Sparsity Analysis

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Generating Random Matrices

```
# generates rows of size P which are valid probability distributions
r_sparse <- function(M,p){
  prob <- runif(M,0,1)
  num_zeros <- rbinom(1,M,p)
  choices <- sample(1:M, num_zeros)
  prob[choices] <- 0
  prob/sum(prob) # return normalized random row vector
}
```

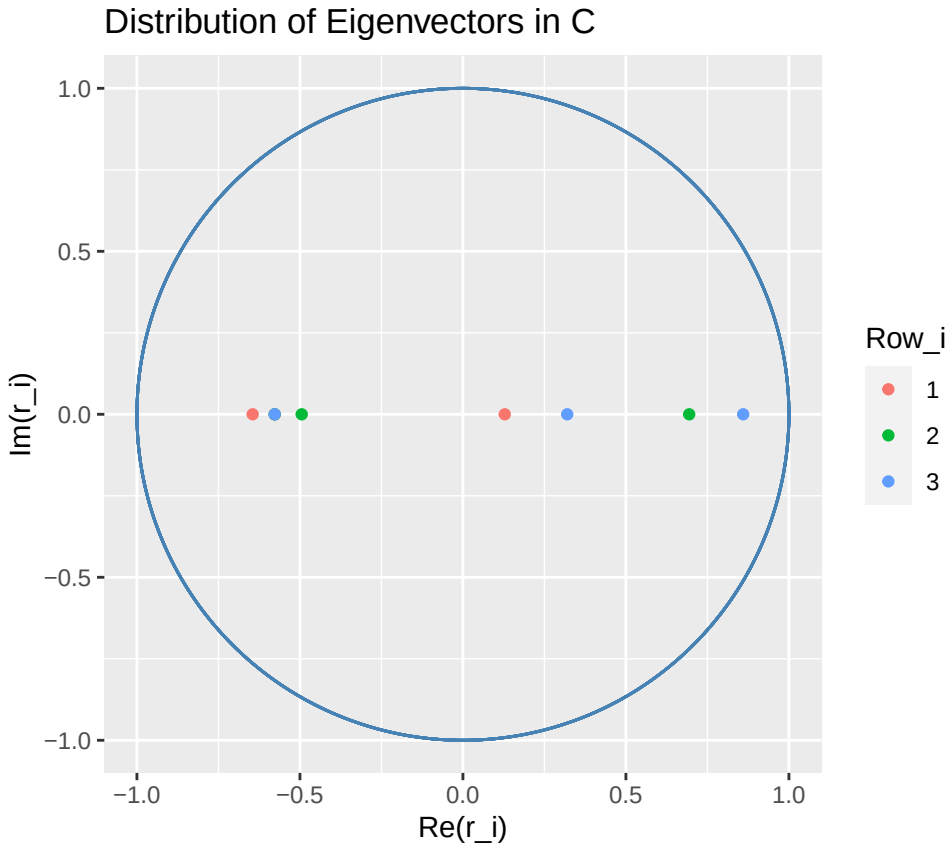
```
# initialize random P
rand_M <- function(M,p,row_fxn){
  P <- matrix(rep(NA, M * M), ncol = M)  # create transition matrix
  for(i in 1:M){P[i,] = row_fxn(M,p)}
  #print(P)
  P
}
```

Eigenvectors

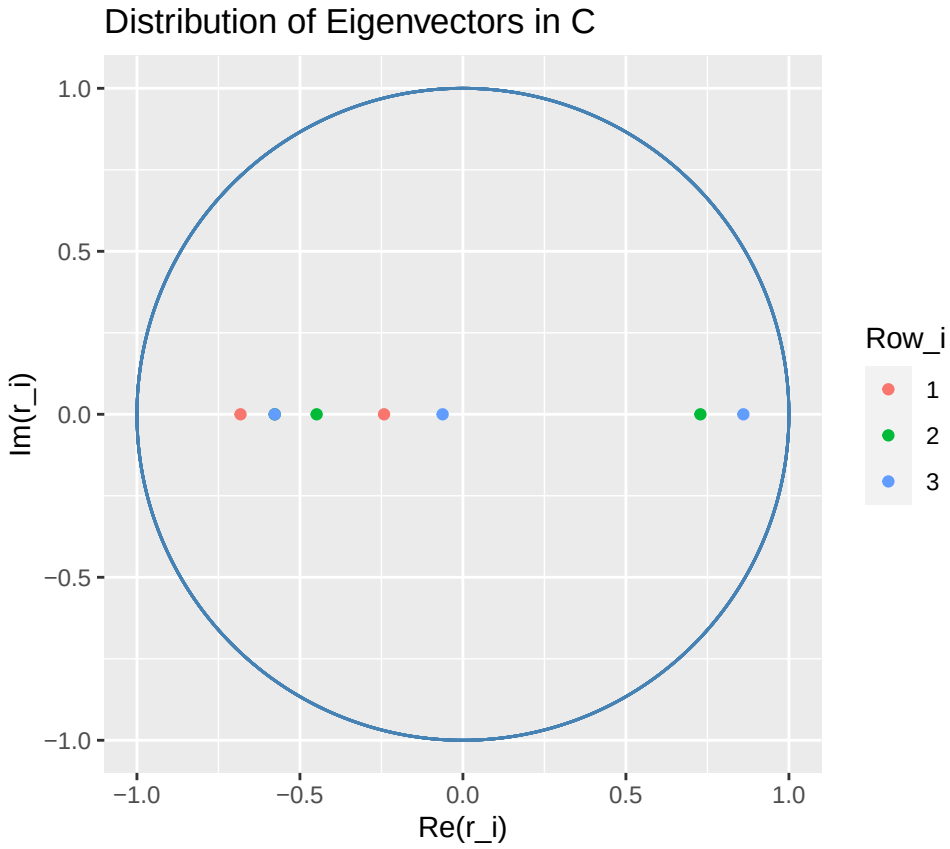
```
eigen_frame <- function(P){
  #print(P)
  M <- length(P[1,])
  eigenvectors <- data.frame(eigen(P)[2])
  complex <- matrix(rep(NA,3*M*M), ncol = 3) # set 3 to hold (re,im) pair and whose row it belongs to
  colnames(complex) <- c("Re","Im","row_i")
  for(i in 1:M){
    for(j in 1:M){
      curr <- eigenvectors[i,j]
      complex[ M*(i-1) + j, ] <- c(round(Re(curr),5),round(Im(curr),5),i)
    }
  }
  data.frame(complex)
}
```

```
M_vec <- c(3,5,10)
p_vec <- c(0.1,0.5,0.6)
c(M1,M2,M3) %<-% M_vec
c(p1,p2,p3) %<-% p_vec
P_vec1 <- matrix(c(rand_M(M1,p1,r_sparse),
                   rand_M(M1,p1,r_sparse),
                   rand_M(M1,p1,r_sparse)),
                 nrow = M_vec[1])
P_vec2 <- matrix(c(rand_M(M2,p2,r_sparse),
                   rand_M(M2,p2,r_sparse),
                   rand_M(M2,p2,r_sparse)),
                 nrow = M_vec[2])
P_vec3 <- matrix(c(rand_M(M3,p3,r_sparse),
                   rand_M(M3,p3,r_sparse),
                   rand_M(M3,p3,r_sparse)),
                 nrow = M_vec[3])
```

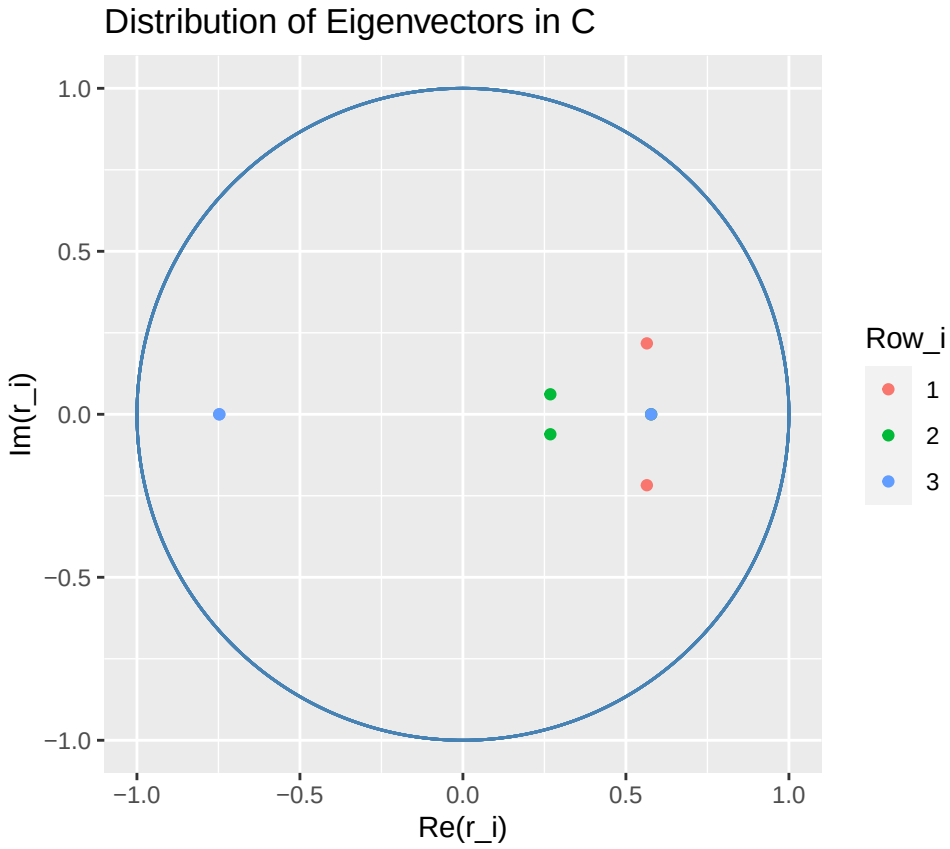
```
##         Re Im row_i
## 1 -0.57735  0     1
## 2 -0.64489  0     1
## 3  0.12835  0     1
## 4 -0.57735  0     2
## 5  0.69411  0     2
## 6 -0.49446  0     2
## 7 -0.57735  0     3
## 8  0.31990  0     3
## 9  0.85967  0     3
##           [,1]      [,2]      [,3]
## [1,] 0.7366127 0.2633873 0.0000000
## [2,] 0.1756440 0.4384649 0.3858911
## [3,] 0.4101231 0.5898769 0.0000000
```



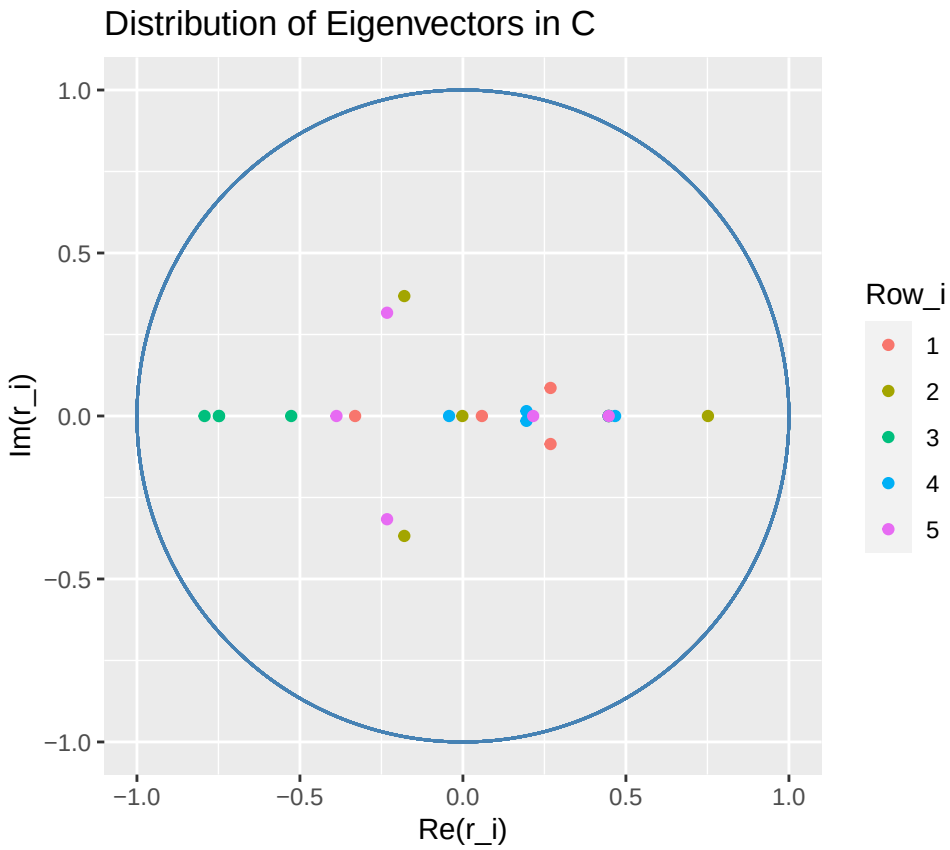
```
##         Re Im row_i
## 1 -0.57735  0     1
## 2 -0.24197  0     1
## 3 -0.68225  0     1
## 4 -0.57735  0     2
## 5 -0.44874  0     2
## 6  0.72845  0     2
## 7 -0.57735  0     3
## 8  0.86028  0     3
## 9 -0.06233  0     3
##           [,1]      [,2]       [,3]
## [1,] 0.2107300 0.4119023 0.37736772
## [2,] 0.3885659 0.2041796 0.40725446
## [3,] 0.4946286 0.4819146 0.02345679
```



