2021

CHEMISTRY — HONOURS

Paper : SEC-A-1

(Mathematics and Statistics for Chemists)

Full Marks : 80

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer **question no. 1** and **any twelve** questions from the rest. 1×20

1. Answer the following questions :

   (a) Find order and degree of the differential equation :

   $$xy\left(\frac{\partial^2 y}{\partial x^2}\right) + x\left(\frac{\partial y}{\partial x}\right)^3 - y\left(\frac{\partial y}{\partial x}\right) = 0$$

   (b) Show that the differential equation $(x - y)\dfrac{\partial y}{\partial x} = x + 2y$ is homogeneous.

   (c) If $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$, find out $\Gamma\left(-\frac{1}{2}\right)$.

   (d) Given $\vec{A} = -2i + j + k$ and $\vec{B} = 3i - j + k$. Determine $\vec{C} = \vec{A} \times \vec{B}$.

   (e) Do the matrices A and B commute if $A = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$?

   (f) In how many ways can 5 differently coloured marbels be arranged in a row?

   (g) Does the limit $f(x, y, z) = \dfrac{x + 2y - 3z}{2x - y}$ as $(x, y, z) \rightarrow (0, 0, 0)$ exist?

   (h) What is the total differential of the pressure for an ideal gas?

   (i) Draw two normal curves having same standard deviation but different mean.

   (j) What do you mean by standard normal distribution?

   (k) What is the difference between the nature of the distribution curves for student's t-distribution and standard normal distribution?

   (l) Find $P\left(-t_{0.25} < t < t_{0.05}\right)$.

Please Turn Over

(m) What do you mean by standard error of regression?

(n) Explain how Q.test is used for rejection of data.

(o) Which of the following could be the medium for a set of integers (97, 98, 56, x, 86), given that $20 < x < 80$?

(p) If the average of 5 numbers is 36 and the average of four of those numbers is 34, then what is the value of the fifth number?

(q) How can you prove that the pressure of an ideal gas is a state function? (Calculation not required)

(r) Find the solution of the differential equation, $y'' + 2y' + 2y = 0$.

(s) Three coins are tossed simultaneously. Derive the sample space.

(t) Find the value of $\overline{\left(C - \overline{C}\right)^2}$ for O_2 gas at 298 K using Maxwell's distribution of molecular speed in 3D, where the terms have their usual significance.

2. (a) Find the value of $\left(\frac{\partial V}{\partial T}\right)_P$ for 1 mol of van der Waals gas.

   (b) Find the Laplace transform of the piecewise continuous function :
   $$f(t) = \begin{cases} 0, & 0 \le t < 2 \\ k, & t \ge 2, \quad k \text{ constant}. \end{cases}$$
   2+3

3. (a) Determine the value of the root mean square speed of the gas molecules obeying Maxwell distribution of molecular speed in 3-dimension. Use Gamma function to evaluate the integral.

   (b) Let A and B be two symmetric matrices of the same order. Show that the matrix AB is symmetric if and only if $AB = BA$, that the matrices A and B commute. 2+3

4. (a) Maxwell Kinetic energy distribution in 3D is given by $P(\epsilon) \ d\epsilon = 2\pi \left(\frac{1}{\pi KT}\right)^{3/2} \sqrt{\epsilon} \ e^{-\epsilon/KT} d\epsilon$.

   where all the terms are of usual significance. Using the concept of co-error function, find the fraction of molecules having kinetic energy greater than ϵ'.

   (b) A loss of 0.4 mg of Zn occurs in the course of a particular analysis of that element. Calculate the percentage of relative error due to this loss, if the weight of Zn in the sample is 400 mg.
   3+2

5. (a) Find the fourier cosine transform of e^{-x^2}.

   (b) The students in a class are selected at random, one after the other, for an examination. Find the probability P that the boys and girls in the class alternate if –

   (i) the class consists of 4 boys and 3 girls.

   (ii) the class consists of 3 boys and 3 girls. 2+3

6. (a) A 2nd order chemical reaction involving one reactant is given by $-dc/dt = kc^2$. Solve the differential equation to find $'c'$ as a function of $'t'$.

   (b) Maxwell's kinetic energy distribution in 2D is given by $P(\epsilon)\,d\epsilon = \dfrac{1}{KT}\,e^{-\epsilon/KT}\,d\epsilon$ where terms are of usual significance. Using Gamma function, calculate $<\epsilon>$ and $<\epsilon^2>$.   2+3

7. A study was made to determine the activation energy E_a for a chemical reaction. The rate constant k was determined as a function of temperature T, and the data in the table below is obtained :

T, K	599	629	647	666	683	700
k, S^{-1}	0.00054	0.0025	0.0052	0.014	0.025	0.064

The data should fit a linear model of the form $\log k = \log A - E_a/(2.303\,RT)$, where A is the pre-exponential factor.

