Sparsity Analysis

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Generating Random Matrices

```r
# generates rows of size P which are valid probability distributions
r_sparse <- function(M,p){
  prob <- runif(M,0,1)
  num_zeros <- rbinom(1,M,p)
  choices <- sample(1:M, num_zeros)
  prob[choices] <- 0
  prob/sum(prob) # return normalized random row vector
}
```

```r
# initialize random P
rand_M <- function(M,p,row_fxn){
  P <- matrix(rep(NA, M * M), ncol = M)  # create transition matrix
  for(i in 1:M){P[i,] = row_fxn(M,p)}
  #print(P)
  P
}
```
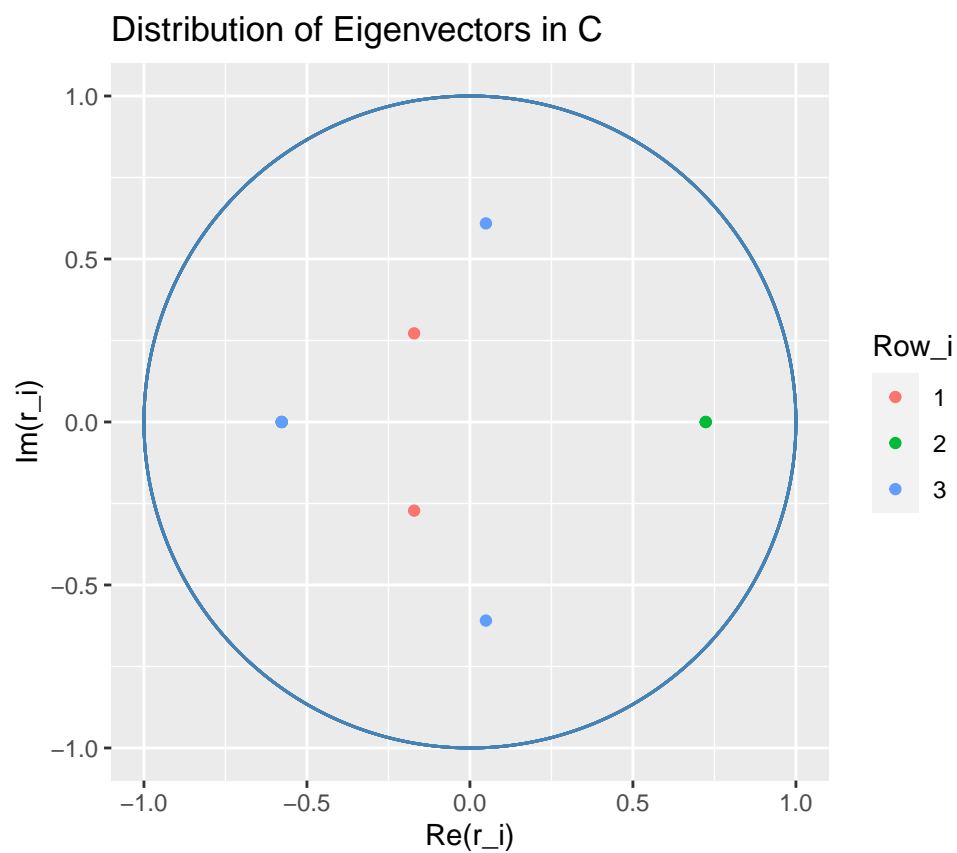
Eigenvectors

```r
eigen_frame <- function(P){
  #print(P)
  M <- length(P[1,])
  eigenvectors <- data.frame(eigen(P)[2])
  complex <- matrix(rep(NA,3*M*M), ncol = 3) # set 3 to hold (re,im) pair and whose row it belongs to
  colnames(complex) <- c("Re","Im","row_i")
  for(i in 1:M){
    for(j in 1:M){
      curr <- eigenvectors[i,j]
      complex[ M*(i-1) + j, ] <- c(round(Re(curr),5),round(Im(curr),5),i)
