

# Computational Eigenvector Simulation

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## Example 1: A Symmetric Stochastic Matrix

### Step 0: Setup the matrix

```
# Set seed
set.seed(23)
# Set parameters
M <- 2
mu <- 0
sd <- 1
# Generate matrix
P <- RM_stoch(M, symm = T, sparsity = F)
```

### The Matrix

```
##           [,1]      [,2]
## [1,] 0.7210461 0.2789539
## [2,] 0.2789539 0.7210461
```

### Eigenvalues of the Symmetric Stochastic Matrix

```
eigen_frame(P)

##           Re Im row_i
## 1  0.70711  0      1
## 2 -0.70711  0      1
## 3  0.70711  0      2
## 4  0.70711  0      2
```

### Step 1: Get the batch

```
# Set batch parameters
B <- 100
# Create batch
batch <- make_batch(M = M, B = B)
head(batch)

##           x1           x2
## 1  0.6388979 -0.1525589
## 2  0.9270891  0.9562608
## 3  0.6810438  0.9932225
## 4  0.7319181  0.4028434
## 5 -0.2190539 -0.3704606
## 6  0.6918946 -0.7214430
```

## Step 2: Evolve the batch

```
# Set evolution parameters
steps <- 20
# Evolve batch
evolved_batch <- evolve_batch(batch, steps, with_steps = T)
head(evolved_batch)
```

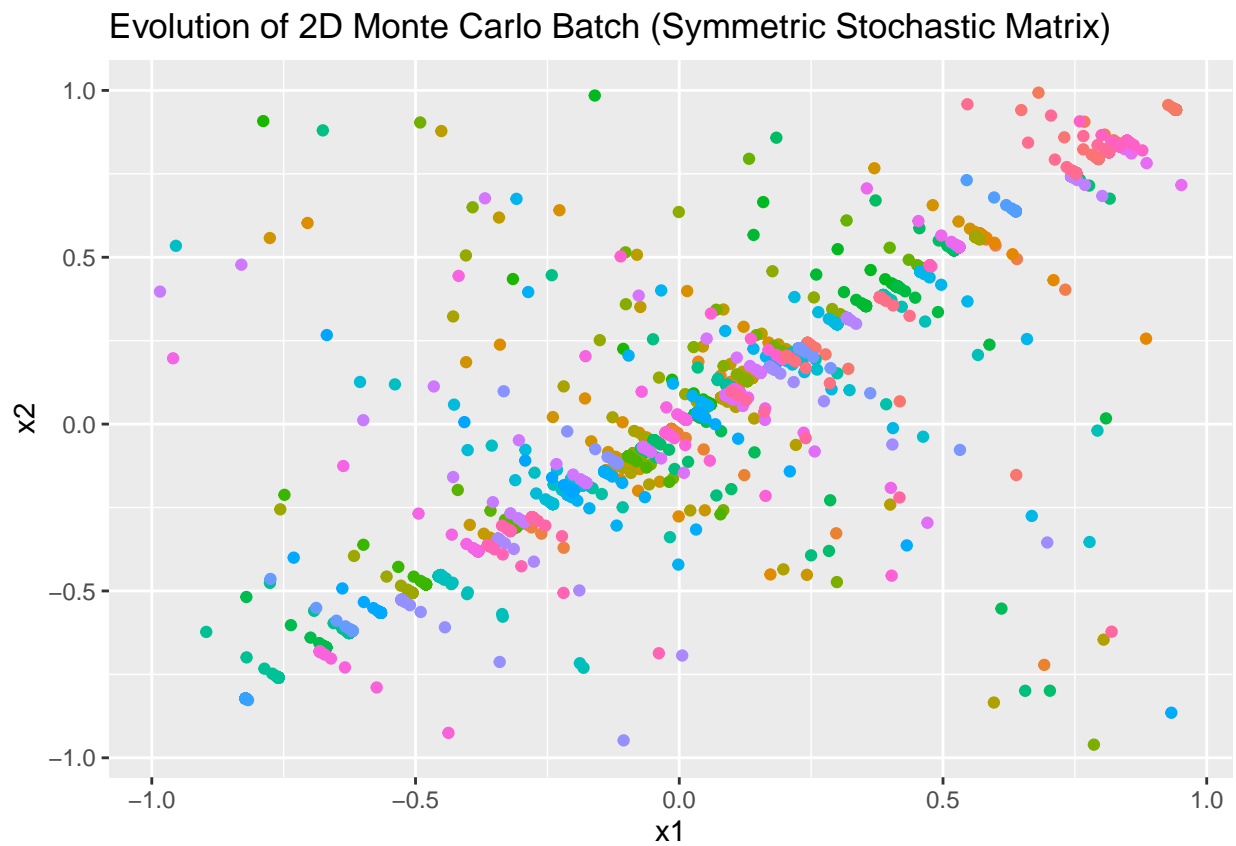
```
##           x1           x2 time element_index
## 1 0.6388979 -0.1525588      0              1
## 2 0.4181180  0.06822105     1              1
## 3 0.3205129  0.16582615     2              1
## 4 0.2773624  0.20897661     3              1
## 5 0.2582859  0.22805310     4              1
## 6 0.2498524  0.23648667     5              1
```

```
tail(evolved_batch)
```

```
##           x1           x2 time element_index
## 2095 0.7946909 0.7946923    15             100
## 2096 0.7946913 0.7946919    16             100
## 2097 0.7946915 0.7946917    17             100
## 2098 0.7946915 0.7946917    18             100
## 2099 0.7946916 0.7946916    19             100
## 2100 0.7946916 0.7946916    20             100
```

