

Q1)

	A	B	C	D
Tuple 1	a1	b1	c1	d1
Tuple 2	a1	b2	c1	d1
Tuple 3	a2	b3	c1	d3
Tuple 4	a2	b3	c2	d3
Tuple 5	a3	b3	c2	d3
Tuple 6	a4	b4	c2	d4

A functional dependency does not hold on a relation if tuples does not agree on same attributes.

a) $A \rightarrow C$ does not hold on R because tuple 3 and 4 cause violation. In these tuples, $A = a2$ but $C = c1$ in tuple 3 and $C = c2$ in tuple 4.

b) $B \rightarrow D$ holds on R. No tuples cause violation.

c) $BCD \rightarrow A$ does not hold on R because tuple 4 and 5 cause violation. In these tuples, $B = b3$, $C = c2$ and $D = d3$ but $A = a2$ for tuple 4 and $A = a3$ for tuple 5.

According to the table, we can only say that $A \rightarrow D$ and $B \rightarrow D$ for A, B and D. Therefore, by applying a short loop of attribute closure algorithm, we get the following result:

d) $A^+ = AD$ $B^+ = BD$

e) AB is not a super key because $AB \rightarrow R$ does not hold. Because of this, we can say that AB is not a candidate key either.

Q2)

$R(A,B,C, D, E, F)$ $\{AB \rightarrow C, A \rightarrow D, F \rightarrow A, D \rightarrow E, BE \rightarrow F, AC \rightarrow B\}$

I used attribute closure algorithm on slides of Chapter 7.

a) A^+

result = A

result = AD ($A \rightarrow D, A \subseteq \text{result}$)

result = ADE ($D \rightarrow E, D \subseteq \text{result}$)

$A^+ = ADE$

b) CF^+

result = CF

result = ACF ($F \rightarrow A, F \subseteq \text{result}$)

result = ABCF ($AC \rightarrow B, AC \subseteq \text{result}$)

result = ABCDEF ($(A \rightarrow D, A \subseteq \text{result}, D \rightarrow E, D \subseteq \text{result})$)

$CF^+ = ABCDEF$

c)

$D \rightarrow E$

$DB \rightarrow BE$ (augmentation)

$BE \rightarrow F$

$F \rightarrow A$

$DB \rightarrow A$ (transitivity)

$DB \rightarrow AB$ (augmentation)

$AB \rightarrow C$

$DB \rightarrow C$ (transitivity)

Q3)

a) Candidate key of the relation is A since $A \rightarrow R$ and there are no attributes that satisfies this condition.

b) Relation does not satisfy BCNF. For $A \rightarrow D$ and $A \rightarrow BD$, A is a super key. Therefore they satisfy a condition for BCNF. However, $B \rightarrow C$ and $D \rightarrow B$ dependency causes a violation because they are not trivial and B for first dependency is not a super key and D for second dependency is not a super key.

c) Relation does not satisfy 3NF. As mentioned in previous part, $A \rightarrow D$ and $A \rightarrow BD$ satisfy BCNF but $B \rightarrow C$ and $D \rightarrow B$ do not. We need to check extra condition of 3NF. $C - B = C$ and not in a candidate key. $B - D = B$ and not in a candidate key either.

Q4)

a) $A \rightarrow D$ and $BC \rightarrow E$ violates BCNF because they are not trivial and they do not have a super key on the left side ($AF \rightarrow BC$ satisfies BCNF because AF is a super key. I found it by calculating AF^+). Therefore, R does not satisfy BCNF.

b) By using $BC \rightarrow E$, R is decomposed to BCE and ABCDF. BCE satisfies BCNF since BC in $BC \rightarrow E$ is a super key. For ABCDF both dependencies violate BCNF. This one is decomposed to AD and ABCF. $A \rightarrow D$ satisfies BCNF for AD and $AF \rightarrow BC$ satisfies BCNF for ABCF. R decomposed to R1, R2 and R3.

R1(B, C, E) R2(A, D) R3(A, B, C, F)

c) Decomposition is dependency preserving because union of closures of decomposed sets involve closure of F.

d) Decomposition is not lossless because their intersection, D, is not a super key for any of the decompositions.

e) We can clearly see that dependencies with AF are lost with this decomposition. Therefore, it is not dependency preserving.

Q5)

a) To know if D is extraneous in $BD \rightarrow C$, I need to check if F implies $B \rightarrow C$.

$B \rightarrow E$

$E \rightarrow AD$

$B \rightarrow AD$ (transitivity)

$B \rightarrow AD$

$AD \rightarrow C$

$B \rightarrow C$ (transitivity)

D is extraneous in $BD \rightarrow C$.

b) To know if C is extraneous in $A \rightarrow BC$, I need to check if A^+ under $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ includes A where A is attribute inspected if it's extraneous.

A^+ :

result = A

result = AB ($A \rightarrow B$)

result = ABE ($B \rightarrow E$)

result = ABED ($E \rightarrow AD$)

result = ABCDE ($AD \rightarrow CE$)

A^+ includes C. C is extraneous.

c) Other than the dependencies above, we need to check other dependencies for extraneous attributes to find the canonical cover.

Form after part a) and b) $\{A \rightarrow B, B \rightarrow CE, AD \rightarrow CE, E \rightarrow AD\}$.

$B \rightarrow CE$:

(Check for C)

$B^+ = BCE$

C is included. C is extraneous.

$\{A \rightarrow B, B \rightarrow E, AD \rightarrow CE, E \rightarrow AD\}$

$AD \rightarrow CE$:

(Check for D)

$A \rightarrow B$

$B \rightarrow E$

$E \rightarrow AD$

$AD \rightarrow CE$ (transitivity)

F implies $A \rightarrow CE$. D is extraneous.

$\{A \rightarrow B, B \rightarrow E, A \rightarrow CE, E \rightarrow AD\}$

$A \rightarrow CE$:

(Check for C)

$A^+ = ABED$

C is not included. C is not extraneous.

(Check for E)

$A^+ = ABCDE$

E is included. E is extraneous.

$\{A \rightarrow B, B \rightarrow E, A \rightarrow C, E \rightarrow AD\} = \{A \rightarrow BC, B \rightarrow E, E \rightarrow AD\}$

$E \rightarrow AD$:

(Check for A)

$E^+ = ED$

A is not included. A is not extraneous.

(Check for D)

$E^+ = ABCE$

D is not included. D is not extraneous.

$\{A \rightarrow BC, B \rightarrow E, E \rightarrow AD\}$

Canonical cover is $\{A \rightarrow BC, B \rightarrow E, E \rightarrow AD\}$