

Sparsity Analysis

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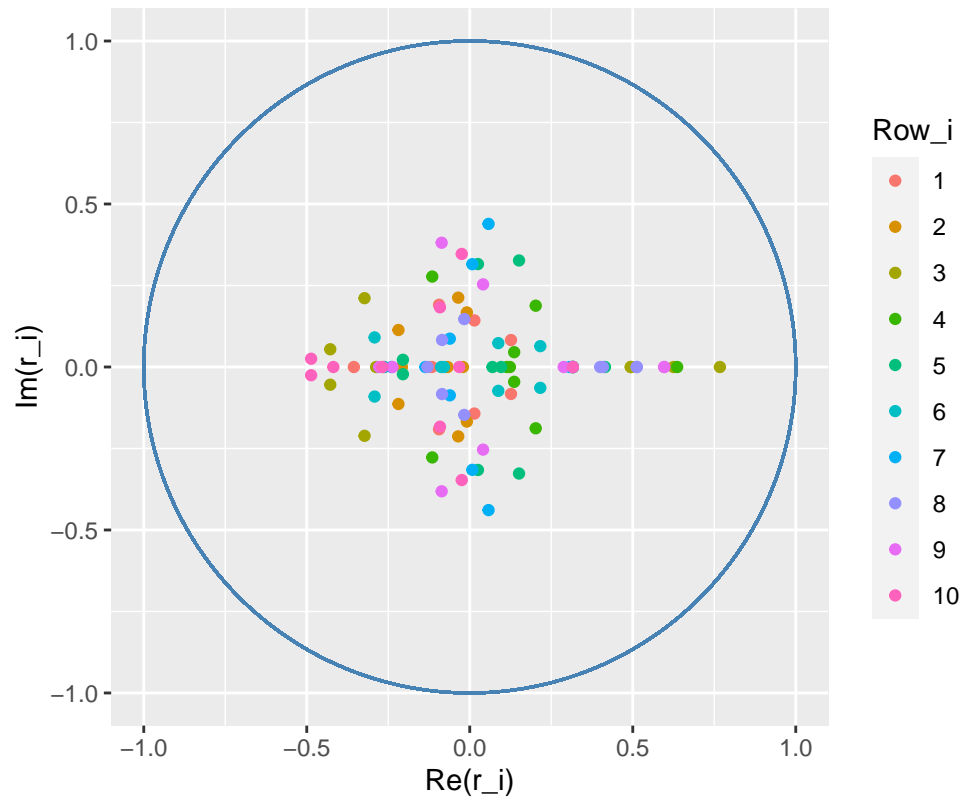
p-Sparse/Erdos-Renyi Matrices

```
M_vec <- c(10,25,50)
p_vec <- c(0.1,0.5,0.8)
c(M1,M2,M3) %<-% M_vec
c(p1,p2,p3) %<-% p_vec
# Generate and concatenate matrices
P_vec1 <- matrix(c(rand_M_erdos(M1,p1),
                        rand_M_erdos(M1,p2),
                        rand_M_erdos(M1,p3)),
                nrow = M_vec[1])
P_vec2 <- matrix(c(rand_M_erdos(M2,p1),
                        rand_M_erdos(M2,p2),
                        rand_M_erdos(M2,p3)),
                nrow = M_vec[2])
P_vec3 <- matrix(c(rand_M_erdos(M3,p1),
                        rand_M_erdos(M3,p2),
                        rand_M_erdos(M3,p3)),
                nrow = M_vec[3])
```

##		Re	Im	row_i
## 1	0.31623	0.00000		1
## 2	0.01424	0.14289		1
## 3	0.01424	-0.14289		1
## 4	-0.11619	0.00000		1
## 5	-0.26436	0.00000		1
## 6	0.12645	-0.08265		1
## 7	0.12645	0.08265		1
## 8	-0.35552	0.00000		1
## 9	-0.09415	-0.19095		1
## 10	-0.09415	0.19095		1
## 11	0.31623	0.00000		2
## 12	-0.21877	0.11353		2
## 13	-0.21877	-0.11353		2
## 14	-0.20818	0.00000		2
## 15	-0.02095	0.00000		2
## 16	-0.00880	0.16744		2
## 17	-0.00880	-0.16744		2
## 18	-0.06808	0.00000		2
## 19	-0.03540	-0.21277		2
## 20	-0.03540	0.21277		2
## 21	0.31623	0.00000		3
## 22	-0.32283	-0.21108		3
## 23	-0.32283	0.21108		3
## 24	0.76730	0.00000		3
## 25	0.62520	0.00000		3
## 26	0.49412	0.00000		3
## 27	0.49412	0.00000		3
## 28	-0.28737	0.00000		3
## 29	-0.42789	-0.05437		3
## 30	-0.42789	0.05437		3
## 31	0.31623	0.00000		4
## 32	-0.11499	-0.27742		4
## 33	-0.11499	0.27742		4
## 34	0.11395	0.00000		4
## 35	0.12310	0.00000		4
## 36	0.13626	0.04546		4
## 37	0.13626	-0.04546		4
## 38	0.63631	0.00000		4
## 39	0.20242	-0.18801		4
## 40	0.20242	0.18801		4
## 41	0.31623	0.00000		5
## 42	0.02511	-0.31577		5
## 43	0.02511	0.31577		5
## 44	0.09623	0.00000		5
## 45	0.41468	0.00000		5
## 46	0.15114	-0.32676		5
## 47	0.15114	0.32676		5
## 48	0.06971	0.00000		5
## 49	-0.20497	0.02213		5
## 50	-0.20497	-0.02213		5
## 51	0.31623	0.00000		6
## 52	0.08764	-0.07320		6
## 53	0.08764	0.07320		6

