Time $2\frac{1}{2}$ hours

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CHENNAI MATHEMATICAL INSTITUTE

Max Score } 60

Subject : Optimization Techniques

I/ Two raw materials, M_1 and M_2 are used to produce interior and exterior paints. The following table provides the basic data.

	Tons of raw material per ton of		Max daily Availability (Tons)
	Exterior paint	Interior paint	
Raw Material M_1	6	4	24
Raw Material M_2	1	2	6
Profit per ton	5 (lac)	4 (lac)	

} ⊛

Market Survey indicates:

⊛ { Maximum demand for interior paint is 2 tons. Also demand for interior paint cannot exceed that for exterior paint by more than 1 ton.

(i) Write down the LP – problem

(ii) Indicate the feasible region in a graph.

(iii) Find an optimal mix so that profit is maximized, subject to above constraints ⊛. [3+2+5]

II/ Let $f(x_1, x_2) = 2x_1 + 6x_2 - 2x_1^2 - 3x_2^2 + 4x_1x_2$

(i) Examine whether f has a unique global maximum in R^2.

(ii) Write down ~~the~~ subgradient vector ξ_0 at $(4.5, 4)$ for the given f. Is it true that $\xi_0^t \begin{pmatrix} x_1 - 4.5 \\ x_2 - 4 \end{pmatrix} \leq 0$ for every $(x_1, x_2) \in R^2$? [5+2+3]

III ✓ minimize $\theta(x_1, x_2) = 2x_1^2 + 2x_2^2 - 2x_1x_2 - 4x_1 - 6x_2$

Subject to

$$\left.\begin{aligned} x_1 + 5x_2 &\leq 5 \\ 2x_1^2 - x_2 &\leq 0 \\ -x_1 &\leq 0 \\ -x_2 &\leq 0 \end{aligned}\right\} \cdots \circledast$$

(i) Is the feasible set convex set in $\mathbb{R}^2$?

(ii) Do you think this problem has an optimal solution? Give reasons. [You need <u>not</u> find the explicit solution]

(iii) If $\bar{x} = (\bar{x}_1, \bar{x}_2)$ solves the above minimization problem then is it true that $(\bar{x}, \bar{u})$ for some $\bar{u} \geq 0$ (coordinatewise) is a saddle point the for the Karush - Kuhn - Tucker function defined by $\psi(x, u) = \theta(x) + g^t u$ when $u \geq 0$, $x = (x_1, x_2)$ and g's are defined through $(\ast)$. [Hint use Slater's constraint qualification or Karlin's constraint qualification]

[2 + 3 + 5]

IV Consider the following dynamic programming problem:

$$\begin{array}{c|c|c} & \underline{\text{State } 1} & \underline{\text{State } 2} \\ a_1 & \frac{3}{(1/2,\,1/2)} & \frac{-3}{(1/2,\,1/2)} \\ a_2 & \frac{6}{(0,\,1)} & \frac{-3}{(1/2,\,1/2)} \end{array}$$

Let $f(1) = a_1$ and $g(1) = a_2$, $f(2)$ & $g(2)$ arbitrary.

(i) Compute $I_\beta(f)(1)$ and $I_\beta(g)(1)$.

(ii) Find an optimal for the undiscounted problem. [5+5]

V Examine whether the following statements are true or false. Give reasons in either case.

(i) Let $S_1 = \{x \in R^3 : x_1^2 + x_2^2 + x_3^2 \leq 1\}$, $S_2 = \{x \in R^3 : x_1 + x_2 + x_3 \geq 1\}$

Then there exists a (hyper) plane which separates S_1 and S_2 $pt's_1 > pts$

(ii) Let $A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & -2 & -2 \\ 2 & -2 & -1 \end{bmatrix}$ Let $q_0 = \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$. Then

LCP (A, q_0) has at most one solution.

(iii) Consider the following preference matrix:

$$\begin{array}{c|ccc} & A & B & C \\ \alpha & (1,3) & (2,2) & (3,1) \\ \beta & (3,1) & (1,3) & (2,2) \\ \gamma & (2,2) & (3,1) & (1,3) \end{array}$$

Then there exists an <u>unstable</u> matching. [7+7+6]