Random Matrix Analysis

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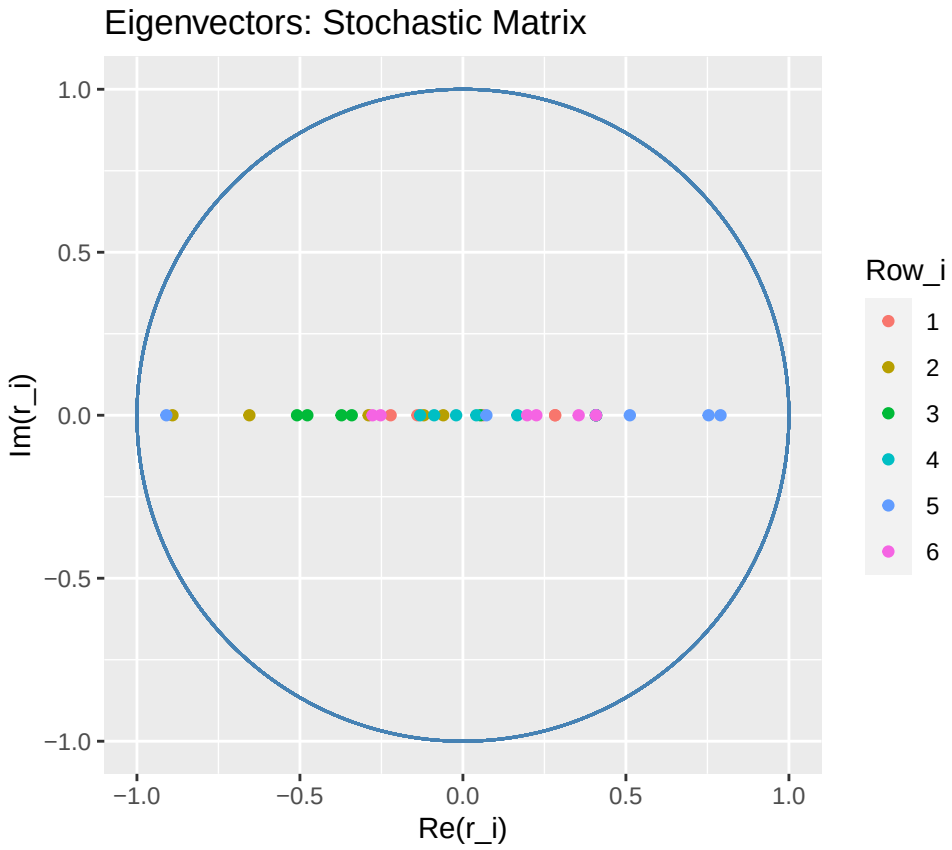
Stochastic Matrices

```
set.seed(23)
P <- rand_M_stoch(M, row_fn = r_zeros)
if(bool_plot){eigen_plot(P, loud = T, "Stochastic")}
```

```
##          Re Im row_i
## 1   0.40825  0     1
## 2   0.28292  0     1
## 3  -0.22174  0     1
## 4   0.28270  0     1
## 5  -0.13952  0     1
## 6   0.06155  0     1
## 7   0.40825  0     2
## 8  -0.11999  0     2
## 9  -0.65502  0     2
## 10 -0.06029  0     2
## 11 -0.28922  0     2
## 12 -0.89095  0     2
## 13  0.40825  0     3
## 14 -0.50889  0     3
## 15 -0.34100  0     3
## 16 -0.47724  0     3
## 17  0.05353  0     3
## 18 -0.37240  0     3
## 19  0.40825  0     4
## 20  0.04118  0     4
## 21 -0.13145  0     4
## 22 -0.02073  0     4
## 23  0.16692  0     4
## 24 -0.08826  0     4
## 25  0.40825  0     5
## 26  0.75361  0     5
## 27  0.51201  0     5
## 28  0.79020  0     5
## 29 -0.90960  0     5
## 30  0.07202  0     5
## 31  0.40825  0     6
## 32 -0.27742  0     6
## 33  0.35504  0     6
## 34 -0.25266  0     6
## 35  0.19696  0     6
## 36  0.22529  0     6
##           [,1]      [,2]      [,3]      [,4]       [,5]      [,6]
```