    }
  }
  data.frame(complex)
}
```

```r
M_vec <- c(3,5,10)
p_vec <- c(0.1,0.5,0.6)
c(M1,M2,M3) %<-% M_vec
c(p1,p2,p3) %<-% p_vec
P_vec1 <- matrix(c(rand_M(M1,p1,r_sparse),
                   rand_M(M1,p1,r_sparse),
                   rand_M(M1,p1,r_sparse)),
               nrow = M_vec[1])
P_vec2 <- matrix(c(rand_M(M2,p2,r_sparse),
                   rand_M(M2,p2,r_sparse),
                   rand_M(M2,p2,r_sparse)),
               nrow = M_vec[2])
P_vec3 <- matrix(c(rand_M(M3,p3,r_sparse),
                   rand_M(M3,p3,r_sparse),
                   rand_M(M3,p3,r_sparse)),
               nrow = M_vec[3])
```

```
##           [,1]      [,2]      [,3]
## [1,] 0.4774419 0.1936672 0.3288909
## [2,] 0.5554326 0.3670818 0.0774856
## [3,] 0.4194536 0.3755488 0.2049976
```
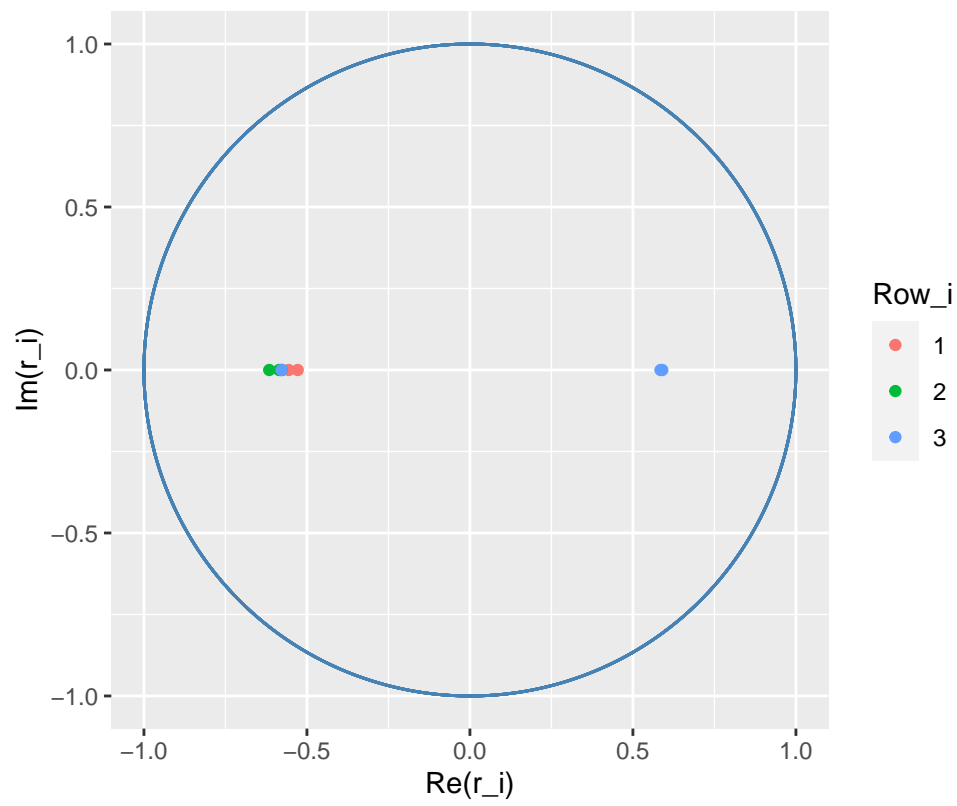
Distribution of Eigenvectors in C
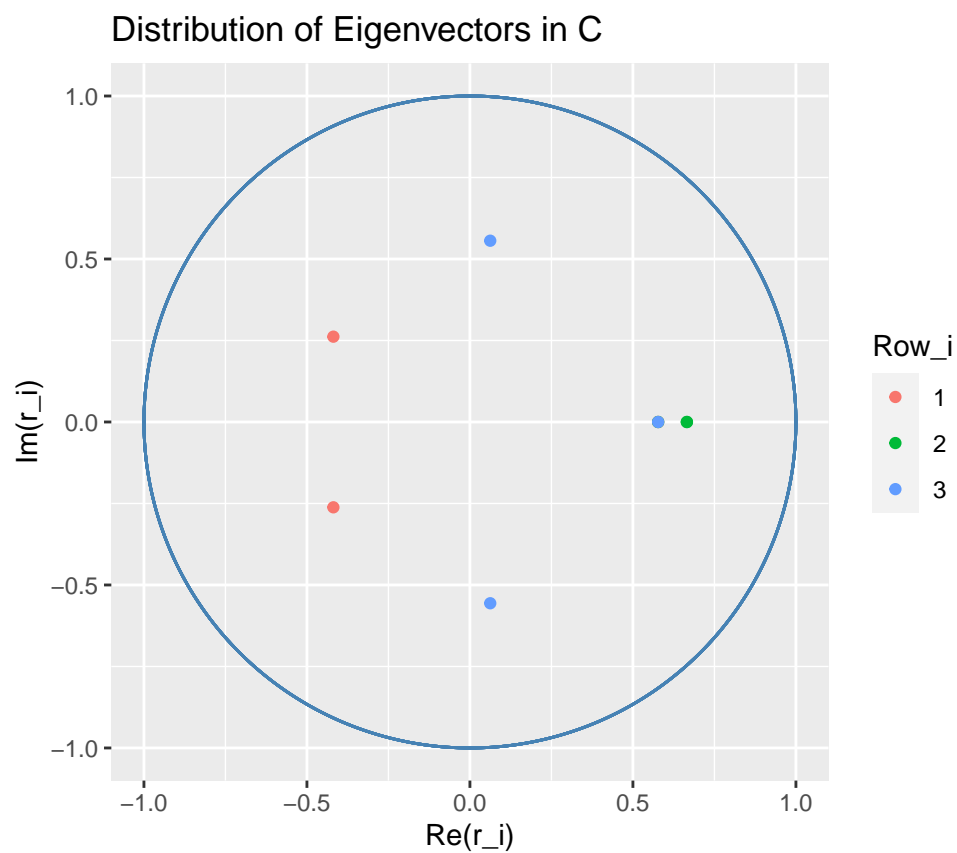

```
