2020-2021 : DS - Analysis

Problem Set - 4: 22 - 01 -2021

In Problems 1- 8, for the given function on the indicated domain, find the critical points, intervals over which the function is increasing/ decreasing, points at which maximum/ minimum are attained. (Note: You may say that maximum does not exist if the concerned limit is $+\infty$; similarly for minimum.)

1. $$f(x) = x^2 - 2x + 1, \; x \in \mathbb{R}.$$

2. $$f(x) = -x^3 + 3x - 5, \; x \in \mathbb{R}.$$

3. $$f(x) = (x-4)^5, \; x \in \mathbb{R}.$$

4. $$f(x) = 2\sin(x) + 3\cos(x), \; x \in [-2\pi, 2\pi].$$

5. $$f(x) = x + \sin(x), \; x \in [-2\pi, 2\pi].$$

6. $$f(x) = \frac{x-3}{x^2+1}, \; x \in [-5, 5].$$

7. $$f(x) = \frac{x^2}{\sqrt{(x+1)}}, \quad -1 < x < \infty.$$

8. $$f(x) = \frac{1}{(x-1)(x-3)}, \; 0 \le x \le 10,$$

   whenever it is defined. (At some points in $[0, 10]$ the function may not be defined. If z is such a point find $f(z-)$, $f(z+)$, which may also have relevance for maxima/ minima.

9. Given $S > 0$. Among all positive numbers x and y with $x + y = S$, show that $x^2 + y^2$ is smallest when $x = y$.

10. Among all rectangles of given area, show that the square has the least perimeter. (Hint: Let x denote the length of the shorter side of a rectangle; note that x varies over $(0, \infty)$.)

11. The cost of producing x units of a product is given by

$$f(x) = 10x^2 + 200x + 6,000.$$

If the company increases the price, then fewer units are sold; the price per unit as a function of units sold is given by

$$p(x) = 1000 - 10x.$$

For what value of x will the profit be maximum? What is the maximum possible profit?