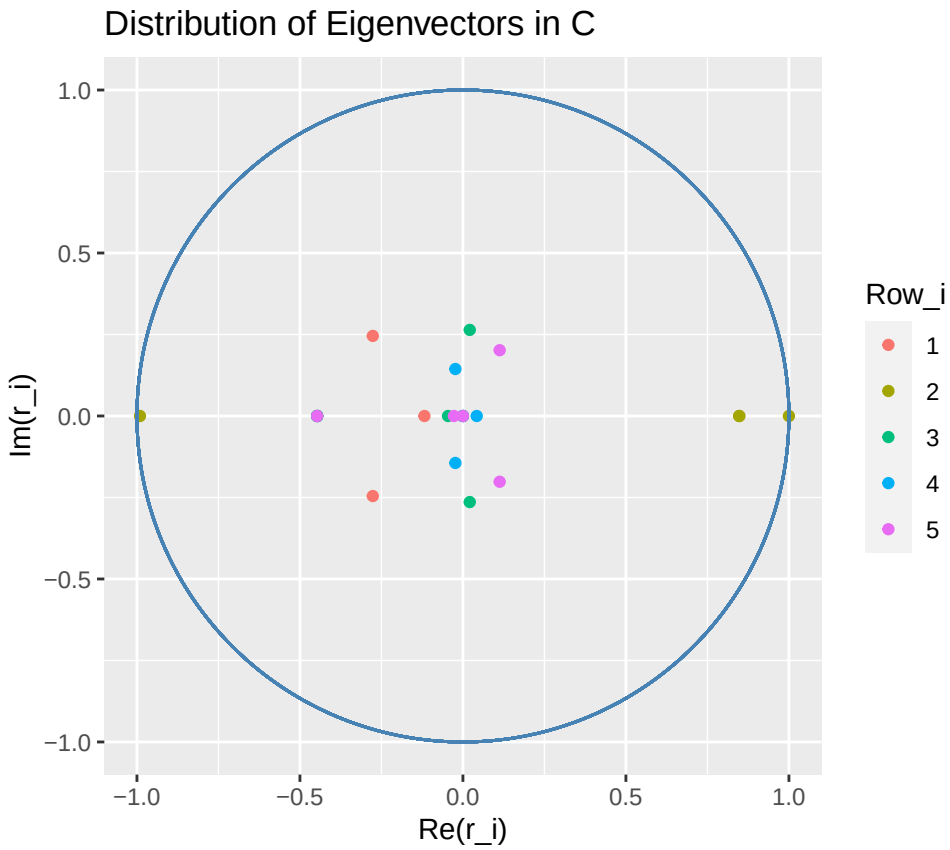
```
##         Re       Im row_i
## 1  0.57735  0.00000     1
## 2  0.56420  0.21758     1
## 3  0.56420 -0.21758     1
## 4  0.57735  0.00000     2
## 5  0.26819 -0.06129     2
## 6  0.26819  0.06129     2
## 7  0.57735  0.00000     3
## 8 -0.74744  0.00000     3
## 9 -0.74744  0.00000     3
##           [,1]      [,2]      [,3]
## [1,] 0.0000000 0.5190446 0.4809554
## [2,] 0.2816669 0.2823760 0.4359572
## [3,] 0.0000000 1.0000000 0.0000000
```



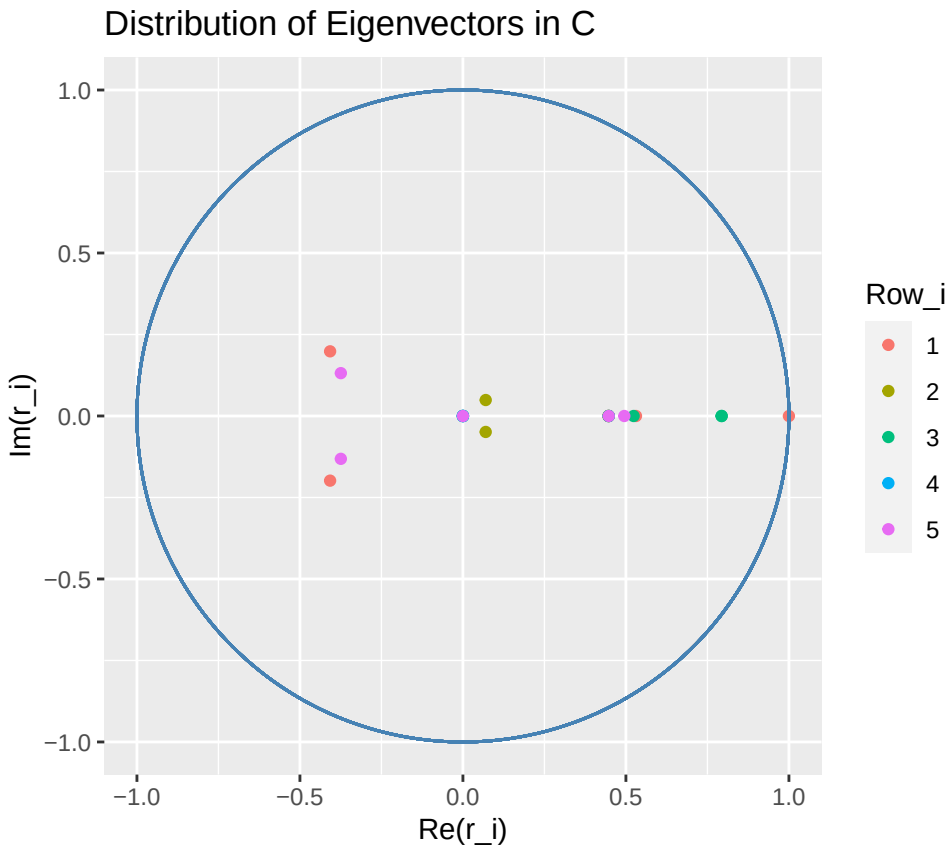
```
##          Re       Im row_i
## 1   0.44721  0.00000     1
## 2   0.26878 -0.08603     1
## 3   0.26878  0.08603     1
## 4  -0.33078  0.00000     1
## 5   0.05822  0.00000     1
## 6   0.44721  0.00000     2
## 7  -0.17984  0.36788     2
## 8  -0.17984 -0.36788     2
## 9   0.75167  0.00000     2
## 10 -0.00179  0.00000     2
## 11  0.44721  0.00000     3
## 12 -0.74833  0.00000     3
## 13 -0.74833  0.00000     3
## 14 -0.52668  0.00000     3
## 15 -0.79259  0.00000     3
## 16  0.44721  0.00000     4
## 17  0.19490  0.01487     4
## 18  0.19490 -0.01487     4
## 19 -0.04242  0.00000     4
## 20  0.46681  0.00000     4
## 21  0.44721  0.00000     5
## 22 -0.23273 -0.31671     5
## 23 -0.23273  0.31671     5
## 24  0.21538  0.00000     5
## 25 -0.38793  0.00000     5
##            [,1]       [,2]      [,3]      [,4]      [,5]
## [1,] 0.58151179 0.37284341 0.0000000 0.0000000 0.0456448
## [2,] 0.00000000 0.00000000 0.3710373 0.6289627 0.0000000
## [3,] 0.01536021 0.00000000 0.0000000 0.1992654 0.7853744
## [4,] 0.25966252 0.22663127 0.1035301 0.4101761 0.0000000
## [5,] 0.57860318 0.06897169 0.0000000 0.0000000 0.3524251
```


Distribution of Eigenvectors in C
1.0
0.5
0.0
−0.5
−1.0
Im(r_i)
−1.0
−0.5
0.0
0.5
1.0
Re(r_i)
Row_i
1
2
3
4
5

```
##          Re       Im row_i
## 1  -0.44721  0.00000     1
## 2  -0.27644  0.24562     1
## 3  -0.27644 -0.24562     1
## 4  -0.11810  0.00000     1
## 5   0.00000  0.00000     1
## 6  -0.44721  0.00000     2
## 7   0.84758  0.00000     2
## 8   0.84758  0.00000     2
## 9  -0.99066  0.00000     2
## 10  1.00000  0.00000     2
## 11 -0.44721  0.00000     3
## 12  0.02093 -0.26401     3
## 13  0.02093  0.26401     3
## 14 -0.04573  0.00000     3
## 15  0.00000  0.00000     3
## 16 -0.44721  0.00000     4
## 17 -0.02313 -0.14409     4
## 18 -0.02313  0.14409     4
## 19  0.04240  0.00000     4
## 20  0.00000  0.00000     4
## 21 -0.44721  0.00000     5
## 22  0.11274  0.20179     5
## 23  0.11274 -0.20179     5
## 24 -0.02765  0.00000     5
## 25  0.00000  0.00000     5
##           [,1] [,2]      [,3]      [,4]      [,5]
## [1,] 0.0000000    0 0.5314554 0.3309245 0.1376201
## [2,] 1.0000000    0 0.0000000 0.0000000 0.0000000
## [3,] 0.0000000    0 0.0000000 0.3169125 0.6830875
## [4,] 0.0000000    0 0.0000000 0.4669041 0.5330959
## [5,] 0.2846967    0 0.0000000 0.7153033 0.0000000
```