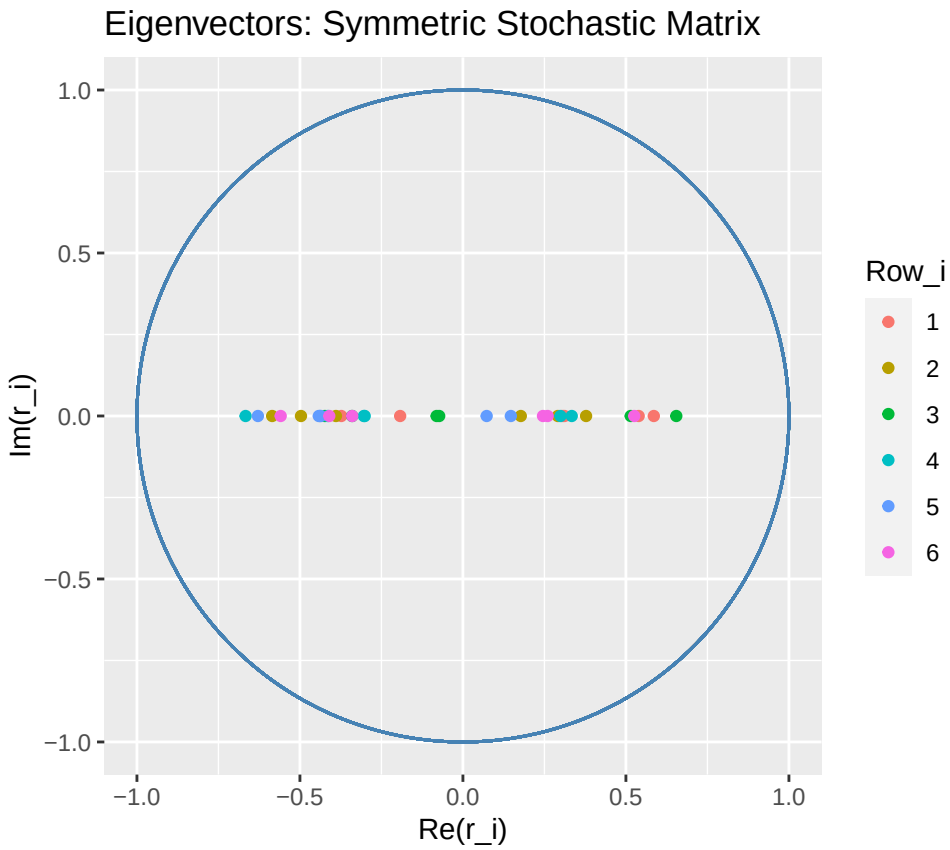
```
## [1,] 0.4328139 0.0000000 0.2491303 0.0000000 0.00000000 0.3180558
## [2,] 0.3766753 0.0000000 0.1690367 0.4542880 0.00000000 0.0000000
## [3,] 0.2796586 0.0000000 0.0000000 0.6393404 0.08100106 0.0000000
## [4,] 0.4300754 0.1458276 0.0000000 0.0000000 0.00000000 0.4240970
## [5,] 0.0000000 0.1960439 0.0000000 0.0000000 0.00000000 0.8039561
## [6,] 0.0000000 0.0000000 0.0000000 0.5648831 0.18254690 0.2525700
```



Symmetric Stochastic Matrices

```
set.seed(23)
P <- rand_M_symm_stoch(M, row_fn = r_zeros)
if(bool_plot){eigen_plot(P, loud = T, "Symmetric Stochastic")}
```

