Due on 13 June 2021

M.Sc. Data Science
LAA - Homework 4

1. Compute the QR factorization of the given matrix A (use the ∞-norm for simplicity in hand calculations) -

$$\begin{pmatrix} 1 & -1 \\ 1 & 4 \\ 1 & 4 \\ 1 & -1 \end{pmatrix}$$

   (a) By hand - using Gram-Schmidt method

   (b) By hand - using Householder matrices

   (c) By hand - using Givens' matrices

   (d) Using numpy.linalg.qr (you can find help at this link). If $\hat{Q}$ and $\hat{R}$ denote your computed matrices, what is $||A - \hat{Q}\hat{R}||_2$?

2. What are the operation counts for QR factorization for each of the following algorithms as done in class:

   (a) MGS

   (b) Givens' matrices

3. Study part of section 3.4 (introductory part on pages 118 - 119) and section 3.4.3 (page 123) of Demmel. You should understand the statement of Lemma 3.1 and Proposition 3.3. Based on this study, write a short note about the stability of QR for each of the methods we have seen. Bonus points for proving Lemma 3.1.

4. Consider the linear system:

$$\begin{aligned} 2x + 8y + 3z &= 2 \\ x + 3y + 2z &= 5 \\ 2x + 7y + 4z &= 8 \end{aligned}$$

   (a) Write down the Jacobi matrix J, the Gauss-Seidel matrix L_1 and the general SOR matrix L_ω for the given system.

   (b) Find the spectral radii, the condition numbers and rates of convergence for each of the above matrices (choose values of ω to be $0.2, 0.8, 1.5$ and 2). You may use numpy linalg modules wherever necessary. Tabulate your answers.