

**2021**  
**MATHEMATICS HONOURS**  
**SEMESTER-5**  
**Internal Assessment**

**Full Marks in each Course: 10**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

**Course: CC11**

**(Probability & Statistics)**

Choose the correct alternative with proper justification.

5x2=10

1. A number is chosen at random among the first 120 natural numbers. The probability of the number chosen being a multiple of 5 or 15 is

- (a)  $\frac{1}{5}$  (b)  $\frac{1}{8}$  (c)  $\frac{1}{16}$  (d) none of (a), (b) or (c)

2. The probability that both S and T occur, the probability that S occurs and T does not, and the probability that T occurs and S does not are all equal to  $p$ . The probability that either S or T occur is:

- (a)  $p$  (b)  $2p$  (c)  $3p^2$  (d) none of (a), (b) or (c)

3. If joint pdf of two random variables X and Y is given by

$$f(x, y) = 2 - x - y; 0 \leq x \leq 1, 0 \leq y \leq 1$$

0, otherwise

conditional pdf of X given Y in the interval  $0 \leq x \leq 1, 0 \leq y \leq 1$  is

- (a)  $\frac{2-x-y}{\frac{3}{2}-x}$  (b)  $\frac{2-x-y}{\frac{3}{2}-y}$  (c)  $\frac{2-x}{\frac{3}{2}-x}$  (d)  $\frac{2-y}{\frac{3}{2}-y}$

4. The annual sales of 10000 firms are normally distributed with mean Rs.50 lakhs and standard deviation Rs.10 lakhs. Given  $\phi(1) = 0.8413$ , the percentage of firms the sales of which are greater than Rs.60 lakhs is

- (a) 15.87 (b) 16.87 (c) 84.13 (d) 95.30

5. Skewness of a binomial distribution with parameter  $p$  is zero if

- (a)  $p < \frac{1}{2}$  (b)  $p = \frac{1}{2}$  (c)  $p > \frac{1}{2}$  (d) none of (a), (b) or (c)

**Course: CC12**

**(Group Theory II & Linear Algebra II)**

Choose the correct alternative with proper justification.

5x2=10

6. The minimal polynomial of  $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$  is

- a.  $(x - 2)(x - 1)^2$
- b.  $(x - 1)(x - 2)^2$
- c.  $(x - 1)(x - 2)$
- d.  $(x - 1)^2(x - 2)^2$

7. If  $A = \begin{pmatrix} 1 & a & b \\ 0 & 10 & c \\ 0 & 0 & 100 \end{pmatrix}$ , then

- a. Both  $A$  and  $A^2$  are diagonalizable
- b.  $A$  is diagonalizable but not  $A^2$
- c.  $A$  and  $A^2$  have the same minimal polynomial
- d.  $A^2$  is diagonalizable but not  $A$

8. Let  $V$  be the vector space of all  $n \times n$  matrices  $A$  and  $T: V \rightarrow V$  is a linear operator defined by  $T(A) = \frac{A+A^T}{4}$ , then nullity of  $T$  is

- a.  $n^2$
- b.  $\frac{n(n-1)}{2}$
- c.  $\frac{n(n+1)}{2}$
- d.  $n$

9. If  $G$  be a finite abelian group of order  $p$  (a prime), then the number of distinct inner automorphism on  $G$  is

- (a) 1 (b)  $p$  (c)  $p!$  (d)  $p(p - 1)$

10. Let  $G$  be a group such that  $Z(G) = \{e\}$ . Then  $|Z(\text{Aut}(G))|$  is

- (a) 1 (b) 2 (c) 0 (d) none of these

**Course: DSE A1**

**(Advanced Algebra)**

Choose the correct alternative with proper justification.

5x2=10

11. The number of Sylow 2-subgroups of  $S_4$

- A. 4                  B. 1                  C. 3                  D. 2 .

12. The number of all groups of order 22

- A. 1                  B. 2                  C. 3                  D. 5

13. Let  $G$  be a group of order 45 then

A.  $G$  has exactly 2 normal subgroups of order 9                  B.  $G$  has normal subgroup of order 9 more than 2

- C.  $G$  has only one normal subgroup of order 9                  D. None of these.

14. The total number of all irreducible polynomial of degree 2 over  $\mathbb{Z}_3$  are

- A. 12                  B. 6                  C. 4                  D. 9 .

15. Which of the following is true

- i. Euclidean domain is always a field
- ii. Principal Ideal domain is always an Unique factorization domain
- iii. Euclidean domain is always Principal Ideal domain.

- A. i and ii are correct                  B. i and iii are correct  
C. i, ii and iii are correct                  D. ii and iii are correct.

**Course: DSE B2**

**(LPP & Game Theory)**

Choose the correct alternative with proper justification.

5x2=10

16. In graphical method of linear programming problem if the cost line coincide with a side of region of basic feasible solutions we get

- i) Unique optimum solution                  ii) Unbounded optimum solution  
iii) No feasible solution                  iv) Infinite number of optimum solutions

17. If the value of the objective function  $z$  can be increased or decreased indefinitely such solution is called

- i) Unbounded Solution                  ii) Unrestricted in sign                  iii) Unique solution                  iv) None of these

18. A  $n$  dimensional convex polyhedron having exactly  $(n+1)$  vertices is called

- i) tetrahedron                      ii) Feasible region                      iii) Simplex                      iv) none of the above

19. The value of the game

		Player B	
		$B_1$	$B_2$
Player A	$A_1$	2	-2
	$A_2$	-2	2

- Is (i)  $1/2$                       (ii) 0                      (iii)  $3/2$                       (iv) none of (i), (ii) and (iii).

Give reason of your answer solving the problem graphically.

20. The minimum cost corresponding to the initial basic feasible solution obtained in North West Corner method of the following Transportation problem

		Destinations				Supply (a <sub>i</sub> )
		D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
Sources	S <sub>1</sub>	7	10	14	8	30
	S <sub>2</sub>	7	11	12	6	40
	S <sub>3</sub>	5	8	15	9	30
Demand (b <sub>j</sub> )		20	20	25	35	

- Is (i) Rs. 750                      (ii) Rs. 850                      (iii) Rs. 950                      (iv) Rs. 1050

Give the solution table only, no explanation is required.