2020

MATHEMATICS HONOURS

Paper: CC-4

Internal Assessment

Full Marks: 10

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Choose the correct alternative.

1. Let a be an element of a group G. $O(a) = n$ and $a^m = e$
   (a) n is prime
   (b) n is a multiple of m
   (c) n is even
   (d) n is a divisor of m

2. The number of generator of the group $(S, .)$ where $S = \{1, I, -1, -i\}$ is
   (a) 1
   (b) 2
   (c) 3
   (d) 4

3. The number of subgroups of a cyclic group of order 12 is
   (a) 4
   (b) 5
   (c) 6
   (d) 7

4. Let $G = (Z_6, +)$ and $\varphi : G \to G$ is defined by $\varphi(\bar{x}) = 2\bar{x},\ \bar{x} \in Z_6$. Then $ker\varphi$ is
   (a) $\{\bar{0}, \bar{1}\}$
   (b) $\{\bar{0}, \bar{2}\}$
   (c) $\{\bar{0}, \bar{3}\}$
   (d) $\{\bar{0}, \bar{4}\}$

5. Let $G = S_3$ and H= A_3 Then $[G : H]$ is
   (a) 1
   (b) 2
   (c) 3
   (d) 0

2020
MATHEMATICS HONOURS
Paper: CC-4
Theory Examination
Full Marks: 32

The figures in the margin indicate full marks.

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Answer any FOUR: 4x8=32

1. Prove that in a group $(G, *)$, $(a * b)^{-1} = b^{-1} * a^{-1}$ for all $a, b \in G$.
2. Let $(G, *)$ be a group and a be an element of G. Define a mapping $\rho_a : G \to G$ by $\rho_a(x) = x * a, x \in G$. Prove that ρ_a is a bijection.
3. Let $(G, *)$ be a group and H be a non empty finite subset of G. Then prove that $(H, *)$ is a sub group of $(G, *)$ if and only if $a, b \in H \Rightarrow a * b \in H$.
4. Prove that every proper subgroup of a group of order 6 is cyclic.
5. If H be a subgroup of a cyclic group G then prove that the quotient group G/H is cyclic.