Distribution of Eigenvectors in C
1.0
0.5
0.0
−0.5
−1.0
Im(r_i)
−1.0
−0.5
0.0
0.5
1.0
Re(r_i)
Row_i
1
2
3
4
5

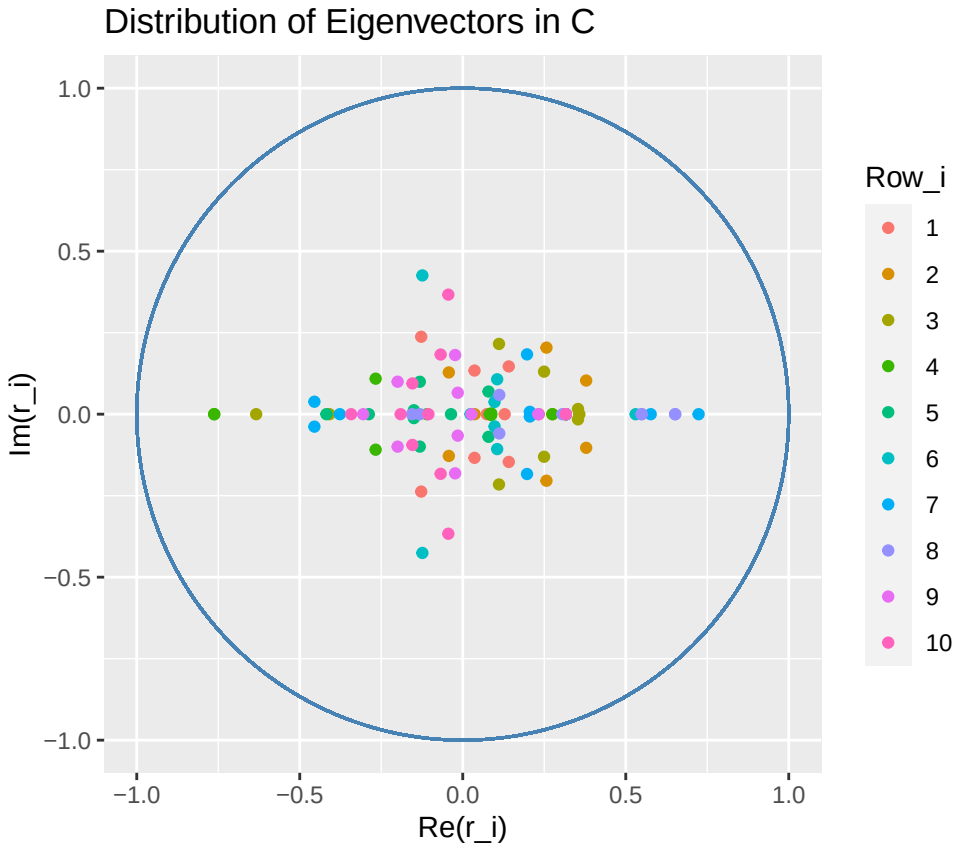
```
##          Re       Im row_i
## 1   0.44721  0.00000     1
## 2   0.53085  0.00000     1
## 3  -0.40746 -0.19831     1
## 4  -0.40746  0.19831     1
## 5   1.00000  0.00000     1
## 6   0.44721  0.00000     2
## 7   0.44620  0.00000     2
## 8   0.06991 -0.04898     2
## 9   0.06991  0.04898     2
## 10  0.00000  0.00000     2
## 11  0.44721  0.00000     3
## 12  0.52353  0.00000     3
## 13  0.79364  0.00000     3
## 14  0.79364  0.00000     3
## 15  0.00000  0.00000     3
## 16  0.44721  0.00000     4
## 17  0.00000  0.00000     4
## 18  0.00000  0.00000     4
## 19  0.00000  0.00000     4
## 20  0.00000  0.00000     4
## 21  0.44721  0.00000     5
## 22  0.49499  0.00000     5
## 23 -0.37446  0.13143     5
## 24 -0.37446 -0.13143     5
## 25  0.00000  0.00000     5
##      [,1]      [,2]       [,3]      [,4]      [,5]
## [1,]    0 0.4686427 0.22439793 0.0000000 0.3069594
## [2,]    0 0.4872468 0.07911052 0.1439473 0.2896954
## [3,]    0 0.4728104 0.00000000 0.0000000 0.5271896
## [4,]    0 0.0000000 0.00000000 1.0000000 0.0000000
## [5,]    0 1.0000000 0.00000000 0.0000000 0.0000000
```


Distribution of Eigenvectors in C
1.0
0.5
0.0
−0.5
−1.0
Im(r_i)
−1.0
−0.5
0.0
0.5
1.0
Re(r_i)
Row_i
1
2
3
4
5

```
##           Re       Im row_i
## 1    0.31623  0.00000     1
## 2   -0.12745 -0.23746     1
## 3   -0.12745  0.23746     1
## 4    0.23357  0.00000     1
## 5    0.12869  0.00000     1
## 6    0.03608  0.13384     1
## 7    0.03608 -0.13384     1
## 8    0.07370  0.00000     1
## 9    0.14122  0.14648     1
## 10   0.14122 -0.14648     1
## 11   0.31623  0.00000     2
## 12  -0.04271  0.12807     2
## 13  -0.04271 -0.12807     2
## 14  -0.15259  0.00000     2
## 15   0.27509  0.00000     2
## 16   0.25682  0.20415     2
## 17   0.25682 -0.20415     2
## 18   0.03666  0.00000     2
## 19   0.37859  0.10321     2
## 20   0.37859 -0.10321     2
## 21   0.31623  0.00000     3
## 22   0.35359 -0.01589     3
## 23   0.35359  0.01589     3
## 24  -0.40893  0.00000     3
## 25   0.35775  0.00000     3
## 26   0.11111  