Instructions

- *The question paper has* ***two sides****. There are six questions.*
- *The duration of the exam is 4 hours. You can refer to the prescribed textbook and lecture notes.*
- *No doubts/clarifications would be entertained during the examination. If needed, make appropriate assumptions and state them in your answers.*

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1. Consider the LP: minimize $c^T x$ subject to $A_i x = b_i$, $i = 1, 2, \ldots, m$, $x \geq 0$. Here we assume that x and c are $n \times 1$ matrices and A_i is $1 \times n$ for every i. **(10 marks)**

   Suppose x^* is an optimum for the above LP. Let y^* be an optimum for the dual.

   Show that x^* is also an optimum to the LP: minimize $(c^T - y_k^* A_k)x$ s.t. $A_i x = b_i$, $i = 1, \ldots, m$, $i \neq k$, $x \geq 0$ where y_k^* is the k^{th} component of y^*.

2. Give an example of A, c, b such that the following two LPs are unbounded: **(10 marks)**

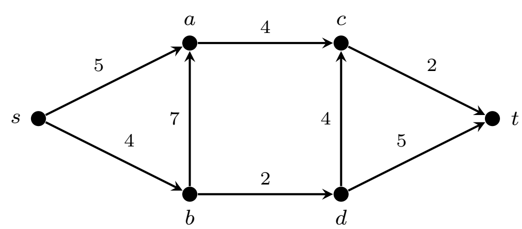
   - maximize $c^T x$ subject to $Ax = b$, $x \geq 0$
   - maximize $-c^T x$ subject to $Ax = b$, $x \geq 0$

   The only difference in the two LPs is in the objective function.

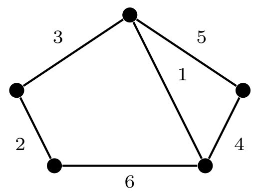
3. For the game below, write the LPs for finding max-min and min-max over mixed strategies: **(5 marks)**

3	-1	2	5
7	13	-2	12
-5	2	0	9

4. Execute the primal-dual algorithm for $s - t$ shortest path that we discussed in class on the following graph. Numbers denote the edge weights. Exhibit the dual feasible solution and the edges that are made tight at each step. **(5 marks)**



5. Execute the primal-dual algorithm for MST discussed in class on the following graph. Numbers denote the edge weights. Exhibit the dual feasible solution and the edges that are made tight at each step. **(5 marks)**



6. Consider the following problem. **(10 marks)**

   Input. A universe D consisting of finite number of elements, and a family $S_1, S_2, \ldots, S_m$ of sets with $S_i \subseteq D$ and $|S_i| \leq 3$ for every $i \in \{1, \ldots, m\}$.

   Goal. Find a minimum size subset $W \subseteq D$ of the universe that intersects with each S_i: that is, $W \cap S_i$ is non-empty for every $i \in \{1, \ldots, m\}$.

   Design a primal-dual algorithm that gives a 3-approximation.