## 54	-0.26206	0.00000	6
## 55	-0.07946	0.00000	6
## 56	0.21634	0.06393	6
## 57	0.21634	-0.06393	6
## 58	-0.08777	0.00000	6
## 59	-0.29197	0.09065	6
## 60	-0.29197	-0.09065	6
## 61	0.31623	0.00000	7
## 62	0.05757	0.43890	7
## 63	0.05757	-0.43890	7
## 64	-0.13631	0.00000	7
## 65	-0.23946	0.00000	7
## 66	0.00783	-0.31544	7
## 67	0.00783	0.31544	7
## 68	0.30651	0.00000	7
## 69	-0.06121	0.08689	7
## 70	-0.06121	-0.08689	7
## 71	0.31623	0.00000	8
## 72	0.51236	0.00000	8
## 73	0.51236	0.00000	8
## 74	0.40840	0.00000	8
## 75	0.40005	0.00000	8
## 76	-0.08491	0.08297	8
## 77	-0.08491	-0.08297	8
## 78	-0.12969	0.00000	8
## 79	-0.01683	-0.14698	8
## 80	-0.01683	0.14698	8
## 81	0.31623	0.00000	9
## 82	0.04080	0.25350	9
## 83	0.04080	-0.25350	9
## 84	-0.27763	0.00000	9
## 85	-0.23706	0.00000	9
## 86	-0.08613	0.38121	9
## 87	-0.08613	-0.38121	9
## 88	0.28825	0.00000	9
## 89	0.59646	0.00000	9
## 90	0.59646	0.00000	9
## 91	0.31623	0.00000	10
## 92	-0.09140	-0.18315	10
## 93	-0.09140	0.18315	10
## 94	-0.03223	0.00000	10
## 95	-0.26792	0.00000	10
## 96	-0.48655	-0.02534	10
## 97	-0.48655	0.02534	10
## 98	-0.41842	0.00000	10
## 99	-0.02451	0.34682	10
## 100	-0.02451	-0.34682	10

Eigenvectors: 0.1-Sparse Matrix



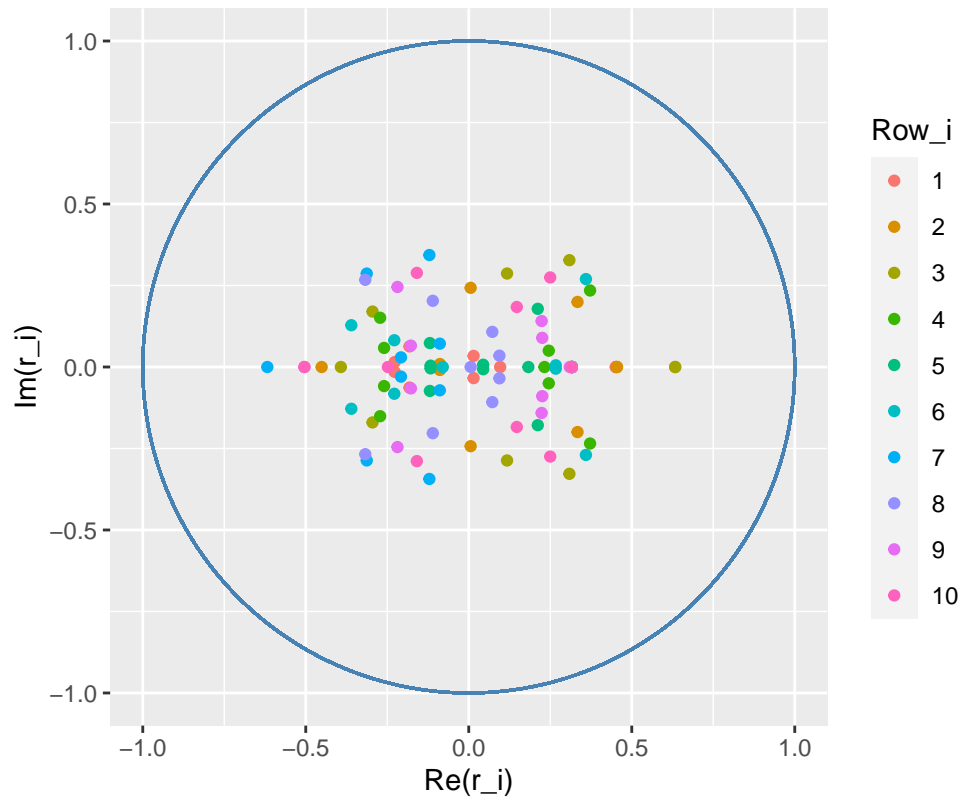
```
## # A tibble: 10 x 3
##   row_i prop_reals is_real
##   <dbl>   <dbl> <lgl>
## 1     1     0.4 FALSE
## 2     2     0.4 FALSE
## 3     3     0.6 FALSE
## 4     4     0.4 FALSE
## 5     5     0.4 FALSE
## 6     6     0.4 FALSE
## 7     7     0.4 FALSE
## 8     8     0.6 FALSE
## 9     9     0.6 FALSE
## 10    10     0.4 FALSE

## [1] "Proportion of real-valued rows: 0"
```

