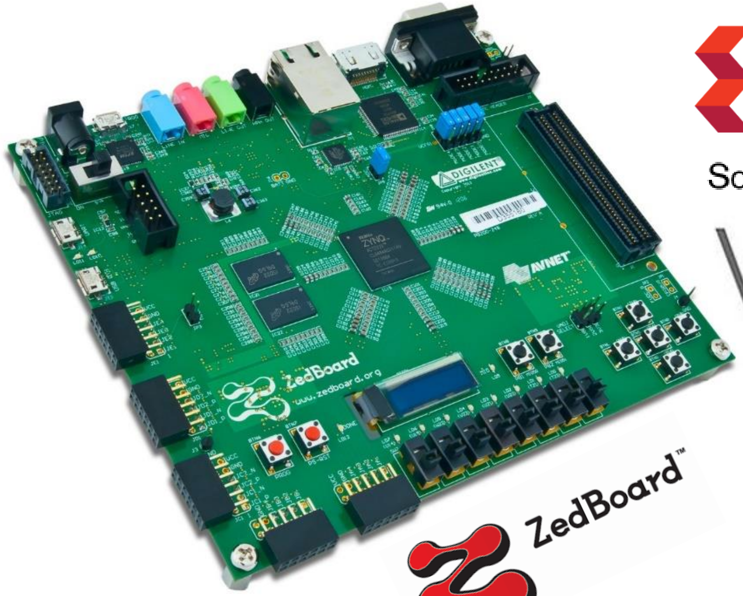
ECE573
Advanced Embedded Logic Design (AELD)



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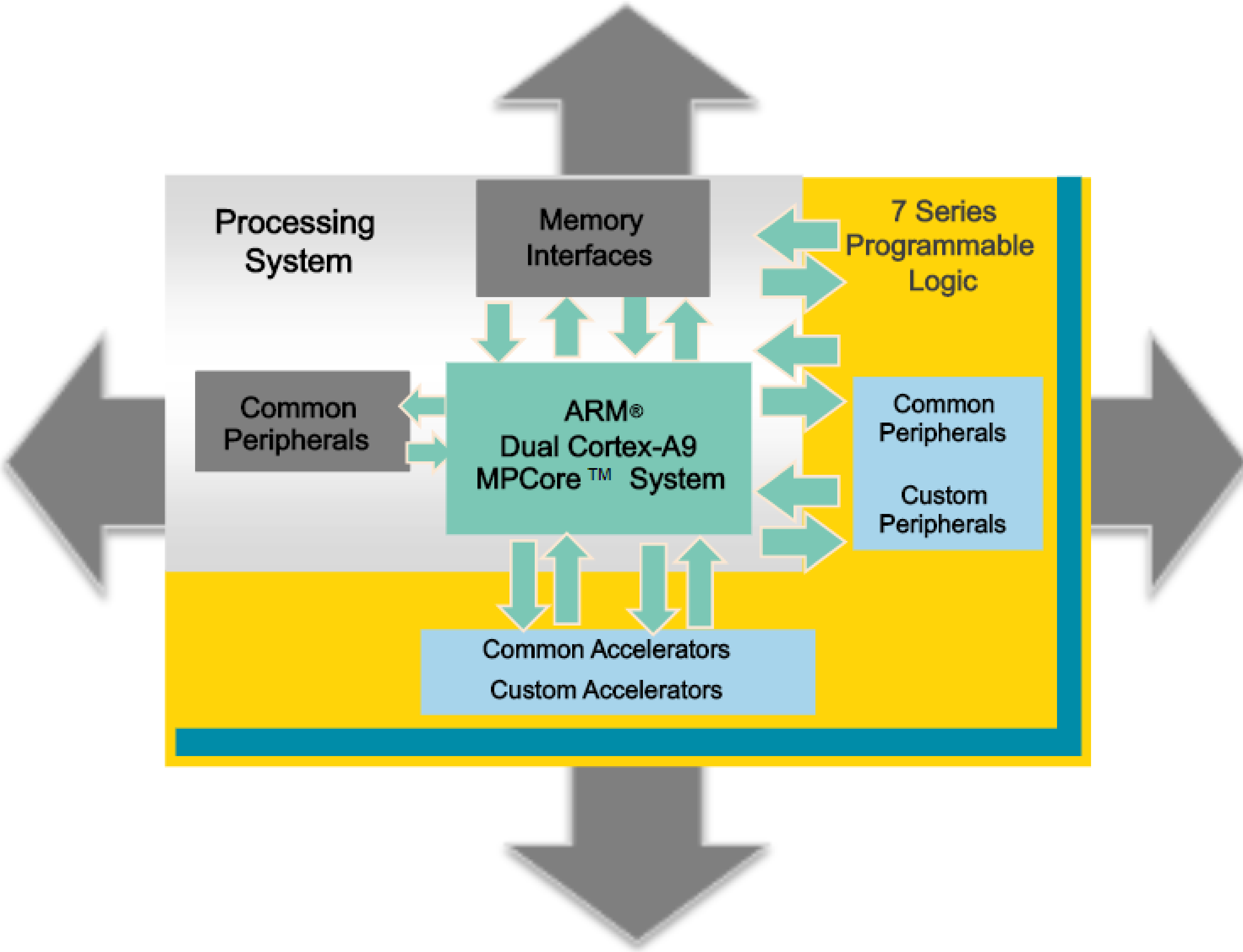
- The material for this presentation is taken from various books, courses and Xilinx/ARM XUP resources. The instructor does not claim ownership of the material presented in this class.

Lab 2

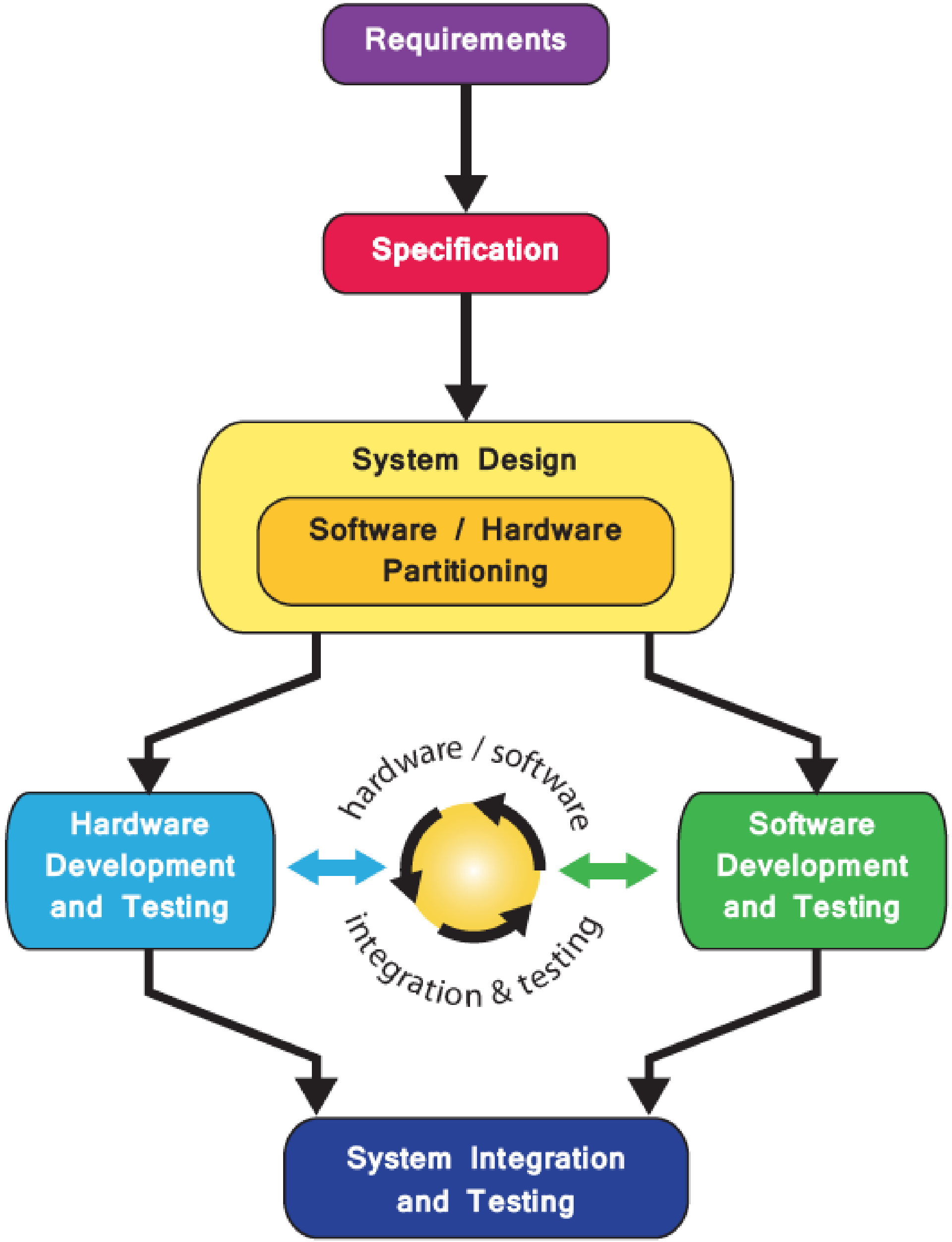
- In this lab, we will discuss the 8-point FFT example with a focus on debugging and execution time calculation. We will also discuss in-built code-optimization feature in SDK. Then, we will discuss generalized FFT code for large dimension
- **Topics to explore:** 1) FFT algorithm, 2) SDK debugging feature, 3) Execution time calculation using in-built timer, 3) Code-optimization on SDK for ARM processor, 4) Large size FFT, 5) Stack and heap size, Header file, Math Library and sleep function
- **Self-study:** Implement floating point arithmetic and matrix multiplication on ARM processor

