LPCO linear algebra refresher

January 17, 2021

Here are some problems to give you a refresher on basic linear algebra: Given an $m \times n$ matrix A,

Problem 1. $\textsc{rowrank}(A) = \textsc{columnrank}(A)$.

Problem 2. $\textsc{rank}(A) + \textsc{nullity}(A) = n$.

Problem 3. Given a subspace $W \subset \mathbb{R}^n$, every $x \in \mathbb{R}^n$ can be written uniquely as $x = x^W + x^\perp$, where $x^W \in W$ and $x^\perp \in W^\perp$.

Problem 4. Suppose $At = b$ for some $t \in \mathbb{R}^m$, then the set of solutions to the equation $Ax = b$ is given by $\{t + y \mid y \in \textsc{nullspace}(A)\}$.

Problem 5. Show that finding A^{-1} can be done in polynomial time (if it exists. Otherwise, return 0).

Problem 6. Show that the intersection of arbitrarily many (not just countable) convex sets is convex.