##           [,1]        [,2]      [,3]
## [1,] 0.6284750 0.008970145 0.3625549
## [2,] 0.8035152 0.196484802 0.0000000
## [3,] 0.4609940 0.351086196 0.1879198
```
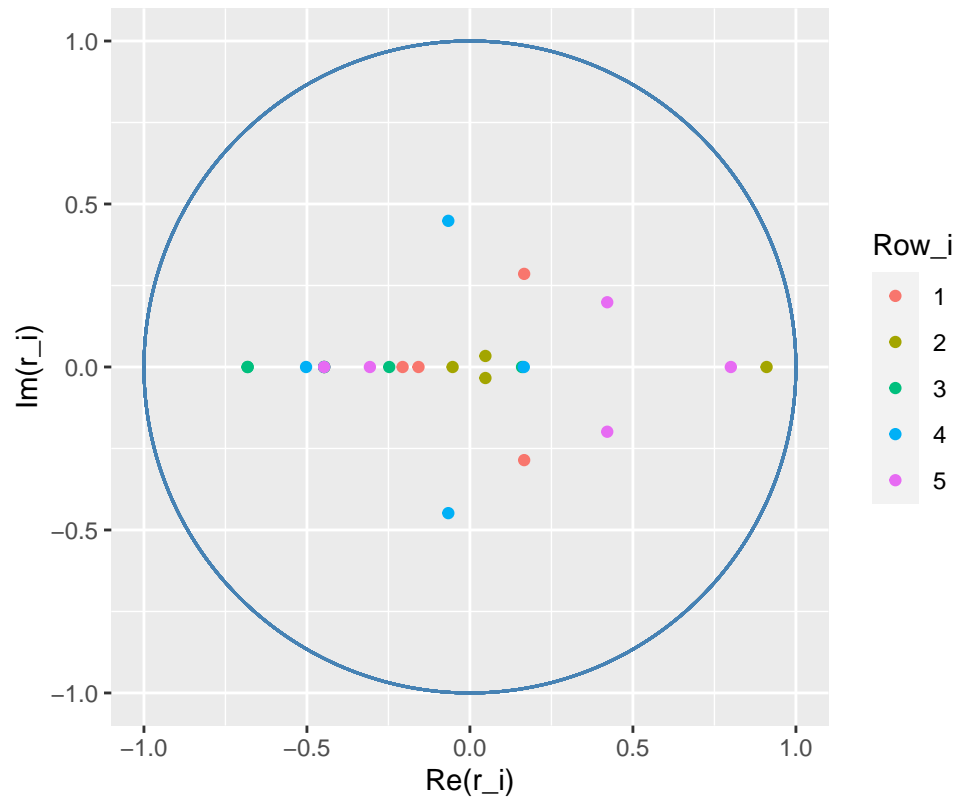
Distribution of Eigenvectors in C

```
##              [,1]        [,2]        [,3]
## [1,] 0.08294734  0.34918366  0.5678690
## [2,] 0.21475033  0.21687195  0.5683777
## [3,] 0.49264028  0.09800048  0.4093592
```

# Distribution of Eigenvectors in C


```
##             [,1]      [,2]      [,3]     [,4]      [,5]
## [1,] 0.2488362 0.0000000 0.4718863 0.000000 0.2792775
## [2,] 0.2616227 0.3831388 0.1867248 0.000000 0.1685137
## [3,] 0.0000000 0.1002873 0.0000000 0.497656 0.4020567
## [4,] 0.4589354 0.2385452 0.0000000 0.000000 0.3025194
## [5,] 0.8062759 0.0000000 0.1937241 0.000000 0.0000000
```

Distribution of Eigenvectors in C
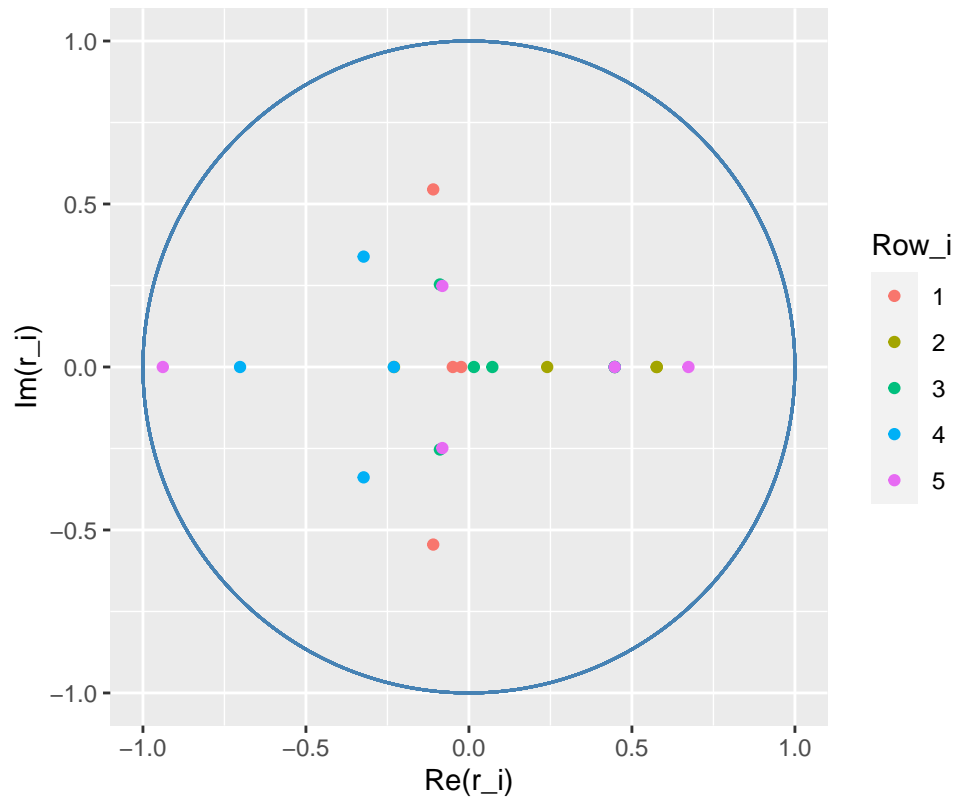
```
##             [,1] [,2]      [,3]      [,4]      [,5]
## [1,] 0.3904948    0 0.0000000 0.2285421 0.3809631
## [2,] 0.0000000    0 0.0000000 1.0000000 0.0000000
## [3,] 