Zynq SoC

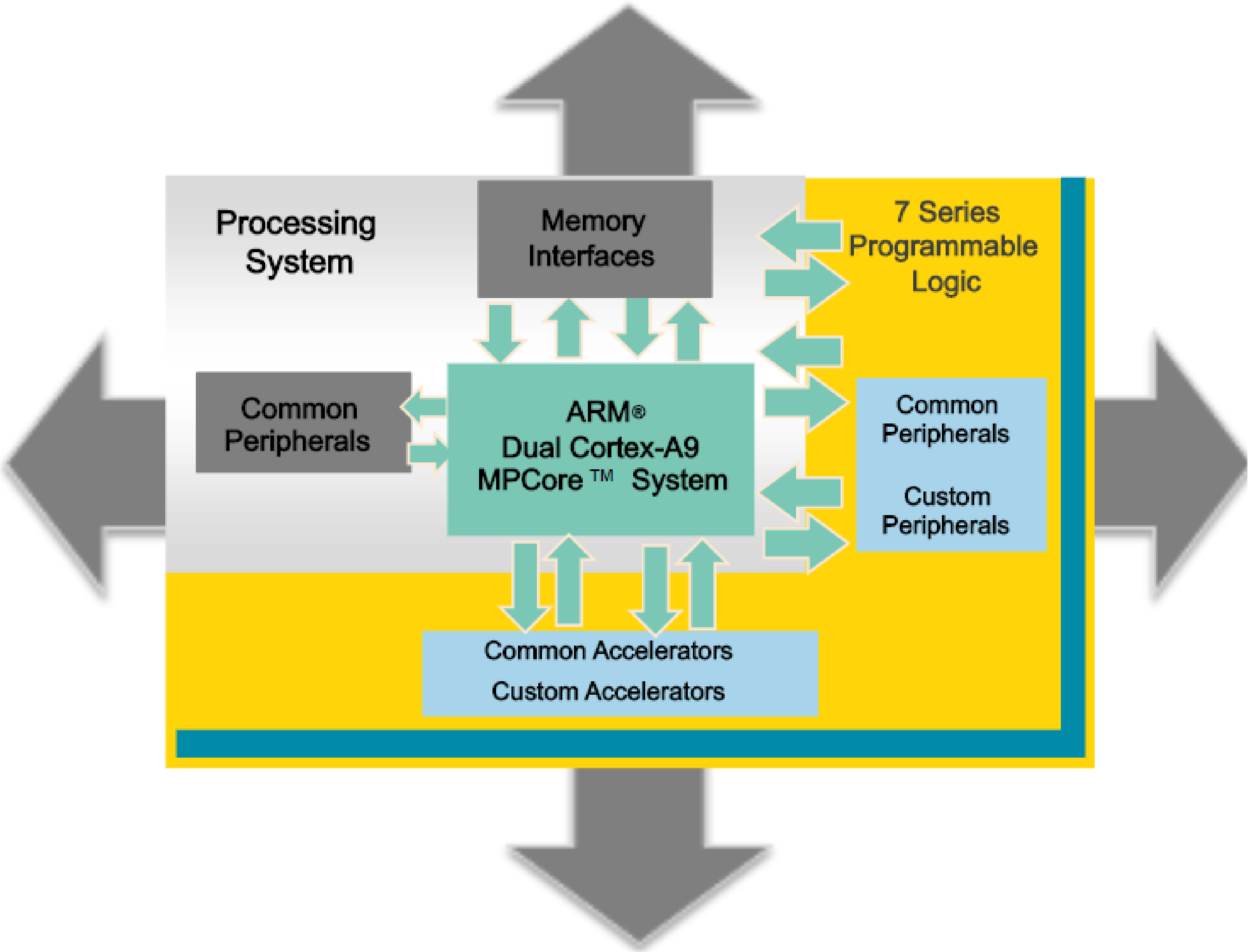


Processing System
Memory Interfaces
7 Series Programmable Logic
Common Peripherals
ARM® Dual Cortex-A9 MPCore ™ System
Common Peripherals
Custom Peripherals
Common Accelerators
Custom Accelerators