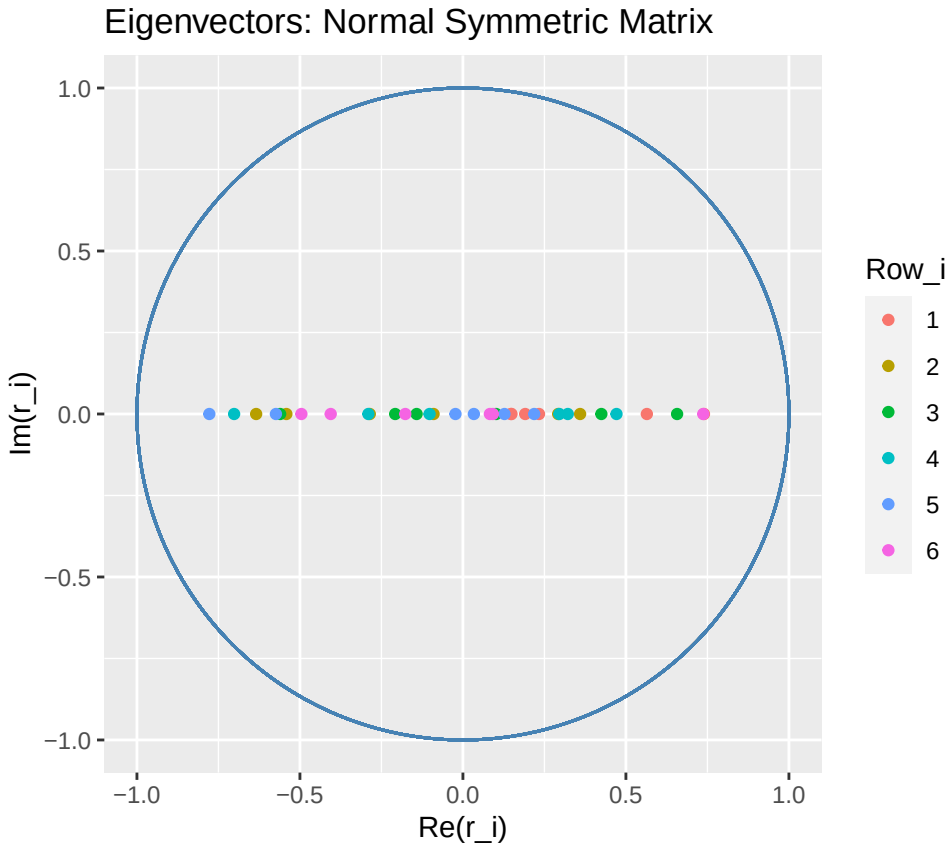
```
##          Re Im row_i
## 1  -0.37305  0     1
## 2  -0.19255  0     1
## 3   0.53909  0     1
## 4   0.58557  0     1
## 5   0.30527  0     1
## 6   0.31153  0     1
## 7  -0.38964  0     2
## 8   0.37783  0     2
## 9   0.29098  0     2
## 10  0.17750  0     2
## 11 -0.58520  0     2
## 12 -0.49678  0     2
## 13 -0.42357  0     3
## 14  0.51465  0     3
## 15 -0.07276  0     3
## 16 -0.33984  0     3
## 17 -0.08061  0     3
## 18  0.65456  0     3
## 19 -0.41414  0     4
## 20 -0.30378  0     4
## 21  0.33381  0     4
## 22 -0.66669  0     4
## 23  0.29917  0     4
## 24 -0.30133  0     4
## 25 -0.43590  0     5
## 26 -0.62886  0     5
## 27 -0.44209  0     5
## 28  0.07281  0     5
## 29 -0.43836  0     5
## 30  0.14705  0     5
## 31 -0.40998  0     6
## 32  0.25990  0     6
## 33 -0.55906  0     6
## 34  0.24562  0     6
## 35  0.52554  0     6
## 36 -0.33955  0     6
##            [,1]      [,2]      [,3]      [,4]      [,5]       [,6]
## [1,] 0.35055326 0.2051425 0.1210401 0.3210291 0.2557029 0.08033136
## [2,] 0.20514247 0.3768353 0.3957851 0.1619988 0.0000000 0.25661961
## [3,] 0.12104012 0.3957851 0.4935262 0.1202743 0.0000000 0.37593904
## [4,] 0.32102911 0.1619988 0.1202743 0.3860888 0.3695440 0.10711420
## [5,] 0.25570290 0.0000000 0.0000000 0.3695440 0.6847786 0.20305521
## [6,] 0.08033136 0.2566196 0.3759390 0.1071142 0.2030552 0.41620788
```


Eigenvectors: Symmetric Stochastic Matrix
1.0
0.5
0.0
−0.5
−1.0
Im(r_i)
−1.0
−0.5
0.0
0.5
1.0
Re(r_i)
Row_i
1
2
3
4
5
6