0.0000000    1 0.0000000 0.0000000 0.0000000
## [4,] 0.0000000    1 0.0000000 0.0000000 0.0000000
## [5,] 0.0000000    0 0.5665908 0.0000000 0.4334092
```

Distribution of Eigenvectors in C
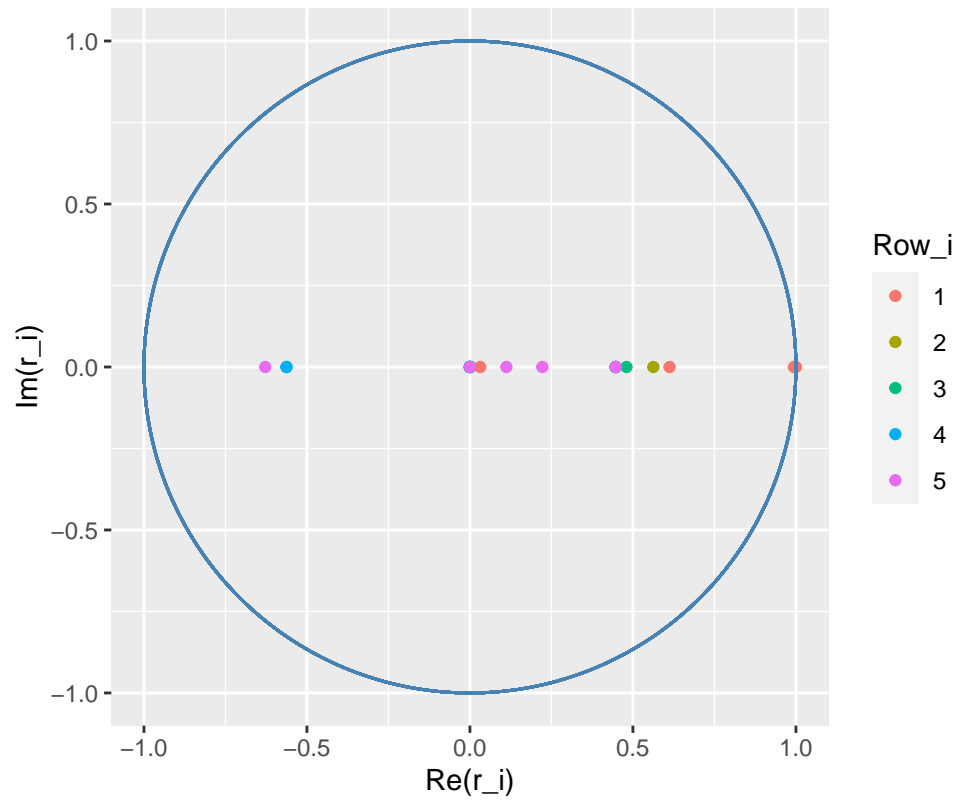
```
##           [,1]      [,2]      [,3]      [,4]       [,5]
## [1,] 0.0000000 0.5172157 0.3110085 0.0000000 0.17177579
## [2,] 0.0000000 0.0000000 1.0000000 0.0000000 0.00000000
## [3,] 0.4152339 0.0000000 0.5847661 0.0000000 0.00000000
## [4,] 0.5116805 0.0000000 0.4104331 0.0000000 0.07788647
## [5,] 0.0000000 0.4036543 0.0000000 0.2569499 0.33939585
```

Distribution of Eigenvectors in C

```
##               [,1]        [,2]      [,3]        [,4]      [,5]      [,6]
##  [1,] 0.0000000000 0.0000000 0.2165838 0.000000000 0.0000000 0.0000000
##  [2,] 0.4272348582 0.0000000 0.0000000 0.175838754 0.0000000 0.3448924
##  [3,] 0.1302868385 0.2830072 0.1357955 0.040106061 0.0000000 0.2556560
##  [4,] 0.0000000000 0.0000000 1.0000000 0.000000000 0.0000000 