Computational Eigenvector Simulation

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Notation

Let $\mathcal{M}_F[M \times N]$ be the set of $M \times N$ matrice over the field F.

- Suppose Q is some $M \times M$ square matrix on a field F. Denote this $Q \in \mathcal{M}_F[M^2]$.
- Suppose v is a row vector on a field F of size M. Denote this by saying $v \in \mathcal{V}_F^{(R)}[M]$.

Note that $\mathcal{V}_F^{(R)}[M]$ is equivalent to $\mathcal{M}_F[1 \times M]$.

- Suppose v is a column vector on a field F of size M. Denote this by saying $v \in \mathcal{V}_F^{(C)}[M]$.

Note that $\mathcal{V}_F^{(C)}[M]$ is equivalent to $\mathcal{M}_F[M \times 1]$.

Definitions

Definition. ($\epsilon -$ **equivalence**) Let F be a field, and $\varepsilon \in \mathbb{R}^+$. Suppose we have vectors $v, v' \in \mathcal{V}_F[M]$. Then, $v \sim_\epsilon v'$ if $||v - v'|| < \epsilon$ where $||\cdot||$ is the norm on F.

Definition. (**Q $-$ evolution sequence**) Let $\pi \in \mathcal{V}[M]$ be a vector of size M. Then, the Q-evolution sequence of π is given by:

$$\text{Seq}(Q, \pi) = \{\pi'_n\}_{n \in \mathbb{N}} \text{ where } \pi'_n = \pi Q^n$$

Definition. ($\epsilon -$ **convergence**) Let $\varepsilon \in \mathbb{R}^+$ Suppose we have a sequence $\text{Seq}(Q, \pi)$. Then, $\text{Seq}(Q, \pi)$ ε-converges at N if:

$$\exists N \in \mathbb{N} : \forall n \geq N \mid \pi'_N \sim_\epsilon \pi'_n$$

Definition. ($\sigma -$ **perturbance**) Let $\pi \in \mathcal{V}[M]$ be a vector of size M. Then, a $\sigma -$ perturbance of π with perturbance vector ε is given by the vector $\pi' = \pi + \varepsilon$ where:

$$\varepsilon = (\varepsilon_i)_{i=1}^M \text{ where } \epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

Definition. (**localized $\sigma -$perturbance**) Let $\pi \in \mathcal{V}[M]$ be a vector of size M. Then, if we have a dimension $d \in \{1, \ldots, M\}$, a localized$\sigma -$perturbance of π in d with perturbance vector ε is given by the vector $\pi' = \pi + \varepsilon$ where:

$$\varepsilon = (\varepsilon_j)_{j=1}^M \text{ where } \varepsilon_d \sim \mathcal{N}(0, \sigma^2) \text{ and } \forall j \neq d : \varepsilon_j = 0$$

MC Simulation

(Monte-Carlo λ Batch) Let $\lambda \in R^+$. Take a batch $\mathcal{B}$ of B row vectors $\pi \sim \text{Unif}(-\lambda, \lambda)$ and denote it $\mathcal{B} = \{\pi_i\}_{i=1}^B$. Then, we have a $\lambda -$ batch denoted $\mathcal{B}_\lambda$.

To simulate the eigenvectors, we will take the $Q -$evolution sequences on each of our $\pi_i \in \mathcal{B}$. So, our simlated batch of evolution sequences is given by:

$$\text{EvolBatch}(Q, \mathcal{B}) = \{\{\pi'_n\}_{n \in \mathbb{N}}\}_{i=1}^N$$

Question: Which λ is sufficient to obtain eigenvectors that yield all the eigenvalues λ_i of Q?