Zynq Design Flow



Lab 2



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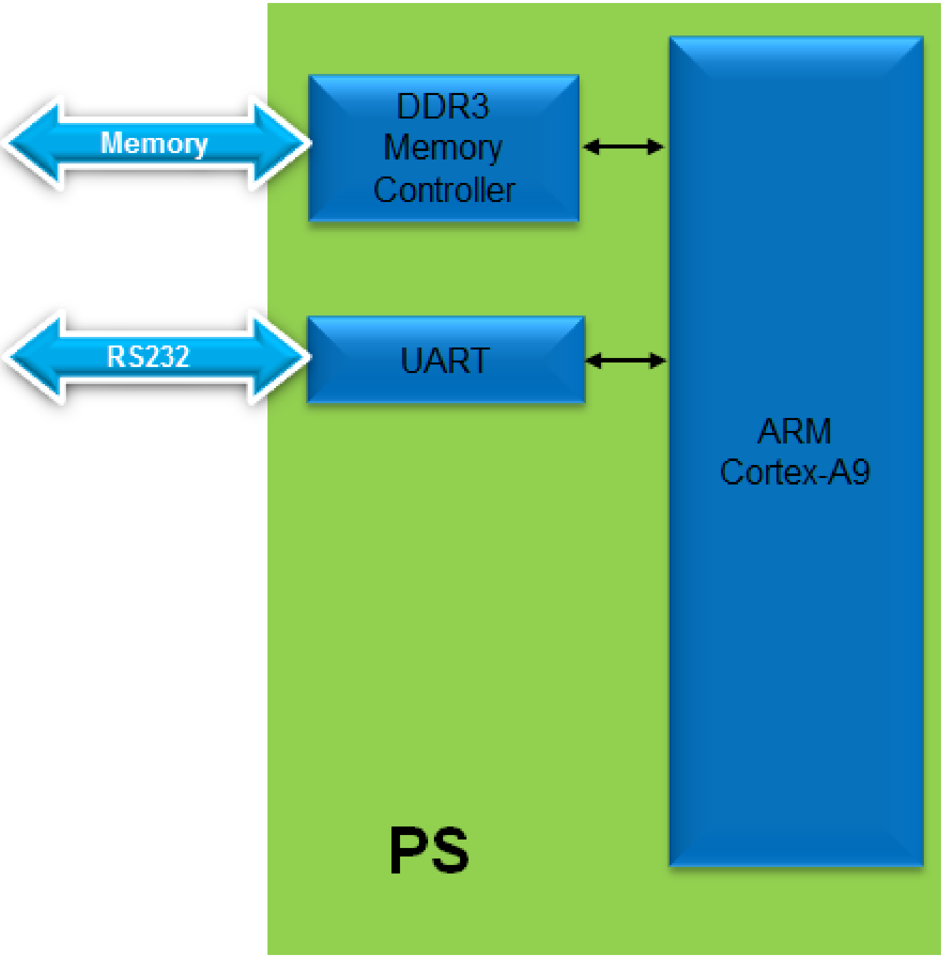
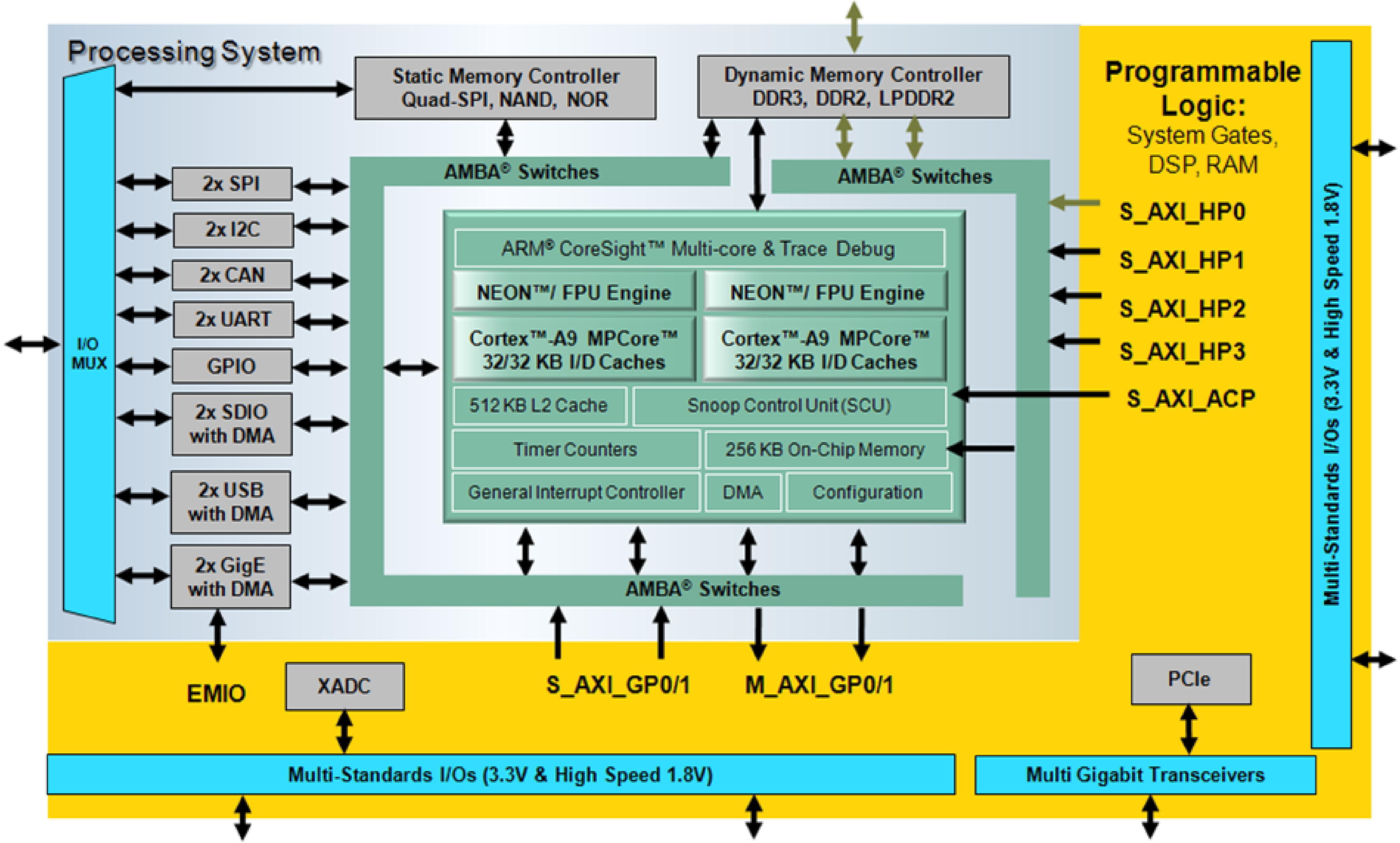


Memory
DDR3 Memory Controller
RS232
UART
ARM Cortex-A9
PS

Zynq Architecture: PS and PL



Vivado Design Flow

Same as Lab 1

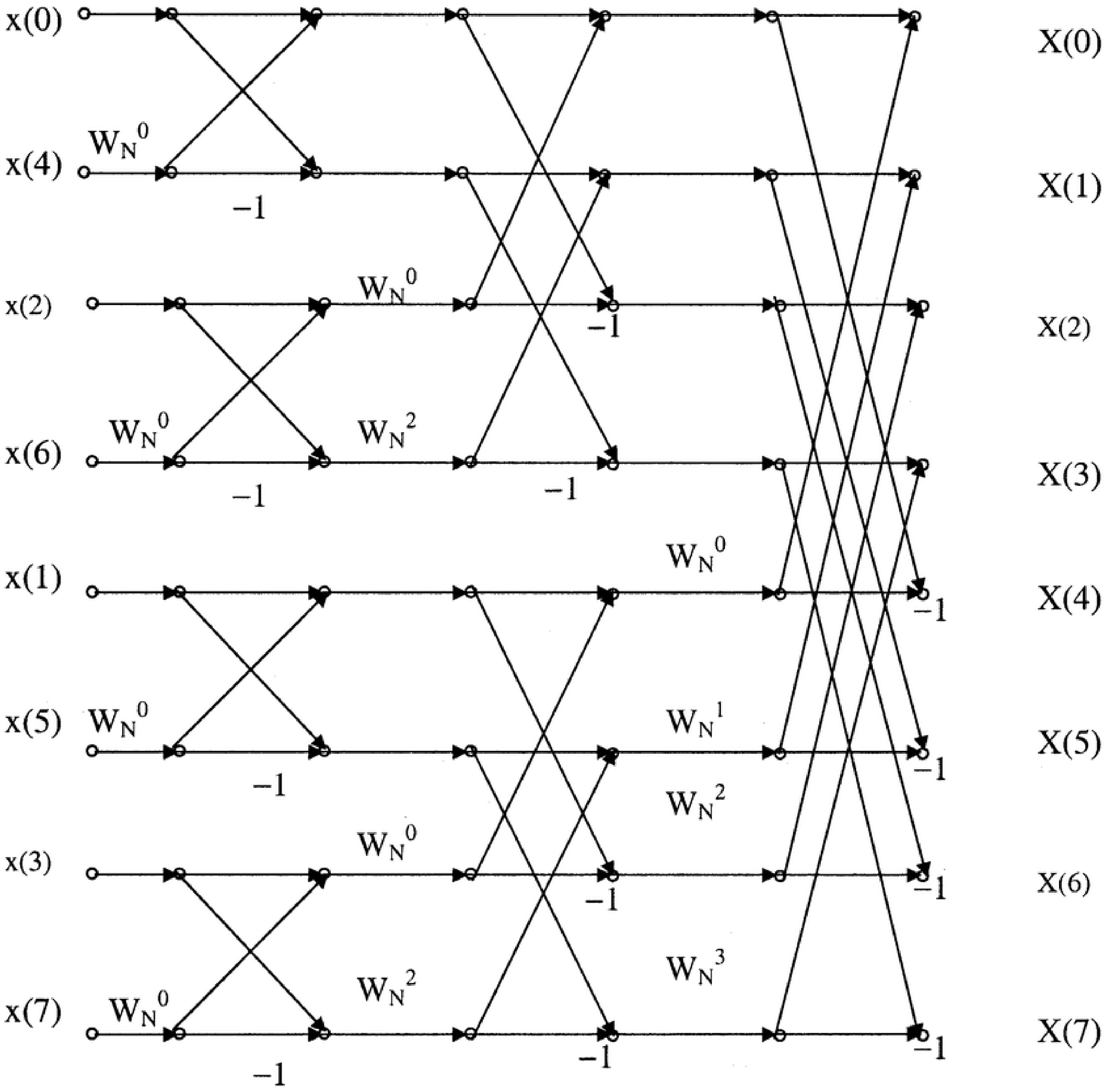
SDK Design Flow

Same as Lab 1 Except application code

FFT on ARM Processor

FFT on ARM Processor

- FFT Size = 8
- No of butter fly stages=3



$W_N^0 = 1$, $W_N^1 = (1-j)/\sqrt{2}$, $W_N^2 = -j$, $W_N^3 = -(1+j)/\sqrt{2}$

FFT on ARM Processor

```
#include <stdio.h>
#include <stdlib.h>
#include <complex.h>
#define FFT_Size 8
const float complex twiddle_factors[FFT_Size/2] = {1-0*I, 0.7071067811865476-0.7071067811865475*I, 0.0-1*I,  -0.7071067811865475-0.7071067811865476*I};
int main()
{
    // For FFT_Size point FFT, define the input.
    // You may modify the code to take the input from user via UART
    float complex FFT_input[FFT_Size] = {11+23*I,32+10*I,91+94*I,15+69*I,47+96*I,44+12*I,96+17*I,49+58*I};
    // FFT output will be stored in this variable
    float complex FFT_output[FFT_Size];
    // Variable for intermediate outputs
    float complex FFT_rev[FFT_Size];

  // Print the FFT input on the UART
  printf("\n FFT input: \r\n");
    for (int i = 0 ; i < FFT_Size ; i++)
    {
        printf("%f %f\n", crealf(FFT_input[i]), cimagf(FFT_input[i]));
    }
```

