Eigenvectors of Symmetric Matrices

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Computational Evidence: Real Symmetric Matrices have Real Eigenvectors

```
unif_fpos <- function(M,f=T,ep){
  # unless specifically initialized, a random fraction will be chosen
  if(f){
    f <- runif(1,0,1)
    paste("f: ",f,sep="")
  }
  b <- f
  a <- (f-1)
  dist <- data.frame(x = runif(M**2, ep*a, ep*b))
  dist <- dist %>% mutate(x_neg = ifelse(x < 0,yes = 1, no = 0))
  dist
}
```

```
make_symm <- function(dist){
  N <- sqrt(length(dist$x))
  P <- matrix(data = dist$x, nrow = N, ncol = N)
  LT <- lower.tri(P)
  UT <- upper.tri(P)
  P[LT] <- P[UT]
  P
}
```

```
# Uniform matrix with symm = T
RM_symm(5,0.5,10)
```

```
##           [,1]      [,2]        [,3]       [,4]      [,5]
## [1,]  2.476556 -3.565007  3.77356628 -1.2593722  3.484690
## [2,] -3.565007  3.923809  1.66278634  1.0079172 -2.694560
## [3,]  3.773566  1.662786 -0.01750584  4.4357300  4.873317
## [4,] -1.259372  1.007917  4.43572997  0.2892768  1.026553
## [5,]  3.484690 -2.694560  4.87331721  1.0265526  2.021992
```

Simulation

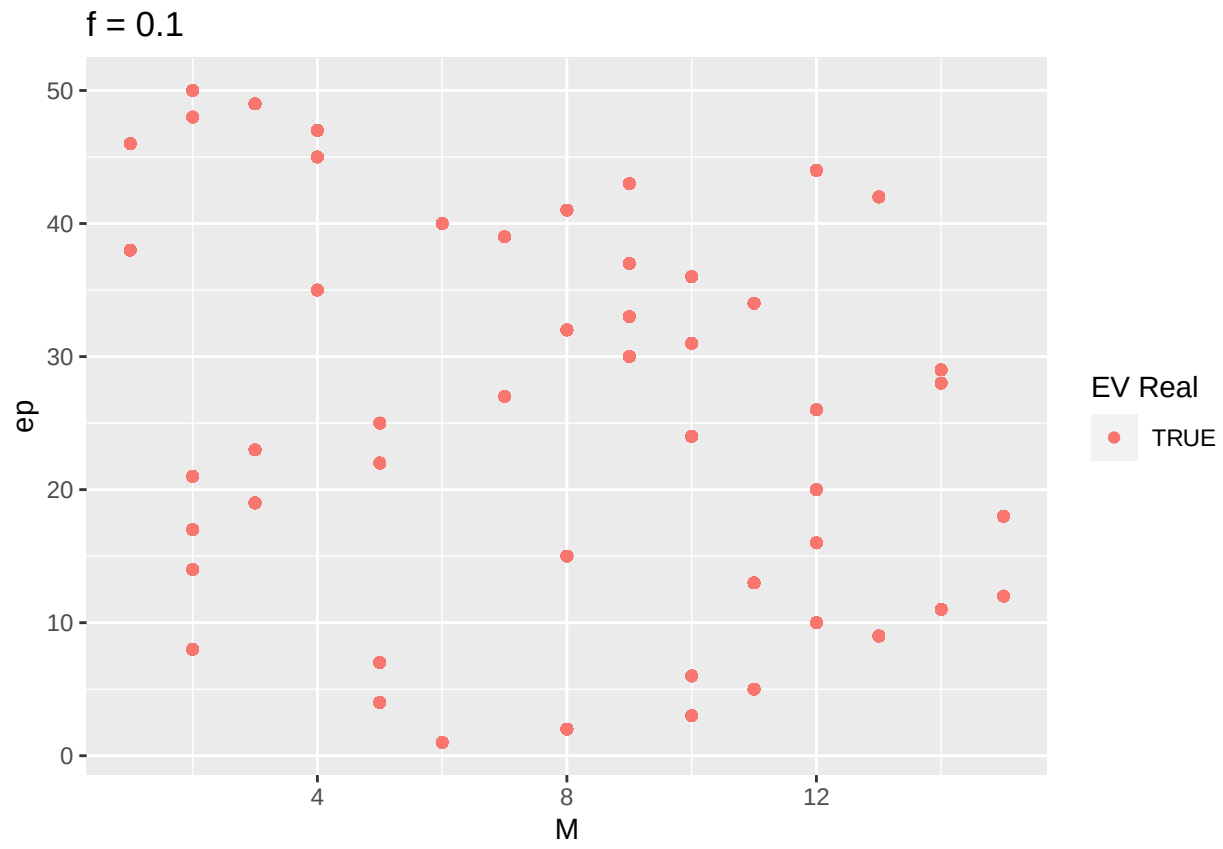
```
simulate_by_f <- function(f,M_max,ep_max,draws){
  M_vec <- sample(1:M_max, draws, replace = T)
  ep_vec <- sample(1:ep_max, draws, replace = F)
  table <- data.frame(M = M_vec, ep = rep(ep_vec,length(M_vec)))
```

```
  bool_vec <- rep(NA, length(table$M))

  for(i in 1:length(table$M)){
    S_curr <- RM_symm(table$M[i],f,table$ep[i])
    bool_vec[i] <- check_real_eigenvectors(evec_frame(S_curr))
  }
  cbind(table,bool_vec)
}

plot_f_table <- function(table, f){
  ggplot() +
    geom_point(data = table, aes(x=M, y=ep, color = factor(bool_vec))) +
    labs(color = "EV Real", title = paste("f = ",f,sep=""))
}
```

```
table <- simulate_by_f(f = 0.1, M_max = 15, ep_max = 50, draws = 50)
head(table)
```

```
##    M ep bool_vec
## 1 12 20     TRUE
## 2 12 44     TRUE
## 3  9 43     TRUE
## 4 11  5     TRUE
## 5 14 28     TRUE
## 6 10 31     TRUE
```

```
plot_f_table(table, f = 0.1)
```



```
#table <- simulate_by_f(f = 0.5, M_max = 15, ep_max = 250, draws = 250)
#head(table)
#plot_f_table(table, f = 0.5)
```

```
#table <- simulate_by_f(f = 0.9, M_max = 15, ep_max = 250, draws = 250)
#head(table)
#plot_f_table(table, f = 0.9)
```