1. An oil refinery can buy two types of oil: light crude oil and heavy crude oil. The cost per barrel of these types of oil is respectively 11 and 9 dollars. The following quantities of gasoline, kerosene and jet fuel are produced per barrel of each type of oil.

Min $11x+9y$

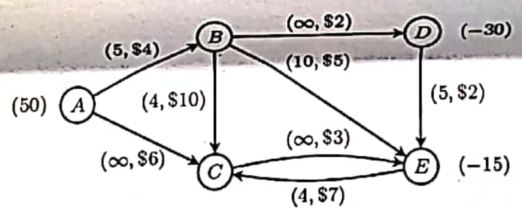
	Gasoline	Kerosene	Jet fuel
Light crude oil	0.4	0.2	0.35
Heavy crude oil	0.32	0.4	0.2

From the above values, note that 5 percent of light crude oil and 8 percent of heavy crude oil are lost in the refining process. The refinery has contracted to deliver 1,000,000 barrels of gasoline, 400,000 barrels of kerosene and 250,000 barrels of jet fuel.

Formulate a linear program for finding the number of barrels of each crude oil that satisfies the demand and minimizes the total cost (assuming fractional barrels are allowed). **(5 marks)**

2. Figure below shows a network-flow problem arising in the distribution of a single product from manufacturing plants (sources) to consumers (sinks). Node A is a source and nodes D and E are sinks. The number in paranthesis next to these nodes shows the product supplied or demanded at the particular node. Each directed edge is marked with two numbers - first coordinate is the maximum capacity of the route (edge) and the second coordinate is the cost per unit for transporting along that edge.

Write an LP to determine the flow of the product along the network, satisfying the supply, demand and capacity constraints with minimum cost. **(5 marks)**



3. Convert the following LP to equational form: **(5 marks)**

$$\begin{array}{lrcrcrl}
\text{Maximize} & 8x_1 & + & 3x_2 & - & 2x_3 & \\
\text{Subject to} & x_1 & - & 6x_2 & + & x_3 & \geq 2 \\
 & 5x_1 & + & 7x_2 & - & 2x_3 & = -4 \\
 & & & & & x_1 & \leq 0 \\
 & & & & & x_2 & \geq 0
\end{array}$$

4. Write the basic feasible solutions of the following LP: **(5 marks)**

$$\begin{array}{rcrcrcrcl}
2x_1 & + & x_2 & + & 4x_3 & & & = & 8 \\
 & & x_2 & & & + & x_4 & = & 4 \\
x_1, & & x_2, & & x_3, & & x_4 & \geq & 0
\end{array}$$

5. Consider the following integer linear program (ILP) over n variables $x_1, x_2, \ldots, x_n$:

$$\text{maximize} \quad \sum_{i=1}^{i=10} x_i$$

$$\begin{array}{lll} \text{subject to:} & x_i + x_j + x_k \leq 1 & \text{for all distinct triples } i, j, k \in \{1, \ldots, 10\} \\ & 0 \leq x_i \leq 1 & \text{for all } i \in \{1, \ldots, 10\} \end{array}$$

(a) What is the ILP optimum?

(b) What is the optimum of its LP relaxation?

(c) In the above program, modify the inequality to: $x_i + x_j + x_k \geq 1$ and the objective function to *minimize* $\sum_{i=1}^{i=10} x_i$.
What are the ILP optimum and the LP-relaxation optimum for this modified program?

Justify your answers. **(10 marks)**