Sparsity Analysis

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Generating Random Matrices

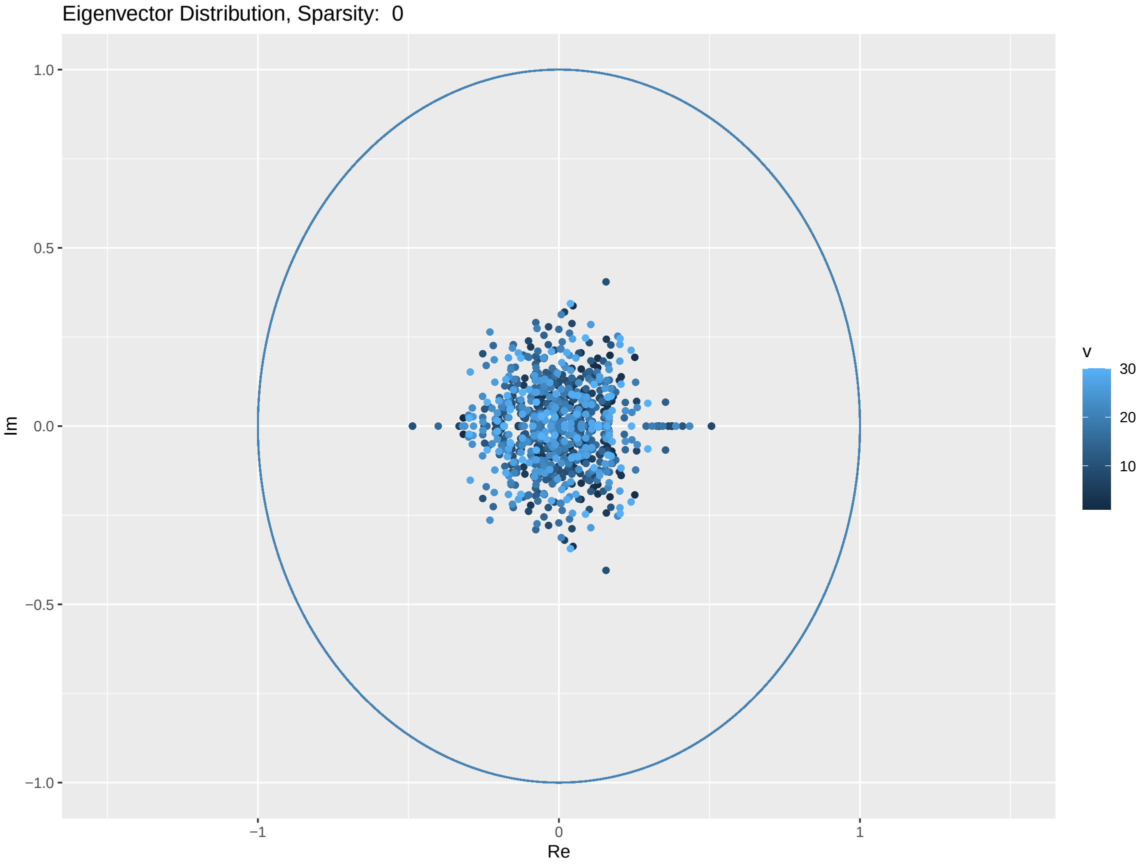
```r
# generates rows of size P which are valid probability distributions
r_sparse <- function(M,p){
  prob <- runif(M,0,1)
  num_zeros <- rbinom(1,M,p)
  choices <- sample(1:M, num_zeros)
  prob[choices] <- 0
  prob/sum(prob) # return normalized random row vector
}
```

