# Computational Eigenvector Simulation

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## Notation

Let $\mathcal{M}_F[M \times N]$ be the set of $M \times N$ matrice over the field $F$.

- Suppose $Q$ is some $M \times M$ square matrix on a field $F$. Denote this $Q \in \mathcal{M}_F[M^2]$.
- Suppose $v$ is a row vector on a field $F$ of size $M$. Denote this by saying $v \in \mathcal{V}_F^{(R)}[M]$.

Note that $\mathcal{V}_F^{(R)}[M]$ is equivalent to $\mathcal{M}_F[1 \times M]$.

- Suppose $v$ is a column vector on a field $F$ of size $M$. Denote this by saying $v \in \mathcal{V}_F^{(C)}[M]$.

Note that $\mathcal{V}_F^{(C)}[M]$ is equivalent to $\mathcal{M}_F[M \times 1]$.

## Definitions

**Definition.** (**$\epsilon -$ equivalence**) Let $F$ be a field, and $\varepsilon \in \mathbb{R}^+$. Suppose we have vectors $v, v' \in \mathcal{V}_F[M]$. Then, $v \sim_\epsilon v'$ if $||v - v'|| < \epsilon$ where $|| \cdot ||$ is the norm on $F$.

**Definition.** (**Q $-$ evolution sequence**) Let $\pi \in \mathcal{V}[M]$ be a vector of size $M$. Then, the $Q$-evolution sequence of $\pi$ is given by:

$$\text{Seq}(Q, \pi) = \{\pi'_n\}_{n \in \mathbb{N}} \text{ where } \pi'_n = \pi Q^n$$

**Definition.** (**$\epsilon -$ convergence**) Let $\varepsilon \in \mathbb{R}^+$ Suppose we have a sequence $\text{Seq}(Q, \pi)$. Then, $\text{Seq}(Q, \pi)$ $\varepsilon$-converges at $N$ if:

$$\exists N \in \mathbb{N} : \forall n \geq N \mid \pi'_N \sim_\epsilon \pi'_n$$

**Definition.** (**$\sigma -$ perturbance**) Let $\pi \in \mathcal{V}[M]$ be a vector of size $M$. Then, a $\sigma -$ perturbance of $\pi$ with perturbance vector $\varepsilon$ is given by the vector $\pi' = \pi + \varepsilon$ where:

$$\varepsilon = (\varepsilon_i)_{i=1}^M \text{ where } \epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

**Definition.** (**localized $\sigma -$perturbance**) Let $\pi \in \mathcal{V}[M]$ be a vector of size $M$. Then, if we have a dimension $d \in \{1, \ldots, M\}$, a localized$\sigma -$perturbance of $\pi$ in $d$ with perturbance vector $\varepsilon$ is given by the vector $\pi' = \pi + \varepsilon$ where:

$$\varepsilon = (\varepsilon_j)_{j=1}^M \text{ where } \varepsilon_d \sim \mathcal{N}(0, \sigma^2) \text{ and } \forall j \neq d : \varepsilon_j = 0$$

## Monte Carlo Batches

**Definition.** (**Monte Carlo $\alpha$-Batch**) Let $\alpha \in \mathbb{R}^+$. Take a batch $\mathcal{B}$ of $B$ row vectors $\pi \sim \text{Unif}(-\alpha, \alpha)$ and denote it as:

$$\mathcal{B} = \{\pi_i\}_{i=1}^B \text{ where } \pi_i \sim \text{Unif}(-\alpha, \alpha)$$

**Definition.** (**Evolved $\alpha$-Batch**) To simulate the eigenvectors, we will take the $Q -$ evolution sequences on each of our $\pi_i \in \mathcal{B}$. So, our simlated batch of evolution sequences is given by:

$$\text{EvolBatch}(Q, \mathcal{B}) = \{\{\pi'_{i,n}\}_{n \in \mathbb{N}}\}_{i=1}^B$$

Question: Which $\alpha$ is sufficient to obtain eigenvectors that yield all the eigenvalues $\lambda_i$ of $Q$?