Q1 Target variable = Return

Total Entropy $= -\frac{4}{10}\log_2\left(\frac{4}{10}\right) - \frac{6}{10}\log_2\left(\frac{6}{10}\right)$

$= 0.971$

Past Trend

Entropy (Positive) $= -\frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\log_2\left(\frac{2}{6}\right)$

$= 0.918$

Entropy (Negative) $= -\frac{0}{4}\log_2\left(\frac{0}{4}\right) - \frac{4}{4}\log_2\left(\frac{4}{4}\right)$

$= 0$

Weighted (Total) Entropy $= \frac{6}{10} \times 0.918 + \cancel{\frac{4}{10} \times 0} \rightarrow 0$

$= 0.551$

Info^n gain $= 0.971 - 0.551$

$= 0.42$

Split Info $= -\frac{6}{10}\log_2\left(\frac{6}{10}\right) - \frac{4}{10}\log_2\left(\frac{4}{10}\right)$

$= 0.971$

Gain Ratio $= \frac{\text{Info}^n \text{ Gain}}{\text{Split Info}} = \frac{0.42}{0.971} = \boxed{0.432}$

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Entropy (low) = $-\frac{2}{6}\log_2\left(\frac{2}{6}\right) - \frac{4}{6}\log_2\left(\frac{4}{6}\right) = 0.918$

Entropy (High) = $-\frac{2}{4}\log_2\left(\frac{2}{4}\right) - \frac{2}{4}\log_2\left(\frac{2}{4}\right) = 1$

Weighted Entropy = 0.951

Info^n gain = 0.971 − 0.951 = 0.02

Split Info^n = 0.971

Gain Ratio = $\frac{0.02}{0.971} = \boxed{0.0201}$

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Entropy (low) = 0

Entropy (High) = $-\frac{4}{7}\log_2\left(\frac{4}{7}\right) - \frac{3}{7}\log_2\left(\frac{3}{7}\right) = 0.985$

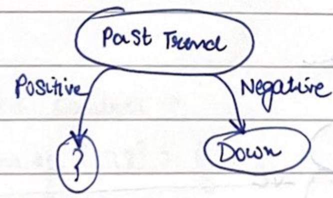
Weighted Entropy = $\frac{7}{10} \times 0.985$
= 0.6895

Info^n gain = 0.971 − 0.6895 = 0.2815

Split Info^n = $-\frac{7}{10}\log_2\left(\frac{7}{10}\right) - \frac{3}{10}\log_2\left(\frac{3}{10}\right) = 0.881$

Gain Ratio = $\frac{0.2815}{0.881} = \boxed{0.32}$

→ Past Trend (0.432) has HIGHEST Gain Ratio. ∴ It is the Root Node.



New data → PAST TREND = POSITIVE

Total Entropy = $-\frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\log_2\left(\frac{2}{6}\right) = 0.918$

Open Interest

Entropy (low) = $-\frac{2}{4}\log_2\left(\frac{2}{4}\right) - \frac{2}{4}\log_2\left(\frac{2}{4}\right) = 1$

Entropy (high) = $-\frac{2}{2}\log_2\left(\frac{2}{2}\right) - \frac{0}{2}\log_2\left(\frac{0}{2}\right) = 0$

Weighted Entropy = $\frac{4}{6}\times 1 + \frac{2}{6}\times 0 = 0.667$

Info gain = $0.918 - 0.667$

$= 0.251$

Split Info = $-\frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\log_2\left(\frac{2}{6}\right) = 0.918$

Gain Ratio = $\frac{0.251}{0.918} = 0.273$

Trading Volume

Entropy (low) = 0

Entropy (high) = 0

Weighted Entropy = 0

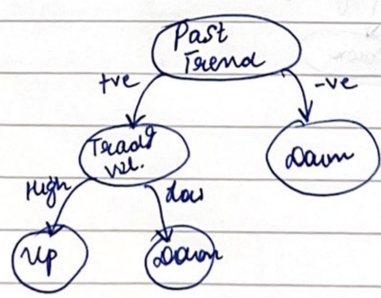
Info gain = $0.918 - 0 = 0.918$

Split Info = $-\frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\log_2\left(\frac{2}{6}\right) = 0.918$