0.0000000
##  [5,] 0.2722762055 0.2415584 0.0000000 0.218760264 0.0000000 0.0000000
##  [6,] 0.0000000000 0.2026573 0.1517395 0.008956059 0.0000000 0.0000000
##  [7,] 0.0290946449 0.0000000 0.0000000 0.087968572 0.4188434 0.0000000
##  [8,] 0.0001737748 0.1091687 0.2228638 0.000000000 0.0000000 0.1905630
##  [9,] 0.1272172839 0.0000000 0.1915671 0.146020312 0.0000000 0.4211869
## [10,] 0.0000000000 0.0000000 0.2308288 0.035843901 0.0000000 0.1435018
##              [,7]       [,8]      [,9]      [,10]
##  [1,] 0.0000000 0.00000000 0.2031556 0.58026067
##  [2,] 0.0000000 0.05203396 0.0000000 0.00000000
##  [3,] 0.0000000 0.00000000 0.1551484 0.00000000
##  [4,] 0.0000000 0.00000000 0.0000000 0.00000000
##  [5,] 0.0000000 0.26740516 0.0000000 0.00000000
##  [6,] 0.1133967 0.00000000 0.3470333 0.17621711
##  [7,] 0.4640934 0.00000000 0.0000000 0.00000000
##  [8,] 0.0000000 0.23977704 0.2374537 0.00000000
##  [9,] 0.0000000 0.00000000 0.0000000 0.11400841
## [10,] 0.0000000 0.19533214 0.3032385 0.09125484
```