Normal Symmetric Matrices

```
set.seed(23)
P <- rand_M_symm_norm(M, mu = 0, sd = 1)
if(bool_plot){eigen_plot(P, loud = T, "Normal Symmetric")}
```

```
##          Re Im row_i
## 1   0.73940  0     1
## 2   0.23345  0     1
## 3   0.14755  0     1
## 4   0.56426  0     1
## 5   0.19105  0     1
## 6   0.14874  0     1
## 7   0.35946  0     2
## 8  -0.63429  0     2
## 9  -0.28566  0     2
## 10 -0.09010  0     2
## 11  0.29224  0     2
## 12 -0.54160  0     2
## 13 -0.14207  0     3
## 14 -0.20779  0     3
## 15 -0.56068  0     3
## 16  0.10180  0     3
## 17  0.42463  0     3
## 18  0.65697  0     3
## 19 -0.10240  0     4
## 20 -0.70187  0     4
## 21  0.47109  0     4
## 22  0.32170  0     4
## 23 -0.29006  0     4
## 24  0.29554  0     4
## 25  0.21867  0     5
## 26 -0.02289  0     5
## 27 -0.57366  0     5
## 28  0.12749  0     5
## 29 -0.77794  0     5
## 30  0.03353  0     5
## 31 -0.49556  0     6
## 32  0.08273  0     6
## 33 -0.17680  0     6
## 34  0.73715  0     6
## 35  0.09197  0     6
## 36 -0.40555  0     6
##           [,1]      [,2]       [,3]       [,4]       [,5]       [,6]
## [1,]  6.496334  1.429240 -1.8316377 -1.4361867  1.1849312 -3.4432263
## [2,]  1.429240  4.744144  1.3507143  2.0244334  1.7631548 -1.9976843
## [3,] -1.831638  1.350714  2.3559787 -0.2885532  1.4290947  1.2432352
## [4,] -1.436187  2.024433 -0.2885532  5.0343555 -1.5300036 -0.1580920
## [5,]  1.184931  1.763155  1.4290947 -1.5300036  2.5533442 -0.5091204
## [6,] -3.443226 -1.997684  1.2432352 -0.1580920 -0.5091204  3.2651919
```


Eigenvectors: Normal Symmetric Matrix
1.0
0.5
0.0
−0.5
−1.0
Im(r_i)
−1.0
−0.5
0.0
0.5
1.0
Re(r_i)
Row_i
1
2
3
4
5
6

Tridiagonal Matrices

```
set.seed(23)
P <- rand_M_trid(M)
P
```

```
##            [,1]       [,2]       [,3]       [,4]      [,5]      [,6]
## [1,]  0.3864247 -0.2780863 0.00000000 0.00000000 0.0000000 0.0000000
## [2,] -0.2780863 -0.8693642 1.01920549 0.00000000 0.0000000 0.0000000
## [3,]  0.0000000  1.0192055 1.82653419 0.04543718 0.0000000 0.0000000
## [4,]  0.0000000  0.0000000 0.04543718 3.58677618 1.5757796 0.0000000
## [5,]  0.0000000  0.0000000 0.00000000 1.57577959 1.9932102 0.2182885
## [6,]  0.0000000  0.0000000 0.00000000 0.00000000 0.2182885 2.2149810
```

```
if(bool_plot){eigen_plot(P, loud = T, "Tridiagonal")}
```

```
##          Re Im row_i
## 1  -0.00019  0     1
## 2   0.00374  0     1
## 3  -0.04988  0     1
## 4  -0.00437  0     1
## 5   0.98602  0     1
## 6   0.15891  0     1
## 7   0.00286  0     2
## 8  -0.02487  0     2
## 9   0.32033  0     2
## 10  0.00958  0     2
## 11 -0.13472  0     2
## 12  0.93729  0     2
## 13  0.01517  0     3
## 14 -0.07482  0     3
## 15  0.94238  0     3
## 16  0.01634  0     3
## 17  0.09802  0     3
## 18 -0.31018  0     3
## 19  0.84942  0     4
## 20 -0.12001  0     4
## 21 -0.01286  0     4
## 22 -0.51371  0     4
## 23 -0.00287  0     4
## 24  0.00346  0     4
## 25  0.52522  0     5
## 26  0.10486  0     5
## 27 -0.01563  0     5
## 28  0.84433  0     5
## 29  0.00293  0     5
## 30 -0.00169  0     5
## 31  0.04885  0     6
## 32  0.98406  0     6
## 33  0.08003  0     6
## 34 -0.15112  0     6
## 35 -0.00036  0     6
## 36  0.00011  0     6
```

```
##            [,1]       [,2]       [,3]       [,4]      [,5]      [,6]
## [1,]  0.3864247 -0.2780863 0.00000000 0.00000000 0.0000000 0.0000000
```

```
## [2,] -0.2780863 -0.8693642 1.01920549 0.00000000 0.0000000 0.0000000
## [3,]  0.0000000  1.0192055 1.82653419 0.04543718 0.0000000 0.0000000
## [4,]  0.0000000  0.0000000 0.04543718 3.58677618 1.5757796 0.0000000
## [5,]  0.0000000  0.0000000 0.00000000 1.57577959 1.9932102 0.2182885
## [6,]  0.0000000  0.0000000 0.00000000 0.00000000 0.2182885 2.2149810
```