FFT on ARM Processor

- Matlab Code

```
// Equivalent Matlab Code for FFT
// clc;
// clear;
// FFT_Size =8;
// input_real =[11,32,91,15,47,44,96,49];
// input_imag =[23,10,94,69,96,12,17,58];
// FFT_input =complex(input_real,input_imag);
// FFT_out=fft(FFT_input,FFT_Size);
// disp(real(FFT_out));
// disp(imag(FFT_out));
```

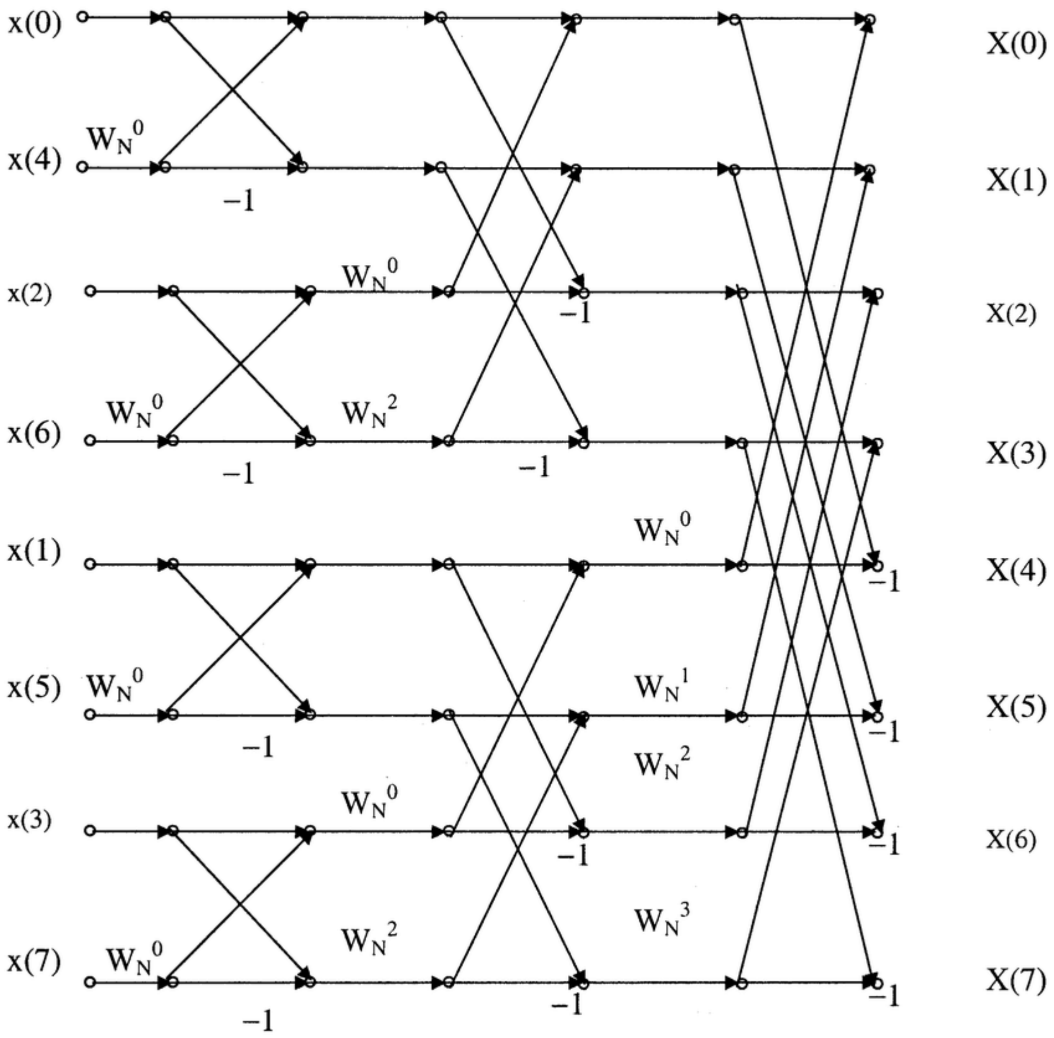
FFT on ARM Processor

```c
    // As discussed in the handout, FFT involves two tasks:
    // 1) Reorder of the inputs to get output in the normal order
    // 2) Multiplications using multi-stage butterfly approach
    InputReorder(FFT_input, FFT_rev); // Task 1
    FFTStages(FFT_rev, FFT_output); // Task 2

    // Print the FFT output on the UART
    printf("\n FFT output: \r\n");
    for (int i = 0 ; i < FFT_Size ; i++)
    {
        printf("%f %f\n", crealf(FFT_output[i]), cimagf(FFT_output[i]));
    }
    // Modify this code for large size FFT
    // How you can generalize the code for any FFT size (limited to power of two)
    // Receive the FFT size and FFT input from User
}
```

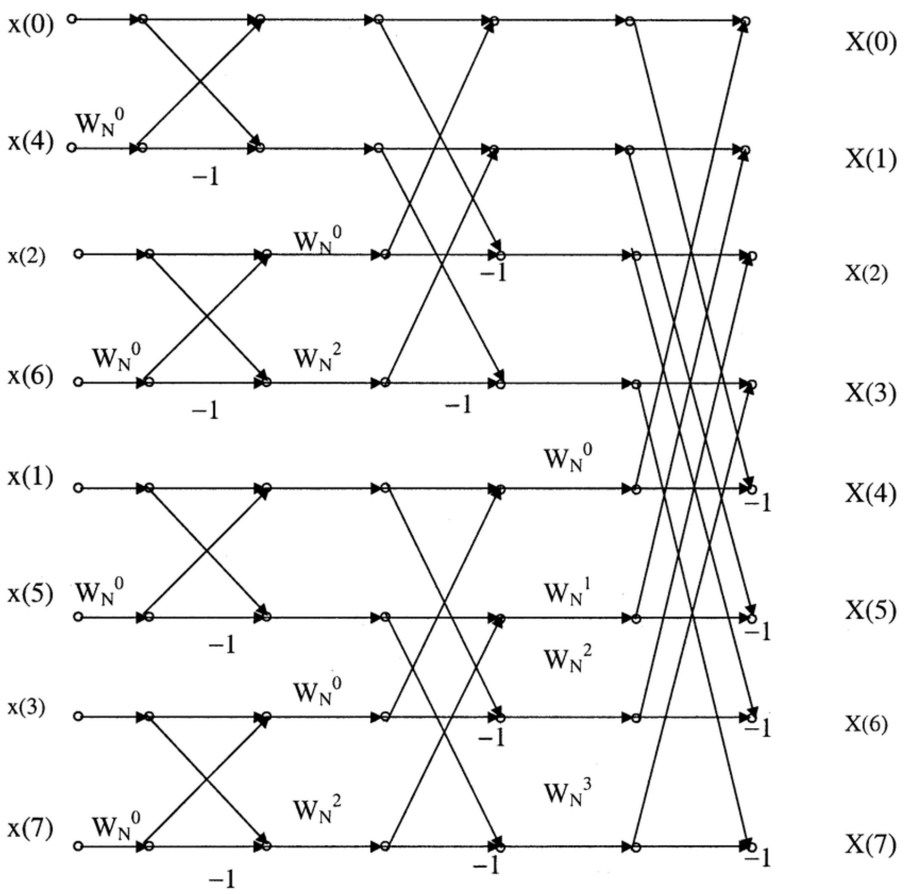
FFT on ARM Processor

```c
// This function reorders the input to get the output in the normal order
// Refer the handout for the desired input order
const int input_reorder[FFT_Size] = {0, 4, 2, 6, 1, 5, 3, 7};
void InputReorder(float complex dataIn[FFT_Size], float complex dataOut[FFT_Size])
{
	for (int i = 0 ; i < FFT_Size ; i++)
	{
		dataOut[i] = dataIn[input_reorder[i]];
	}
}
```



