

**2021**  
**MATHEMATICS – HONOURS**  
**SEMESTER-4**  
**INTERNAL ASSESSMENT**  
**Full Marks of each Course: 10**

*The figures in the margin indicate full marks.*  
*Symbols and notations used here carry their usual meaning.*  
*Candidates are required to give their answers in their own words as far as practical.*

**Course: CC8 (Riemann Integration & Series of functions)**

Choose the correct alternative with proper justification:

5x2=10

1. If  $f$  is  $R$ -integrable on  $[a, b]$  then
  - (a)  $|f|$  may not be  $R$ -integrable on  $[a, b]$
  - (b)  $|f|$  is  $R$ -integrable on  $[a, b]$  and converse is also true
  - (c)  $|f|$  is  $R$ -integrable on  $[a, b]$  but converse is not true
  - (d)  $|f|$  is never  $R$ -integrable on  $[a, b]$
2. If  $f$  be a function defined on  $[0, \frac{\pi}{4}]$  by  $f(x) = \begin{cases} \cos x, & \text{if } x \in Q \\ \sin x, & \text{if } x \in [0, \frac{\pi}{4}] \setminus Q \end{cases}$ 
  - (a)  $f$  is not  $R$ -integrable over  $[0, \frac{\pi}{4}]$
  - (b)  $f$  is  $R$ -integrable over  $[0, \frac{\pi}{4}]$
  - (c)  $f$  is  $R$ -integrable over  $[0, 1]$
  - (d)  $f$  is not  $R$ -integrable over  $[0, 1]$
3. Which of the following improper integrals is/ are convergent
  - (a)  $\int_0^\infty x^3 e^{-x^2} dx$
  - (b)  $\int_0^\infty \frac{dx}{1+x^2}$
  - (c) both (a) and (b)
  - (d) neither (a) nor (b)
4. If the power series  $\sum a_n x^n$  has radius of convergence  $R$  then the radius of convergence of the power series  $\sum a_{2n} x^{4n}$  is
  - (a)  $R$
  - (b)  $2R$
  - (c)  $\sqrt[4]{R}$
  - (d) none of these
5. Let for all  $n \in \mathbb{N}$ ,  $f_n(x) = x - \frac{1}{n}$  and  $g_n(x) = x + \frac{2}{n}$ ,  $x \in [0, \infty)$ , then  $\{f_n g_n\}$  is
  - (a) Not point-wise convergent on  $[0, \infty)$
  - (b) Uniform convergent on  $[0, \infty)$
  - (c) Point-wise convergent but not uniform convergent on  $[0, \infty)$
  - (d) None of these

**Course: CC9 (PDE & Multivariate Calculus II)**

Choose the correct alternative with proper justification:

5x2=10

6. A tightly stretched homogeneous string of length  $l$  with its fixed ends at  $x = 0$  and  $x = l$  executes transverse vibrations. The motion is started with zero initial velocity by displacing the string into the form  $f(x) = a \sin^2 \pi x$ . Then the required deflection  $u(x, t)$  at any time  $t$  is
- $\frac{1}{4}a(1 - \cos 2\pi x \cos 2\pi ct)$
  - $\frac{1}{4}a(1 - \sin 2\pi x \cos 2\pi ct)$
  - $\frac{1}{2}a(1 - \sin 2\pi x \cos 2\pi ct)$
  - $\frac{1}{2}a(1 - \cos 2\pi x \cos 2\pi ct)$

7. The surface whose tangent planes cut off an intercept of constant length  $k$  from the axis of  $z$  is
- $\varphi\left(\frac{1}{y}, \frac{1}{kx}\right) = 0$
  - $\varphi\left(\frac{x}{y}, \frac{x}{z-k}\right) = 0$
  - $\varphi\left(\frac{x}{z}, \frac{z}{z-k}\right) = 0$
  - $\varphi\left(\frac{1}{z}, \frac{xz}{ky}\right) = 0$