0.21555     3
## 27   0.11111 -0.21555     3
## 28  -0.63366  0.00000     3
## 29   0.24940  0.13056     3
## 30   0.24940 -0.13056     3
## 31   0.31623  0.00000     4
## 32   0.08742 -0.00244     4
## 33   0.08742  0.00244     4
## 34   0.08264  0.00000     4
## 35   0.27493  0.00000     4
## 36  -0.76243  0.00000     4
## 37  -0.76243  0.00000     4
## 38  -0.10997  0.00000     4
## 39  -0.26721  0.10893     4
## 40  -0.26721 -0.10893     4
## 41   0.31623  0.00000     5
## 42   0.07904  0.06946     5
## 43   0.07904 -0.06946     5
## 44  -0.41823  0.00000     5
## 45  -0.28815  0.00000     5
## 46  -0.14996 -0.01206     5
## 47  -0.14996  0.01206     5
## 48  -0.03611  0.00000     5
## 49  -0.13175 -0.09945     5
## 50  -0.13175  0.09945     5
## 51   0.31623  0.00000     6
## 52  -0.12388 -0.42567     6
## 53  -0.12388  0.42567     6
```

```
## 54   0.21364  0.00000     6
## 55   0.53024  0.00000     6
## 56   0.09759  0.03753     6
## 57   0.09759 -0.03753     6
## 58   0.02367  0.00000     6
## 59   0.10557  0.10709     6
## 60   0.10557 -0.10709     6
## 61   0.31623  0.00000     7
## 62   0.19711  0.18349     7
## 63   0.19711 -0.18349     7
## 64   0.57654  0.00000     7
## 65  -0.37719  0.00000     7
## 66   0.20578 -0.00718     7
## 67   0.20578  0.00718     7
## 68   0.72348  0.00000     7
## 69  -0.45505 -0.03824     7
## 70  -0.45505  0.03824     7
## 71   0.31623  0.00000     8
## 72   0.65157  0.00000     8
## 73   0.65157  0.00000     8
## 74  -0.15545  0.00000     8
## 75   0.30561  0.00000     8
## 76   0.11210 -0.05910     8
## 77   0.11210  0.05910     8
## 78  -0.13165  0.00000     8
## 79   0.54880  0.00000     8
## 80   0.54880  0.00000     8
## 81   0.31623  0.00000     9
## 82  -0.02340  0.18147     9
## 83  -0.02340 -0.18147     9
## 84   0.23168  0.00000     9
## 85  -0.30535  0.00000     9
## 86  -0.01528 -0.06578     9
## 87  -0.01528  0.06578     9
## 88   0.02674  0.00000     9
## 89  -0.19978 -0.09961     9
## 90  -0.19978  0.09961     9
## 91   0.31623  0.00000    10
## 92  -0.06781  0.18294    10
## 93  -0.06781 -0.18294    10
## 94  -0.34251  0.00000    10
## 95  -0.10532  0.00000    10
## 96  -0.04425 -0.36675    10
## 97  -0.04425  0.36675    10
## 98  -0.19044  0.00000    10
## 99  -0.15425 -0.09435    10
## 100 -0.15425  0.09435    10
##             [,1]       [,2]      [,3]      [,4]       [,5]      [,6]      [,7]
##  [1,] 0.28517139 0.00000000 0.0000000 0.0000000 0.36261875 0.2255371 0.0000000
##  [2,] 0.06968884 0.30106848 0.0000000 0.1817860 0.00000000 0.0775623 0.0000000
##  [3,] 0.00000000 0.10686865 0.0000000 0.1949979 0.00000000 0.1469841 0.1804226
##  [4,] 0.00000000 0.05420101 0.0000000 0.0000000 0.00000000 0.2548443 0.1409879
##  [5,] 0.29032275 0.00000000 0.0000000 0.0000000 0.00000000 0.0000000 0.0000000
##  [6,] 0.00000000 0.00000000 0.0000000 0.0000000 0.45428636 0.5457136 0.0000000
```

