Chapter 2

Random Matrices

(Definition) Random Matrix. Suppose we have an arbitrary matrix, $P \in \mathcal{M}_{\mathbb{R}}[M, N]$ for some $M, N \in \mathbb{N}^+$. Then, we call P a random matrix is all of its entries p_{ij} are random variables.

(Definition) Ergodic Matrix. A matrix is ergodic if it is aperiodic, and positive irreducible.

Ergodicity

- .

Notation

Suppose we have a $M \times M$ square matrix $\mathbf{P}$ (for some $M \in \mathbb{N}$) on a field F. We notate $\mathbf{P} \in \mathcal{M}_F[M^2]$.

Take $\mathbf{P} \in \mathcal{M}_F[M^2]$.

Structural Properties of Matrices

If $\mathbf{P}$ is symmetric, then its upper triangle is equal to the lower triangle.

If $\mathbf{P}$ is tridiagonal, then it is a band matrix of width 1.

Entry-wise Properties of Matrices

If $\mathbf{P}$ is row-stochastic, then $\forall i : \sum_j p_{ij} = 1$.

```
RM_stoch <- function(M, symm = F, sparsity = F){...}
```

If $\mathbf{P}$ is $\mathcal{N}(\mu, \sigma^2)$, then its entries satisfy $p_{ij} \sim \mathcal{N}(\mu, \sigma^2)$.

```
RM_normal <- function(M, normal_args = c(0,1), symm = F){...}
```

If $\mathbf{P}$ is $p-$sparse, then $\forall i, j \in S_M : p_{ij}/c \sim \mathrm{Bern}(p)$ for some $c \in \mathbb{R}$.

```
RM_erdos <- function(M, p_sparse){...}
```