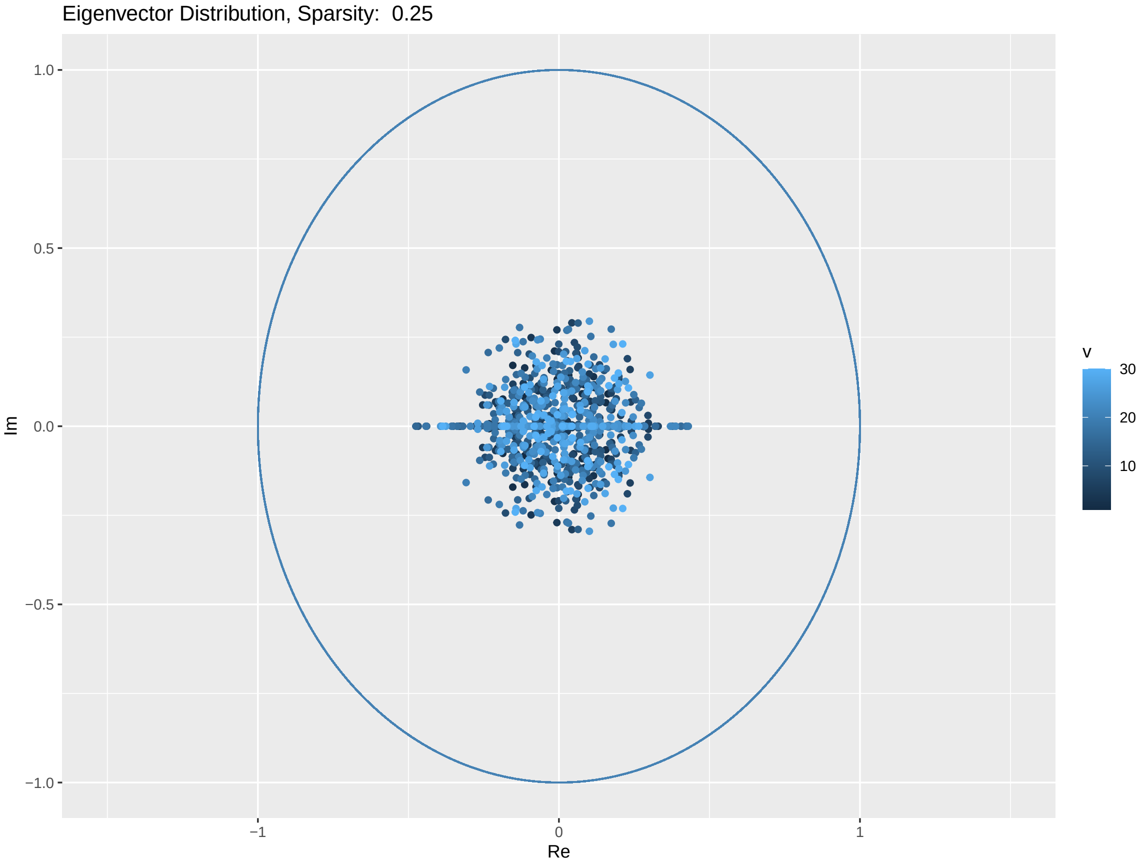
```r
# initialize random P
rand_M <- function(M,p,row_fxn){
  P <- matrix(rep(NA, M * M), ncol = M)  # create transition matrix
  for(i in 1:M){P[i,] = row_fxn(M,p)}
  P
}
```

Eigenvectors

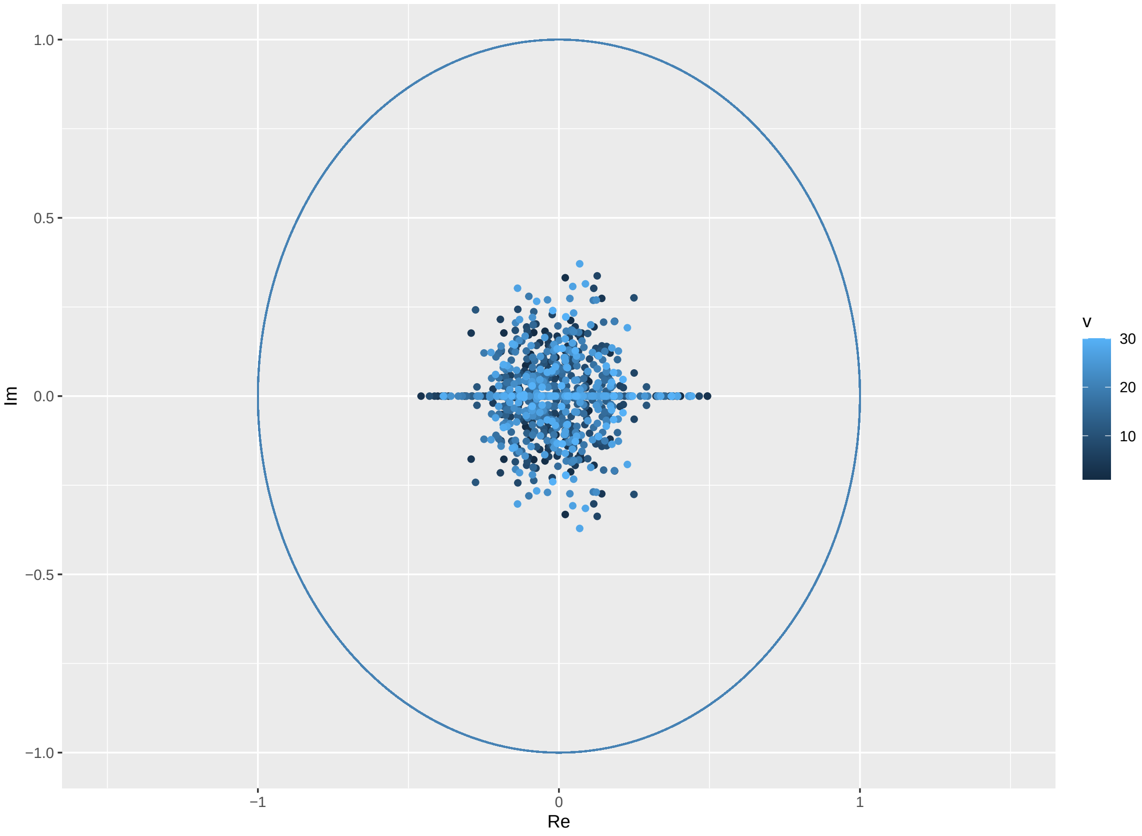
```r
complex_df <- function(eigenvectors){
  cols <- 3 # set 3 to hold (re,im) pair and whose row it belongs to
  complex <- matrix(rep(NA,cols*M*M), ncol = cols)
  colnames(complex) <- c("Re","Im","v")
  for(i in 1:M){
    for(j in 1:M){
      curr <- eigenvectors[i,j]
      complex[M*(i-1) + j, ] <- c(Re(curr),Im(curr),i)
    }
  }
  complex
}
```

```r
eigen_plot <- function(P,sparsity){
  eigen_vecs <- data.frame(eigen(P)[2])
  complex <- complex_df(eigen_vecs)
  r <- 1
  ep <- 0.5
  plot <- ggplot(complex) +
    geom_point(aes(x = Re, y = Im, color = v)) +
    labs(x = "Re", y = "Im", title = paste("Eigenvector Distribution, Sparsity: ",sparsity)) +
    xlim(-(r+ep),r+ep) + ylim(-r,r) +
    ggforce::geom_circle(aes(x0=0,y0=0,r=r), color = "steelblue")
  plot
}
```

