

Course Title: Calculus of One Variable

Course Level: 200 Level

Credits: 3

Prerequisite: Mathematics at 10+2 level, or both Precalculus and Introduction to Calculus at FLAME.

Course Description:

Calculus is a branch of mathematics in which we study the rate of change and area under curves. It finds applications in Physics, Engineering, Economics, and Finance, among many others. In this course, students will learn the key concepts and techniques of single-variable differential and integral calculus. The course balances theory and application, leading to a mastery of calculus of one variable.

Course Objectives:

1. To understand the concepts underlying limits, continuity, and discontinuity of functions.
2. To understand the applications of differential calculus in modelling of real world problems.
3. To analyze convergence criterion of sequences and series through various tests.
4. To gain expertise on various techniques of integration.
5. To apply integration techniques in estimating length, area, and volume of smooth objects.

Course Outcomes (CO):

Upon successful completion of the course, students will be able to:

1. Understand the definitions, theorems, and proofs related to limits and continuity.
2. Understand, state, and prove theorems of differential calculus and solve problems.
3. Perform convergence tests on sequences and series.
4. Understand the concept of Riemann integration and properties of integrals.
5. Apply various techniques of integration to solve application-based problems.

Course Content Structure:

Module 1: Review of functions and their graphic visualization (4 Hours)

- Motivation behind calculus
- Functions, their domain and range
- Composition of functions and inverse functions
- Graphing of polynomials and functions, families of functions, scaling, stretching, compressing, shifting, and symmetries of curves, polar coordinate system

Module 2: Limits and continuity of functions (6 Hours)

- Limits, sidedness of limits, and simple examples of their numerical estimation
- An intuitive understanding of epsilon-delta definition of limits
- Limit laws and squeeze theorem (sandwich theorem)
- Asymptotes and limits at infinity
- Continuity of functions, Types of discontinuity, Uniform continuity
- Intermediate value theorem

Module 3: Derivatives (6 Hours)

- Differentiability of functions and derivatives of elementary functions from first principles
- Geometrical interpretation of derivatives, Slopes and tangent lines
- Product rule, quotient rule, and chain rule
- Leibnitz's theorem of successive differentiation
- Implicit differentiation
- Indeterminate forms and L'Hôpital's rule

Module 4: Applications of derivatives (6 Hours)

- Tangents and normals
- Increasing and decreasing functions, convexity and concavity of graphs, inflection points

- Critical points, local and global extrema, and simple optimization problems
- Extreme value theorem, Rolle's theorem, and Lagrange's mean value theorem

Module 5: Sequences, series, and their convergence (6 Hours)

- Sequences, limits theorems on sequences, and monotone sequences
- Infinite series, geometric series, harmonic series, p-series, and alternating series
- Convergence tests - comparison test, limit comparison test, ratio test, and root test
- Absolute convergence and conditional convergence
- Power series, Taylor series, Maclaurin series, and their convergence criterion

Module 6: Integrals (8 Hours)

- Antiderivatives, indefinite integrals, and their physical interpretation
- Techniques of integration - substitution method, by parts, partial fraction decomposition, and Leibniz's integral rule (Feynman's technique)
- Riemann sums, Definite integrals, Fundamental theorem of calculus, and area under a curve
- Improper integrals

Module 7: Applications of integrals (6 Hours)

- Area between two curves
- Volume of solids by slicing, volume of solids of revolution, and volumes by cylindrical shells
- Length of a plane curve and area of a surface of revolution

Suggested Readings and Textbooks:

Textbooks:

1. G. B. Thomas and R. L. Finney, *Calculus and Analytic Geometry*, 9th Edition, Addison-Wesley Publishing Company, 1991.
2. H. Anton, I. C. Bivens, and S. Davis, *Calculus*, 10th Edition, John Wiley & Sons Inc., 2012.

Reference Readings:

1. J. Hass, C. Heil, and M. D. Weir, *Thomas' Calculus*, 14th Edition, Pearson India, 2017.

2. M. Kline, *Calculus: An Intuitive and Physical Approach*, 2nd Edition, Dover Publications, 1998.
 3. J. Stewart, *Calculus*, 8th Edition, Cengage Learning, 2015.
 4. N. Piskunov, *Differential and Integral Calculus*, Volume 1, 4th Edition, Mir Publishers, 1981.
-

Suggested Assessment Plan:

1. Quiz#1 (25%) - open book, MCQs (15%) & descriptive (10%), covering Modules 1, 2, & half of Module 3.
2. Quiz#2 (30%) - closed book, MCQs (20%) & descriptive (10%), covering Modules 3, 4, & 5.
3. Class participation (5%): For attendance, and engagement in discussions and activities.
4. Final examination (40%): closed book, MCQs (25%) & descriptive (15%), covering entire syllabus with an emphasis (> 50%) on Modules 6 & 7.

Note:

- 1) Both the quizzes would be conducted within a week of completing the assigned syllabus.
- 2) Time duration: Quiz 1 – 90 minutes, Quiz 2 – 100 minutes, Final exam – 120 minutes
- 3) A scientific (non-graphic) calculator is permitted in both open book and closed book exams.
- 4) Open book format implies that a student is allowed to use either a textbook/reference book of one's choice or a one-page formula sheet on A4 size paper. Both sides of the paper may be utilized for that. Class notebooks and soft copies of the texts are not allowed. Further, access to the internet is not allowed.
- 5) There would be no retest of any of the evaluation components. If you miss/skip any of the quizzes/exams, your score for the respective component would be assigned zero.

Suggested Pedagogy:

The course will employ a mix of lectures, interactive discussions, problem-solving sessions, and flipped-classrooms. Technology-enhanced learning tools, simulation software and online resources, will be used to facilitate understanding. Experiential learning strategies, including group projects will be incorporated to foster collaboration and critical thinking.