### Step 3: Analyze the batch

```
# Plot the evolution arrays of the batch elements  
batch_data <- evolved_batch  
# 2d plot  
batch_2d_plot(batch_data, "(Symmetric Stochastic Matrix)")
```



## Example 2: A Symmetric Normal Matrix

### Step 0: Setup the matrix

```
# Set seed
set.seed(6)
# Set parameters
M <- 9
mu <- 0
sd <- 1
# Generate matrix
P <- RM_normal(M, c(mu,sd), symm = T)
```

### The Matrix

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,]  0.26960598 -0.6299854  0.86865983  1.7271955  0.02418764  0.36802518
## [2,] -0.62998541  1.7278511 -1.17859974  0.6532067 -0.36856649 -0.59955464
## [3,]  0.86865983  2.0148645  0.51874901 -1.4049179  2.01486448 -1.18815834
## [4,] -1.17859974  0.4924386 -1.30920430 -0.6714594  0.49243855 -1.17939052
## [5,]  1.72719552  0.3680252  0.05460517  0.7386219 -0.88516413 -0.43233430
## [6,]  0.65320671 -0.5995546  0.19038081  1.7076774 -0.59677030 -0.05413369
## [7,] -1.40491794 -1.1881583 -1.05871745 -1.1697359  0.55377583 -0.03808156
## [8,]  0.02418764 -1.1793905 -0.42162386  1.1379026  0.04487299 -0.16026528
## [9,] -0.36856649 -0.4323343  0.46161167 -0.1704941 -1.09437298  0.24581094
##           [,7]      [,8]      [,9]
## [1,] -1.30920430  0.7386219  0.04487299
## [2,]  0.05460517  1.7076774 -1.09437298
## [3,]  0.19038081 -1.1697359 -0.03808156
## [4,] -1.05871745  1.1379026 -0.16026528
## [5,] -0.42162386 -0.1704941  0.24581094
## [6,]  0.46161167 -0.5967703  1.26325824
## [7,]  0.21413762  0.5537758 -1.05951952
## [8,]  1.26325824 -0.3891779 -0.44724564
## [9,] -1.05951952 -0.4472456  2.60809809
```

### The Eigenvalues