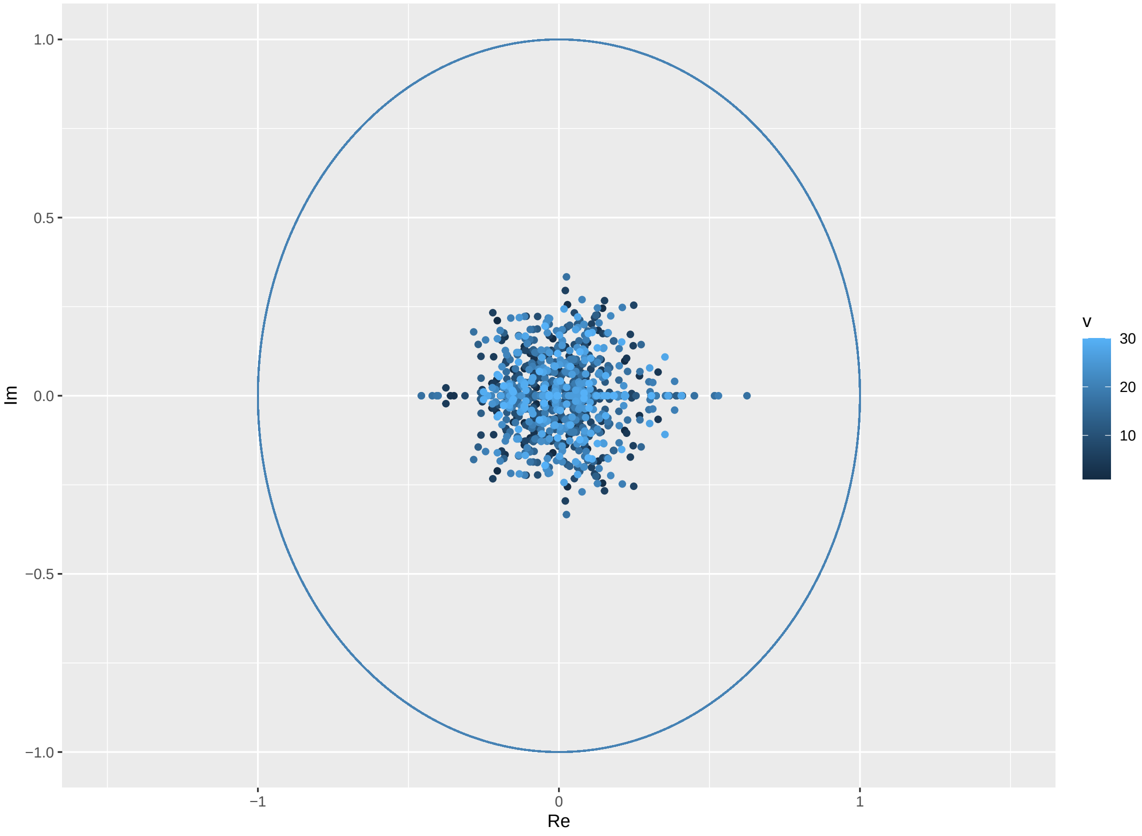

Eigenvector Distribution, Sparsity: 0
1.0
0.5
0.0
−0.5
−1.0
Im
−1
0
1
Re
v
30
20
10

