or more components, which are its maximal connected subgraphs; two vertices are in the same component if and only if there is a path between them.



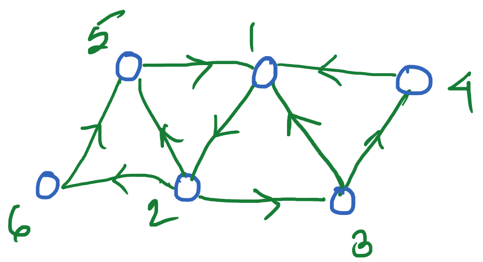
disconnected graph.

connected component 2.

connected component 1

Closed walk

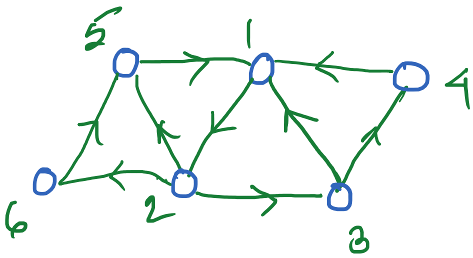
A walk is closed if it starts and ends at the same vertex



closed walk 1 2 6 5 1 2 3 4 1

Cycle

a cycle is a closed walk that enters and leaves each vertex at most once.



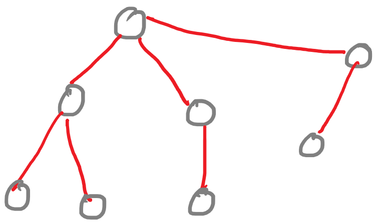
1 2 3 1

1 2 5 1

cycle

Acyclic

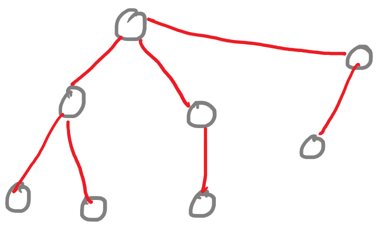
An undirected graph is acyclic if no subgraph is a cycle.



no cycle. so acyclic.

Tree

A tree is a connected acyclic graph.

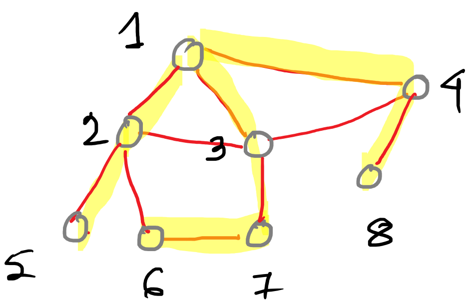


Forests

Acyclic graphs are also called forests. Multiple trees makes a forest.

Spanning Tree

A spanning tree of an undirected graph G is a subgraph that is a tree and contains every vertex of G. A graph has a spanning tree if and only if it is connected.
A graph has multiple spanning trees. Spanning trees of a graph may not be unique.



125 + 1376 + 148

Directed Walk

A directed walk is a sequence of vertices v0 ---> v1 ---> …. ---->vl such that v_{i-1} ---> v_i is a directed edge for every index i. Vertex v is reachable from vertex u in a directed graph G if and only if G contains a directed walk (and therefore a directed path) from u to v.

A directed graph is **strongly connected** if every vertex is reachable from every other vertex.

A directed graph is **acyclic** if it does not contain a directed cycle

Directed acyclic graphs are often called **dags**.