Gain Ratio = $\frac{0.918}{0.918}$

$= 1$

→ Trading Volume is chosen



Final

Decision Tree

Q2 Points = $\{1, 2, 3, 4\}$

Centres given = $(2, 4)$; $\mu_1 = 2$ & $\mu_2 = 4$

Assign pts. to nearest centre → use $|x - \mu_j|$

Pt-1 → μ_1 dist. $= |1-2| = 1$ ⇒ Assign $\{1\}$ to cluster-1

μ_2 dist. $= |1-4| = 3$

Pt-2 → $\mu_1 = |2-2| = 0$ ⇒ Assign $\{2\}$ to cluster 1

$\mu_2 = |2-4| = 2$

Pt-3 → $\mu_1 = |3-2| = 1$ ⇒ Equal dist., prefer smaller index ∴ cluster-1

$\mu_2 = |3-4| = 1$

Pt-4 → $\mu_1 = |4-2| = 2$

$\mu_2 = |4-4| = 0$ ⇒ cluster 2

⇒ Finally, we have cluster 1 = $\{1, 3, 3\}$ with centre $\mu_1 = \{2\}$

" 2 = $\{4\}$ " " $\mu_2 = \{4\}$

Recomputing the centres →

New μ_1 = Mean of $\{1, 3, 3\} = \frac{6}{3} = 2$

New $\mu_2 = 4$

New $(\mu_1, \mu_2) = (2, 4)$

∴ clusters also remain same

⇒ K-Means clustering algo. has converged & will not further change

Checking for local Minima,
consider alternate cluster assignment.

eg-

$C_1 = \{1, 2\}$ $\quad C_2 = \{3, 4\}$

$\mu_1 = \frac{3}{2} = 1.5$ $\quad \mu_2 = \frac{7}{2} = 3.5$

Now we will compare costs of 2 cluster assignments →

$$COST = \sum_{i=1}^{n} |x_i - \mu_j|^2$$

1st $\{1,2,3\}$ $\{4\}$ $[2]$ $[4]$

$$(1-2)^2 + (2-2)^2 + (3-2)^2 + (4-4)^2 = 2$$

2nd $\{1,2\}$ $\{3,4\}$ $[1.5]$ $[3.5]$

$$(1-1.5)^2 + (2-1.5)^2 + (3-3.5)^2 + (4-3.5)^2 = 1$$

→ The alternate cluster assignment has a lower cost,
∴ the initial one was not a local Minimum.

⇒ K-Means algorithm might converge to a ~~[illegible]~~ Solution which is not a local Min. [due to it's Iterative Nature].

Q3 $P(Yes) = \frac{9}{14}$

$P(No) = \frac{5}{14}$

computing likelihoods →

Outlook

	Yes	No	P(Y)	P(N)
Sunny	2	3	2/9	3/5
Overcast	4	0	4/9	0/5
Rainy	3	2	3/9	2/5
Total	9	5	100%	100%

Temperature

	Yes	No	P(Y)	P(N)
Hot	2	2	2/9	2/5
Mild	4	2	4/9	2/5
cool	3	1	3/9	1/5
Total	9	5	100%	100%

Humidity

	Yes	No	P(Y)	P(N)
High	3	4	3/9	4/5
Normal	6	1	6/9	1/5
Total	9	5	100%	100%

Wind

	Yes	No	P(Y)	P(N)
Weak	6	2	6/9	2/5
Strong	3	3	3/9	3/5
Total	9	5	100%	100%

→ Sunny, cool, high, strong

⇓

$P(Yes \mid Sunny, cool, high, Strong) = P(Yes) \times P(Sunny \mid Yes) \times P(cool \mid Yes) \times P(high \mid Yes) \times P(Strong \mid Yes)$

$= \frac{9}{14} \times \frac{2}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{3}{9} = 0.0053$