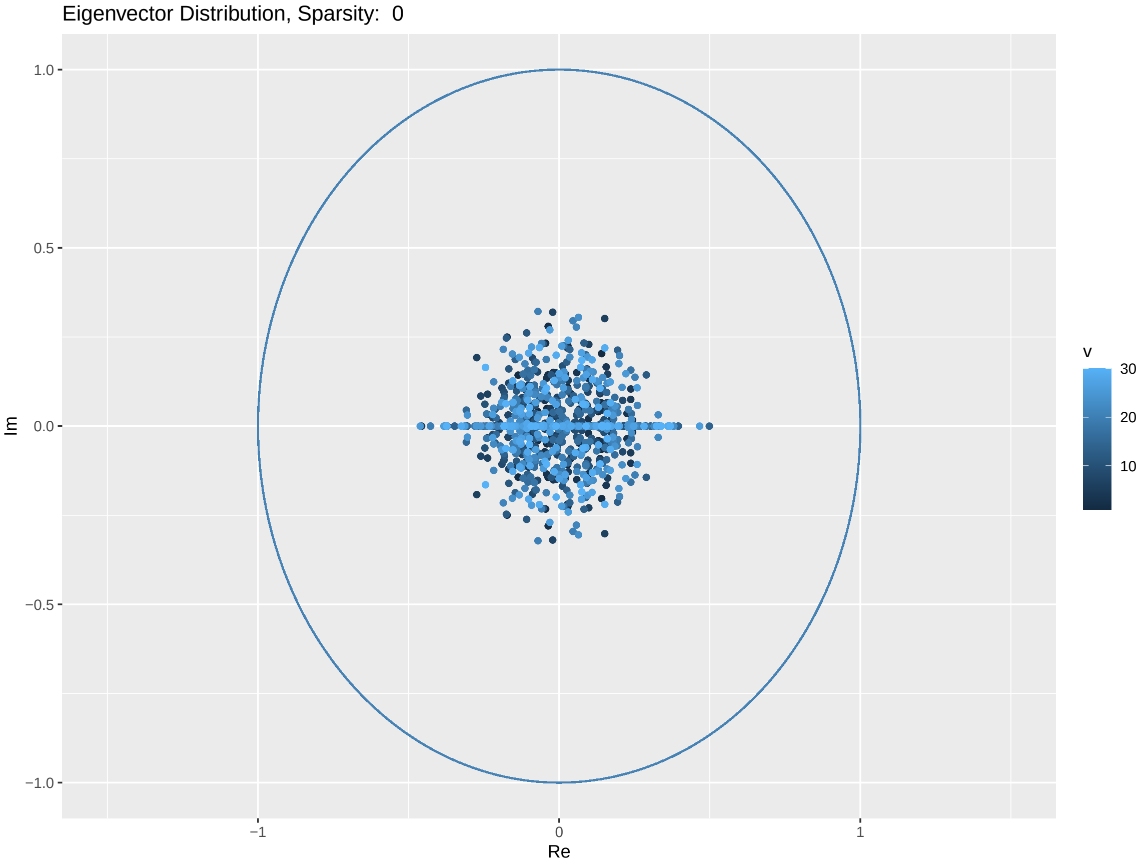

Eigenvector Distribution, Sparsity: 0.25
1.0
0.5
0.0
−0.5
−1.0
Im
−1
0
1
Re
v
30
20
10

