

2020
MATHEMATICS HONOURS
Paper: 1
Internal Assessment
SET-3

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Full Marks-100

Time – 3 Hours

MODULE-I

Group – A (Marks – 20)

Answer any TWO

1. (i) Show that all values of i^i are real. 5
- (ii) Is the matrix $\begin{pmatrix} 1 & -2 \\ 6 & 0.4 \end{pmatrix}$ orthogonal? Justify. 5
2. Use De Moivre's theorem to show that $a^n + b^n = 2^{\frac{n}{2}+1} \cos \frac{n\pi}{4}$ where a and b are roots of $x^2 - 2x + 2 = 0$ and n is a positive integer. 10
3. Solve by Cardon's method: $x^3 - 3x + 1 = 0$ 10
4. Find the rank of the matrix $\begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ 10

Group – B (Marks – 15)

Answer Question No. 5 and any ONE from the rest

5. Write the condition that a general equation of 2nd degree represents a pair of straight lines.

Write the equation of a tangent to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) . 9

OR

Reduce the equation $3x^2 - 2y^2 + 6x - 8y - 17 = 0$ to its canonical form and find the nature of the conic. 9

6. Prove that the product of perpendiculars from the point (x_1, y_1) on the straight lines $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}} \quad 6$$

7. Find the condition that the line $\frac{l}{r} = A \cos \theta + B \sin \theta$ will touch the conic $\frac{l}{r} = 1 + e \cos \theta$. 6

8. Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant. 6

Group – C (Marks – 15)

Answer Question No. 9 and any ONE from the rest

9. (a) Find the equation of the straight line passing through $(1, 3, -2)$ and $(-8, -2, -9)$. 9

OR

- (b) Find the unit vector perpendicular to both $2\vec{i} + \vec{j} - 3\vec{k}$ and $5\vec{i} + 3\vec{j} - 2\vec{k}$. 9

10. Prove by vector method that perpendicular bisectors of a triangle are concurrent. 6

11. (i) Find the angle between the vectors and $5\vec{i} + 3\vec{j} - \vec{k}$ and $2\vec{i} - 3\vec{j} + 5\vec{k}$. 3

- (ii) What is the geometrical interpretation of vector product and scalar triple product? 3

MODULE-II
Group – A (25 marks)

Differential Calculus

12. (a) Answer any **one**.

5

(i) Show that the sequence $\left\{ \frac{1}{n^2} \right\}$ is convergent.

(ii) Define strictly monotonically decreasing function.

13. Answer any **ONE**.

(a) (i) Find the equation of the tangent to the curve $(x^2 - 5x + 4)^2 + (y - 3)^2 = 2$ at the point (1, -2). 8

(ii) State Cauchy's mean value theorem.

4

(iii) If the sequence $\{u_n\}$ and $\{v_n\}$ be convergent and converge to u and v respectively. Then prove that $\{u_n + v_n\}$ is convergent and converges to u+v. 8

(b) (i) State D'Alembert ratio test. Test the convergence of the series $1 + \frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots$

8

(ii) Prove or disprove that the necessary condition for the convergence of a series $\sum u_n$ is

$$\lim_{n \rightarrow \infty} u_n = 0.$$

8

(iii) State the root tests for convergence of a series of positive terms.

4

(d) (i) Show that the maximum value $x + \frac{1}{x}$ is less than its minimum value. Explain the reason. 10

(ii) Find the equation of pedal of any parabola with respect to its vertex.

10

Group-B
Full Marks-10

14. Answer ANY ONE question:

a) Evaluate $\int \frac{e^x dx}{e^x - 3e^{-x} + 2}$

b) Evaluate: $\int e^x \frac{2 - \sin 2x}{1 + \cos 2x}$

c) Using the definition of definite integral, evaluate $\int_a^b \log(kx) dx$.

Group-C
Full Marks-15

Answer any Q.15 & and any **ONE**

15. Answer any **one**:

i) Find the degree of the differential equation $\left(\frac{d^4 y}{dx^4}\right)^3 + 5x \sqrt{\left(\frac{dy}{dx}\right)^7} + y = 4$. Find the differential equation of the curve $y = a \sin 2x + b \cos 2x$ where a, b are parameters. 7

ii) Define singular solution. Explain it geometrically. 7

16. Solve: $\frac{dy}{dx} + \left(\frac{x}{1-x^2}\right)y = x\sqrt{y}$ 8

17. Obtain the general and singular solution of $y = px + \sqrt{4p^2 + 9}$ 8

18. Find the equation of orthogonal trajectories of $y^2 = a^2(x^2 + 4y^2)$ 8