Illy for No →

$P(No \mid Sunny, cool, high, Strong) = P(No) \times P(Sunny \mid No) \times P(cool \mid No) \times P(high \mid No) \times P(Strong \mid No)$

$= \frac{5}{14} \times \frac{3}{5} \times \frac{1}{5} \times \frac{4}{5} \times \frac{3}{5}$

$= 0.0021$

∴ Prob^y for No is More, ∴ for given example, PLAY TENNIS = NO

Q.4 $k = 2$

Pt. $(x, y) = (170, 57)$

Euclidean distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

x (Height)	y (Weight)	E.D.	Class
167	51	6.7	Underweight
182	62	13	Normal
176	69	13.4	"
173	64	7.6	"
172	65	8.2	"
174	56	4.1	Underweight
* 169	58	1.4	~~Underweight~~ Normal
173	57	3	"
* 170	55	2	"

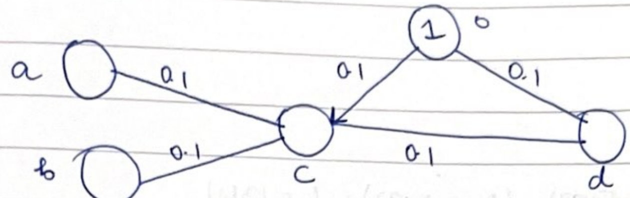
Min. dist. $(k=2)$ are ⇒ 1.4 & 2 that correspond to (169, 58) & (170, 55) whose classes are Normal & Normal

∴ (170, 57) belongs to Normal class

Ans

Q5 $\eta = 0.3$ Iterations $= 2$

$\alpha = 0.9$



Activation fn. $\Rightarrow$ Sigmoid $\Rightarrow \sigma(x) = \frac{1}{1+e^{-x}}$ $\quad$ $MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$

$= \frac{1}{2}\sum_{i=1}^{2}(y_i - \hat{y}_i)^2$

(1,0)

$\rightarrow net_c = 1\times 0.1 + \cancel{0\times 0.1}^{\,0} + 0.1\times 1$

$= 0.2$

$out_c = \frac{1}{1+e^{-0.2}} = 0.5498$

$net_d = 0.1 \times 0.5498 + 0.1\times 1$

$= 0.15498$

$out_d = \frac{1}{1+e^{-0.15498}} = 0.53867$

$$\cancel{\frac{\partial L}{\partial w} = \frac{\partial L}{\partial out} \times \frac{\partial out}{\partial net} = (y_i - \hat{y}_i) \times}$$

$\delta_d = (d_{target} - o_d) \times o_d (1 - o_d)$

$= (1 - 0.54) \times 0.54 \times (1 - 0.54) = 0.114$

$\delta_c = o_c \times (1 - o_c) \times w_{cd} \times \delta_d$

$= 0.0028$

Updating weights $\rightarrow$

$w_{do} = w_{do} + \eta\, \delta_d \times 1 = 0.1 + 0.3 \times 0.114 \times 1 = 0.1342$

$w_{dc} = w_{dc} + \eta\, \delta_d \times out_c = 0.1189$

$w_{co} = w_{co} + \eta \times \delta_c \times 1 = 0.10085$

$w_{ca} = w_{ca} + \eta \times \delta_c \times a = 0.10085$

$w_{cb} = w_{bc} + \eta \times \delta_c \times b = 0.1$

[0,1]

$net_c = 0.2008$

$out_c = 0.55$

$net_d = 0.1996$

$out_d = 0.5497$

$\delta d = 0.5497 \times (1 - 0.5497) \times (0 - 0.5497) = (-0.1361)$

$\delta_c = 0.55 \times (1 - 0.55) \times 0.1189 \times (-0.1361) = (-0.004)$

Updating weights $\rightarrow$

$w_{do} = 0.1342 - 0.01 = 0.1242$

$w_{dc} = 0.1189 - 0.0055 = 0.1134$

$w_{co} = 0.10085 - 0.0004 = 0.10085$

$w_{ca} = 0.10085 + 0.00086 = 0.1016$

$w_{cb} = 0.1 - 0.0012 = 0.0988$