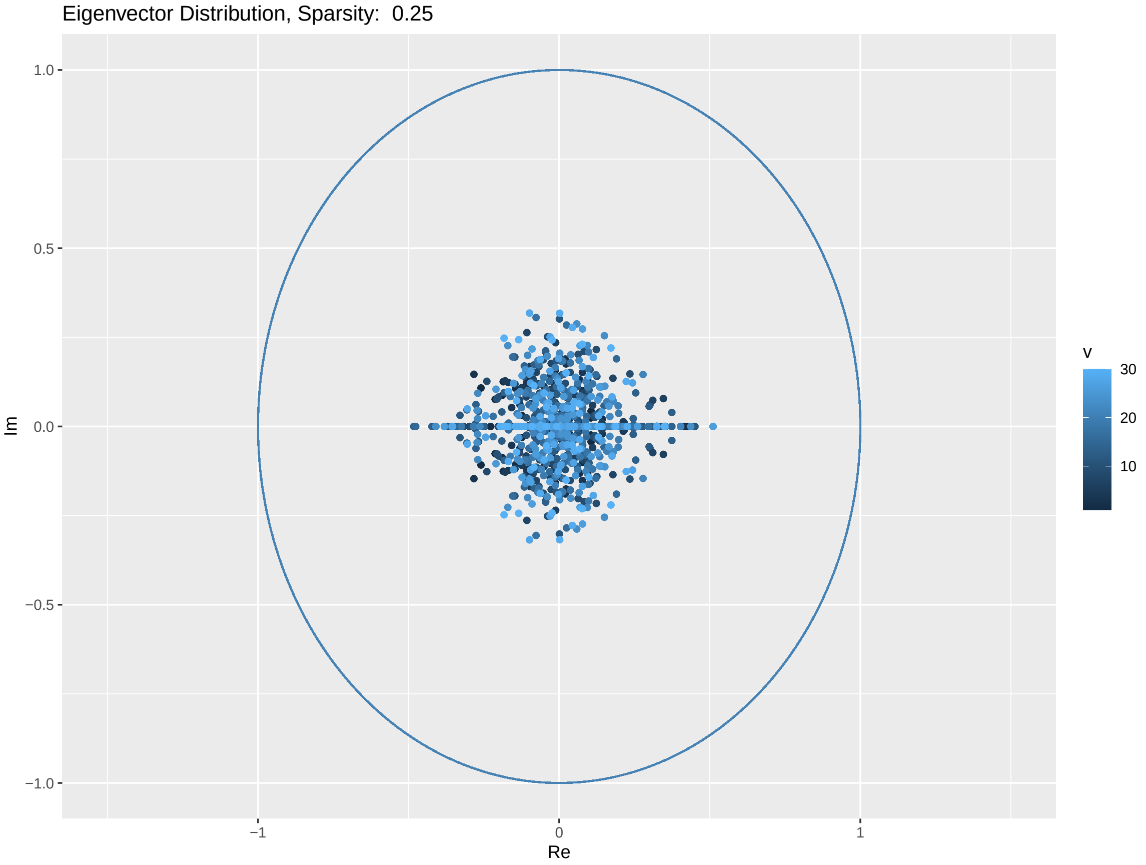
Eigenvector Distribution, Sparsity: 0.5

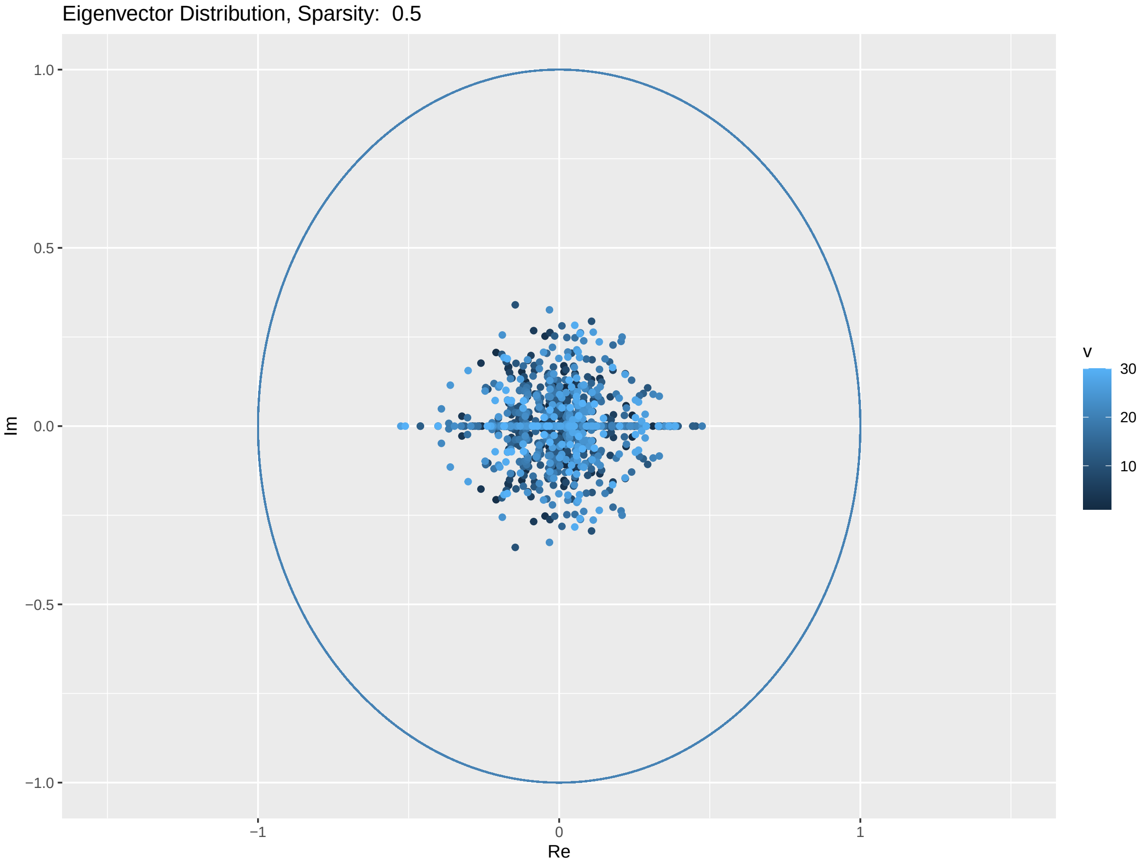


Eigenvector Distribution, Sparsity: 0.75

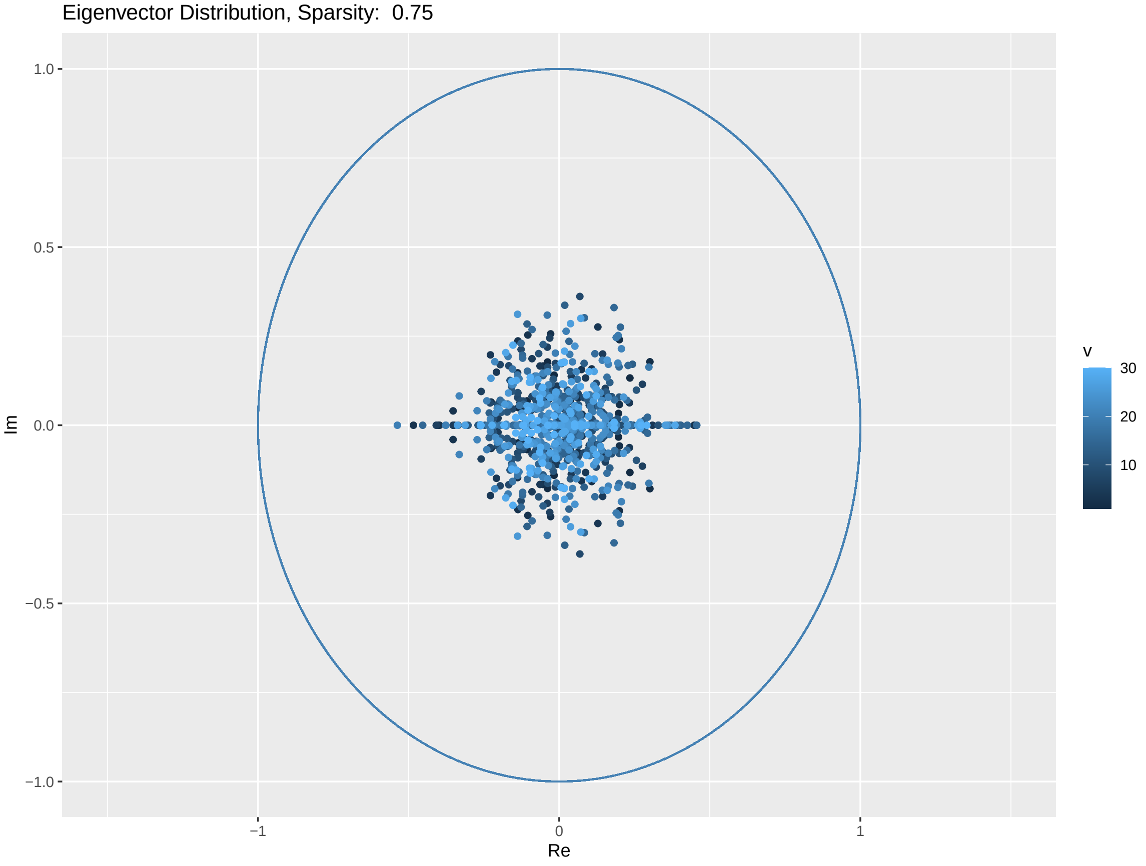



