Introuduction: Lydian-Groups

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Groups

Group G is a set of elements g_i satisfying the four conditions below, relative to some binary operation. We often use multiplicative notation ($g_1 g_2$) or additive notation ($g_1 + g_2$) to represent the binary operation. For definiteness, we use multiplicative notation below; however, one could replace xy with $b(x, y)$ below.

If the elements of G satisfy the following four properties, then G is a group.

1. $\exists e \in G \mid \forall g \in G : eg = ge = g$. (Identity.) We often write $e = 1$ for multiplicative groups, and $e = 0$ for additive groups.
2. $\forall x, y, z \in G :\ (xy)z = x(yz)$. (Associativity.)
3. $\forall x \in G, \exists y \in G \mid xy = yx = e$. (Inverse.) We write $y = x^{-1}$ for multiplication, $y = -x$ for addition.
4. $\forall x, y \in G : xy \in G$. (Closure.)

Paper outline

- Prelude: The audience is both mathematicians and musicians.
- Introduction: Mathematical notation and Tonality
- $\mathbb{Z}/12\mathbb{Z}$ and the Chromatic Scale