$W_N^0 = 1$, $W_N^1 = (1-j)/\sqrt{2}$, $W_N^2 = -j$, $W_N^3 = -(1+j)/\sqrt{2}$

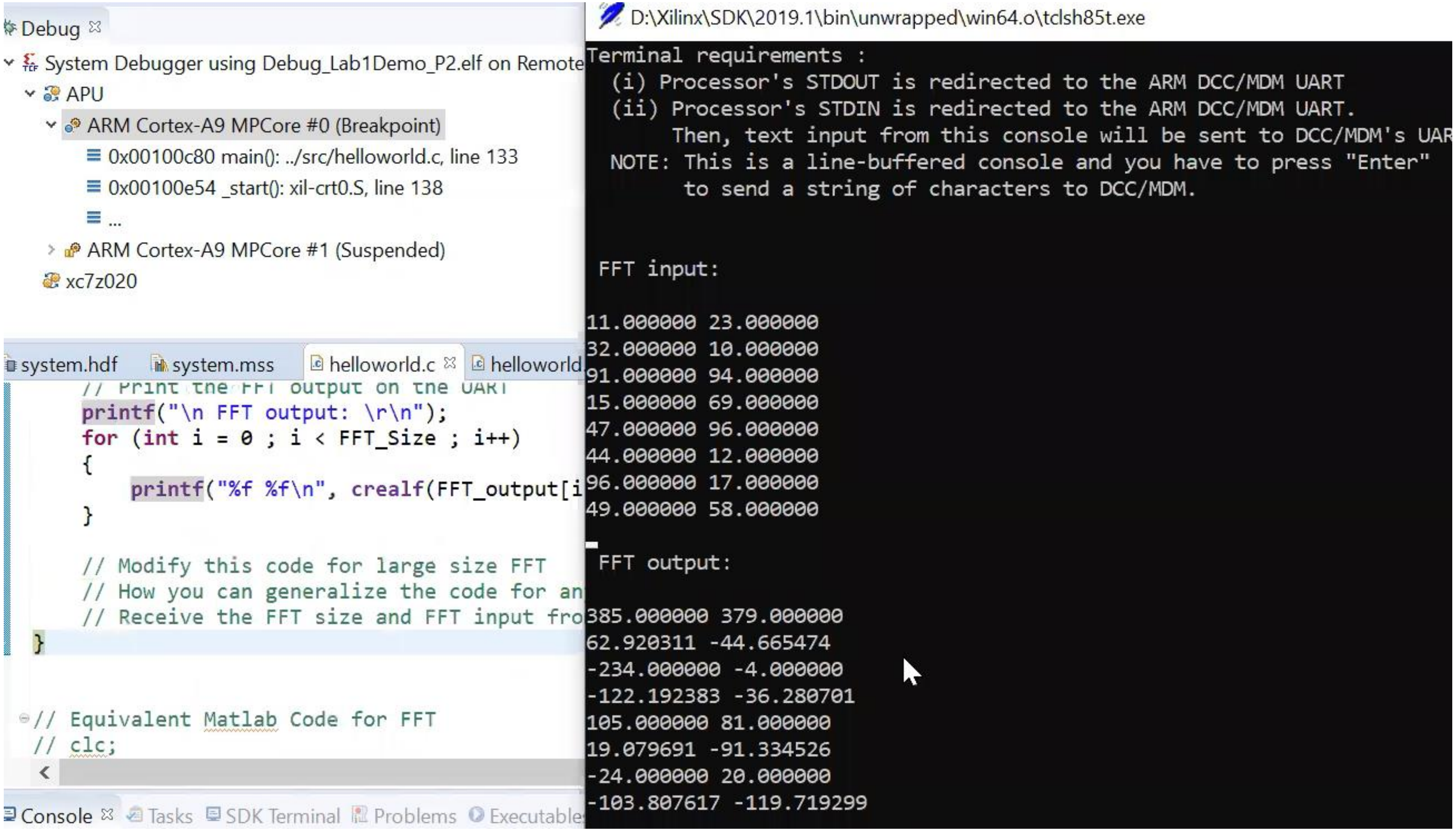
FFT on ARM Processor



$W_N^0 = 1$, $W_N^1 = (1-j)/\sqrt{2}$, $W_N^2 = -j$, $W_N^3 = -(1+j)/\sqrt{2}$

```
// For FFT of size FFT_Size, the number of butterfly stages are 2^stages = FFT_Size.
// For 8-point FFT, there are three butterfly stages.
void FFTStages(float complex FFT_input[FFT_Size], float complex FFT_output[FFT_Size])
{
    float complex stage1_out[FFT_Size], stage2_out[FFT_Size];
    // Stage 1
    for (int i = 0 ; i < FFT_Size ; i=i+2)
    {
        stage1_out[i] = FFT_input[i] + FFT_input[i+1];
        stage1_out[i+1] = FFT_input[i] - FFT_input[i+1];
    }
    // Stage 2
    for (int i = 0 ; i < FFT_Size ; i=i+4)
    {
        for (int j = 0; j < 2; ++j)
        {
            stage2_out[i+j] = stage1_out[i+j] + twiddle_factors[2*j]*stage1_out[i+j+2];
            stage2_out[i+2+j] = stage1_out[i+j] - twiddle_factors[2*j]*stage1_out[i+j+2];
        }
    }
    // Stage 3
    for (int i = 0 ; i < FFT_Size/2 ; i++)
    {
        FFT_output[i] = stage2_out[i] + twiddle_factors[i]*stage2_out[i+4];
        FFT_output[i+4] = stage2_out[i] - twiddle_factors[i]*stage2_out[i+4];
    }
}
```