##		Re	Im	row_i
## 1	0.31623	0.00000		1
## 2	0.45119	0.00000		1
## 3	0.45119	0.00000		1
## 4	0.01479	-0.03404		1
## 5	0.01479	0.03404		1
## 6	-0.18176	-0.06317		1
## 7	-0.18176	0.06317		1
## 8	0.09637	0.00000		1
## 9	-0.22715	0.01517		1
## 10	-0.22715	-0.01517		1
## 11	0.31623	0.00000		2
## 12	0.00578	-0.24291		2
## 13	0.00578	0.24291		2
## 14	0.45465	0.00000		2
## 15	0.45465	0.00000		2
## 16	-0.08841	0.00913		2
## 17	-0.08841	-0.00913		2
## 18	-0.45126	0.00000		2
## 19	0.33369	0.19955		2
## 20	0.33369	-0.19955		2
## 21	0.31623	0.00000		3
## 22	0.11744	-0.28679		3
## 23	0.11744	0.28679		3
## 24	0.30872	-0.32766		3
## 25	0.30872	0.32766		3
## 26	-0.29517	-0.17011		3
## 27	-0.29517	0.17011		3
## 28	-0.39234	0.00000		3
## 29	0.63328	0.00000		3
## 30	0.63328	0.00000		3
## 31	0.31623	0.00000		4
## 32	0.24551	0.05002		4
## 33	0.24551	-0.05002		4
## 34	-0.27186	0.15097		4
## 35	-0.27186	-0.15097		4
## 36	0.37209	0.23474		4
## 37	0.37209	-0.23474		4
## 38	0.23231	0.00000		4
## 39	-0.25970	-0.05851		4
## 40	-0.25970	0.05851		4
## 41	0.31623	0.00000		5
## 42	0.04422	0.00691		5
## 43	0.04422	-0.00691		5
## 44	-0.11922	-0.07345		5
## 45	-0.11922	0.07345		5
## 46	0.21175	-0.17843		5
## 47	0.21175	0.17843		5
## 48	0.18295	0.00000		5
## 49	-0.11699	-0.00411		5
## 50	-0.11699	0.00411		5
## 51	0.31623	0.00000		6
## 52	-0.36022	0.12817		6
## 53	-0.36022	-0.12817		6

## 54	0.35945	-0.26975	6
## 55	0.35945	0.26975	6
## 56	-0.22843	0.08211	6
## 57	-0.22843	-0.08211	6
## 58	-0.07965	0.00000	6
## 59	0.26690	0.00509	6
## 60	0.26690	-0.00509	6
## 61	0.31623	0.00000	7
## 62	-0.12086	0.34347	7
## 63	-0.12086	-0.34347	7
## 64	-0.20787	-0.02954	7
## 65	-0.20787	0.02954	7
## 66	-0.31288	-0.28659	7
## 67	-0.31288	0.28659	7
## 68	-0.61790	0.00000	7
## 69	-0.08872	0.07153	7
## 70	-0.08872	-0.07153	7
## 71	0.31623	0.00000	8
## 72	-0.31748	-0.26746	8
## 73	-0.31748	0.26746	8
## 74	0.09409	0.03474	8
## 75	0.09409	-0.03474	8
## 76	0.07233	-0.10780	8
## 77	0.07233	0.10780	8
## 78	0.00601	0.00000	8
## 79	-0.11027	-0.20305	8
## 80	-0.11027	0.20305	8
## 81	0.31623	0.00000	9
## 82	0.22358	0.14104	9
## 83	0.22358	-0.14104	9
## 84	-0.21869	0.24558	9
## 85	-0.21869	-0.24558	9
## 86	0.22542	-0.08949	9
## 87	0.22542	0.08949	9
## 88	0.30981	0.00000	9
## 89	-0.17745	-0.06567	9
## 90	-0.17745	0.06567	9
## 91	0.31623	0.00000	10
## 92	0.14739	-0.18403	10
## 93	0.14739	0.18403	10
## 94	-0.15916	0.28874	10
## 95	-0.15916	-0.28874	10
## 96	-0.50414	0.00000	10
## 97	-0.50414	0.00000	10
## 98	-0.24806	0.00000	10
## 99	0.24990	0.27477	10
## 100	0.24990	-0.27477	10

Eigenvectors: 0.5-Sparse Matrix



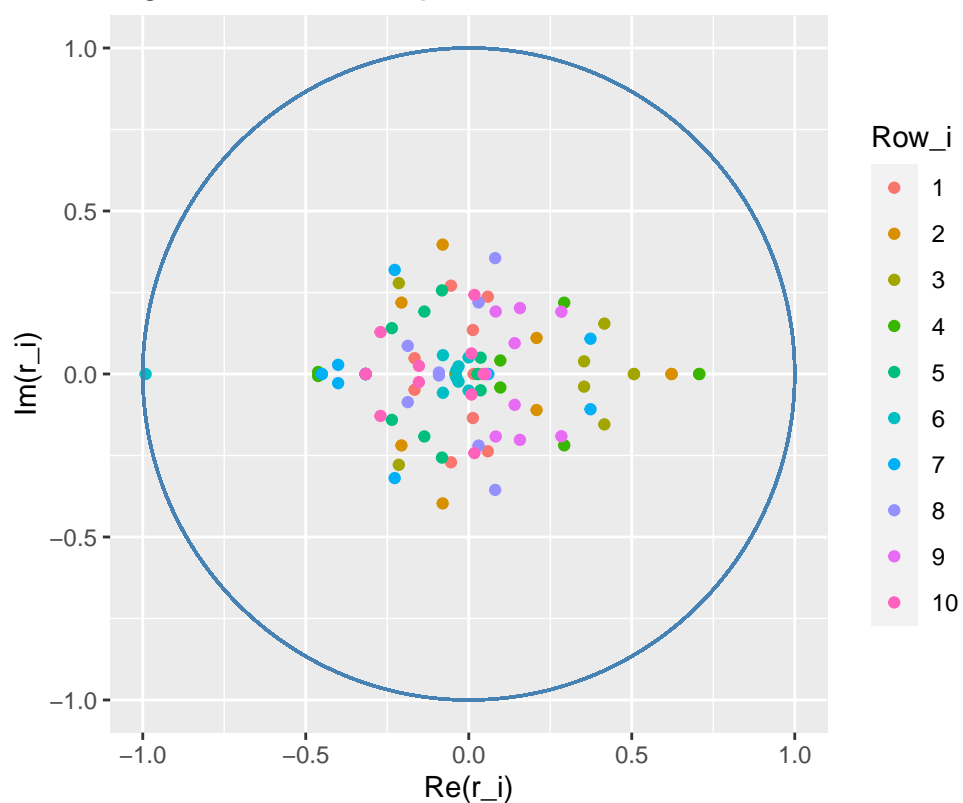
```
## # A tibble: 10 x 3
##   row_i prop_reals is_real
##   <dbl>   <dbl> <lgl>
## 1     1     0.4 FALSE
## 2     2     0.4 FALSE
## 3     3     0.4 FALSE
## 4     4     0.2 FALSE
## 5     5     0.2 FALSE
## 6     6     0.2 FALSE
## 7     7     0.2 FALSE
## 8     8     0.2 FALSE
## 9     9     0.2 FALSE
## 10    10     0.4 FALSE

## [1] "Proportion of real-valued rows: 0"
```

