

Trigonometric Formula Sheet

1. Basic Identities

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ 1 + \tan^2 \theta &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta\end{aligned}$$

2. Reciprocal Relations

$$\begin{aligned}\sin \theta &= \frac{1}{\csc \theta}, & \cos \theta &= \frac{1}{\sec \theta}, & \tan \theta &= \frac{1}{\cot \theta} \\ \csc \theta &= \frac{1}{\sin \theta}, & \sec \theta &= \frac{1}{\cos \theta}, & \cot \theta &= \frac{1}{\tan \theta}\end{aligned}$$

3. Quotient Relations

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

4. Cofunction Identities

$$\begin{aligned}\sin(90^\circ - \theta) &= \cos \theta, & \cos(90^\circ - \theta) &= \sin \theta \\ \tan(90^\circ - \theta) &= \cot \theta, & \cot(90^\circ - \theta) &= \tan \theta \\ \sec(90^\circ - \theta) &= \csc \theta, & \csc(90^\circ - \theta) &= \sec \theta\end{aligned}$$

5. Sum and Difference Formulas

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}\end{aligned}$$

6. Double Angle Formulas

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A \\ \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}\end{aligned}$$

7. Triple Angle Formulas

$$\begin{aligned}\sin 3A &= 3 \sin A - 4 \sin^3 A \\ \cos 3A &= 4 \cos^3 A - 3 \cos A \\ \tan 3A &= \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}\end{aligned}$$

8. Half Angle Formulas

$$\begin{aligned}\sin \frac{A}{2} &= \pm \sqrt{\frac{1 - \cos A}{2}}, & \cos \frac{A}{2} &= \pm \sqrt{\frac{1 + \cos A}{2}} \\ \tan \frac{A}{2} &= \frac{\sin A}{1 + \cos A} = \frac{1 - \cos A}{\sin A}\end{aligned}$$

9. Product-to-Sum Formulas

$$\begin{aligned}\sin A \sin B &= \frac{1}{2} [\cos(A - B) - \cos(A + B)] \\ \cos A \cos B &= \frac{1}{2} [\cos(A - B) + \cos(A + B)] \\ \sin A \cos B &= \frac{1}{2} [\sin(A + B) + \sin(A - B)]\end{aligned}$$

10. Sum-to-Product Formulas

$$\begin{aligned}\sin A + \sin B &= 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} \\ \sin A - \sin B &= 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \\ \cos A - \cos B &= -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}\end{aligned}$$