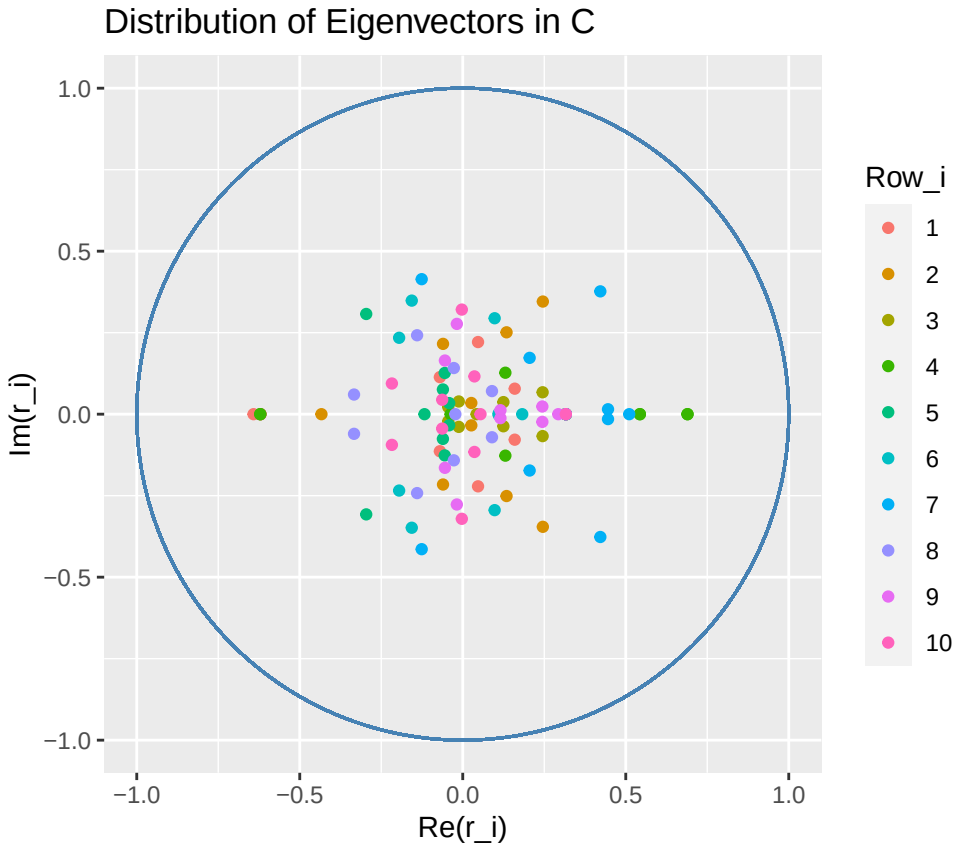
```
##  [7,] 0.00000000 0.13218316 0.1638585 0.0000000 0.42162057 0.0000000 0.0000000
##  [8,] 0.07205520 0.00000000 0.2132999 0.1435303 0.00000000 0.0000000 0.2833922
##  [9,] 0.14382362 0.21643049 0.0000000 0.0000000 0.25579432 0.0000000 0.0000000
## [10,] 0.00000000 0.26650855 0.0000000 0.0000000 0.01387118 0.1423476 0.0000000
##            [,8]      [,9]      [,10]
##  [1,] 0.0000000 0.0000000 0.12667275
##  [2,] 0.0000000 0.3085931 0.06130130
##  [3,] 0.2082290 0.1624979 0.00000000
##  [4,] 0.1145780 0.0192952 0.41609362
##  [5,] 0.1310493 0.5128155 0.06581251
##  [6,] 0.0000000 0.0000000 0.00000000
##  [7,] 0.0000000 0.2823377 0.00000000
##  [8,] 0.2518258 0.0000000 0.03589660
##  [9,] 0.0000000 0.2757209 0.10823071
## [10,] 0.0000000 0.5772727 0.00000000
```



```
##           Re       Im row_i
## 1    0.31623  0.00000     1
## 2   -0.07021 -0.11369     1
## 3   -0.07021  0.11369     1
## 4    0.04688  0.22128     1
## 5    0.04688 -0.22128     1
## 6    0.15946  0.07827     1
## 7    0.15946 -0.07827     1
## 8   -0.62042  0.00000     1
## 9   -0.62042  0.00000     1
## 10  -0.64173  0.00000     1
## 11   0.31623  0.00000     2
## 12   0.02623  0.03435     2
## 13   0.02623 -0.03435     2
## 14  -0.06056 -0.21571     2
## 15  -0.06056  0.21571     2
## 16   0.13435 -0.25109     2
## 17   0.13435  0.25109     2
## 18   0.24569  0.34549     2
## 19   0.24569 -0.34549     2
## 20  -0.43351  0.00000     2
## 21   0.31623  0.00000     3
## 22   0.24497  0.06725     3
## 23   0.24497 -0.06725     3
## 24   0.12467 -0.03705     3
## 25   0.12467  0.03705     3
## 26  -0.04433  0.02172     3
## 27  -0.04433 -0.02172     3
## 28  -0.01216 -0.03895     3
## 29  -0.01216  0.03895     3
## 30   0.04242  0.00000     3
## 31   0.31623  0.00000     4
## 32  -0.62121  0.00000     4
## 33  -0.62121  0.00000     4
## 34   0.68944  0.00000     4
## 35   0.68944  0.00000     4
## 36   0.54336  0.00000     4
## 37   0.54336  0.00000     4
## 38   0.13070  0.12720     4
## 39   0.13070 -0.12720     4
## 40  -0.03731  0.00000     4
## 41   0.31623  0.00000     5
## 42  -0.04161 -0.03403     5
## 43  -0.04161  0.03403     5
## 44  -0.05519  0.12646     5
## 45  -0.05519 -0.12646     5
## 46  -0.29609 -0.30737     5
## 47  -0.29609  0.30737     5
## 48  -0.06089  0.07577     5
## 49  -0.06089 -0.07577     5
## 50  -0.11735  0.00000     5
## 51   0.31623  0.00000     6
## 52   0.11029  0.00243     6
## 53   0.11029 -0.00243     6
```

