Problems: Domian, Range and Graphs

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Domain using Wavy Curve Method

Problem

Find the domain of

$$f(x) = \sqrt{(x-2)(x+3)}.$$

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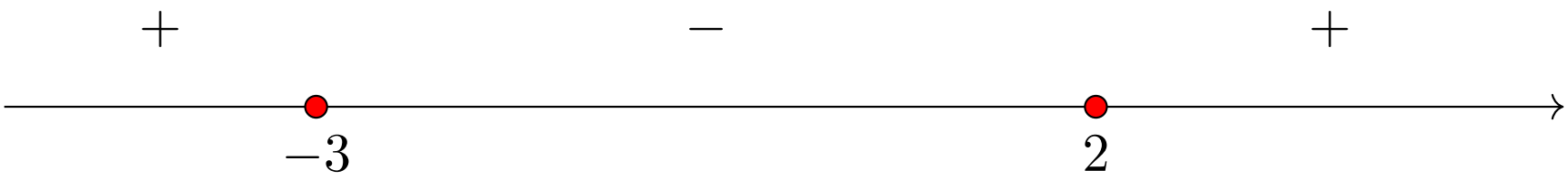
Solution

Condition:

$$(x-2)(x+3) \geq 0$$

Critical points: $x = -3,\ 2$.

Domain using Wavy Curve Method



Result

Keep intervals where product ≥ 0:

$$\text{Domain} = (-\infty, -3] \cup [2, \infty).$$

Problems: Domain and Range

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$$\text{Domain} = [-2, 2]$$

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Range: Let $y = f(x)$.

$$y = \sqrt{4 - x^2}$$

Again, since it's a square root, $y \geq 0$. Square both sides:

$$y^2 = 4 - x^2$$

Rearrange to solve for x^2:

$$x^2 = 4 - y^2$$

Problems: Domain and Range

For x^2 to be a real number, it must be non-negative:

$$x^2 \geq 0 \Rightarrow 4 - y^2 \geq 0$$

$$4 \geq y^2 \Rightarrow -2 \leq y \leq 2$$

Combining with $y \geq 0$, the range is $[0, 2]$.

$$Range = [0, 2]$$

Cubic Polynomial (Vanishing Method)

Problem

Sketch the graph of

$$f(x) = x^3 - 2x^2 - 5x + 6.$$

Finding factors

Try integer values: $x = -3, -2, -1, 0, 1, 2, 3, \ldots$

$$f(1) = 1^3 - 2(1)^2 - 5(1) + 6 = 1 - 2 - 5 + 6 = 0$$

Therefore, $(x - 1)$ is a factor of $f(x)$.

Cubic Polynomial (Vanishing Method)

Factorisation

$$f(x) = x^3 - 2x^2 - 5x + 6 = x^2(x-1) - x(x-1) - 6(x-1)$$

$$f(x) = (x-1)(x^2 - x - 6).$$

Factor the quadratic:

$$x^2 - x - 6 = (x-3)(x+2).$$

So,

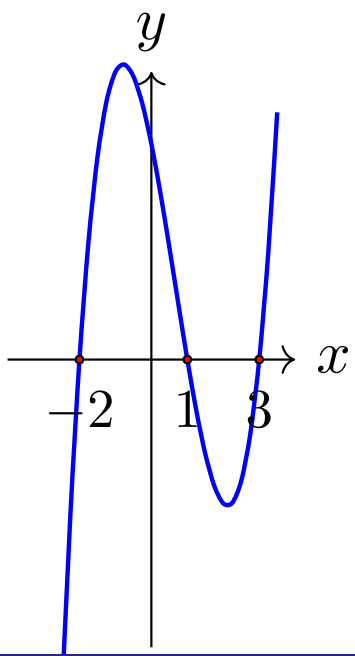
$$f(x) = (x-1)(x-3)(x+2).$$

Zeros: $x = -2,\ 1,\ 3.$

Cubic graph

Note

You can decide the sign of $f(x)$ in each interval using the **wavy curve method**, or by checking the value of $f(x)$ at a test point in every interval.



Quadratic Polynomial (Middle-term Splitting)

Example

Sketch the graph of

$$f(x) = x^2 + 5x + 6.$$

Factorisation: $(x + 2)(x + 3)$.

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Quadratic Polynomial (Completing the Square)

Problem

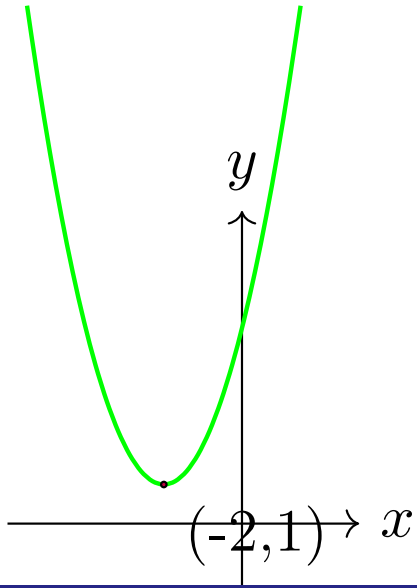
Sketch the graph of

$$f(x) = x^2 + 4x + 5.$$

Completing square:

$$f(x) = x^2 + 4x + 5 = (x^2 + 2x * 2 + 2^2) + 1 = (x + 2)^2 + 1$$

Draw x^2 then vertical shift 1, horizontal shift -2 . Vertex: $(-2, 1)$



Graphical Problems: Quadratic Cubic

Questions:

1. Sketch the graph of $y = x^2 - 4x + 5$ and determine the number of real roots.
2. Sketch the graph of $y = -x^2 + 2x + 3$ and find the points where it intersects the x-axis.
3. Sketch the graph of $y = x^3 - 3x^2 + 2x$ and determine the number of real roots.
4. Sketch the graph of $y = x^3 + x^2 - x - 1$ and find where it crosses the x-axis.

Solutions: Quadratic Cubic Graphs

Solutions:

1. $y = x^2 - 4x + 5$ Discriminant:
   $D = (-4)^2 - 4(1)(5) = 16 - 20 = -4 < 0 \Rightarrow$ No real roots, parabola lies above x-axis.
2. $y = -x^2 + 2x + 3$ Discriminant:
   $D = 2^2 - 4(-1)(3) = 4 + 12 = 16 > 0$ Roots: $x = -1, 3$
3. $y = x^3 - 3x^2 + 2x = x(x-1)(x-2)$ Roots: $x = 0, 1, 2$ (3 real roots)
4. $y = x^3 + x^2 - x - 1 = (x^2 - 1)(x + 1) = (x - 1)(x + 1)^2$ Roots:
   $x = 1, -1$ (x = -1 is repeated)