Matrix Refactor

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```
# Global parameters
M <- 3
```

Normal Matrices

```
# Set seed
set.seed(23)
# Set parameters
mu <- 1
sd <- 2
# Generate matrixx
P <- RM_normal(M, normal_args = c(mu, sd), symm = F)
P

## [,1] [,2] [,3]
## [1,] 1.3864247 0.1306358 2.826534
## [2,] 4.5867762 2.9932102 3.214981
## [3,] 0.4438274 3.0384110 1.090874</pre>
```

Symmetric Normal Matrices

```
# Set seed
set.seed(23)
# Set parameters
mu <- 1
sd <- 2
# Generate matrixx
P <- RM_normal(M, normal_args = c(mu, sd), symm = T)
P

## [,1] [,2] [,3]
## [1,] 9.928535 15.83749 4.095652
## [2,] 15.837494 40.33393 14.637480
## [3,] 4.095652 14.63748 10.618931</pre>
```

Stochastic Matrices

Sparse

```
# Set seed
set.seed(23)
# Set parameters
mu <- 1
sd <- 2
# Generate matrixx
P <- RM_stoch(M, symm = F, sparsity = T)</pre>
     [,1] [,2]
                          [,3]
##
## [1,] 0 0.4019552 0.5980448
## [2,] 0 1.0000000 0.0000000
## [3,] 1 0.0000000 0.0000000
```

Non-sparse

```
# Set seed
set.seed(23)
# Set parameters
mu <- 1
sd <- 2
# Generate matrixx
P <- RM_stoch(M, symm = F, sparsity = F)</pre>
##
           [,1] [,2]
                                 [,3]
## [1,] 0.5095594 0.1971352 0.2933055
## [2,] 0.3637477 0.4193927 0.2168595
## [3,] 0.3463251 0.3515677 0.3021073
```

Symmetric Stochastic Matrices

mu <- 1

```
# Set seed
set.seed(23)
# Set parameters
mu <- 1
sd <- 2
# Generate matrixx
P <- RM_stoch(M, symm = T, sparsity = T)</pre>
##
            [,1]
                      [,2] [,3]
## [1,] 0.5192256 0.4019552 0
## [2,] 0.4019552 1.0000000 0
## [3,] 0.0000000 0.0000000 1
Non-sparse
# Set seed
set.seed(23)
# Set parameters
```

```
sd <- 2
# Generate matrixx
P <- RM_stoch(M, symm = T, sparsity = F)
P

## [,1] [,2] [,3]
## [1,] 0.3845411 0.3316342 0.3343892
## [2,] 0.3316342 0.3552307 0.3389347</pre>
```

[3,] 0.3343892 0.3389347 0.3348097

Tridiagonal Matrices

```
# Set seed
set.seed(23)
# Generate matrix
P <- RM_trid(M)
P

## [,1] [,2] [,3]
## [1,] 0.3864247 1.7933881 0.0000000
## [2,] 1.7933881 -0.8693642 0.9966051
## [3,] 0.0000000 0.9966051 1.8265342</pre>
```

p-Sparse Matrices

```
# Set seed
set.seed(23)
# Set parameters
p <- 0.2
# Generate matrix
P <- RM_erdos(M, p, stoch = F)</pre>
```

Stochastic p-Sparse Matrices

```
# Set seed
set.seed(23)
# Set parameters
p <- 0.2
# Generate matrix
P <- RM_erdos(M, p, stoch = T)</pre>
```

Suppose we have a $M \times M$ square matrix \mathbf{P} (for some $M \in \mathbb{N}$) on a field F. We notate $\mathbf{P} \in \mathcal{M}_F[M^2]$. Take $\mathbf{P} \in \mathcal{M}_F[M^2]$.

Structural Properties of Matrices

If P is symmetric, then its upper triangle is equal to the lower triangle.

If P is tridiagonal, then it is a band matrix of width 1.

Entry-wise Properties of Matrices

```
If \mathbf{P} is row-stochastic, then \forall i: \sum_j p_{ij} = 1. 
 \mathsf{RM\_stoch} \leftarrow \mathsf{function}(\mathsf{M}, \mathsf{symm} = \mathsf{F}, \mathsf{sparsity} = \mathsf{F}) \{ \dots \} 
 If \mathbf{P} is \mathcal{N}(\mu, \sigma^2), then its entries satisfy p_{ij} \sim \mathcal{N}(\mu, \sigma^2). 
 \mathsf{RM\_normal} \leftarrow \mathsf{function}(\mathsf{M}, \mathsf{normal\_args} = \mathsf{c}(\mathsf{0}, \mathsf{1}), \mathsf{symm} = \mathsf{F}) \{ \dots \} 
 If \mathbf{P} is p-\mathsf{sparse}, then \forall i,j \in S_M: p_{ij}/c \sim \mathsf{Bern}(p) for some c \in \mathbb{R}. 
 \mathsf{RM\_erdos} \leftarrow \mathsf{function}(\mathsf{M}, \mathsf{p\_sparse}) \{ \dots \}
```