



$$\begin{aligned}
 \mathbf{C}_{AD} &= \mathbf{C}_{AB} \mathbf{C}_{BC} \mathbf{C}_{CD} \Rightarrow {}_A \mathbf{r} = \mathbf{C}_{AD} {}_D \mathbf{r} \\
 &= \begin{pmatrix} \cos z & -\sin z & 0 \\ \sin z & \cos z & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos y & 0 & \sin y \\ 0 & 1 & 0 \\ -\sin y & 0 & \cos y \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos x & -\sin x \\ 0 & \sin x & \cos x \end{pmatrix} \\
 &= \begin{pmatrix} \cos(y) \cos(z) & \cos(z) \sin(x) \sin(y) - \cos(x) \sin(z) & \sin(x) \sin(z) + \cos(x) \cos(z) \sin(y) \\ \cos(y) \sin(z) & \cos(x) \cos(z) + \sin(x) \sin(y) \sin(z) & \cos(x) \sin(y) \sin(z) - \cos(z) \sin(x) \\ -\sin(y) & \cos(y) \sin(x) & \cos(x) \cos(y) \end{pmatrix}
 \end{aligned}$$