##		Re	Im	row_i
## 1	-0.31623	0.00000		1
## 2	0.01490	0.00000		1
## 3	-0.05448	-0.27093		1
## 4	-0.05448	0.27093		1
## 5	-0.16626	0.04840		1
## 6	-0.16626	-0.04840		1
## 7	0.05843	-0.23711		1
## 8	0.05843	0.23711		1
## 9	0.01278	-0.13497		1
## 10	0.01278	0.13497		1
## 11	-0.31623	0.00000		2
## 12	0.05560	0.00000		2
## 13	-0.20637	0.21907		2
## 14	-0.20637	-0.21907		2
## 15	0.62213	0.00000		2
## 16	0.62213	0.00000		2
## 17	-0.08004	0.39691		2
## 18	-0.08004	-0.39691		2
## 19	0.20815	0.11059		2
## 20	0.20815	-0.11059		2
## 21	-0.31623	0.00000		3
## 22	-0.04173	0.00000		3
## 23	0.41597	0.15456		3
## 24	0.41597	-0.15456		3
## 25	-0.21447	-0.27866		3
## 26	-0.21447	0.27866		3
## 27	0.50675	0.00000		3
## 28	0.50675	0.00000		3
## 29	0.35371	0.03879		3
## 30	0.35371	-0.03879		3
## 31	-0.31623	0.00000		4
## 32	0.03084	0.00000		4
## 33	0.29282	-0.21881		4
## 34	0.29282	0.21881		4
## 35	0.09694	-0.04144		4
## 36	0.09694	0.04144		4
## 37	-0.46201	-0.00619		4
## 38	-0.46201	0.00619		4
## 39	0.70710	0.00000		4
## 40	0.70710	0.00000		4
## 41	-0.31623	0.00000		5
## 42	0.02685	0.00000		5
## 43	-0.08212	0.25643		5
## 44	-0.08212	-0.25643		5
## 45	0.03630	0.05001		5
## 46	0.03630	-0.05001		5
## 47	-0.13624	-0.19162		5
## 48	-0.13624	0.19162		5
## 49	-0.23582	0.14071		5
## 50	-0.23582	-0.14071		5
## 51	-0.31623	0.00000		6
## 52	-0.99122	0.00000		6
## 53	-0.03150	-0.02344		6

## 54	-0.03150	0.02344	6
## 55	-0.00026	0.05100	6
## 56	-0.00026	-0.05100	6
## 57	-0.07925	-0.05765	6
## 58	-0.07925	0.05765	6
## 59	-0.03807	-0.01069	6
## 60	-0.03807	0.01069	6
## 61	-0.31623	0.00000	7
## 62	0.06007	0.00000	7
## 63	-0.45048	0.00000	7
## 64	-0.45048	0.00000	7
## 65	-0.22715	-0.31924	7
## 66	-0.22715	0.31924	7
## 67	0.37352	0.10808	7
## 68	0.37352	-0.10808	7
## 69	-0.40067	0.02812	7
## 70	-0.40067	-0.02812	7
## 71	-0.31623	0.00000	8
## 72	0.04841	0.00000	8
## 73	0.03016	-0.21963	8
## 74	0.03016	0.21963	8
## 75	0.08097	0.35555	8
## 76	0.08097	-0.35555	8
## 77	-0.18706	0.08635	8
## 78	-0.18706	-0.08635	8
## 79	-0.09143	0.00454	8
## 80	-0.09143	-0.00454	8
## 81	-0.31623	0.00000	9
## 82	0.05253	0.00000	9
## 83	0.28423	-0.19098	9
## 84	0.28423	0.19098	9
## 85	0.15707	-0.20208	9
## 86	0.15707	0.20208	9
## 87	0.08267	0.19168	9
## 88	0.08267	-0.19168	9
## 89	0.14083	-0.09457	9
## 90	0.14083	0.09457	9
## 91	-0.31623	0.00000	10
## 92	0.04520	0.00000	10
## 93	0.01733	0.24254	10
## 94	0.01733	-0.24254	10
## 95	-0.27066	0.12865	10
## 96	-0.27066	-0.12865	10
## 97	0.00810	-0.06293	10
## 98	0.00810	0.06293	10
## 99	-0.15290	-0.02502	10
## 100	-0.15290	0.02502	10

