PHSA SEM-1 Internal Examination

Paper : CC1 Time : 30 min Full marks : 20

Answer any ten of the following questions

Each question carries 2 marks

1. Find a Unit vector parallel to the resultant of vectors $\vec{A} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{B} = 2\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$.

2. Find the projection of the vector $\vec{A} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$ on the vector $\vec{B} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

3. Show that $\vec{A} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{B} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$ are parallel to each other.

4. If $\varphi(x,y,z) = xy^3z^2$, find $\vec{\nabla}\varphi$ at the point (1,-2,-1).

5. Find the value of $(\vec{\nabla} \times \vec{r})$ where $\vec{r}$ is the position vector.

6. What is the difference between a series and a sequence ?

7. How is the comparison test used to determine the nature of an infinite series ?

8. Why it is not important to consider the lower limit of the integral in case of integral test ?

9. show that the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ is divergent in nature.

10. For what value of k, the series $\sum_{n=1}^{\infty} \frac{1}{k^{\ln n}}$ is convergent ?

11.What is the condition under which four vectors ***a***, ***b***, ***c*** and ***d*** are linearly dependent ?

12.If a vector ***g*** is represented as a linear combination of the basis vectors ***{z***$\boldsymbol{\{z_k\}}$, how is it possible to evaluate the constants of the linear combination ?

13.A vector ***a*** is represented in three dimensional Cartesian system as

$$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

Interpret this in the language of vector space theory.

14.The differential equation

$$y''(x) + p(x)y'(x) + qy(x) = 0$$

has the solutions $y_1(x)$ and $y_2(x)$ which are linearly independent. Interprete the solutions in the laguage of vector space theory.

15.Two vectors ***a*** and ***b*** are reresented as linear combinations of the basis vectors $\boldsymbol{\{e_i\}}$. Evaluate the inner product of ***a*** and ***b*** in terms of the constants of the linear combinations.

INTERNAL EXAMINATION -2021

PHSA - SEM 1- CC2

20 MARKS

Answer ***any ten*** from the following questions.

1. State Newton's first law of motions.

2. Define inertial frame of reference.

3. Define Centre of Mass. Show that its position is unique for a system of particles.

4. Explain stable and unstable equilibrium with respect to potential energy curves.

5. Show that in a central force field angular momentum of a particle about the centre of the force is conserved.

6. Prove that for a particle moving under the influence of a central force field the path is planar.

7. Prove that for a particle moving under the influence of a central force field the areal velocity is constant.

8. What is conservative force field?

9. For what kind of a rigid body will the angular momentum and angular velocity always be parallel?

10. Let S' be a reference frame which is rotating with respect to a frame S with an angular velocity $\vec{\omega}$. Prove that for an arbitrary vector $\vec{A}$

$$\frac{d\vec{A}}{dt} = \frac{d'\vec{A}}{dt} + \vec{\omega} \times \vec{A}$$

Where $\frac{d}{dt}$ and $\frac{d'}{dt}$ refers to a time derivatives with respect to S and S' respectively.

11. Assuming that a rigid reference frame fixed at the centre of the earth is inertial, set up the equations of motion with respect to a frame fixed on the surface of the earth for a particle of mass m moving under the gravitational force of the earth and the other forces $\vec{F}_{other}$.

12. What is moment of inertia tensor?

13. A rigid body is rotating with an angular velocity $\vec{\omega}$, about an axis through the origin O and having direction cosines $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$. Calculate the moment of inertia of the said body about the axis.

14. For a fluid at rest in a non-conservative force field, show that Pascal's law is not valid.

15. State the equation of continuity for the motion of an ideal fluid.