Eigenvector Distribution, Sparsity: 0
1.0
0.5
0.0
−0.5
−1.0
Im
−1
0
1
Re
v
30
20
10

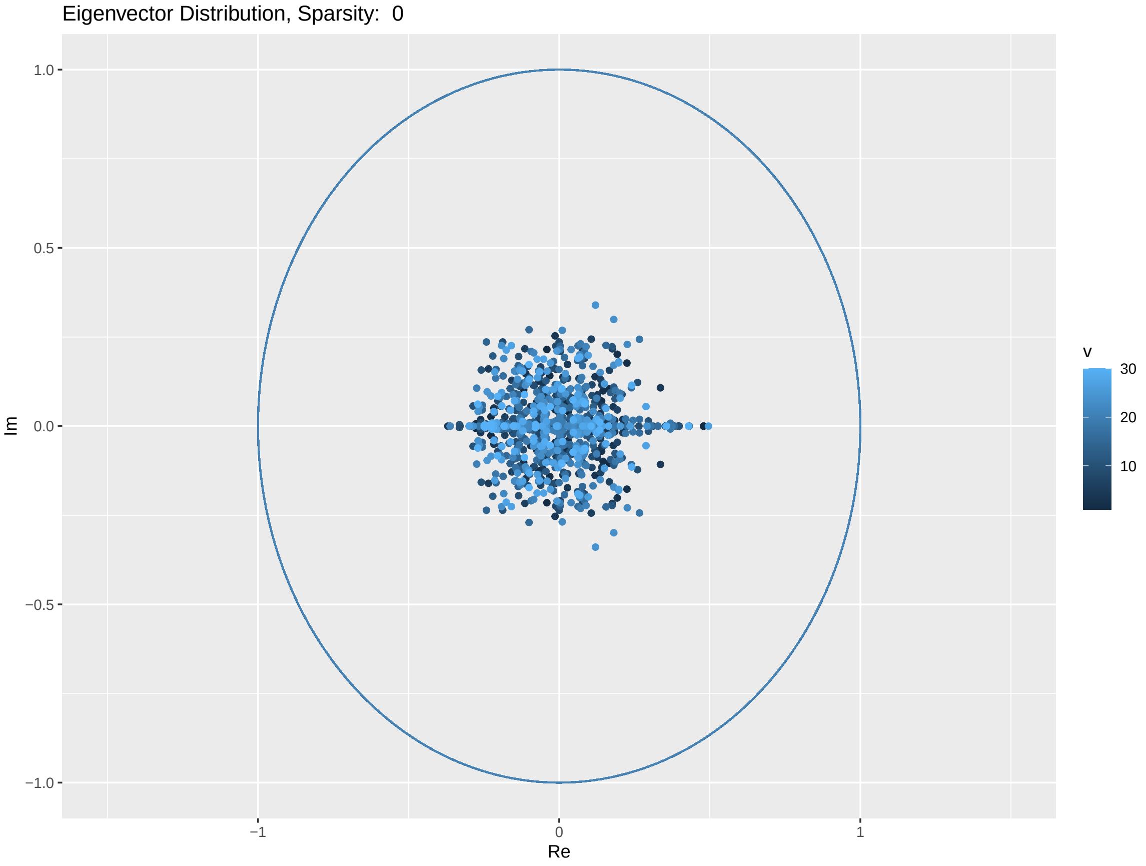

Eigenvector Distribution, Sparsity: 0.25
1.0
0.5
0.0
−0.5
−1.0
Im
−1
0
1
Re
v
30
20
10



