2020
MATHEMATICS (HONOURS)
Paper: 4
Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meaning.

(Module-VII)

Group-A

(Marks 15)

1. Answer any two questions

(a)State and prove Euler's theorem on homogeneous function of three variables.

(b) Justify the Schwarz theorem for the function

$$f(x,y)=\frac{x^2y^2}{x^2+y^2}\ ,\ (x,y)\neq(0,0)$$

(c)If $=\tan^{-1}\frac{x^2+y^2}{(x-y)}$, prove that

$$x^2\frac{\partial^2u}{\partial x^2}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial y^2}=(1-4sin^2u)sin2u$$

Group-B

(Marks 10)

2. Answer any one of the following **10**

(a) Find the envelope of the family of co-axial ellipse$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where the parameters are connected by the relation $a+b=c$ (c being fixed)

(b) Show that the curve $y=\begin{cases}\sqrt{1+x^2}sin\frac{1}{x} & ,\ x\neq 0\\ 0 & x=0\end{cases}$ has no asymptote parallel to the y-axis and its only asymptotes are $y=\pm 1$

(Module-VIII)

Group-A

(Marks 8)

3. Answer any one of the following

(a) A sphere of weight , is equilibrium on a smooth plane of inclination α to the horizontal, being supported by a string , which is of length equal to the radius and is fastened to two points, one on the sphere and one on the plane. Prove that the tension of the string is $\frac{2}{3}\sqrt{3}\omega \sin\alpha$ **8**

(b) A uniform ladder of weight W' rests on a rough horizontal ground and against a smooth vertical wall inclined at an angle α to the horizon. Prove that a man of weight W can climb to the top of the ladder withoutthe ladder slipping if

$\frac{W'}{W} > \frac{2(1-\mu\tan\alpha)}{2\mu\tan\alpha-1}$, μ being the coefficient of the friction. **8**

Group-B

(Marks 5)

4. Answer any one of the following

(a) Show that the centres of all sections of the sphere $x^2+y^2+z^2=a^2$ by the planes through the point (α,β,γ) lie on the sphere $x^2+y^2+z^2-\alpha x-\beta y-\gamma z=0$. **5**

(b) Reduce the equation $11x^2+10y^2+6z^2-8yz+4zx-12xy+72x-72y+36z+150=0$ to its canonical form and determine the type of the quadric represented by it. **5**

Group-C

(Marks 12)

5. Answer any two questions

(a) A particle is projected vertically upwards from the earth's surface just sufficient with a velocity to carry it to infinity. Show that the time it takes in reaching a height is $\frac{1}{3}\sqrt{\frac{2a}{g}}\left\{\left(1+\frac{h}{a}\right)^{3/2}-1\right\}$, where a is the radius of the earth. **6**

(b)A particle is projected vertically upward with a velocity with u in a medium whose resistance varices of the square of the velocity. Investigate the motion. Show that the particle come to the rest at a height $\frac{v^2}{2g}\log\left(1+\frac{u^2}{v^2}\right)$, where v is the terminal velocity. **6**

(c) Two perfectly inelastic bodies of masses $m_1 and\ m_2$ moving with velocities $u_1 and\ u_2$ in the same direction impinge directly. Show that the loss of kinetic energy due to the impact is $\frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(u_1 - u_2)^2$ **6**