Distribution of Eigenvectors in C
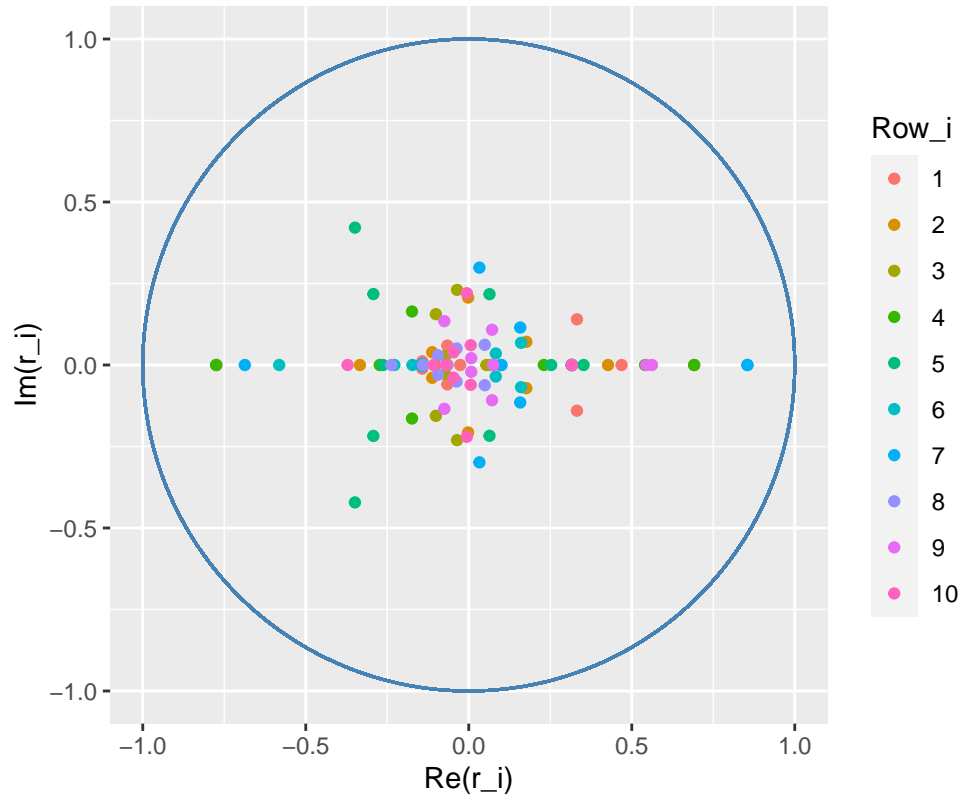
```
##              [,1]       [,2]       [,3]       [,4]       [,5]       [,6]       [,7]
##  [1,]  0.0000000 0.00000000 0.2749984 0.00000000 0.00000000 0.21723228 0.0000000
##  [2,]  0.0000000 0.18283420 0.0000000 0.45167913 0.05913724 0.00000000 0.0000000
##  [3,]  0.0000000 0.00000000 0.2519458 0.00000000 0.00000000 0.00000000 0.3070667
##  [4,]  0.0000000 0.34737534 0.0000000 0.00000000 0.00000000 0.00000000 0.0000000
##  [5,]  0.0000000 0.06409141 0.1472019 0.16184537 0.00000000 0.01963845 0.1763508
##  [6,]  0.2626660 0.00000000 0.0000000 0.07509928 0.14949647 0.13978114 0.0000000
##  [7,]  0.0000000 0.00000000 0.1534744 0.00000000 0.24892443 0.23024963 0.0000000
##  [8,]  0.0000000 0.46790438 0.0000000 0.09591908 0.43617654 0.00000000 0.0000000
##  [9,]  0.2336707 0.27401055 0.0000000 0.12993232 0.00000000 0.36238639 0.0000000
## [10,]  0.1219123 0.00000000 0.3567102 