# 2020-2021 : DS - Analysis

## Problem Set - 7: 24 - 02 - 2021

1. Let $f$ and $g$ be differentiable functions from $U$ into $\mathbb{R}$, where $U$ is an open set in $\mathbb{R}^n$. Show that for all $x \in U$, the following hold:

   (i) $\bigtriangledown(af + bg)(x) = a \bigtriangledown f(x) + a \bigtriangledown g(x)$, where $a, b \in \mathbb{R}$ are constants.

   (ii) $\bigtriangledown(fg)(x) = g(x) \bigtriangledown f(x) + f(x) \bigtriangledown g(x)$.

2. Let $f(x, y, z) = x^2y^7 + 18,\ (x, y, z) \in \mathbb{R}^3$. Find $\bigtriangledown f(x, y, z)$; also find $\bigtriangledown f(1, 2, 3)$, and the directional derivative at $(1, 2, 3)$ in the direction of $(-2, 0, 7)$.

3. Let $f(x, y, z) = \sin(xyz),\ (x, y, z) \in \mathbb{R}^3$. Find $\bigtriangledown f(x, y, z)$; also find $\bigtriangledown f(1, \pi, \pi)$, and the directional derivative at $(1, \pi, \pi)$ in the direction of $(1, 3, 2)$.

4. Let $f(x, y, z) = e^{3x+y} \sin(7z),\ (x, y, z) \in \mathbb{R}^3$. Find $\bigtriangledown f(x, y, z)$, and $\bigtriangledown f(0, 2, (\pi/7))$.

5. Let $f(x, y, z) = e^{xyz},\ (x, y, z) \in \mathbb{R}^3$. Find $\bigtriangledown f(x, y, z)$, and $H(x, y, z)$.

6. Let $f(x, y) = \sin(x^2+y),\ (x, y) \in \mathbb{R}^2$. Find the gradient and the Hessian of $f$. Also find $\bigtriangledown f(-\sqrt{\pi}, \pi)$ and $H(-\sqrt{\pi}, \pi)$.

7. Let $f(x, y) = x^2 - 15xy - y^2,\ (x, y) \in \mathbb{R}^2$. Find the gradient and the Hessian of $f$. Also find $\bigtriangledown f(-3, 5)$ and $H(-3, 5)$.