Indian Institute of Technology, Jodhpur

Department of Computer Science & Engineering

CSL 7020: Machine Learning

Assignment 4

Topic: Image Compression and Decompression using PCA

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1. Dataset:

We analyze an image for compression and decompression. We have used RGB model for processing the model. The key to understanding RGB image processing is recognizing that an RGB image is simply a composite of three independent grayscale images that correspond to the intensity of red, green, and blue light. These three images can be processed separately and then recombined into a single image that human beings will perceive as having color.
We have used an image of size (583, 875) for the compression. We will compress it with PCA and later reconstruct the image from the Principal Components.

2. Approach:

I. **Pre-processing:** Since we are using RGB model, we have separated the R, G, B channels and applied the image compression procedure and later merged the three channels to get the decompressed image.

II. **Model – Description:** The goal is to reduce the dimensions of a d-dimensional dataset by projecting it onto a (k)-dimensional subspace (where k<d) in order to increase the computational efficiency while retaining most of the information.
We will compute eigenvectors (the principal components) of a data and collect them in a projection matrix. Each of those eigenvectors is associated with an eigenvalue. If some eigenvalues have a significantly larger magnitude than others, then the reduction of the dataset via PCA onto a smaller dimensional subspace by dropping the “less informative” Eigen pairs is reasonable.

The PCA Steps are as follows:

- Obtain the Eigenvectors and Eigenvalues from the covariance matrix or correlation matrix, or perform Singular Value Decomposition.
- Sort eigenvalues in descending order and choose the k eigenvectors that correspond to the k largest eigenvalues where k is the number of dimensions of the new feature subspace (k≤d).

- Construct the projection matrix W from the selected k eigenvectors.
- Transform the original dataset X via W to obtain a k-dimensional feature subspace Y.

1. **Covariance Matrix:**
   After transforming the R, G and B channel images into arrays of range [0,255], we have created the covariance matrices. The goal is to perform the Eigen decomposition on the covariance matrix Σ, which is a d×d matrix where each element represents the covariance between two features. The covariance between two features is calculated as follows:

$$\sigma_{jk} = \frac{1}{n-1} \sum_{i=1}^{n} \left(x_{ij} - \bar{x}_j \right) \left(x_{ik} - \bar{x}_k \right) .$$

   We can summarize the calculation of the covariance matrix via the following matrix equation:

$$\Sigma = \frac{1}{n-1} \left((\mathbf{X} - \bar{\mathbf{x}})^T \; (\mathbf{X} - \bar{\mathbf{x}}) \right)$$

   Where $\bar{x}$ is the mean vector x.

   The mean vector is a d-dimensional vector where each value in this vector represents the sample mean of a feature column in the dataset.

2. **Selecting Principal Components or Eigen pairs with maximum variance:** The typical goal of a PCA is to reduce the dimensionality of the original feature space by projecting it onto a smaller subspace, where the eigenvectors will form the axes. In order to decide which eigenvector(s) can dropped without losing too much information for the construction of lower-dimensional subspace, we need to inspect the corresponding eigenvalues: The eigenvectors with the lowest eigenvalues bear the least information about the distribution of the data; those are the ones can be dropped. In order to do so, the common approach is to rank the eigenvalues from highest to lowest in order choose the top k eigenvectors.
   The number k can be found out by checking the data variance that can represent approximately the original data.

3. **Projection Matrix:**

   Now we reconstruct the image by the selected Principal Components. Here, we use the projection matrix W to transform our samples onto the new subspace via the equation

   Y=X×W, Where X is mean-shifted matrix

4. Now from Projection matrices of R, G & B, we recreate the image.

3. Results:

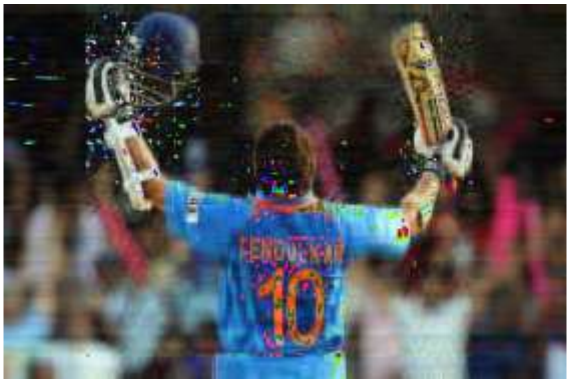
The main part in reconstructing image from PCA is choosing the number of Principle components which can represent maximum variance of the dataset.

We have used different number of principle components for this experiments and the results are as follows.

We have used the following formula for calculating the **compression ratio**:

((Original File Size – Compressed File Size)/ Original File Size) * 100

K= 30 (Compression ratio: 61.33%)



K = 60 (Compression ratio: 58.80%)

K = 80(Compression ratio: 58.56%)

K=100(Compression ratio: 59.09%)

K=120(Compression ratio: 59.61%)

K=125(Compression ratio: 59.67%)

K = 150 (Compression ratio: 59.91%)

4. **Analysis:**
PCA is an analysis approach. We have used covariance matrix method for doing PCA. Covariance method is basically a QR method of decomposition of a matrix. By finding out principle components, we can compress an image and later decompress to the same resolution but in smaller size.
We can also use Singular Value Decomposition (SVD) technique to do PCA analysis.