where  $\varphi$  being arbitrary

8. The nature of the following partial differential equation  $2(x^2 + y^2)z_{xx} + 2(x + y)z_{xy} + z_{yy} = 0$  is
- Elliptic in  $\Omega = \{(x, y) \in \mathbb{R}^2 : x \neq y\}$   
Parabolic in  $\Omega = \{(x, y) \in \mathbb{R}^2 : x = y\}$
  - Hyperbolic in  $\Omega = \{(x, y) \in \mathbb{R}^2 : x \neq y\}$   
Parabolic in  $\Omega = \{(x, y) \in \mathbb{R}^2 : x = y\}$
  - Parabolic in  $\Omega = \{(x, y) \in \mathbb{R}^2 : x \neq y\}$   
Elliptic in  $\Omega = \{(x, y) \in \mathbb{R}^2 : x = y\}$
  - Elliptic in  $\Omega = \{(x, y) \in \mathbb{R}^2 : x \neq y\}$   
Hyperbolic in  $\Omega = \{(x, y) \in \mathbb{R}^2 : x = y\}$
9. The value of  $\oint_C \vec{F} \cdot d\vec{r}$  over  $C$  by Stoke's theorem, where  $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x + z)\hat{k}$  and  $C$  is the boundary of the triangle with vertices at  $(0,0,0)$ ,  $(1,0,0)$  and  $(1,1,0)$  is
- 1/2
  - 1/3
  - 1/4
  - 1/5
10. Let  $E = [0,2; 0,3]$  and  $f(x, y) = \begin{cases} 3 & , x \text{ rational} \\ y^2 & , x \text{ irrational} \end{cases}$ . Then
- $\int_0^3 dy \int_0^2 f dx$  exists and equals to 0.18.
  - $\int_0^3 dy \int_0^2 f dx$  exists and equals to 1.8.
  - $\int_0^3 dy \int_0^2 f dx$  exists and equals to 18.
  - $\int_0^3 dy \int_0^2 f dx$  does not exist.

**Course: CC10 (Mechanics)**

Choose the correct alternative with proper justification:

5x2=10

11. Which of the following conditions do not change the effect of couple

- a) Shifting of couple to a new position in the same plane      b) Rotation of couple in its plane  
c) Shifting of couple to a parallel plane      d) All of the above

12. A block of mass 4kg rests on a horizontal plane. The plane is gradually inclined until at an angle  $\alpha=15^\circ$  with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface.

- a) 0.814      b) 0.27      c) 1.5      d) 3.5

13. If for a particle moving in SHM, there is a sudden increase of 1% in restoring force just while particle passing through mean position, percentage change in amplitude will be

- a) 1%      b) 2%      c) 0.5%      d) 4%

14. A gardener pushes a lawn roller through a distance of 20m. If he applies a force of 20 kg-wt in a direction inclined at  $60^\circ$  to the ground find the work done by him. [ $g=9.8\text{m/s}^2$ ]

- a) 400 J      b) 1960 J      c) 250 J      d) 2514 J

15. If  $v_0$  be the orbital velocity of a satellite close to earth's surface and  $v_e$  be the escape velocity for the Earth, then

- a)  $v_e = \sqrt{2}v_0$       b)  $v_e = v_0$       c)  $v_e = 2v_0$       d) None of these

**Course: SEC-B2 (Scientific computing with R)**

Choose the correct alternative with proper justification:

5x2=10

16. Which one is the correct output of the following command while executing in R console?

`c(1/81,1/64,1/8)^c(-1/4,-1/3,-1/3)^c(1/4,1/3,1/3)`

- (a) 3 4 2      (b) 1.316074 1.587401 1.259921      (c) NaN NaN NaN      (d) Error

17. What will be output for the following code?

```
x<- LETTERS[1:"C"]
```

```
for ( i in x) {
```

```
print(x)
}
```

- (a) C            (b) NaN            (c) NA            (d) Error

18. Which one of the following is the correct outcome of  
For (i in 10:12) {print("marks of student i=", 2\*i)}?

(a) [1] marks of student 10= 20  
[1] marks of student 11= 22  
[1] marks of student 12= 24

(b) [1] marks of student i= 20  
[1] marks of student i= 22  
[1] marks of student i= 24

(c) [1] marks of student i= 2\*10  
[1] marks of student i= 2\*11  
[1] marks of student i= 2\*12

(d) [1] "marks of student i="  
[1] "marks of student i="

Error

19. What will be the output of the following R code?

```
f <- function(num = 1) {  
  hello <- "Hello, world!\n"  
  for(i in seq_len(num)) {  
    cat(hello)  
  }  
  chars <- nchar(hello) * num  
  chars  
}  
f()
```

(a) Hello, world!  
[1] 10

(b) Hello, world!  
[1] 12

(c) Hello, world!  
[1] 14

(d) Error

20. What will be the output of the following R code?

```
g = function(x) {
```

```
      a == 3
x+a+y
}
a=5
y <- 3
g(2)
```

- (a) 8      (b) 10      (c) error      (d) None of these