   (a) Find the slope, intercept and standard error of the estimate using least square analysis.

   (b) Find the activation energy and its standard deviation.   5

8. (a) A pair of dice is tossed twice, Find the probability of scoring 7 points, (i) onces, (ii) at least once.

   (b) Six portions of a well mixed sample are analyzed separately for chlorides. The data obtained are: 20.36, 19.98, 20.26, 20.18, 19.72 and 19.86% chloride. Find 95% confidence limit for the true chloride contain. Given, $t_{0.95,\,5} = 2.571$.   2+3

9. (a) Out of 5 mathematicians and 7 physicists, a committee consisting of 2 mathematicians and 3 physicists is to be formed. In how many ways this can be done if,

   (i) any mathematician and physicist can be included

   (ii) one particular physicist must be on the committee?

   (b) For an ideal gas $pV = nRT$. Taking $V = f(n, p, T)$, verify that dV is an exact differential. 2+3

10. The homogeneity of the chloride level in a water sample from a lake was tested by analyzing portions drawn from the top and from near the bottom of the lake, with the following results in ppm of Cl.

Top	26.30	26.43	26.28	26.19	26.49
Bottom	26.22	26.32	26.20	26.11	26.42

   (i) Apply the *t-test* at 95% confidence level to determine if the chloride level from the top of the lake is different from that at the bottom.

   [Given : Spooled $=$ 0.1199

   t_{crit} $=$ 2.31   for 8 degrees of freedom at 95% confidence.]

Please Turn Over

(ii) Use the paired *t-test* to determine whether there is a significant difference between the top and bottom values at the 95% confidence level.

[Given : For 4 degrees of freedom at 95% confidence, $t = 2.75$] 5

11. (a) Use the Taylor series to evaluate $\int_0^1 \dfrac{\sin x}{x}\, dx$ to four place accuracy.

(b) Batteries in many new cases are warranted for 36 months. Assume this duration of warranty was chosen by the battery manufacturers so that only 5 in 1000 batteries will fail during the warranty period, whereas 25 in 1000 will fail in 38 months. What is the average life of the batteries?

[Given, t_{99}, infinity = 2.58; t_{95}, infinity = 1.96] 2+3

12. What is the objective of a "F test"? A given sample was analysed by two chemists with the following results :

| Chemist 1 (g/L) | 1.90 | 1.82 | 1.70 | 1.94 | 1.85 | 1.40 |
| Chemist 2 (g/L) | 1.35 | 1.65 | 1.76 | 1.41 | 1.80 | 1.33 |

Calculate the F-Statistics for these two sets of data. 5

13. Explain the term 'Linear correlation coefficient'. Using the following data, calculate slope and intercept for a best fit straight line :

| Age (in week) | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| Length of tree (cm) | 1.4 | 1.5 | 2.2 | 2.4 | 3.1 | 3.2 | 3.2 |

| Age (in week) | 11 | 12 | 14 | 15 | 16 | 17 |
| Length of tree (cm) | 3.9 | 4.1 | 4.7 | 4.5 | 5.2 | 5.0 |

5

14. A new automated procedure for determination of glucose in serum (method A) is to be compared to the established method (method B). Both methods are performed on serum from the same six patients in order to eliminate patient to patient variability. Do the following results confirm a difference in the two methods at the 95% confidence level? 5

	Patient 1	Patient 2	Patient 3	Patient 4	Patient 5	Patient 6
Method A glucose (mg/L)	1044	720	845	800	957	650
Method B glucose (mg/L)	1028	711	820	795	935	639

(Given critical *t* value 2.57 for 95% confidence level and 5 degree of freedom.)

15. Five analysts determined calcium by a volumetric method and obtained the amounts (in m mol Ca) shown in the table below :

Trial No.	Analyst 1	Analyst 2	Analyst 3	Analyst 4	Analyst 5
1	10.3	9.5	12.1	9.6	11.6
2	9.8	8.6	13.0	8.3	12.5
3	11.4	8.9	12.4	8.2	11.4

(Given : The critical value of F at the 95% confidence level for 4 and 10 degrees of freedom is 3.48.)

Do the means differ significantly at the 95% confidence level?   5

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