```
eigen_frame(P)
```

```
##           Re           Im row_i
## 1  -0.10399 -0.15539      1
## 2  -0.10399  0.15539      1
## 3  -0.18463  0.00000      1
## 4  -0.05121  0.00000      1
## 5  -0.35573 -0.28293      1
## 6  -0.35573  0.28293      1
## 7   0.31997  0.06061      1
## 8   0.31997 -0.06061      1
## 9  -0.01229  0.00000      1
## 10  0.60610  0.00000      2
## 11  0.60610  0.00000      2
## 12 -0.02246  0.00000      2
## 13 -0.17749  0.00000      2
```

## 14	-0.03028	0.05576	2
## 15	-0.03028	-0.05576	2
## 16	0.02410	-0.09115	2
## 17	0.02410	0.09115	2
## 18	0.18149	0.00000	2
## 19	0.06305	-0.51889	3
## 20	0.06305	0.51889	3
## 21	0.03536	0.00000	3
## 22	-0.56805	0.00000	3
## 23	0.59469	0.00000	3
## 24	0.59469	0.00000	3
## 25	-0.59349	0.00000	3
## 26	-0.59349	0.00000	3
## 27	-0.56756	0.00000	3
## 28	0.20238	0.07579	4
## 29	0.20238	-0.07579	4
## 30	-0.33936	0.00000	4
## 31	0.05530	0.00000	4
## 32	0.30963	0.20013	4
## 33	0.30963	-0.20013	4
## 34	0.09201	0.02117	4
## 35	0.09201	-0.02117	4
## 36	0.19351	0.00000	4
## 37	-0.00467	-0.12897	5
## 38	-0.00467	0.12897	5
## 39	0.43150	0.00000	5
## 40	-0.04679	0.00000	5
## 41	-0.07943	0.32180	5
## 42	-0.07943	-0.32180	5
## 43	0.33236	-0.01130	5
## 44	0.33236	0.01130	5
## 45	0.14426	0.00000	5
## 46	-0.07983	0.12924	6
## 47	-0.07983	-0.12924	6
## 48	0.55629	0.00000	6
## 49	0.37590	0.00000	6
## 50	-0.02645	-0.24221	6
## 51	-0.02645	0.24221	6
## 52	0.54578	-0.07201	6
## 53	0.54578	0.07201	6
## 54	0.51046	0.00000	6
## 55	-0.05409	0.30697	7
## 56	-0.05409	-0.30697	7
## 57	-0.40167	0.00000	7
## 58	0.03077	0.00000	7
## 59	0.14718	0.11009	7
## 60	0.14718	-0.11009	7
## 61	-0.06862	0.01561	7
## 62	-0.06862	-0.01561	7
## 63	-0.32781	0.00000	7
## 64	-0.02756	0.23133	8
## 65	-0.02756	-0.23133	8
## 66	0.43810	0.00000	8
## 67	0.06047	0.00000	8

```
## 68  0.07405 -0.28178      8
## 69  0.07405  0.28178      8
## 70  0.11579  0.10815      8
## 71  0.11579 -0.10815      8
## 72 -0.46349  0.00000      8
## 73 -0.25873  0.14019      9
## 74 -0.25873 -0.14019      9
## 75 -0.00891  0.00000      9
## 76  0.70145  0.00000      9
## 77 -0.09166  0.05922      9
## 78 -0.09166 -0.05922      9
## 79  0.27357  0.07315      9
## 80  0.27357 -0.07315      9
## 81 -0.06062  0.00000      9
```

## Step 1: Get the batch

```
# Set batch parameters
B <- 100
# Create batch
batch <- make_batch(M = M, B = B)
head(batch)
```

```
##           x1           x2           x3           x4           x5           x6
## 1 -0.04070632  0.8922378  0.9740657  0.4485278 -0.01106800 -0.6037098
## 2  0.64676080  0.8610923 -0.9942109 -0.2012166  0.35944675 -0.2557878
## 3 -0.94910317 -0.7664982  0.3276610  0.9985725  0.76266919  0.5934003
## 4  0.14872891 -0.9079017 -0.8628192  0.6779868 -0.39852092 -0.5209503
## 5 -0.71352980 -0.9320286 -0.6531758  0.9893077  0.07481969 -0.2443750
## 6 -0.40847367 -0.0574896 -0.5034317 -0.1766644 -0.89289404 -0.1135333
##           x7           x8           x9
## 1 -0.87823312  0.9018124 -0.83237921
## 2 -0.19575995 -0.5325527  0.31532668
## 3  0.64135987 -0.9057326  0.07605654
## 4  0.98524870 -0.4475074  0.98107982
## 5  0.05282391 -0.6270022 -0.67814647
## 6  0.99590605  0.4076312 -0.15609459
```

## Step 2: Evolve the batch

```
# Set evolution parameters
steps <- 20
# Evolve batch
evolved_batch <- evolve_batch(batch, steps, with_steps = T)
# View
head(evolved_batch)
```

