1. (9 points) Consider the LP: Minimize $c^T x$ subject to $Ax = b, x \geq 0$ where there are n variables and m constraints. Assume that the LP has a unique minimum q, and the feasible region forms a polytope $\mathcal{P}$. We have seen that q is a vertex of $\mathcal{P}$. Let p be the vertex with the second best objective function value. Prove that q is adjacent to p on the polytope.

2. (7 points) Consider the following tableau and answer the questions given below:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
-1	3	0	0	0	-4	$-\frac{7}{2}$	33	0
2	4	0	0	1	-32	-4	36	0
0	-2	1	0	0	4	$\frac{3}{2}$	-15	0
3	0	0	1	0	0	1	0	1

   1. Which columns form the current basis and what is the current bfs?
   2. Is the current bfs optimal? Is it degenerate?
   3. List the candidate columns which can enter the basis at this point (assuming that we are executing the simplex algorithm). Also mention the candidate leaving columns corresponding to each candidate entering column.
   4. Hence demonstrate one step of simplex on this tableau.

3. (7 points) Write the dual of the following LP:

$$\begin{aligned} \text{Minimize } x_1 + 3x_2 - 4x_3 & \\ \text{Subject to} & \\ 2x_1 + x_2 + 2x_3 &\geq 5 \\ 3x_1 + 2x_2 - 2x_3 &\geq 2 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

4. (4 points) Suppose you have an LP formulation with n sign-unconstrained variables. Show that one can reformulate the LP with $n+1$ non-negative variables (as opposed to the $2n$ variables formulation we have seen in class).

5. (7 points) *Def: A face of a convex polytope is any intersection of the polytope with a halfspace such that none of the interior points of the polytope lie on the boundary of the halfspace.*

   Prove that if F is a face of a convex polytope $\mathcal{P}$ in $\mathcal{R}^n$, then F is also a convex polytope. Further, prove that every vertex of F is also a vertex of $\mathcal{P}$.