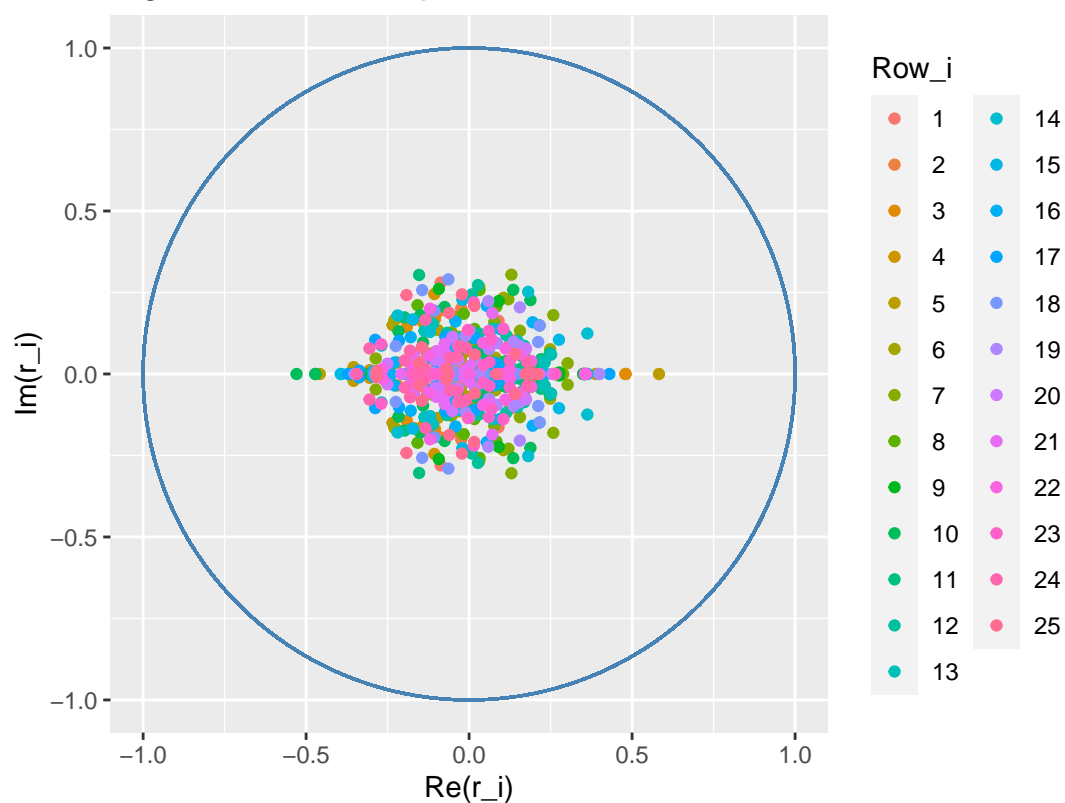
Eigenvectors: 0.8-Sparse Matrix



```
## # A tibble: 10 x 3
##   row_i prop_reals is_real
##   <dbl>   <dbl> <lgl>
## 1     1     0.2 FALSE
## 2     2     0.4 FALSE
## 3     3     0.4 FALSE
## 4     4     0.4 FALSE
## 5     5     0.2 FALSE
## 6     6     0.2 FALSE
## 7     7     0.4 FALSE
## 8     8     0.2 FALSE
## 9     9     0.2 FALSE
## 10    10     0.2 FALSE

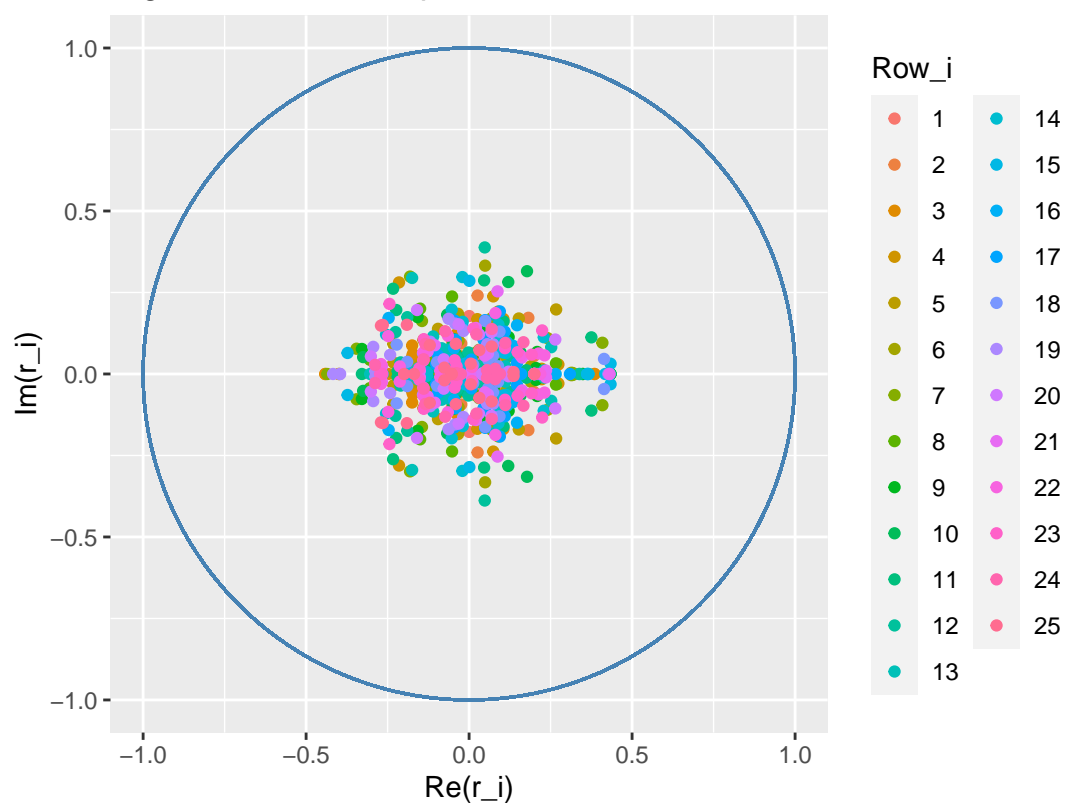
## [1] "Proportion of real-valued rows: 0"
```