FFT on ARM Processor

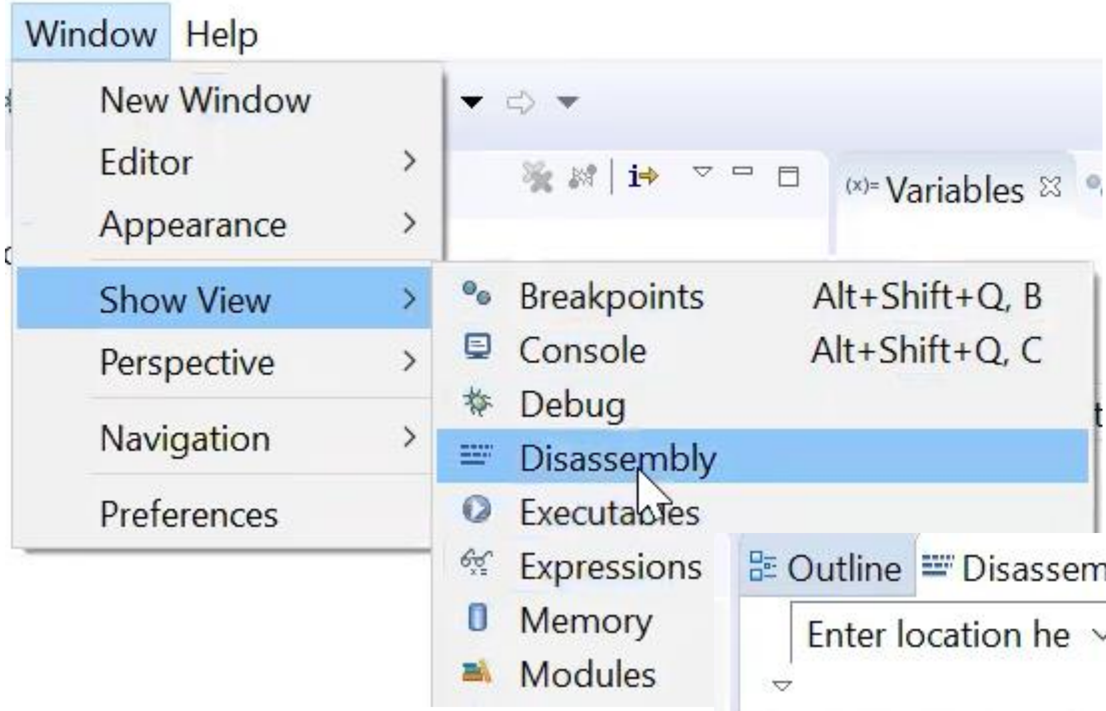


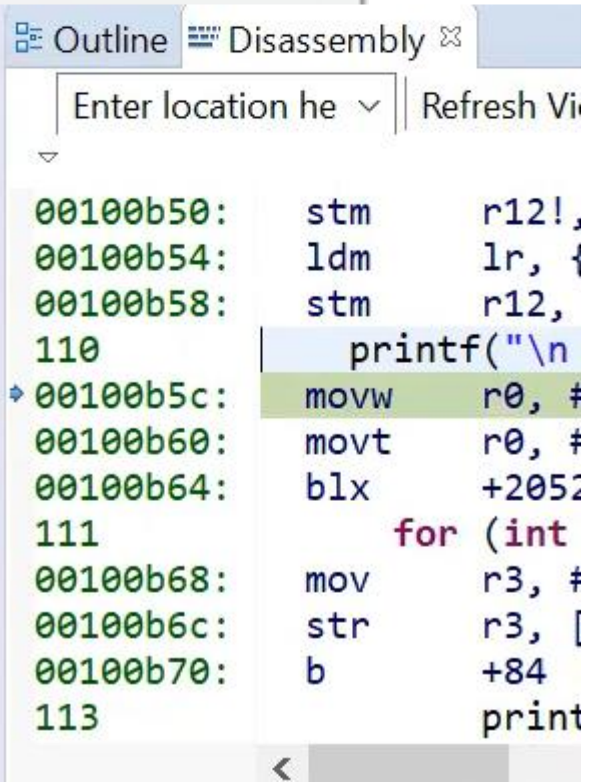
SDK Debugging

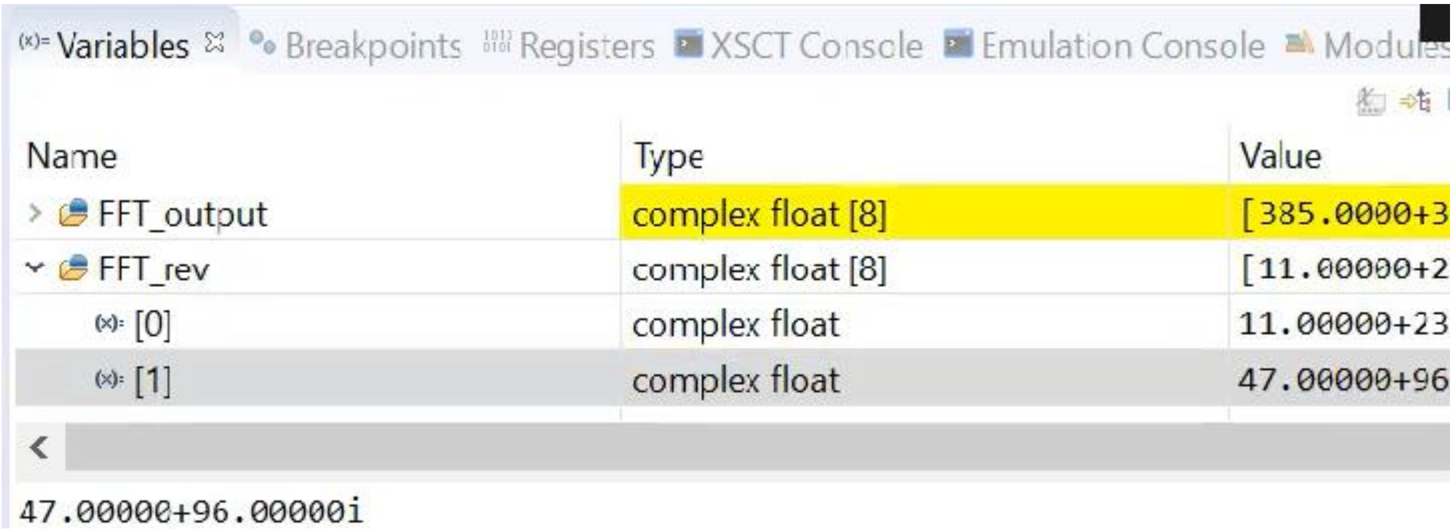
Detailed Handout is attached at the end

SDK Debugging

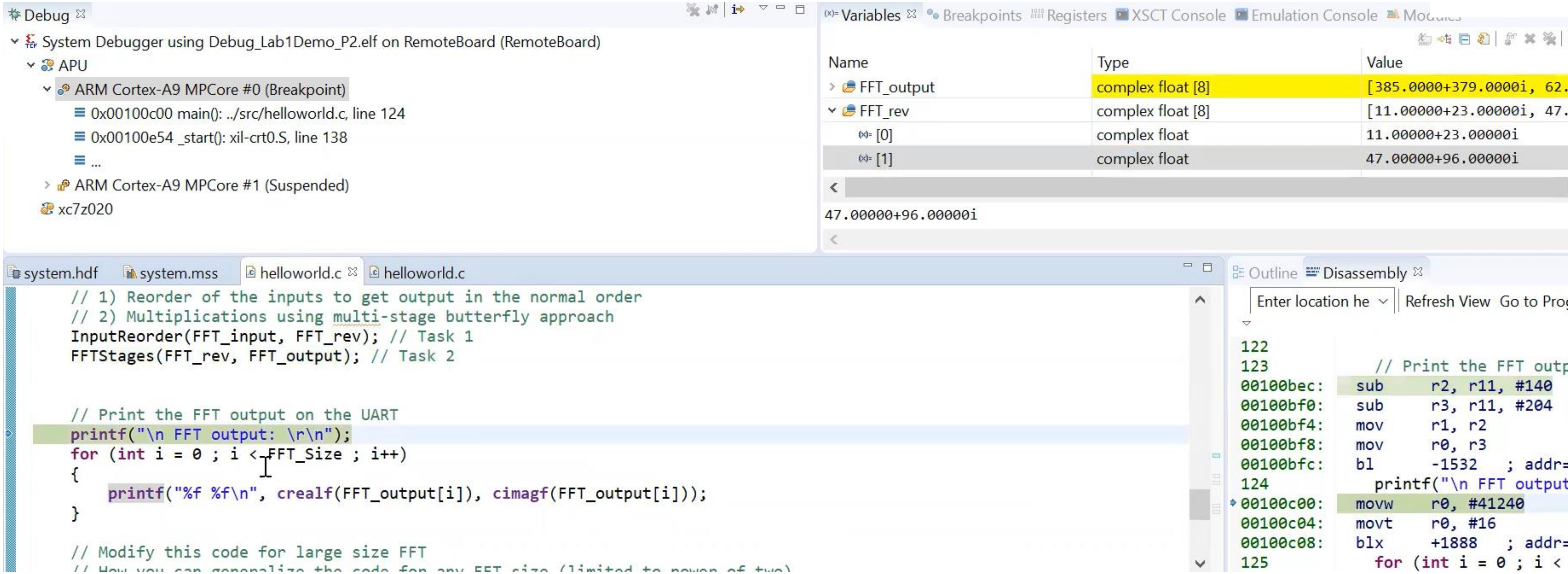
- Explore Variable, Breakpoints, Registers tab and step-by-step debugging.





