2020-2021 : DS - Analysis

Problem Set - 2: 09 - 01 -2021

In Problems 2 - 9, test the given series for convergence/ divergence, giving justifications for your conclusions.

1. (*Telescoping series*) Let $a_n = b_n - b_{n+1},\ n = 1, 2, 3, \cdots$. Show that the series $\sum_{n=1}^{\infty} a_n$ converges if and only if the sequence $\{b_n : n, 1, 2, \cdots\}$ converges; in such a case, show that

$$\sum_{n=1}^{\infty} a_n = b_1 - (\lim_{n\to\infty} b_n).$$

Using the above show that

(i)

$$\sum_{n=1}^{\infty} \frac{1}{n^2+n} = 1.$$

(ii)

$$\sum_{n=1}^{\infty} \log\left(\frac{n}{n+1}\right)$$

diverges.

2.

$$\sum_{n=1}^{\infty} \frac{a^n}{10^n},$$

where $0 < a < 10$.

3.

$$\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}.$$

4.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+5)}}.$$

5.

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}.$$

6.

$$\sum_{n=1}^{\infty} \frac{n\cos^2(n\pi/3)}{2^n}.$$

7.

$$\sum_{n=1}^{\infty} n^2 e^{-n^3}.$$

8.

$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}.$$

9.

$$\sum_{n=1}^{\infty} \frac{n^n}{(n!)3^n}.$$

(Hint: What is $\lim_{n\to\infty}(1+\frac{1}{n})^n$?)