Eigenvectors: 0.1-Sparse Matrix



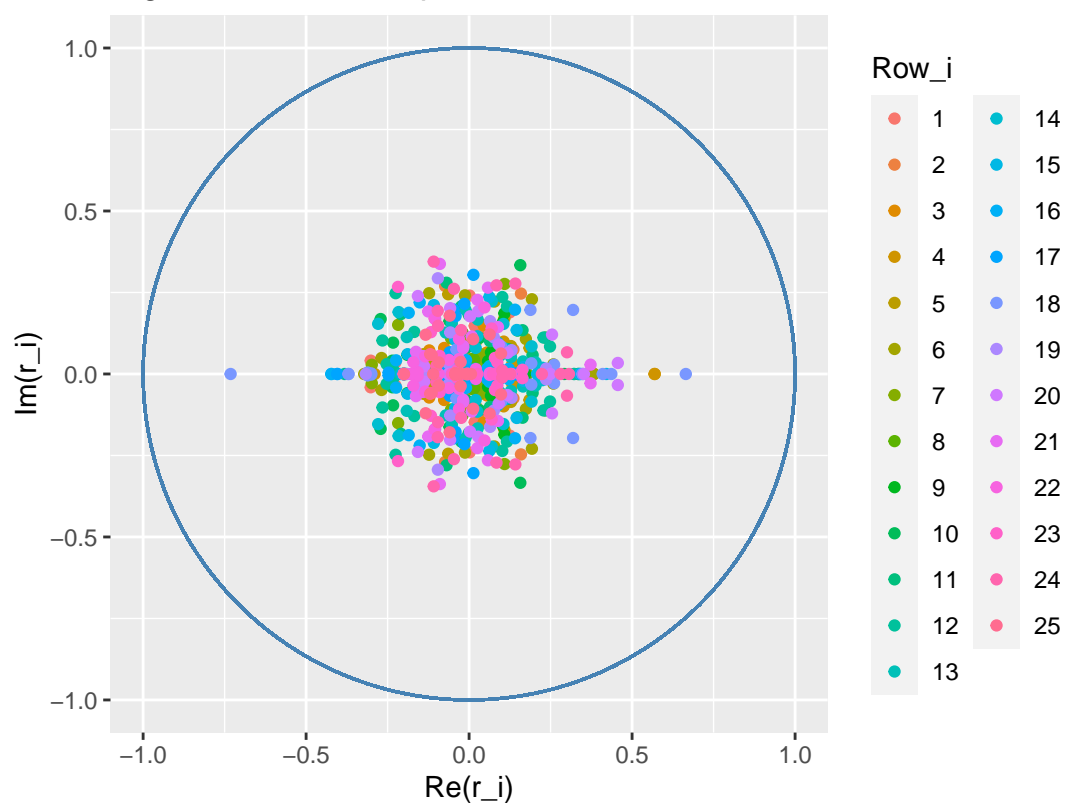
```
## # A tibble: 25 x 3
##   row_i prop_reals is_real
##   <dbl>   <dbl> <lgl>
## 1     1     0.2 FALSE
## 2     2     0.2 FALSE
## 3     3     0.36 FALSE
## 4     4     0.2 FALSE
## 5     5     0.28 FALSE
## 6     6     0.2 FALSE
## 7     7     0.2 FALSE
## 8     8     0.2 FALSE
## 9     9     0.2 FALSE
## 10    10     0.28 FALSE
## # ... with 15 more rows
## [1] "Proportion of real-valued rows: 0"
```

