2020-2021 : DS - Analysis

Problem Set - 6: 15 - 02 - 2021

1. Show that the improper integral
$$\int_0^\infty \sin(x)dx$$
diverges.

2. Does the improper integral
$$\int_0^\infty \frac{1}{\sqrt{e^x}}dx$$
converge? If yes, is it possible to find it?

3. Find
$$\int_1^4 (x-1)^{-2/3}dx.$$

4. Let $B > 2$. Find the area under the curve $y = e^{-2x}$ between 2 and B. Does this area approach a limit when $B \to \infty$? If so, find the limit.

5. Let $\mu \in \mathbb{R},\ \sigma^2 > 0$. Let g be given by
$$g(x) \quad = \quad \frac{1}{\sigma\sqrt{2\pi}} \exp\big(-\frac{(y-\mu)^2}{2\sigma^2}\big), \quad -\infty < y < \infty,$$
where σ is the positive square root of σ^2. Evaluate
$$\int_{\mathbb{R}} xg(x)dx, \text{ and } \quad \int_{\mathbb{R}} x^2 g(x)dx.$$

In Problems 6 - 8, express the given function f (defined on the indicated domain around 0,) in the form
$$f(x) = P_n(x) + R_{n+1}(x),$$

where P_n is the Taylor polynomial of degree n for f, and R_{n+1} the corresponding remainder; also find an error estimate of the form

$$|R_{n+1}(x)| \leq \frac{M_{n+1}|x|^{n+1}}{(n+1)!}$$

wherever possible. (In some cases, you may need to take n to be odd or even.)

6.

$$f(x) = \cos x, \ x \in \mathbb{R}.$$

7.

$$f(x) = \frac{1}{1+x}, \ |x| < 1.$$

8.

$$f(x) = \frac{1}{2-x}, \ |x| < 2.$$