Statistics of the Ratios in the CRS

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Possible Distributions

Recall that we are considering the entries of the consecutive ratio sequence. One possible distribution of these entries is a Cauchy distribution.

A Cauchy(x_0, γ) distribution has the following:

PDF: $\frac{1}{\pi\gamma\left[1+\left(\frac{x-x_0}{\gamma}\right)^2\right]}$
Median: x_0
Mean: Undefined
Variance: Undefined

A Log-Cauchy(μ, σ) distribution has the following:

PDF: $\frac{1}{\pi x}\left[\frac{\sigma}{\ln(x-\mu)^2+\sigma^2}\right]$
Median: e^{μ}
Mean: ∞
Variance: ∞

Variance by Power

Additionally, we also consider the plots of the variance of the CRS by a given time. In clearer terms, we also consider the following:

$$V(n) = \text{Var}\left(\frac{\vec{a}Q^{n+1}}{\vec{a}Q^n}\right) \text{ for } \vec{a} = (a_i) \text{ where } a_i \sim \text{Unif}(-\lambda, \lambda)$$