Eigenvector Distribution, Sparsity: 0.5



Eigenvector Distribution, Sparsity: 0.75


Eigenvector Distribution, Sparsity: 0
1.0
0.5
0.0
−0.5
−1.0
Im
−1
0
1
Re
v
30
20
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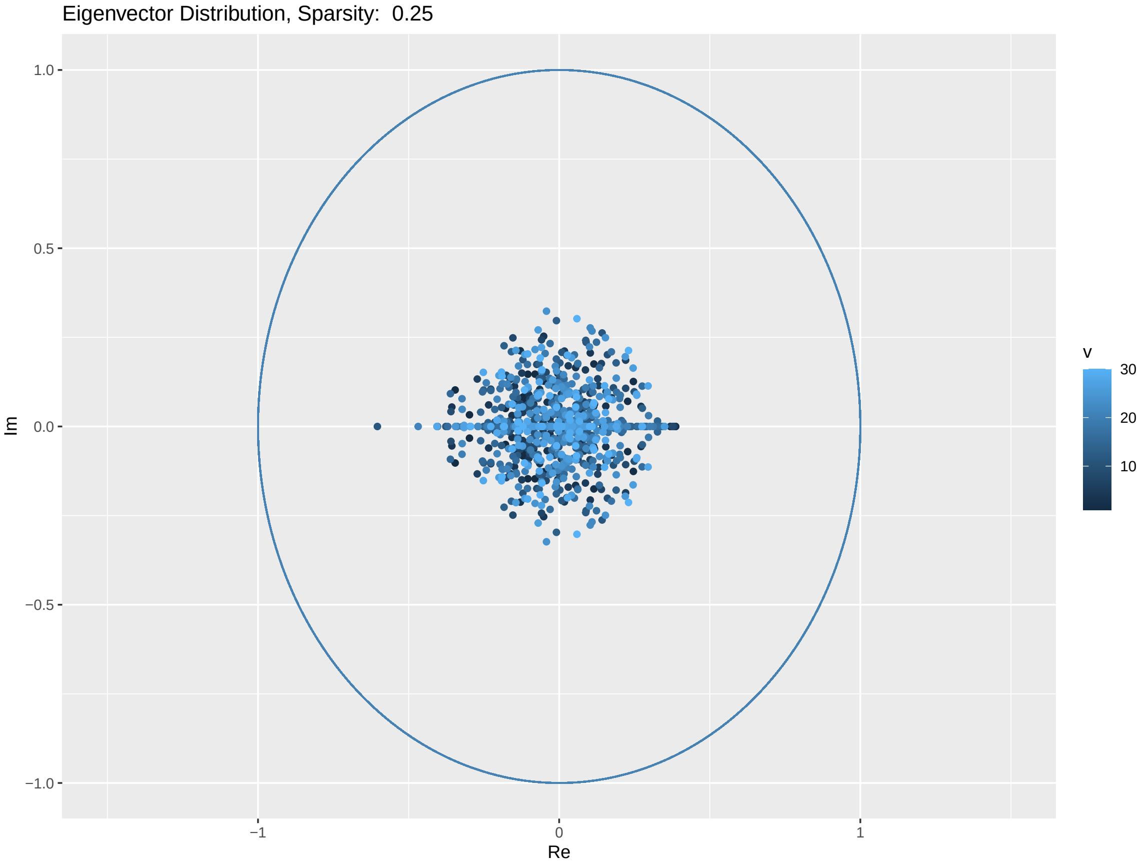
