2020
MATHEMATICS HONOURS
Paper: I
Full Marks: 100

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meaning.

MODULE–I (Full Marks–50)
Group-A (Marks–36)
Answer any *FOUR* questions

1. Let a and b be two integers with $b>0$. Then show that there exist unique integers q and r such that $a=bq+r$ where $0\le r<b$. 9

2.(i) For two integers m and n the relation $am+bn=1$ holds, where a and b are non-zero integers. Prove that a and b are prime to each other. 4

(ii) If $\gcd(a,b)=1$, show that $\gcd(a+b,a-b)=1$ or 2 where a and b are two non-zero integers. 5

3. Show that the ratio of the principal values of $(1+i)^{1-i}$ and $(1-i)^{1+i}$ is $\sin(log2)+icos(log2)$. 9

4. If n is a positive integer, prove that $\dfrac{1}{\sqrt{4n+1}}<\dfrac{3.7.11.\ \ldots\ \ldots\ .(4n-1)}{5.9.13.\ \ldots\ \ldots\ .(4n+1)}<\sqrt{\dfrac{3}{4n+3}}$. 9

5. If α,β,γ be the roots of the equation $x^3+px^2+qx+r=0$, then find, in terms of p,q,r the value of $\dfrac{\alpha}{\beta}+\dfrac{\alpha}{\gamma}+\dfrac{\beta}{\alpha}+\dfrac{\beta}{\gamma}+\dfrac{\gamma}{\alpha}+\dfrac{\gamma}{\beta}$. 9

6. Find, by Sturm's method, the number and positions of the real roots of $x^4-2x^3-7x^2+10x+10=0$.

7. Solve the following equation by Ferrari's method $x^4-2x^3-5x^2+10x-3=0$ 9

Group – B
Marks – 14
Answer any *TWO*

8. A mapping $f:S\to R,$ where $S=\{x:-1<x<1\}$, is defined by $f(x)=\dfrac{x}{1-|x|}, x\in S.$ Show that f is bijective. 7

9. Let $f:A\to B$ be a mapping. A relation ρ is defined on

"$x\rho y$ if and only if $f(x)=f(y),\ x,y\in A$". Show that ρ is an equivalence relation on A. 7

10. Let $(S,\circ)$ be a semigroup. If for $a,b\in S,\ a^2\circ b=b=b\circ a^2,$ Prove that $(S,\circ)$ is an abelian group. 7

11. Let H be a non-empty subset of a group $(G,*)$ such that $a\in H, b\in H \Rightarrow a*b^{-1}\in H.$ Prove that $(H,*)$ is a subgroup of the group $(G,*)$. 7

MODULE–II (Full Marks–50)

Group-A

Full Marks-20

Answer any TWO questions:

1. Reduce the equation $5x^2-7y^2+2x-3y=0$ in the form $ax^2+by^2=1$ by proper translation of axes without rotation. 10
2. Show that the equation $7x^2-48xy-7y^2-20x+140y+300=0$ represents a hyperbola and find its canonical equation. 10
3. Prove that the equation of the chord joining the points $\theta=\theta_1$ and $\theta=\theta_2$ on the circle $r=2a\cos\theta$ $r\cos(\theta-\theta_1-\theta_2)=2a\cos\theta_1\cos\theta_2$ 10
4. If by a rotation of rectangular axes about origin, the expression $(ax^2+2hxy+by^2)$ changes to $(a'x'^2+2h'x'y'+b'y'^2)$, then prove that $a+b=a'+b'$ and $ab-h^2=a'b'-h'^2$ 10
5. Find the equation and length of the common tangent to the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{b^2}-\frac{y^2}{a^2}=-1$. 10

Group-B

(Marks – 15)

Answer question No. 6 and any ONE from the rest

6. Find the angle between the two straight lines whose direction cosines are give by $l+m+n=0,\ l^2+m^2-n^2=0.$ 5

OR

Show that the lines $\frac{x+3}{2}=\frac{y+5}{3}=\frac{z-7}{-3}, \frac{x+1}{4}=\frac{y+1}{5}=\frac{z+1}{-1}$ are coplanar and find the equation of the plane containing them. 5

7. A point P moves on the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, which is fixed and the plane through P perpendicular to OP meets the axes in A , B, C. If the planes through A,B,C parallel to the co-ordinate planes meet in point Q then show that the locus of Q is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}$. 10

8. A variable plane which remains at a constant distance 3p from the origin cuts the co-ordinate axes at A. B, C. Show that the locus of the centroid of the triangle ABC is $x^{-2}+y^{-2}+z^{-2}=p^{-2}$ 10

9. Show that the equation of the plane containing the straight line $\frac{y}{b}+\frac{z}{c}=1, x=0$ and parallel to the straight line $\frac{x}{a}-\frac{z}{c}=1, y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if $2d$ be the shortest distance between the lines, show that $\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$. 10

Group-C

Full Marks-15

Answer any ONE questions `

10. For any triangle ABC, with usual notations, prove that $a=b\cos C+c\cos B$ and $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

.

11. Show that the shortest distance between two opposite edges of a regular tetrahedron is equal to half the diagonal of the square described on an edge.
12. A person travels due East at the rate of 6 miles per hour and observes that the wind seems to blow directly from the North; he then doubles his speed and the wind appears to come from the North-East. Determine the direction and the velocity of the wind.