```
## 54  -0.19517  0.23434     6
## 55  -0.19517 -0.23434     6
## 56  -0.15641  0.34834     6
## 57  -0.15641 -0.34834     6
## 58   0.09787 -0.29433     6
## 59   0.09787  0.29433     6
## 60   0.18245  0.00000     6
## 61   0.31623  0.00000     7
## 62   0.42202 -0.37674     7
## 63   0.42202  0.37674     7
## 64  -0.12596  0.41414     7
## 65  -0.12596 -0.41414     7
## 66   0.20519  0.17278     7
## 67   0.20519 -0.17278     7
## 68   0.44555 -0.01512     7
## 69   0.44555  0.01512     7
## 70   0.51057  0.00000     7
## 71   0.31623  0.00000     8
## 72  -0.14006 -0.24211     8
## 73  -0.14006  0.24211     8
## 74  -0.02744 -0.14153     8
## 75  -0.02744  0.14153     8
## 76  -0.33355  0.06022     8
## 77  -0.33355 -0.06022     8
## 78   0.08963  0.07079     8
## 79   0.08963 -0.07079     8
## 80  -0.02271  0.00000     8
## 81   0.31623  0.00000     9
## 82   0.11563  0.01214     9
## 83   0.11563 -0.01214     9
## 84  -0.05476 -0.16452     9
## 85  -0.05476  0.16452     9
## 86   0.24418  0.02359     9
## 87   0.24418 -0.02359     9
## 88  -0.01812 -0.27698     9
## 89  -0.01812  0.27698     9
## 90   0.29300  0.00000     9
## 91   0.31623  0.00000    10
## 92  -0.00283  0.32091    10
## 93  -0.00283 -0.32091    10
## 94  -0.21739 -0.09416    10
## 95  -0.21739  0.09416    10
## 96   0.03584  0.11586    10
## 97   0.03584 -0.11586    10
## 98  -0.06303  0.04419    10
## 99  -0.06303 -0.04419    10
## 100  0.05438  0.00000    10
##             [,1]        [,2]       [,3]      [,4]      [,5]      [,6]
##  [1,] 0.06446350 0.350036001 0.00000000 0.0000000 0.0000000 0.4110942
##  [2,] 0.12060958 0.059879711 0.21860653 0.1478217 0.1911012 0.0000000
##  [3,] 0.00000000 0.000000000 0.00000000 0.1840847 0.3376753 0.1945722
##  [4,] 0.44287349 0.004385332 0.00000000 0.0000000 0.0000000 0.0000000
##  [5,] 0.00000000 0.153681185 0.00000000 0.0000000 0.4717977 0.0000000
##  [6,] 0.00000000 0.000000000 0.10700417 0.0000000 0.0000000 0.0000000
```

```
