Problem Set - 8: 05 - 03 - 2021

In Problems 1 - 7, find the critical points; also, indicate if the function has a local minimum/ maximum, global minimum/ maximum, whenever possible, at each of the critical points.

1.
$$f(x, y) = x^2 + 4xy - y^2 - 8x - 6y, \ (x, y) \in \mathbb{R}^2.$$

2.
$$f(x, y) = x^4 + y^2, \ (x, y) \in \mathbb{R}^2.$$

3.
$$f(x, y) = (x + y)e^{-xy}, \ (x, y) \in \mathbb{R}^2.$$

4.
$$f(x, y) = x^2 - xy + y^2 - 2x + y, \ (x, y) \in \mathbb{R}^2.$$

5.
$$f(x, y) = 9x^2 - 12xy + 3y^2, \ (x, y) \in \mathbb{R}^2.$$

6.
$$f(x, y) = (x^2 + y^2)e^{-(x^2+y^2)}, \ (x, y) \in \mathbb{R}^2.$$

7.
$$f(x, y) = x^3 y^2 (1 - x - y), \ x \geq 0, \ y \geq 0.$$

8. Let $S = \{(x, y) \in \mathbb{R}^2 : -2 \leq x \leq 0, \ 0 \leq y \leq 1\}$. Let $f(x, y) = 3xy^3, \ (x, y) \in S\}$. Find the maximum and minimum of f on S.

9. Let S denote the closed square with corners at the points $(\pm 1, \ \pm 1)$. Let $f(x, y) = x - y, \ (x, y) \in S$. Find the maximum and minimum of f on S.

10. Find the maxima and minima of the function $f(x, y, z) = 2x^2 + y^2 + z^2, \ (x, y, z) \in \mathbb{R}^3$, on the surface $x^2 + y^2 + 2z^2 = 2$.

11. The utility obtained when a person works for x hours at job A, and for y hours at job B is given by

$$f(x, y) = 2\sqrt{x} + \sqrt{y}, \ x \geq 0, \ y \geq 0.$$

How many hours should the person work on each job to maximise the utility, if the person works for a total of 10 hours ?