Free Parameters: Thesis Direction

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Currently, the thesis loosely focuses on “Spectral Statistics of Random Matrices”. To narrow down our focus, this document will outline and organize the potential avenues of focus. Listed below are the different types of matrices to explore. For each matrix, there is a list of ‘free parameters’. Simulations controlling and varying these parameters provide viable avenues of exploration.

Stochastic Matrices

Stochastic matrices are ergodic if they have a limiting distribution that is also the stationary distribution. This is achieved when the matrix represents a Markov chain that is aperiodic and irreducible.

- What happens to the mixing time t if we force the matrix to be symmetric?

Free Parameters:

1. Boolean: Symmetric
2. Boolean: Sparse/Fully Connected
3. Integer: $M \in \mathbb{N}$ (Matrix Dimension)

Erdos-Renyi Graphs

Walks on an ERG represent a parameterizable class of stochastic matrices. Specifically, we can parameterize its sparsity or connectedness with a parameter p.

- How does p impact the mixing time of the Markov Chain?
- How does p impact the spectrum of the random matrix ensemble?

Free Parameters:

1. Real Number: $p_{\text{sparsity}} \in [0, 1]$
2. Integer: $M \in \mathbb{N}$ (Matrix Dimension)

Normal Matrices

For normal matrices, we study a hidden markov chain in the CRS. Because this is no longer a traditional stochastic setting, the term “mixing time” is overloaded. Let the mixing time represent the power needed such that a row vector is ε-convergent to an eigenvector of the matrix's largest eigenvalue.

- How do the parameters μ, σ of the matrix ensemble affect mixing time?
- What properties exist in the entries of the CRS? How are they distributed and are they i.i.d entrywise?
- What happens to the mixing time t if we force the matrix to be symmetric?
- Do the signs of the entries in the matrix impact mixing time, and how so?

Free Parameters:

1. Real Numbers: $\mu, \sigma \in \mathbb{R}$
2. Boolean: Symmetric
3. Integer: $M \in \mathbb{N}$ (Matrix Dimension)

Band Matrices

A band matrix is a matrix parameterizable by band length, an integer $k \in \mathbb{N}^+$ than stipulates all entries of the matrix be within k entries away from the main diagonal. Let the convention be that $k = 0$ is a diagonal matrix and $k = 1$ a tridiagonal matrix.

- What about the mixing times stochastic band matrices?
- What about the mixing times of normal band matrices?

Free Parameters:

1. Integer: $k \in \mathbb{N}^+$
2. Boolean: Symmetric
3. Integer: $M \in \mathbb{N}$ (Matrix Dimension)

So, all in all, we have two notions of mixing time. One is the official mixing time of stochastic matrices. The other is one for approximating eigenvectors of the matrix.

Goals

- Create package for random matrices and random ensemble.