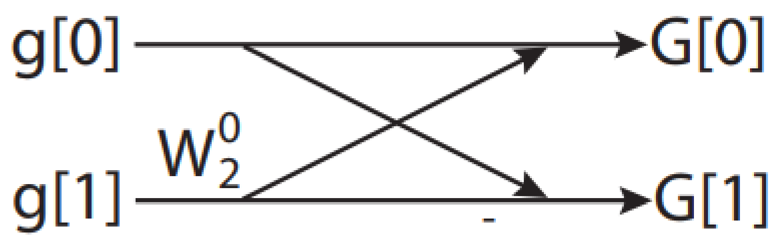


SDK Debugging



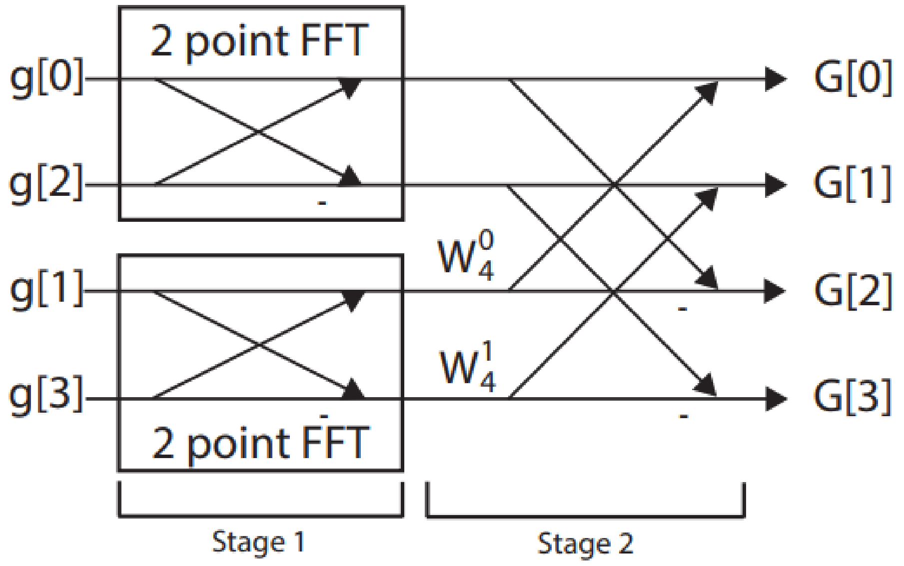
Large Size Fast Fourier Transform

Fast Fourier Transform



- Reduces the complexity of the DFT
- 2-point FFT: $S = \begin{bmatrix} W_2^{00} & W_2^{01} \\ W_2^{10} & W_2^{11} \end{bmatrix}$ where $W = e^{-j2\pi}$
- For inputs, $g[0]$ and $g[1]$, outputs are $G[0] = g[0] + g[1]$ and $G[1] = g[0] - g[1]$

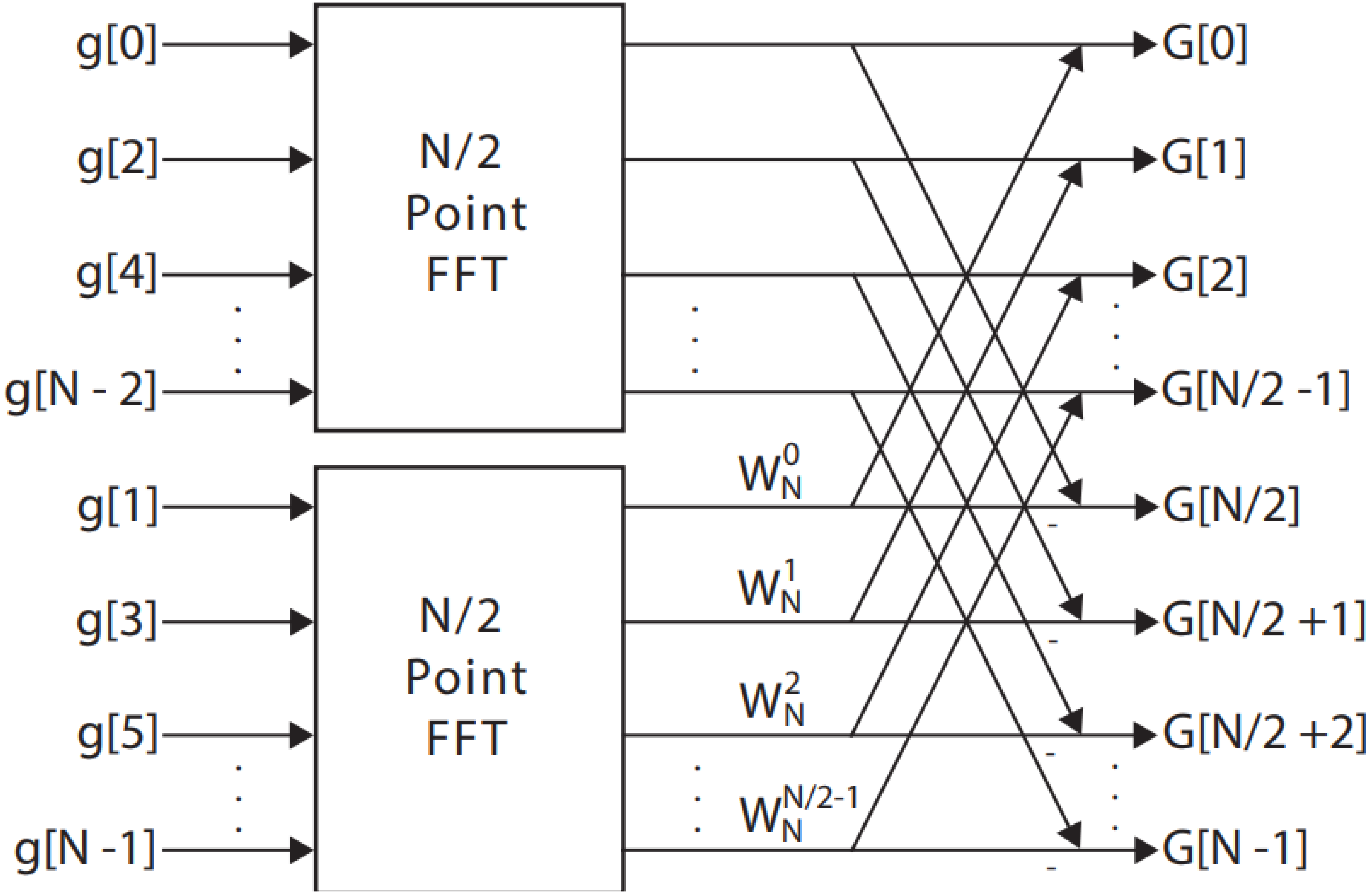
Fast Fourier Transform



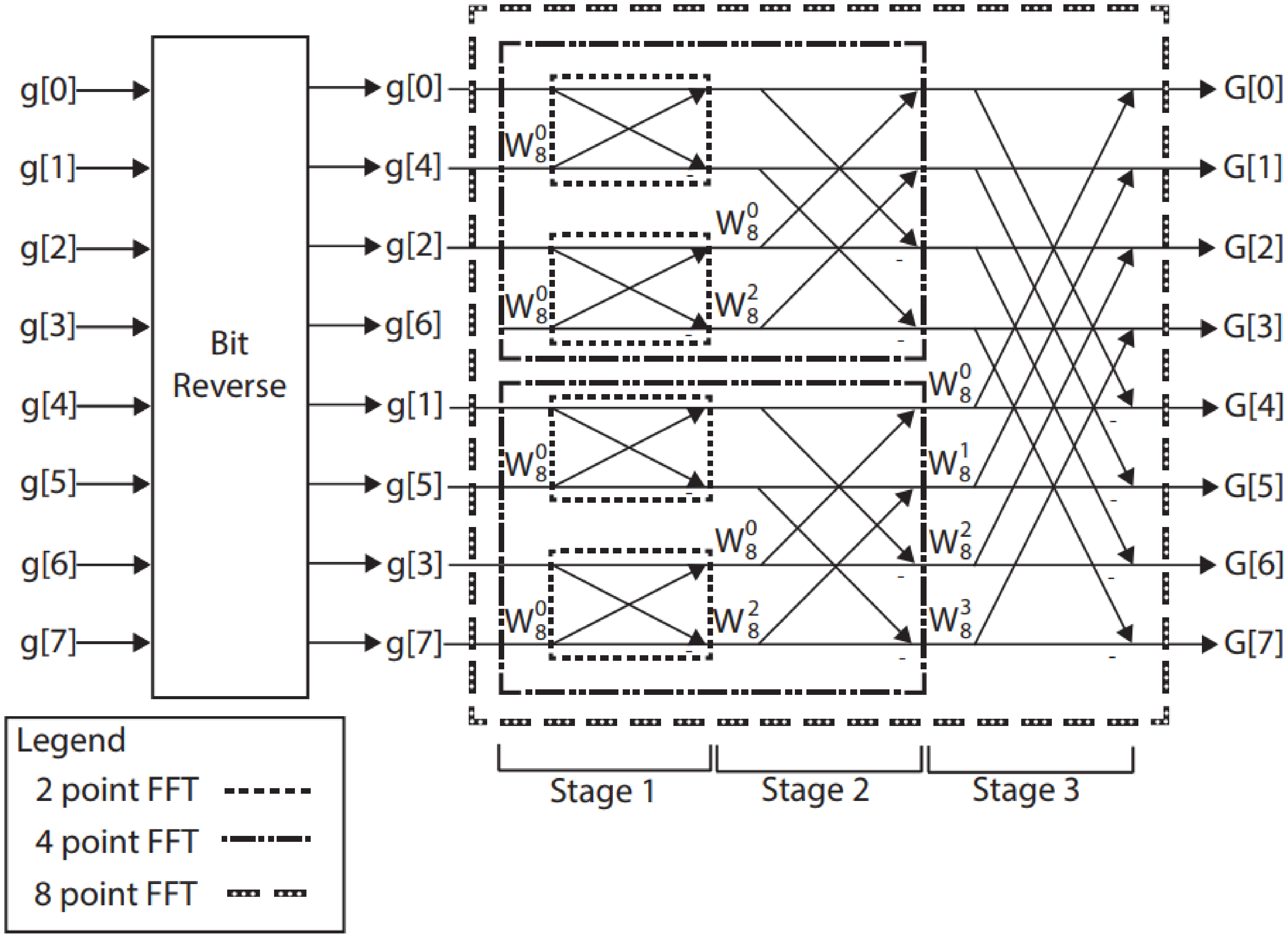
$$
\begin{aligned}
G[0] &= (g[0]+g[2]) + e^{\frac{-j2\pi 0}{4}}(g[1]+g[3]) \\
G[1] &= (g[0]-g[2]) + e^{\frac{-j2\pi 1}{4}}(g[1]-g[3]) \\
G[2] &= (g[0]+g[2]) + e^{\frac{-j2\pi 2}{4}}(g[1]+g[3]) \\
G[3] &= (g[0]-g[2]) + e^{\frac{-j2\pi 3}{4}}(g[1]-g[3])
\end{aligned}
$$