0.17563171 0.34574580 0.00000000 0.0000000
##              [,8]       [,9]      [,10]
##  [1,] 0.00000000 0.00762614 0.50014318
##  [2,] 0.02850141 0.27784801 0.00000000
##  [3,] 0.27246036 0.08412790 0.08439918
##  [4,] 0.65262466 0.00000000 0.00000000
##  [5,] 0.22224396 0.20862813 0.00000000
##  [6,] 0.12829329 0.00000000 0.24466378
##  [7,] 0.19439249 0.17295903 0.00000000
##  [8,] 0.00000000 0.00000000 0.00000000
##  [9,] 0.00000000 0.00000000 0.00000000
## [10,] 0.00000000 0.00000000 0.00000000
```

Distribution of Eigenvectors in C
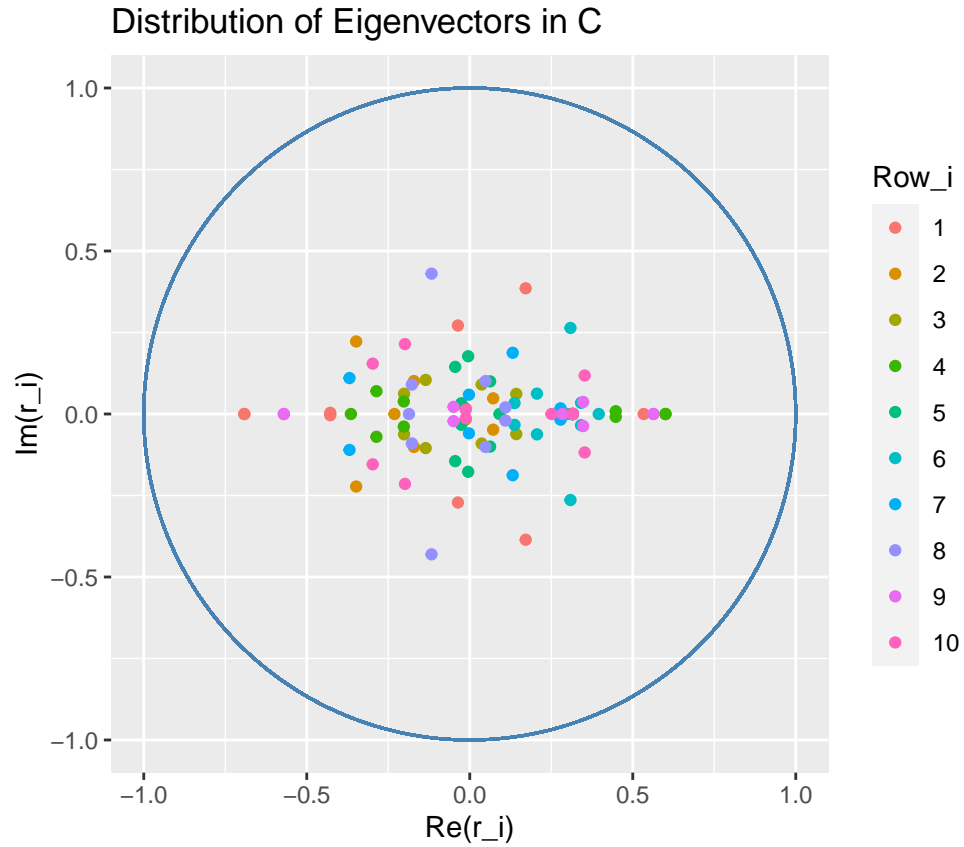
```
##            [,1]      [,2]      [,3]      [,4]      [,5]       [,6]      [,7]
##  [1,] 0.288821060 0.0000000 0.0000000 0.0000000 0.2873682 0.00000000 0.0000000
##  [2,] 0.000000000 0.3049687 0.0000000 0.0189521 0.0000000 0.00000000 0.1943063
##  [3,] 0.037417968 0.0000000 0.1847922 0.0000000 0.0000000 0.23881747 0.0000000
##  [4,] 0.000000000 0.0000000 0.0000000 0.0000000 0.3706734 0.08876368 