Eigenvectors: 0.5-Sparse Matrix

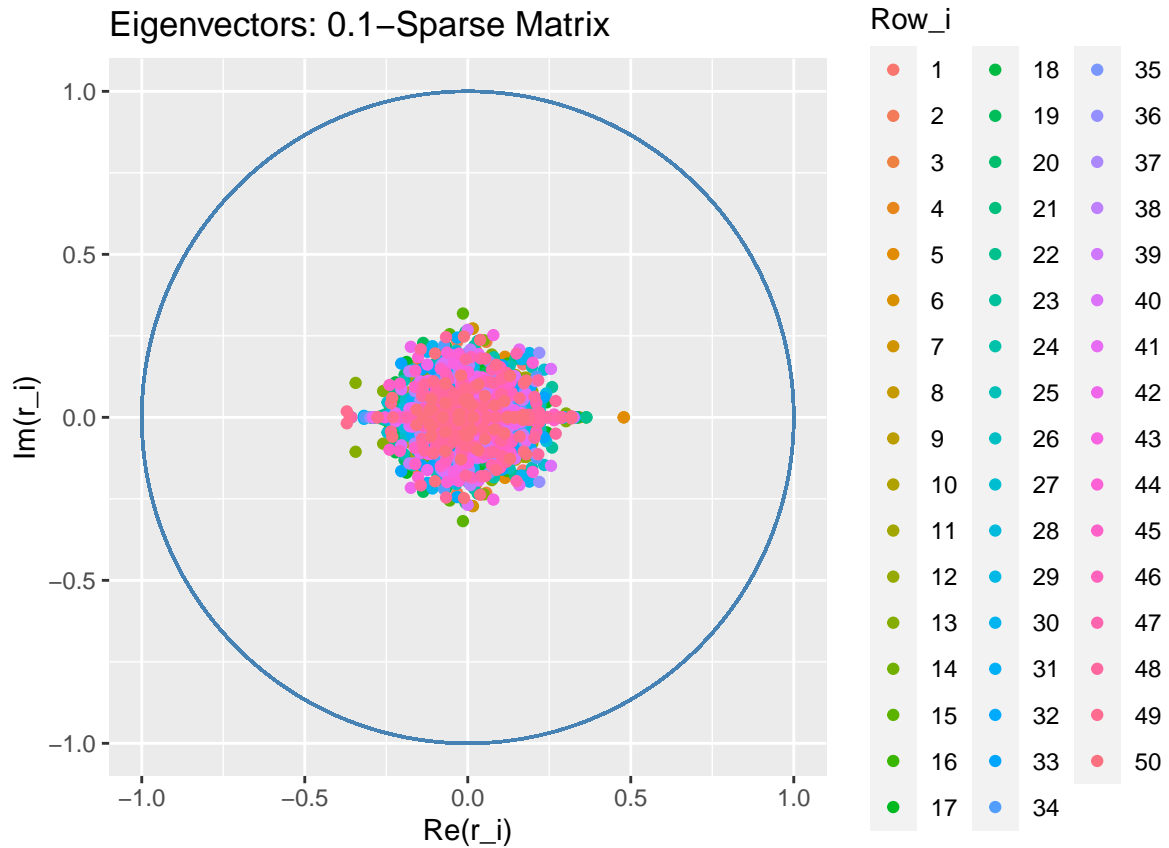


```
## # A tibble: 25 x 3
##   row_i prop_reals is_real
##   <dbl>   <dbl> <lgl>
## 1     1     0.12 FALSE
## 2     2     0.12 FALSE
## 3     3     0.28 FALSE
## 4     4     0.2  FALSE
## 5     5     0.12 FALSE
## 6     6     0.12 FALSE
## 7     7     0.12 FALSE
## 8     8     0.12 FALSE
## 9     9     0.28 FALSE
## 10    10     0.12 FALSE
## # ... with 15 more rows
## [1] "Proportion of real-valued rows: 0"
```