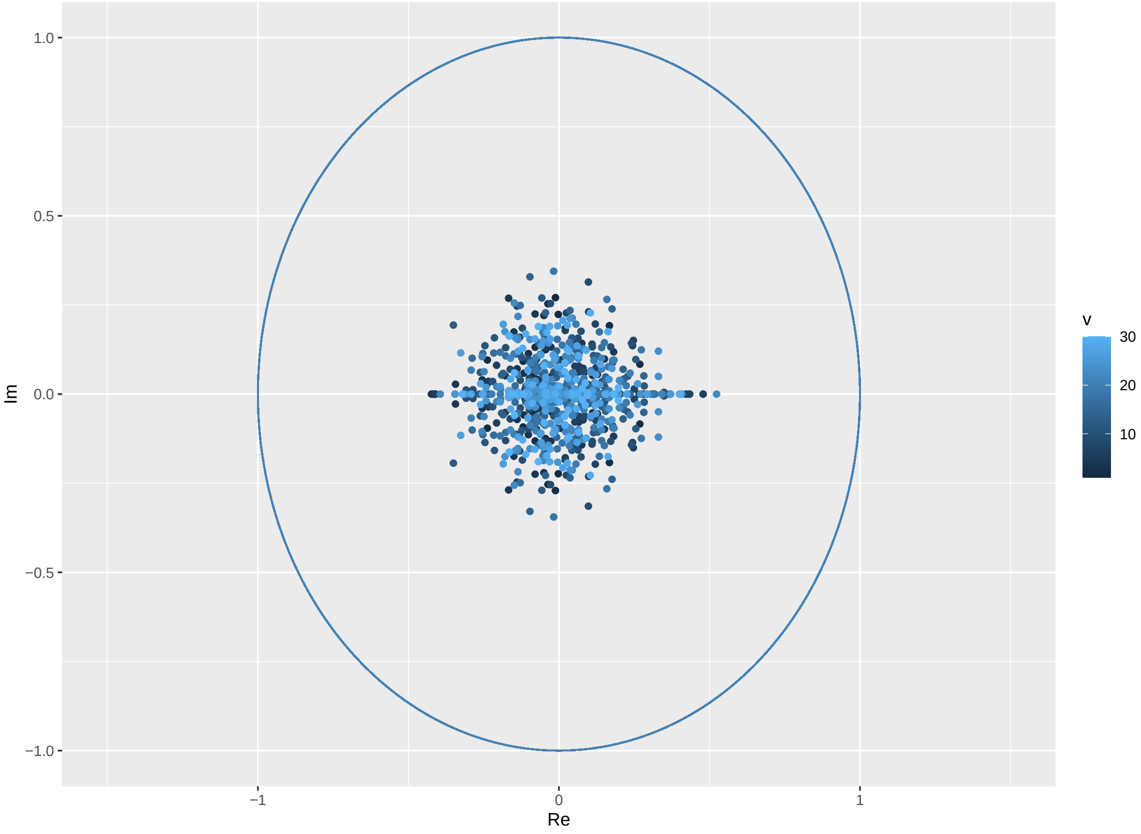
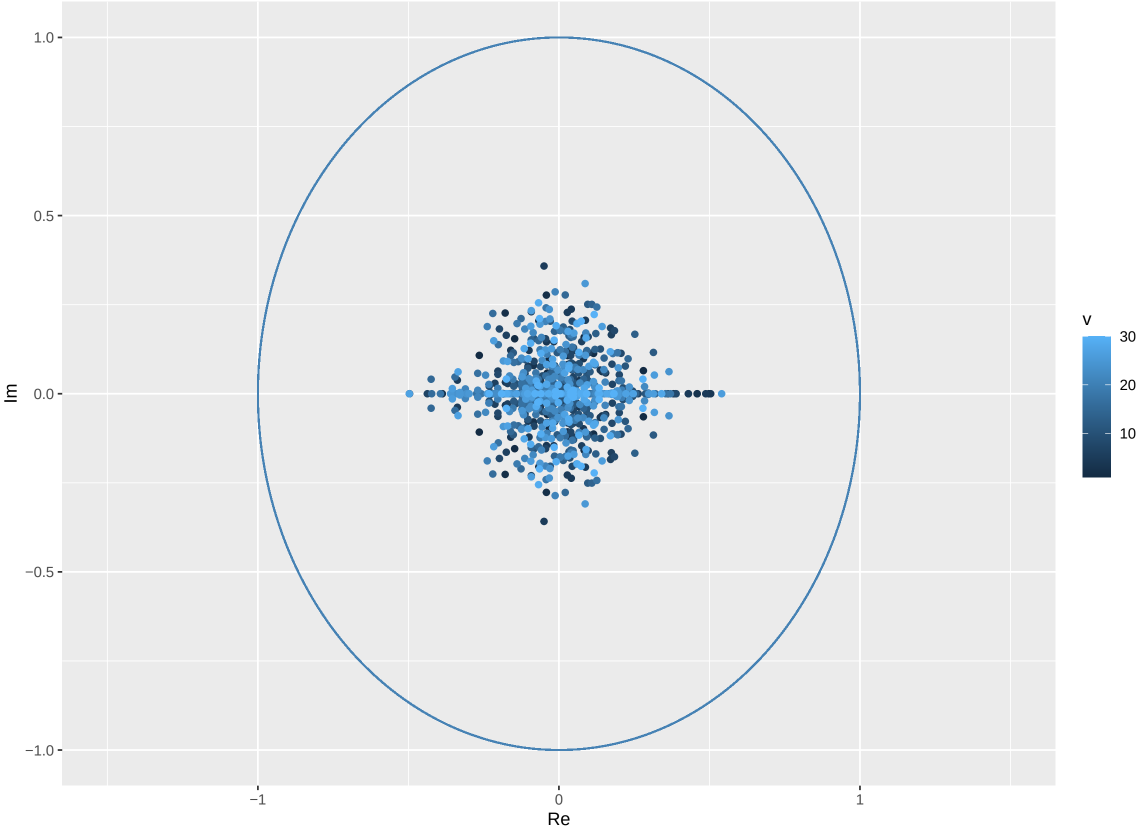

Eigenvector Distribution, Sparsity: 0.25
1.0
0.5
0.0
−0.5
−1.0
Im
−1
0
1
Re
v
30
20
10

Eigenvector Distribution, Sparsity: 0.5



Eigenvector Distribution, Sparsity: 0.75


Im
Re
v