##  [7,] 0.00000000 0.000000000 0.08933514 0.0000000 0.0000000 0.0000000
##  [8,] 0.00000000 0.000000000 0.67924109 0.0000000 0.2525487 0.0000000
##  [9,] 0.04316237 0.000000000 0.25108409 0.1851979 0.0000000 0.0000000
## [10,] 0.07880158 0.000000000 0.00000000 0.1382138 0.0000000 0.0000000
##             [,7]       [,8]      [,9]      [,10]
##  [1,] 0.00000000 0.00000000 0.1744063 0.00000000
##  [2,] 0.00000000 0.00000000 0.2619813 0.00000000
##  [3,] 0.00000000 0.00000000 0.0000000 0.28366772
##  [4,] 0.55274118 0.00000000 0.0000000 0.00000000
##  [5,] 0.00000000 0.00000000 0.3745211 0.00000000
##  [6,] 0.00000000 0.44168294 0.0000000 0.45131289
##  [7,] 0.00000000 0.00000000 0.0000000 0.91066486
##  [8,] 0.00000000 0.06821022 0.0000000 0.00000000
##  [9,] 0.08104249 0.13914134 0.0000000 0.30037182
## [10,] 0.00000000 0.48828993 0.2366454 0.05804934
```



```
##           Re       Im row_i
## 1   -0.31623  0.00000     1
## 2   -0.25850  0.00000     1
## 3    0.04102  0.14982     1
## 4    0.04102 -0.14982     1
## 5   -0.14317  0.29277     1
## 6   -0.14317 -0.29277     1
## 7   -0.41927  0.00000     1
## 8   -0.02574  0.00000     1
## 9    0.17274  0.19137     1
## 10   0.17274 -0.19137     1
## 11  -0.31623  0.00000     2
## 12   0.68164  0.00000     2
## 13  -0.15104  0.06281     2
## 14  -0.15104 -0.06281     2
## 15  -0.16609  0.09177     2
## 16  -0.16609 -0.09177     2
## 17  -0.21775  0.00000     2
## 18   0.01676  0.00000     2
## 19  -0.08110  0.08217     2
## 20  -0.08110 -0.08217     2
## 21  -0.31623  0.00000     3
## 22   0.42922  0.00000     3
## 23  -0.01950 -0.35041     3
## 24  -0.01950  0.35041     3
## 25   0.10204  0.25756     3
## 26   0.10204 -0.25756     3
## 27  -0.43950  0.00000     3
## 28   0.47192  0.00000     3
## 29   0.50859  0.00000     3
## 30   0.50859  0.00000     3
## 31  -0.31623  0.00000     4
## 32  -0.21315  0.00000     4
## 33   0.47826  0.00000     4
## 34   0.47826  0.00000     4
## 35   0.07436 -0.15777     4
## 36   0.07436  0.15777     4
## 37   0.18625  0.00000     4
## 38  -0.28859  0.00000     4
## 39  -0.32368  0.07603     4
## 40  -0.32368 -0.07603     4
## 41  -0.31623  0.00000     5
## 42   0.10174  0.00000     5
## 43  -0.16355  0.43106     5
## 44  -0.16355 -0.43106     5
## 45   0.52354  0.00000     5
## 46   0.52354  0.00000     5
## 47   0.31368  0.00000     5
## 48   0.48784  0.00000     5
## 49  -0.36528 -0.25303     5
## 50  -0.36528  0.25303     5
## 51  -0.31623  0.00000     6
## 52   0.20840  0.00000     6
## 53  -0.27099 -0.10885     6
```

```