$$
\begin{aligned}
G[0] &= (g[0]+g[2]) + e^{\frac{-j2\pi 0}{4}}(g[1]+g[3]) \\
G[1] &= (g[0]-g[2]) + e^{\frac{-j2\pi 1}{4}}(g[1]-g[3]) \\
G[2] &= (g[0]+g[2]) - e^{\frac{-j2\pi 0}{4}}(g[1]+g[3]) \\
G[3] &= (g[0]-g[2]) - e^{\frac{-j2\pi 1}{4}}(g[1]-g[3])
\end{aligned}
$$

Fast Fourier Transform

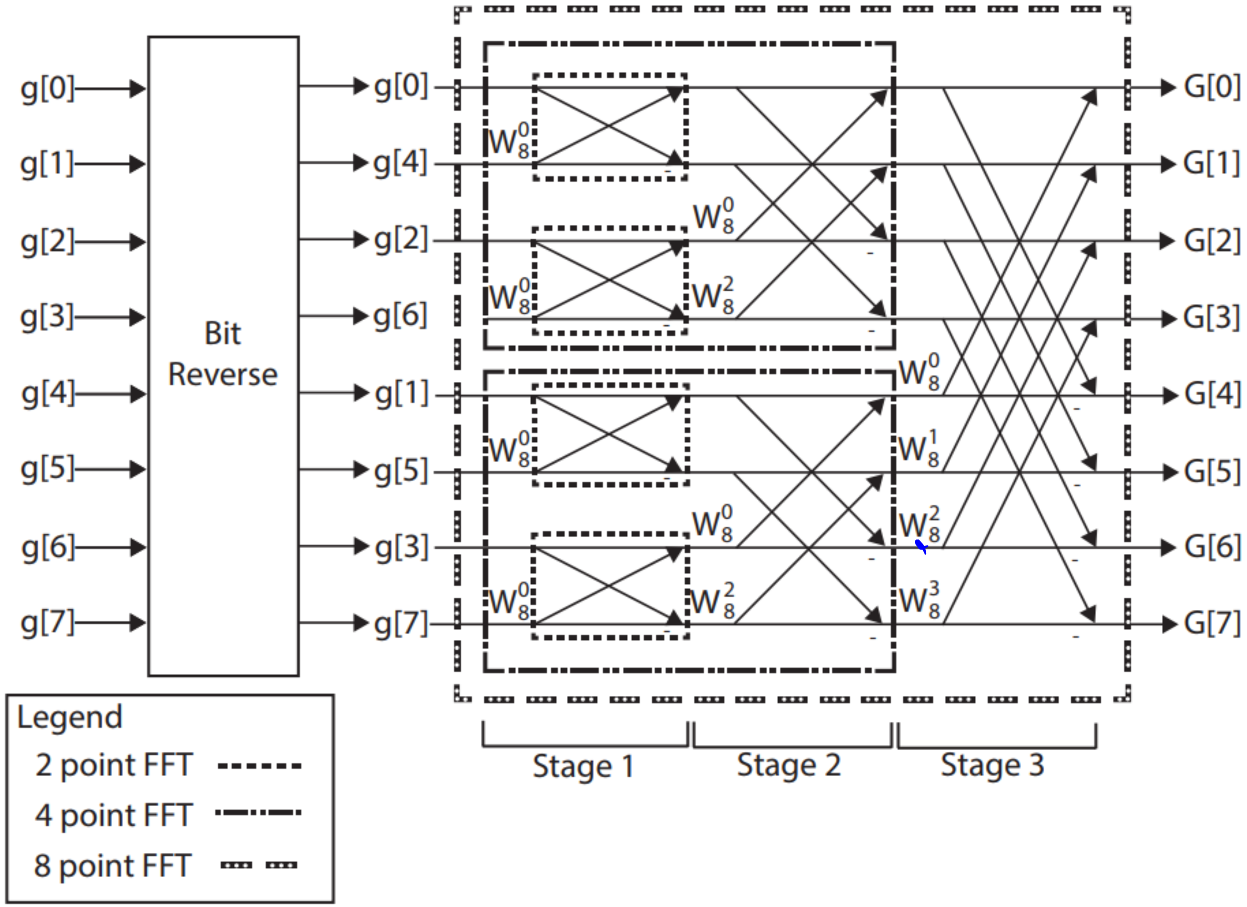


Fast Fourier Transform

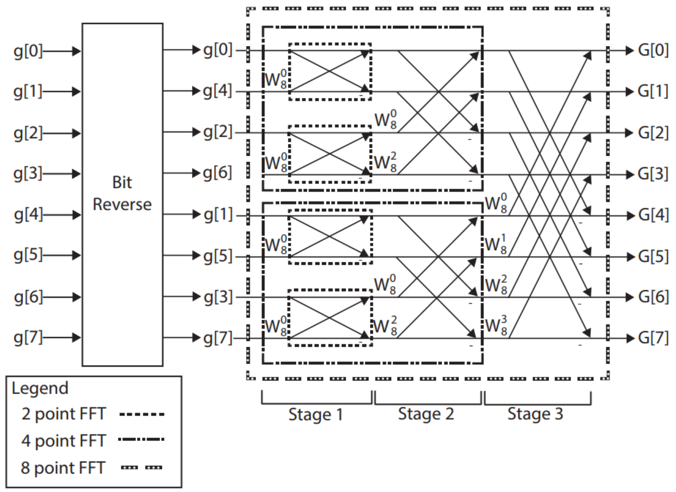


Fast Fourier Transform: Bit Reverse

Index	Binary	Reversed Binary	Reversed Index
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7



Fast Fourier Transform



- Consider each stage separately
- In each stage, fixed number of weights are used for each butterfly. For stage 1, there is only one weight. For stage two, there are two weights….
- In a stage, each weight is being used for (FFTSIZE/No. of Distinct Weights of the stage) times. In stage 1, there is only weight and it is reused 4 times to get 8 outputs. In stage 2, there are 2 weights which are reused 2 times to get 8 outputs…
- We can consider three FOR loops: 1) Outer For loops: Number of stages, 2) Middle For loop: Number of weights in a stage, 3) Inner For loop: Number of times a weight is reused i.e. (FFTSIZE/No. of Distinct Weights of the stage)

HomeWork

- Compare the execution time of FFT of sizes 512 and 1024
- Realize the following function on PS:

$$Q = \frac{X}{T} + \sqrt{\frac{2 * N}{T}}$$

  - **Assume X,T and N are vectors of size 3 and take these values from user. Show the output on UART along with execution time.**
  - **Analyze the execution times for different optimization settings and different vector sizes (say 5, 10, 15).**
  - Do explore various arithmetic functions on your own. It is not possible to cover all aspects in lectures and labs. Similarly, you can explore SW debugging functionality for these functions.

Math Library

- If math.h is not available, add it using following steps: