Chapter 5

Fuzzy Logic

(Knowledge-Based Systems; R Akerkar, P Sajja)

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Fuzzy Logic

- Flexible machine learning technique
- Mimicking the logic of human thought
- Logic may have two values and represents two possible solutions
- Fuzzy logic is a multi valued logic and allows intermediate values to be defined
- Provides an inference mechanism which can interpret and execute commands
- Fuzzy systems are suitable for uncertain or approximate reasoning

Fuzzy Logic Vs Bivalued Logic

- Bivalued logic can have only two possible values as 0/1, yes/no, right/wrong etc
- Fuzzy logic can be multi valued. It can have relative values like yes, not, not so much, a little bit etc.

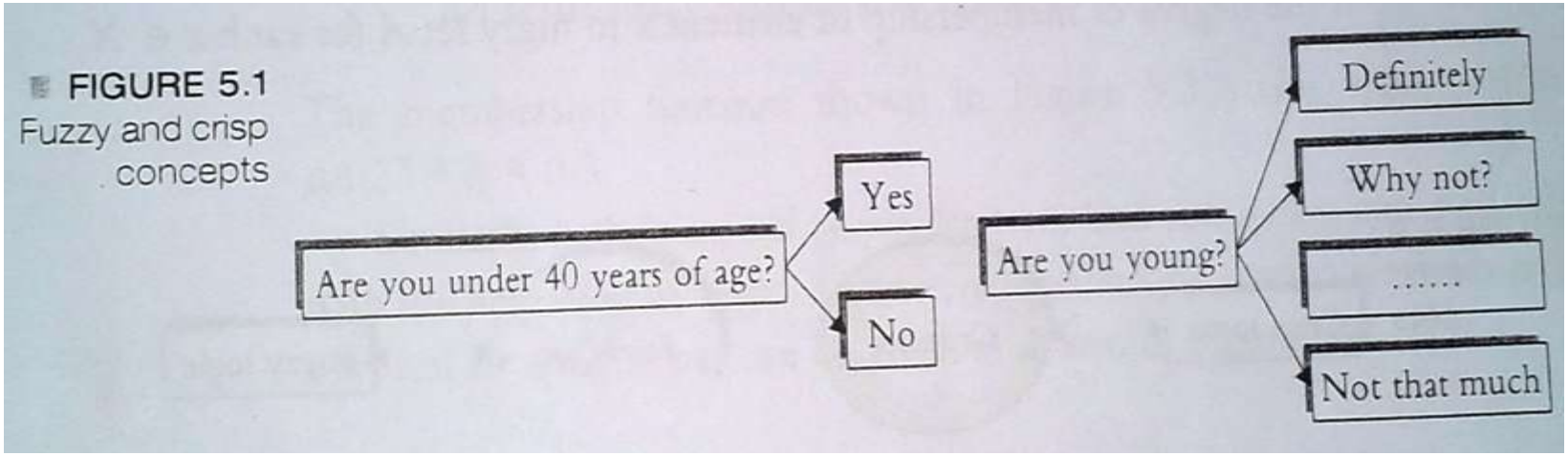


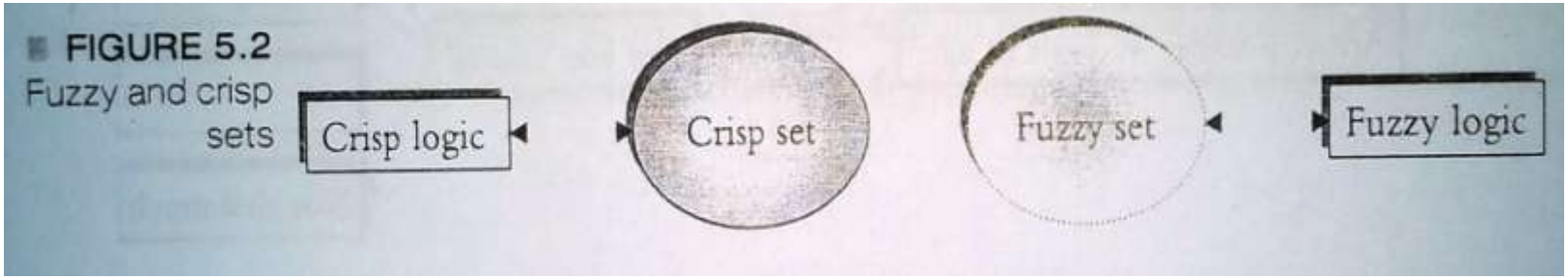
FIGURE 5.1
Fuzzy and crisp concepts

Characteristics of Fuzzy Logic

- Exact reasoning is viewed as a limiting case of approximate reasoning
- Everything is a matter of degree
- Knowledge is interpreted as a collection of elastic or equivalently fuzzy constraints on a collection of variables
- Inference is viewed as a process of propagating elastic constraints
- Any logical system can be fuzzified

Fuzzy Set

- Let X be a non empty set, A fuzzy set A in X is characterized by its membership function μA: X -> [0,1], where μA(x) is the degree of membership of element x in fuzzy set A for each x ∈ X



Membership Function

- Maps elements of a fuzzy set to real numbered values in the interval 0 to 1.
- The curve representing the mathematical function is a membership function that determines the degree of belonging of member x to the fuzzy set T.

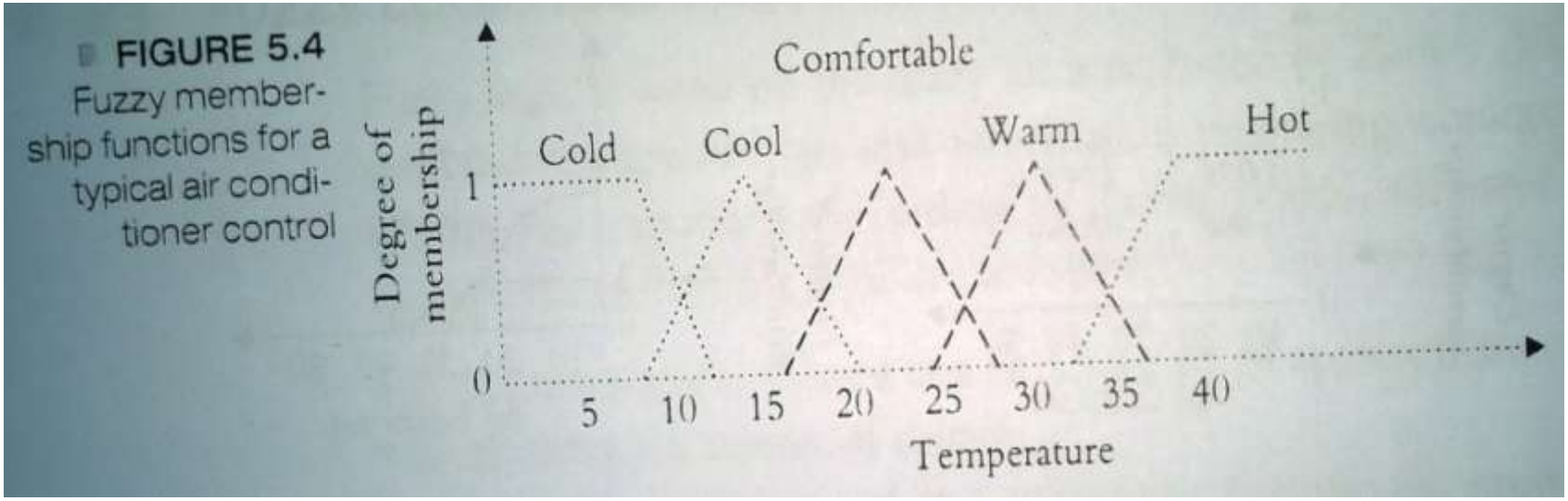


FIGURE 5.4 Fuzzy membership functions for a typical air conditioner control

Fuzzification

The process of transforming crisp(bivalued) input values into linguistic values is called fuzzification

Steps of Fuzzification:

Step 1: Input values are translated into linguistic concepts, which are represented by fuzzy set.

Step 2: Membership functions are applied to the measurements, and the degree of membership is determined

Defuzzification

Defuzzification converts the fuzzy values into crisp (bivalued) value.

Example methods of defuzzification:

- **Max-membership method:** This method chooses the elements with maximum value
- **Centroid method:** This method find the centre point of the targeted fuzzy region by calculating the weighted mean of the output fuzzy region
- **Weighted average method:** Assigns weight to each membership function in the output by its respective maximum membership value

Operations on Fuzzy set

Intersection of Fuzzy Set

The intersection of A and B is defined as $(A \cap B)(x) = \min\{A(x), B(x)\} = A(x) \cap B(x), \forall x \in X$, as demonstrated in figure

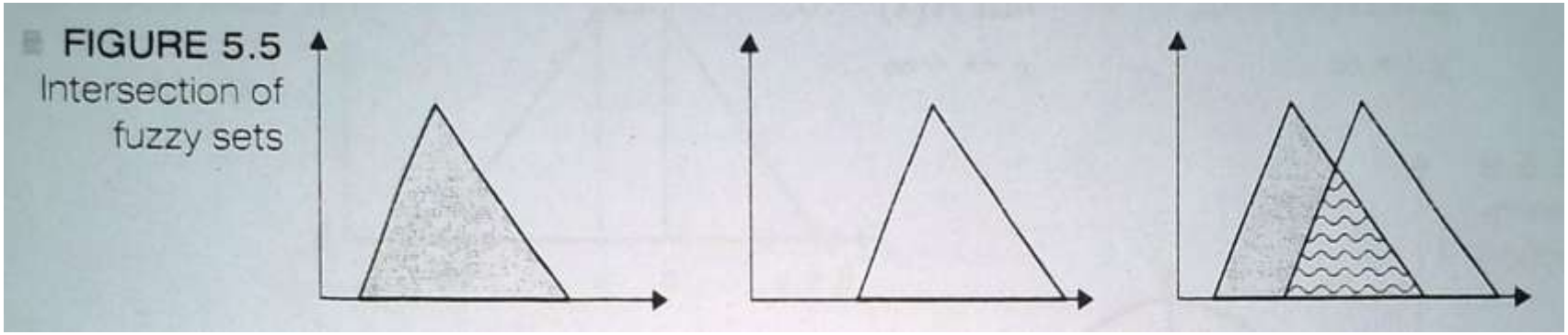
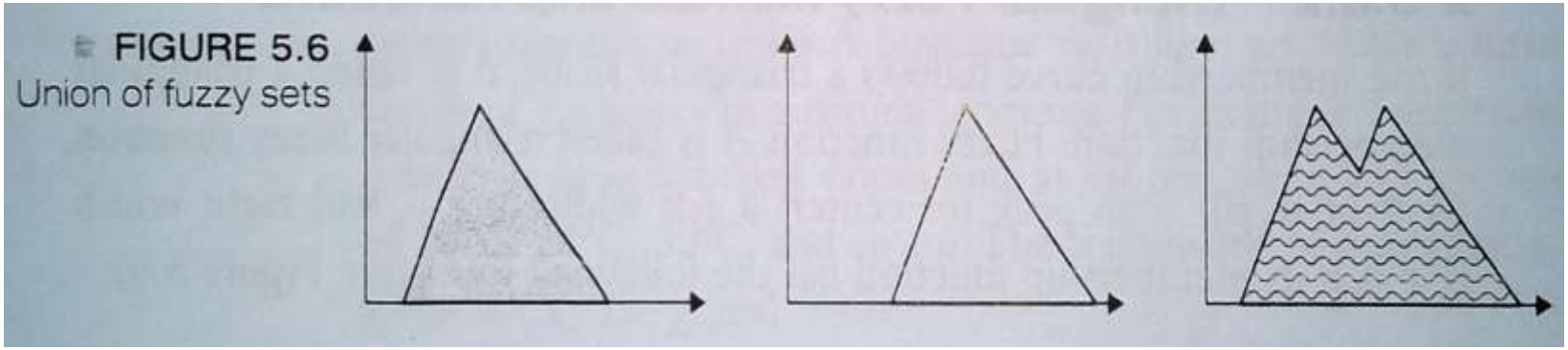


FIGURE 5.5
Intersection of fuzzy sets

Operations on Fuzzy set

Union of Fuzzy Set

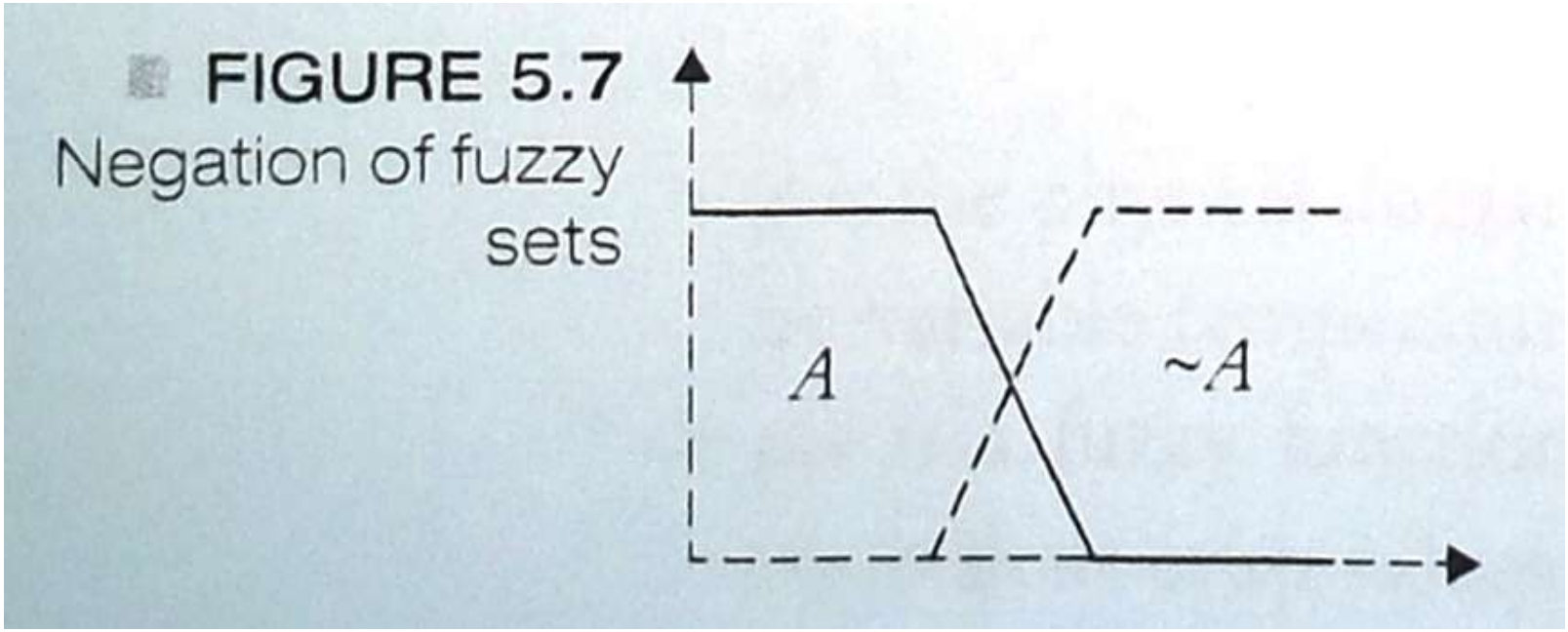
The union of A and B is defined as $(A \cup B)(x) = \max\{A(x), B(x)\} = A(x) \cup B(x), \forall x \in X$, as demonstrated in figure



Operations on Fuzzy set

Complement of Fuzzy Set

The complement of a fuzzy set A is defined as (~ A)(x) = 1 - A(x) as demonstrated in figure



Operations on Fuzzy set

Equality of Fuzzy Sets

Let A and B are fuzzy sets on classical set X. A and B are said to be equal, denoted as $A = B$ if $A \subset B$ and $B \subset A$. That is $A = B$, if and only if $A(x) = B(x) \; \forall \; x \in X$

Types of Fuzzy Function

Quasi-Fuzzy Membership Functions:

The membership function follows a quasi curve. A quasi curve is a real line with a normal fuzzy convex and a continuous membership function satisfying the limit conditions as below:

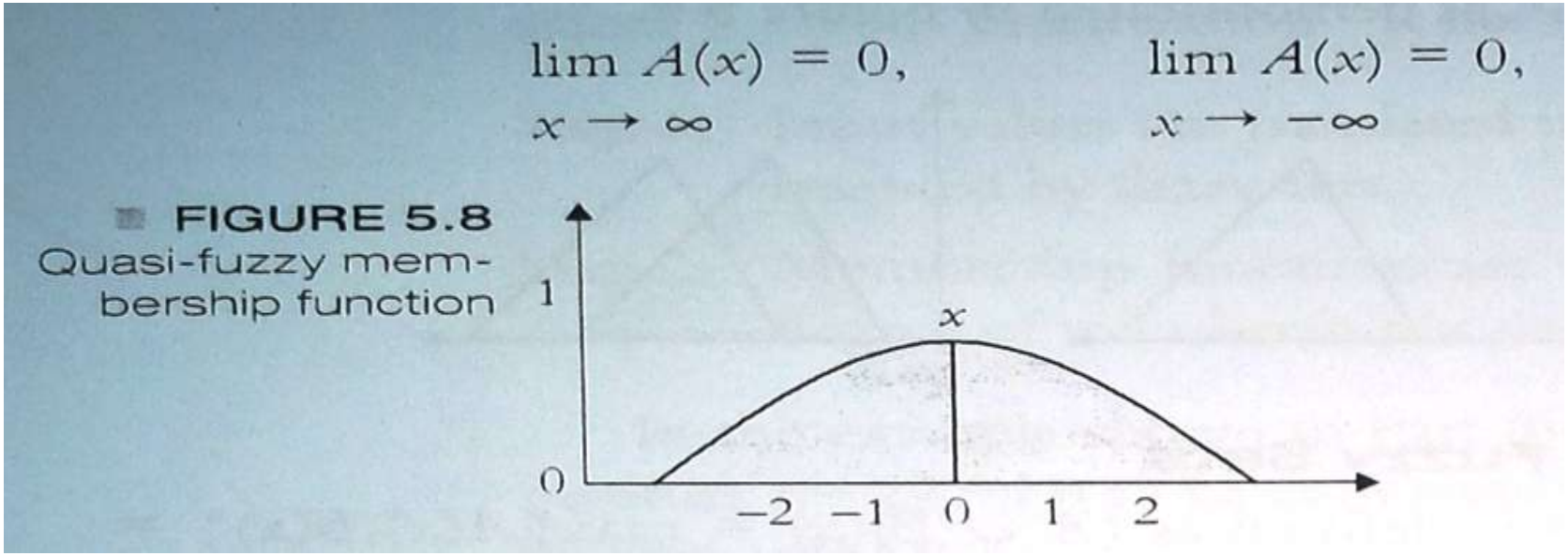


FIGURE 5.8
Quasi-fuzzy membership function

Types of Fuzzy Function

Triangular Fuzzy Membership Functions:

The membership curve follows a triangular shape then it is triangular membership function. Fuzzy function A is called triangular fuzzy function(A=a,α,β) with peak a, left width α>o and right width β>o

$$A(x) = 1 - (a - x)/\alpha \text{ is if } a - \alpha \le x \le a$$
$$= 1 - (x - a)/\beta \text{ if } a \le x \le a + \beta$$
$$= 0 \text{ otherwise.}$$

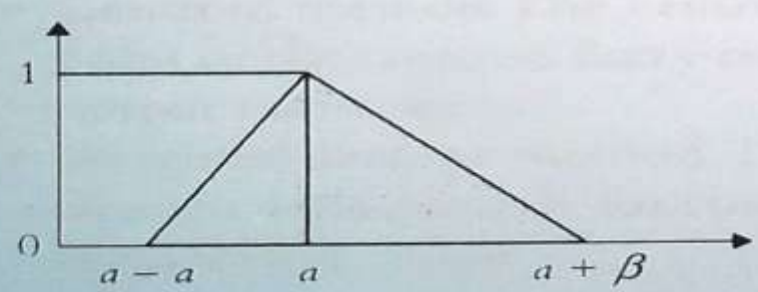


FIGURE 5.9
Triangular fuzzy membership function

Types of Fuzzy Function

Trapezoidal Fuzzy Membership Functions:

The membership curve follows a trapezoidal shape. Fuzzy function A is called triangular fuzzy function(A=a,α,β) with tolerance interval [a, b], left width α and right width β

$$A(x) = 1 - (a - x)/\alpha \text{ if } a - \alpha \le x \le a$$
$$= 1 \text{ if } a \le x \le b$$
$$= 1 - (x - b)/\beta \text{ if } a \le x \le b + \beta$$
$$= 0 \text{ otherwise.}$$

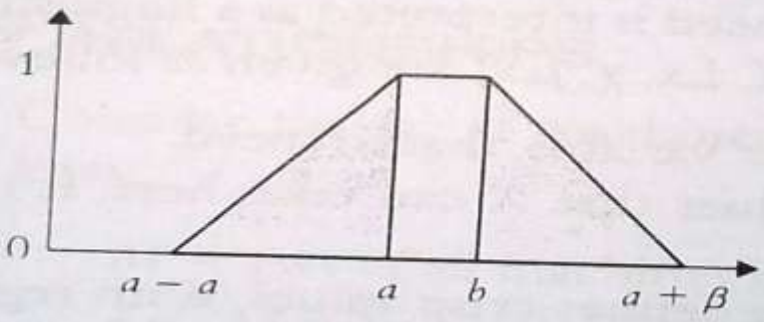


FIGURE 5.10
Trapezoidal fuzzy membership function

Linguistic Variable

A variable whose values are words or sentences in natural language. Example: Temperature is linguistic variable if it takes values hot, cool, warm, comfortable etc.

The framework of linguistic variable is given as (X, Lx, χ, μx) where

- X denotes the symbolic name of linguistic variable
- Lx is a set of linguistic values that X can take
- χ is the physical domain that defines crisp values
- μx is a fuzzy function that maps linguistic terms of variables to the equivalent crisp values

Fuzzy Propositions

A fuzzy proposition is a statement that drives a fuzzy truth value.

- **Fuzzy Connectives:** Fuzzy connectives are used to join simple fuzzy propositions to make compound propositions. Examples of fuzzy connectives are:
- Negation
- Disjunction
- Conjunction
- Imlication

Fuzzy Rules

The power and flexibility of simple If-Then-Else logic rules is enhanced by adding linguistic parameter.

Fuzzy rules are expressed in the form:

IF variable IS set THEN action

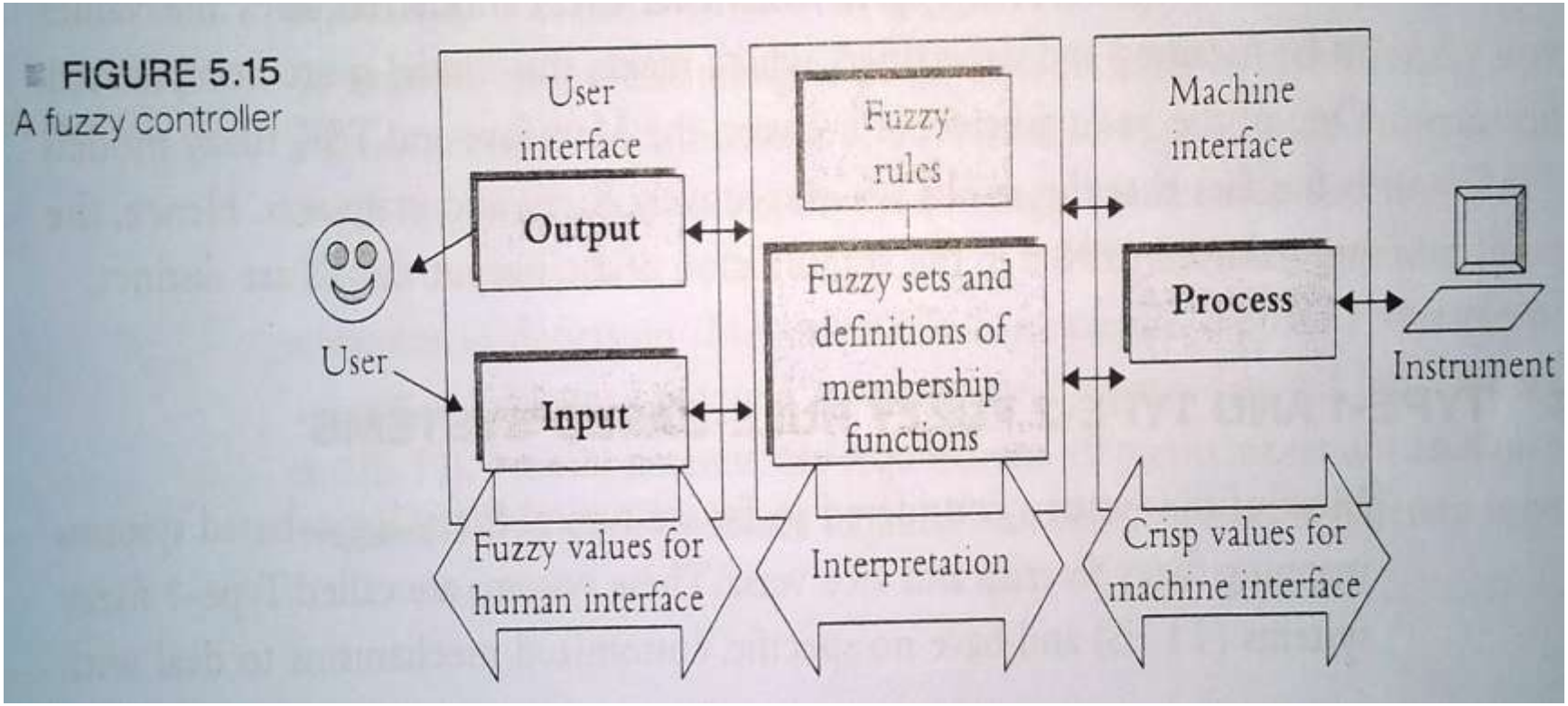
- **Examples:**
  - IF temperature is very cold THEN stop air conditioner
  - IF temperature is normal THEN adjust air conditioner
  - IF temperature is hot THEN start air conditioner

Fuzzy Control System

A fuzzy control system is based on Fuzzy Logic. The process of designing fuzzy control system can be described using following steps

- **Step 1:** Identify the principal input, output and process tasks
- **Step 2:** Identify linguistic variables used and define fuzzy sets and memberships accordingly
- **Step 3:** Use these fuzzy sets and linguistic variables to form procedural rules
- **Step 4:** Determine the defuzzification method
- **Step 5:** Test the system and modify if necessary

Fuzzy Control System



Modeling Fuzzy Systems

Fuzzy system modeling can be pursued using the following steps

- **Step 1:** Choose the relative input and output variables
- **Step 2:** Determine the number of linguistic terms associated with each input/output variables
- **Step 3:** Select a specific type of fuzzy system
- **Step 4:** Design a collection of fuzzy if-then rules. To formulate initial rule base, the input space is divided into multi dimensional partitions and then actions are assigned to each of the partitions

Limitations of Fuzzy Systems

- Fuzzy systems lack the capability of machine learning as-well-as neural network type pattern recognition
- Verification and validation of a fuzzy knowledge-based system require extensive testing with hardware
- Determining exact fuzzy rules and membership functions is a hard task
- Stability is an important concern for fuzzy control

Applications of Fuzzy Logic

- Automatic control system
- Prediction, diagnostic and advisory systems
- User interface and neural language processing
- Domestic appliances and embedded systems
- Soft computing and hybrid systems with artificial neural networks
- Very Large Scale Integrated circuits (VLSI) micro controller
- Fuzzy expert system and fuzzy inference