2020
MATHEMATICS HONOURS
Paper: II
Full Marks: 100

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meaning.

MODULE–I (Full Marks–50)

Group-A

Full Marks-40

Answer any **four** : $4\times10=40$

1. **a**. Let S be a bounded set, then prove that $T=\{-x : x\in S\}$ is bounded and also prove that $\sup T=-\inf S$.

OR

State and prove the Archimedean property. 10

2. a. Define open set. Prove that the set $\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},.....,\frac{1}{n},........\right\}$ is not an open set.

Give an example of an infinite family of open sets $\{O_n : n\in N\}$ where $\bigcap_{n\in N} O_n$ is an open set. 10

3. a. Let S be a set of real numbers and $\sup S$ exists. If $\sup S\notin S$ then prove that $\sup S$ is a limit point of S. Also if $\sup S\in S$ then by example establish that $\sup S$ may or may not be a limit point of S. 10

4. a. Define convergence of a sequence of real numbers. Verify that the harmonic sequence $\left\{\frac{1}{n}\right\}$ converges to 0. 6

b. Prove that every convergent sequence is bounded. 4

5. a. State and prove the Sandwich theorem for sequences of real numbers. Hence prove that the sequence $\left\{\frac{x^n}{n}\right\}$ where $0 < x \le 1$ is convergent to 0. 10

6. Define sub-sequential limit of a sequence. Prove that a sequence $\{x_n\}_{n\in N}$ is convergent iff $\overline{\lim} x_n = \underline{\lim} x_n$. 10

7. a. Prove that every subset of an infinite countable set is countable. 6

b. Define uniform continuity of a function at a point. 4

8. Let [a, b] be a closed and bounded interval. And a function $f:[a,b] \to R$ be continuous on [a, b]. If $f(a) \ne f(b)$ then f attains every value between $f(a)$ and $f(b)$ at least once in the open interval (a, b). 10

Group-B

Full Marks-10

Answer any **ONE** questions:

9. Work out the integrals: i) $\int_0^{\pi/2} \frac{dx}{4+5\sin x}$. 5

ii) $\int \frac{\cos x dx}{\sin x + \cos x}$ 5

10. Show that $\int \frac{\sin^5 x}{\cos^4 x} dx = \frac{1}{3\cos^3 x} - \frac{1}{\cos x} - \cos x$ 10

11. Find $\lim_{n\to\infty} \frac{1+2^{10}+3^{10}+......+n^{10}}{n^{11}}$. 10

Module IV (Full Marks – 50)

Group – A
(Linear Algebra –I)
Full Marks – 35

Answer Question No. 1 and any three from the rest

1. If $x = a^2 + 2bc,\ y = b^2 + 2ca,\ z = c^2 + 2ab,$ prove that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} x & y & z \\ y & x & z \\ z & y & x \end{vmatrix}$. 8

2. Determine the value of $\begin{vmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{vmatrix}$ by Laplace's expansion in terms of minors of order 2 obtained from the first two rows. 9

3. Determine the row rank and the column rank of the matrix A and verify that the row rank of A=column rank of A, where $A = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 3 & 2 & 6 & 9 \\ 1 & 1 & 2 & 6 \end{pmatrix}$ 9

4. Find $\dim S \cap T$ where S and T are subspaces of the vector space R^4 given by $S = \{(x, y, z, w) \in R^4 : 2x + y + 3z + w = 0\}$, $T = \{(x, y, z, w) \in R^4 : x + 2y + z + 3w = 0\}$. 9

5. Find a non-singular matrix P such that $P^t AP$ is the normal form of A under congruence, where $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$ 9

6. Prove that for all α, β in a Euclidean space V, $(\alpha + \beta, \alpha - \beta) = 0$ if and only if $\|\alpha\| = \|\beta\|$. 9

Group-B
Vector Calculus
Full Marks-15
Answer any ONE question.

7. Prove that the necessary and sufficient condition for a vector $\vec{r} = \vec{f}(t)$ to have a constant direction is $\vec{f}(t) \times \frac{d\vec{f}}{dt} = 0$.

8. If $\vec{r} = (a\cos t)\vec{i} + (a\sin t)\vec{j} + (at\tan\alpha)\vec{k}$, then prove that $\left[\frac{d\vec{r}}{dt} \quad \frac{d^2\vec{r}}{dt^2} \quad \frac{d^3\vec{r}}{dt^3}\right] = a^3\tan\alpha$.

9. Find the values of the constants a, b, c so that the directional derivative of the function $f = axy^2 + byz + cz^2x^3$ at the point (1, 2, -1) has maximum magnitude 64 in the direction parallel to z-axis.

10. Show that the vector $\vec{V} = (4xy - z^3)\vec{i} + 2x^2\vec{j} - 3xz^2\vec{k}$ is irrotational. Show that $\vec{V}$ can be expressed as the gradient of some scalar function .