```
##           x1           x2           x3           x4           x5           x6
## 1 -0.04070632  0.8922378  0.9740657  0.4485278479 -0.0110680 -0.6037098
## 2  0.89340682  4.4484221 -1.1190702 -0.0008681338  2.6887784 -2.5144866
## 3 -1.15108119  6.3487892 -8.7654245  3.5720391666 -0.1324768 -3.0677049
## 4 -13.49382004 -11.2565447 -33.6484867 22.4332115687  5.0232651 -4.3648379
## 5 -64.39240983 -106.7865663 -121.4386480 7.6282426263 35.6864319 -8.8460320
## 6 -65.32396004 -475.1807328 -234.4972195 0.2264540528 60.5696915 93.1006388
```

##		x7	x8	x9	time	element_index
## 1	-0.8782331	0.9018124	-0.8323792	0		1
## 2	1.3716611	0.7617005	-3.4963601	1		1
## 3	1.5271654	12.6334262	-18.2137218	2		1
## 4	30.6269924	30.2377006	-65.9176159	3		1
## 5	97.3596707	72.1196516	-212.7728560	4		1
## 6	365.5374035	41.0172012	-575.3679760	5		1

```
tail(evolved_batch)
```

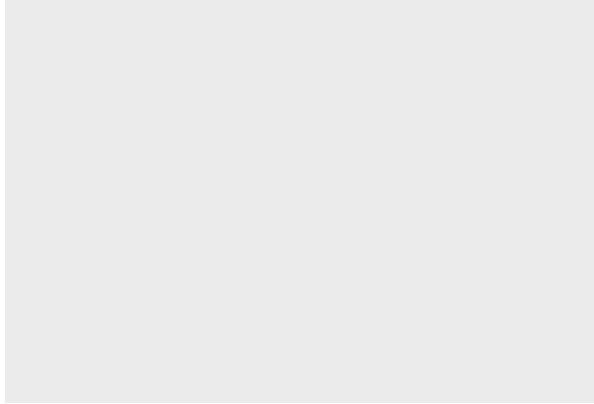
##		x1	x2	x3	x4	x5	x6
## 2095	-728700.1	4513747	-1182398	1538295	-1158882.5	-1590139.1	
## 2096	-7542272.2	5293629	-8523404	2734045	-2524878.5	-2993915.7	
## 2097	-21795794.0	-11448084	-29582611	3963894	-6493426.4	-794137.8	
## 2098	-62249787.5	-101996034	-73886776	-14048050	549063.2	20623805.3	
## 2099	-61222516.4	-386197571	-95907369	-74473824	29625674.8	108126909.1	
## 2100	89843542.8	-1011872549	135806059	-306344254	209117638.9	338027813.9	

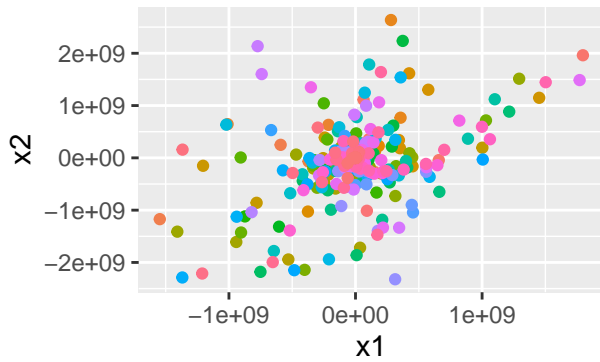
