## 2020 MATHEMATICS – HONOURS SEMESTER-1 TUTORIAL Full Marks In each Course: 15

The figures in the margin indicate full marks .

Symbols and notations used here carry their usual meaning.

Candidates are required to give their answers in their own words as far as practical.

Course: CC1 (Calculus, Geometry & Vector Analysis) 5×3

- 1. Reduce the following equations to the canonical form and determine the nature of the conic:  $6x^2 5xy 6y^2 + 14x + 5y + 4 = 0$
- 2. Determine the volume of the solid obtained by rotating the region bounded by  $y = \sqrt[3]{x}$  and  $y = \frac{x}{4}$  that lies in the first quadrant about the y-axis.
- 3. Find the equation of the common tangent to the hyperbolas  $\frac{x^2}{a^2} \frac{y^2}{b^2} = 1$  and  $\frac{x^2}{b^2} \frac{y^2}{a^2} = 1$ . Also find the length of the common tangent between the points of contact.

Course: CC2 (Algebra) 5×3

- 4. If  $(x^2+3px+q)$  has a factor of the form  $(x-a)^2$ , then Prove that  $q^2+4p^3=0$ .
- 5. Prove that an equivalence relation R on a set S determines a Partition of S. Conversely, each partition of S provides an equivalence relation on S.
- 6. Find the rank of the matrix =  $\begin{bmatrix} 1 & 0 & 3 \\ 4 & -1 & 5 \\ 2 & 0 & 6 \end{bmatrix}.$