0.2698086
##  [5,] 0.000000000 0.0000000 0.0000000 0.5307530 0.0000000 0.46924696 0.0000000
##  [6,] 0.000000000 0.0000000 0.0000000 0.4043257 0.0000000 0.00000000 0.4082731
##  [7,] 0.000000000 0.0000000 0.0000000 0.0000000 0.0000000 0.00000000 0.4773937
##  [8,] 0.003077017 0.0000000 0.0000000 0.0000000 0.0000000 0.48966326 0.0000000
##  [9,] 0.145891352 0.0000000 0.0000000 0.0000000 0.0000000 0.19835819 0.4510186
## [10,] 0.358971382 0.2711791 0.0000000 0.0459161 0.0000000 0.00000000 0.0000000
##            [,8]       [,9]      [,10]
##  [1,] 0.03454322 0.00000000 0.3892675
##  [2,] 0.36975247 0.00000000 0.1120204
##  [3,] 0.10033243 0.21229328 0.2263466
##  [4,] 0.27075437 0.00000000 0.0000000
##  [5,] 0.00000000 0.00000000 0.0000000
##  [6,] 0.12649593 0.06090526 0.0000000
##  [7,] 0.52260632 0.00000000 0.0000000
##  [8,] 0.00000000 0.00000000 0.5072597
##  [9,] 0.00000000 0.20473185 0.0000000
## [10,] 0.00000000 0.15769217 0.1662412
```

Distribution of Eigenvectors in C