##		x7	x8	x9	time	element_index
## 2095	-1104567	2463490	632684.1	15		100
## 2096	1306505	9596460	-5748924.5	16		100
## 2097	23822360	18327250	-31317969.2	17		100
## 2098	81887931	25113401	-105675021.5	18		100
## 2099	247247907	-79277300	-233522876.5	19		100
## 2100	357293387	-474641783	-256198784.0	20		100

### Step 3: Analyze the batch

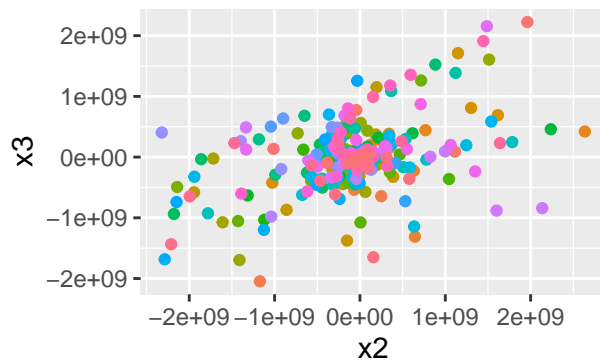
```
# Plot the evolution arrays of the batch elements
batch_data <- evolved_batch
# 3d plot
batch_3d_plot(batch_data, "(Symmetric Normal Matrix)")
```



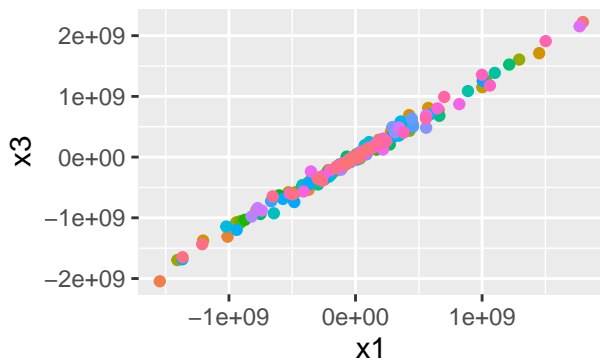
Evolution of Monte Carlo Batch



Evolution of Monte Carlo Batch



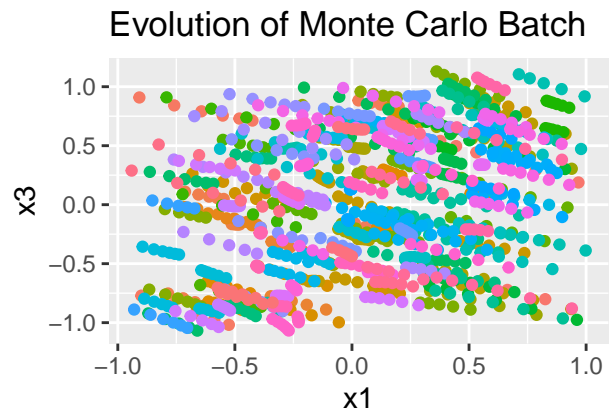
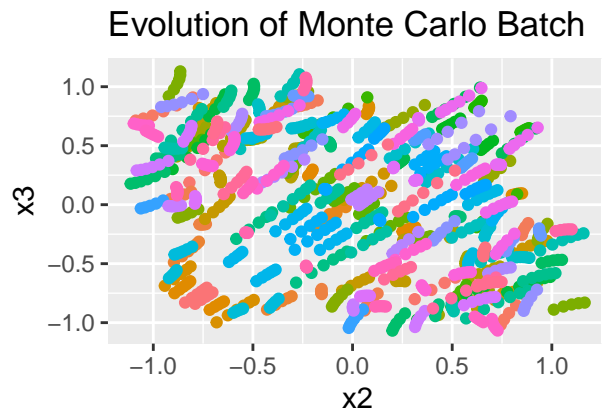
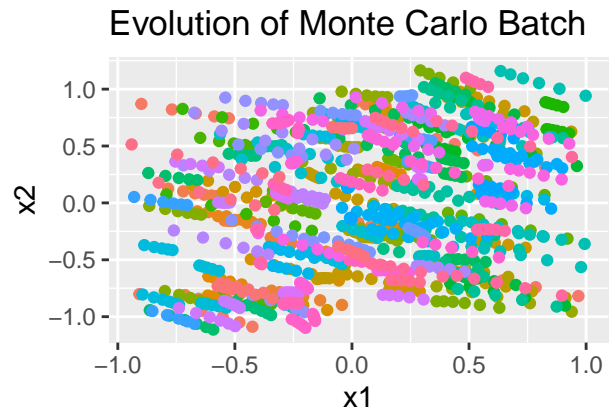
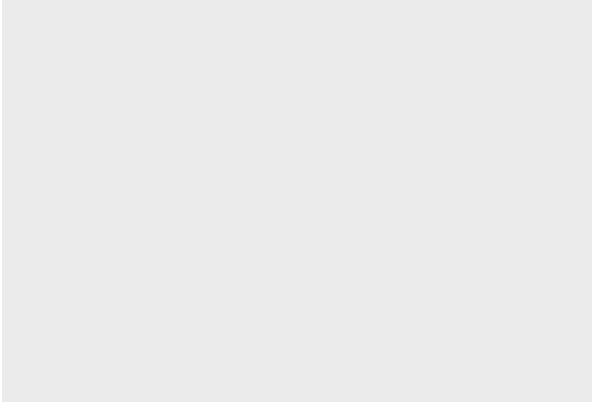
Evolution of Monte Carlo Batch



```
set.seed(27)
M <- 3
P <- RM_stoch(M, symm = T, sparsity = T)
# Set batch parameters
B <- 100
# Create batch
batch <- make_batch(M = M, B = B)
# Set evolution parameters
steps <- 10
# Evolve batch
evolved_batch <- evolve_batch(batch, steps, with_steps = T)

batch_3d_plot(evolved_batch)
```





```
#batch_2d_customplot(evolved_batch, n1 = 1, n2 = 2) + transition_time(V4)
```