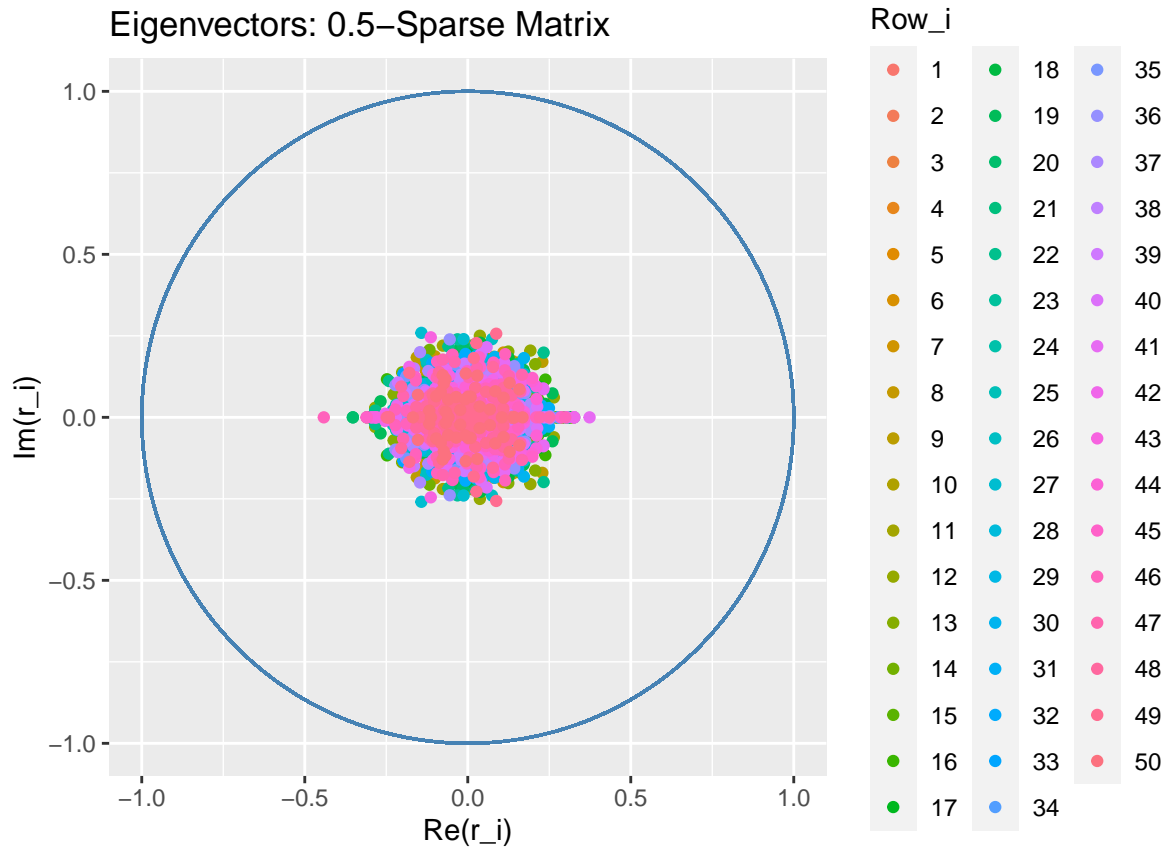
Eigenvectors: 0.8-Sparse Matrix



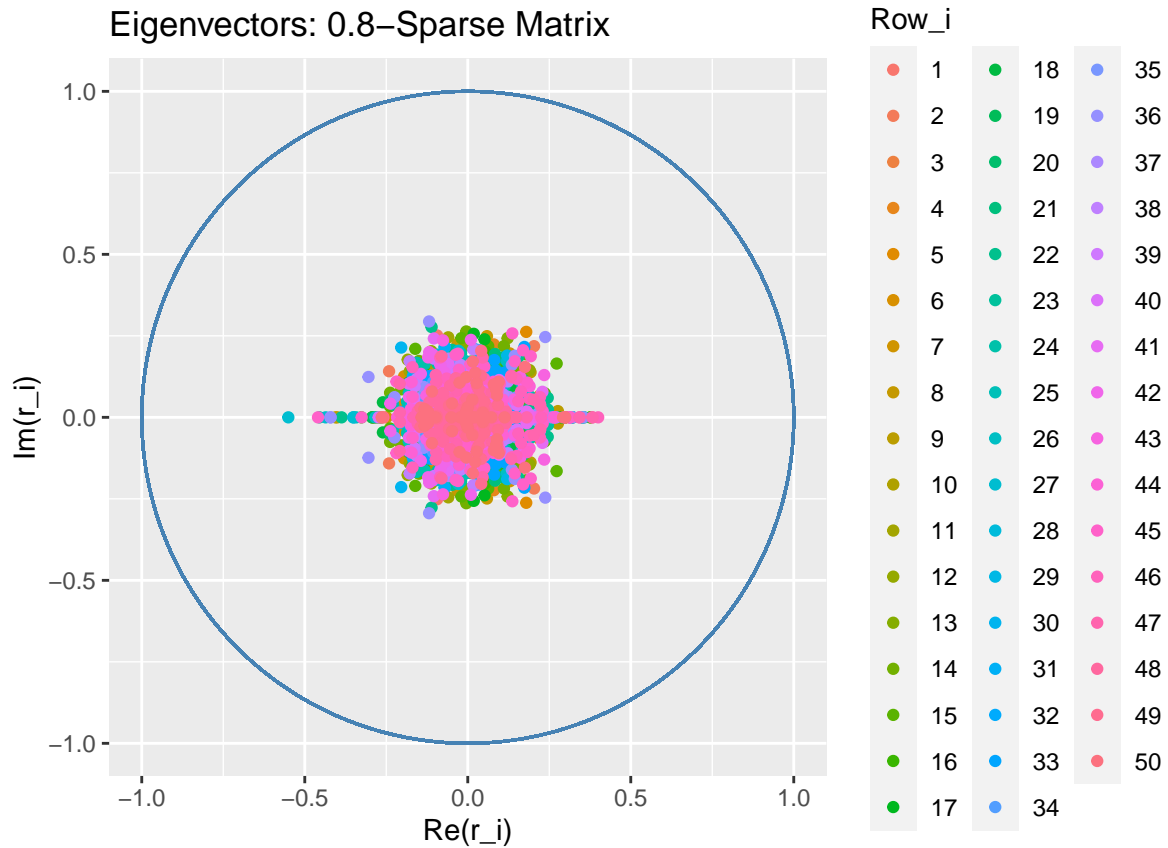
```
## # A tibble: 25 x 3
##   row_i prop_reals is_real
##   <dbl>   <dbl> <lgl>
## 1     1     0.28 FALSE
## 2     2     0.2  FALSE
## 3     3     0.2  FALSE
## 4     4     0.28 FALSE
## 5     5     0.2  FALSE
## 6     6     0.28 FALSE
## 7     7     0.2  FALSE
## 8     8     0.2  FALSE
## 9     9     0.2  FALSE
## 10    10     0.28 FALSE
## # ... with 15 more rows
## [1] "Proportion of real-valued rows: 0"
```



```
## # A tibble: 50 x 3
##   row_i prop_reals is_real
##   <dbl>   <dbl> <lgl>
## 1     1     0.16 FALSE
## 2     2     0.16 FALSE
## 3     3     0.12 FALSE
## 4     4     0.12 FALSE
## 5     5     0.16 FALSE
## 6     6     0.12 FALSE
## 7     7     0.12 FALSE
## 8     8     0.12 FALSE
## 9     9     0.12 FALSE
## 10    10     0.12 FALSE
## # ... with 40 more rows
## [1] "Proportion of real-valued rows: 0"
```



```
## # A tibble: 50 x 3
##   row_i prop_reals is_real
##   <dbl>   <dbl> <lgl>
## 1     1     0.12 FALSE
## 2     2     0.12 FALSE
## 3     3     0.16 FALSE
## 4     4     0.12 FALSE
## 5     5     0.12 FALSE
## 6     6     0.12 FALSE
## 7     7     0.12 FALSE
## 8     8     0.12 FALSE
## 9     9     0.12 FALSE
## 10    10     0.12 FALSE
## # ... with 40 more rows
## [1] "Proportion of real-valued rows: 0"
```



```
## # A tibble: 50 x 3
##   row_i prop_reals is_real
##   <dbl>   <dbl> <lgl>
## 1     1     0.2 FALSE
## 2     2     0.2 FALSE
## 3     3     0.2 FALSE
## 4     4     0.2 FALSE
## 5     5     0.24 FALSE
## 6     6     0.24 FALSE
## 7     7     0.2 FALSE
## 8     8     0.24 FALSE
## 9     9     0.24 FALSE
## 10    10     0.2 FALSE
## # ... with 40 more rows
## [1] "Proportion of real-valued rows: 0"
```