Q-1 Let (X1, X2, ---) be a random sample of size n taken from a Normal population with parameters mean $= \theta_1$ and variance $= \theta_2$. Find the Maximum likelihood estimates of these two parameters

Sol →
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$x_1, x_2 --- x_n$ → sample of size n

$$L(x_1, x_2, --- x_n) = f(x_1) \cdot f(x_2) \cdot --- \cdot f(x_n)$$

$$L \Rightarrow \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Taking ln on both sides

$$\ln(L) = \frac{-n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma^2} \right) \quad -(1)$$

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Take partial derivative w.r.t μ of eq (1)

$$\frac{d\ln(L)}{d\mu} = 0 + \sum_{i=1}^{n} -2\frac{(x_i-\mu)}{2\sigma^2} = 0$$

$$= \sum_{i=1}^{n}(x_i-\mu) = 0$$

$$\Rightarrow n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

Hence $\theta_1 = \bar{x}$ is therefore sample mean.

Taking derivate w.r.t σ^2 of eq (1)

$$\frac{d\ln(L)}{d\sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2\sigma^2} = 0$$

$$-n + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{\sigma^2} = 0$$

$$n = \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n}(x_i-\mu)^2\right)$$

Hence $\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$

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Q-2 Let $x_1, x_2, \ldots x_n$ be a random sample from $B(m, \theta)$ distribution, where $\theta \in \Theta = (0,1)$ is unknown and 'm' is a known positive integer. Compute value of θ using M.L.E.

Sol → Binomial Distribution → $^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

Taking log on both sides

$$\log L = \sum_{i=1}^{n} \left(\log({}^nC_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right)$$

$$\log L = \sum_{i=1}^{n} \log({}^nC_{x_i}) + \log\theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum_{i=1}^{n} (n-x_i)$$

Diff w.r.t θ

$$\frac{d(\log L)}{d\theta} = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{1}{1-\theta} \sum (n-x_i) = 0$$

$$\frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta}$$

$$\boxed{\theta = \frac{\sum x_i}{n^2}}$$