## 54  -0.27099  0.10885     6
## 55   0.04118 -0.44779     6
## 56   0.04118  0.44779     6
## 57   0.45443  0.00000     6
## 58   0.60881  0.00000     6
## 59   0.21461  0.36418     6
## 60   0.21461 -0.36418     6
## 61  -0.31623  0.00000     7
## 62   0.01459  0.00000     7
## 63   0.06578  0.01925     7
## 64   0.06578 -0.01925     7
## 65  -0.05738  0.22501     7
## 66  -0.05738 -0.22501     7
## 67   0.25169  0.00000     7
## 68   0.03591  0.00000     7
## 69  -0.05697  0.03611     7
## 70  -0.05697 -0.03611     7
## 71  -0.31623  0.00000     8
## 72   0.09808  0.00000     8
## 73  -0.16134  0.28685     8
## 74  -0.16134 -0.28685     8
## 75  -0.05168 -0.36888     8
## 76  -0.05168  0.36888     8
## 77   0.41730  0.00000     8
## 78  -0.17922  0.00000     8
## 79   0.25056 -0.25242     8
## 80   0.25056  0.25242     8
## 81  -0.31623  0.00000     9
## 82  -0.40442  0.00000     9
## 83  -0.00193 -0.08310     9
## 84  -0.00193  0.08310     9
## 85  -0.06872 -0.02239     9
## 86  -0.06872  0.02239     9
## 87  -0.07771  0.00000     9
## 88  -0.00425  0.00000     9
## 89  -0.01553 -0.02117     9
## 90  -0.01553  0.02117     9
## 91  -0.31623  0.00000    10
## 92  -0.10820  0.00000    10
## 93  -0.32707 -0.26922    10
## 94  -0.32707  0.26922    10
## 95   0.25954  0.09520    10
## 96   0.25954 -0.09520    10
## 97   0.02278  0.00000    10
## 98  -0.22583  0.00000    10
## 99   0.15101 -0.14250    10
## 100  0.15101  0.14250    10
##            [,1]       [,2]       [,3]       [,4]       [,5]      [,6]
##  [1,] 0.0000000 0.51358552 0.00000000 0.00000000 0.00000000 0.0000000
##  [2,] 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000
##  [3,] 0.5044577 0.00000000 0.00000000 0.33818914 0.00000000 0.0000000
##  [4,] 0.2126427 0.18644633 0.02052775 0.03672315 0.25728741 0.0000000
##  [5,] 0.0000000 0.00000000 0.00000000 0.00000000 0.00000000 0.0000000
##  [6,] 0.0000000 0.00000000 0.00000000 0.52343303 0.00000000 0.1247182
```

```
##  [7,] 0.0000000 0.15785978 0.00000000 0.00000000 0.00000000 0.0000000
##  [8,] 0.0000000 0.03472049 0.00000000 0.16400350 0.00000000 0.2724730
##  [9,] 0.0000000 0.32608218 0.17427955 0.25762141 0.03299422 0.0000000
## [10,] 0.0000000 0.00000000 0.31815995 0.36145811 0.00000000 0.0000000
##             [,7]      [,8]       [,9]      [,10]
##  [1,] 0.00000000 0.0000000 0.48641448 0.00000000
##  [2,] 0.00000000 0.0000000 1.00000000 0.00000000
##  [3,] 0.00000000 0.0000000 0.11865741 0.03869574
##  [4,] 0.01089868 0.2754740 0.00000000 0.00000000
##  [5,] 0.38954924 0.0000000 0.00000000 0.61045076
##  [6,] 0.00000000 0.0000000 0.00000000 0.35184881
##  [7,] 0.37580955 0.1330263 0.33330440 0.00000000
##  [8,] 0.03119174 0.0000000 0.16979631 0.32781493
##  [9,] 0.00000000 0.1571197 0.05190293 0.00000000
## [10,] 0.32038195 0.0000000 0.00000000 0.00000000
```