2020
MATHEMATICS – HONOURS
SEMESTER-3
TUTORIAL
Full Marks In each Course: 15

The figures in the margin indicate full marks .
Symbols and notations used here carry their usual meaning.
Candidates are required to give their answers in their own words as far as practical.

Course: CC5 (Theory of Real Functions) ***5×3***

1. Show that $\lim_{x\to\infty} xsinx$ does not exists in R^*.

2. A function $f: R \to R$ satisfies the condition $f(x+y) = f(x)f(y) for\ all\ x, y \in R$. If f is continuous at $x = 0$ then prove that f is continuous at every point in R.

3. A function f is twice differentiable on $[a, b]$ and $f(a) = f(b) = 0$ and $f(c) < 0$ for some $c \in (a, b)$. Prove that there is at least one point $p \in (a, b)$ for which $f''(p) > 0$.

Course: CC6 (Ring Theory & Linear Algebra I) ***5×3***

4. Prove that a square matrix A is orthogonally diagonalisable iff A is symmetric.

5. Use Cayley-Hamilton theorem to find A^{100} , where $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

6. In a commutative ring with unity, an ideal P is a prime ideal iff the quotient ring R/P is an integral domain.

Course: CC7 (ODE & Multivariate Calculus-I) ***5×3***

7. Solve the ordinary differential equation by changing independent variable:
$$x\frac{d^2y}{dx^2} + \left(4x^2 - 1\right)\frac{dy}{dx} + 4x^3 y = 2x^3$$

8. Find the series solution of the ordinary differential equation $\left(1+x^2\right)\frac{d^2y}{dx^2} + 10x\frac{dy}{dx} + 20y = 0$.

9. Find the nature of the stationary points of the function
$$f(x,y) = (3-x)(3-y)(x+y-3).$$