1 Energy Flow Optimization Problem & Solution

1.1 Decision Variables

$q_t^{pv\to c}$: Energy (kWh) sent from the PV system to the consumer at time t.

$q_t^{pv\to b}$: Energy (kWh) sent from the PV system to the battery at time t.

$q_t^{pv\to g}$: Energy (kWh) sold from the PV system back to the grid at time t.

$q_t^{b\to c}$: Energy (kWh) discharged from the battery to the consumer at time t.

$q_t^{b\to g}$: Energy (kWh) discharged from battery and sold to the grid at time t.

$q_t^{g\to c}$: Energy (kWh) bought from the grid to serve the consumer at time t.

$q_t^{g\to b}$: Energy (kWh) bought from the grid to charge the battery at time t.

s_t : State of charge of the battery (kWh) at the end of hour t.

y_t : Binary, 1 if any grid-buy occurs at time t (else 0). –for part B

z_t : Binary, 1 if any grid-sell occurs in hour t (else 0). – for part B

1.2 Parameters

T : Index set of hours, $T = \{0, 1, \ldots, n-1\}$.

PV_t : PV generation available (kWh) at time t.

D_t : Consumer demand (kWh) at time t.

c_t^{buy} : Buy price (c/kWh) at time t.

c_t^{sell} : Sell price (c/kWh) at time t.

c_t^{LCOS} : Battery discharge cost (c/kWh) to each kWh discharged.

B : Battery capacity (kWh).

R : Maximum battery charge or discharge rate (kW or kWh per hour).

G : Grid buy/sell capacity (kW or kWh per hour).

η : Battery charging efficiency (=0.92).

s_0 Initial battery charge (kWh) (=0).

1.3 Mathematical formulation of the problem:

1.3.1 part A

$$\min_{q,s} \quad \sum_{t\in T}\Big[(q_t^{g\to c}+q_t^{g\to b})\,c_t^{\text{buy}}-(q_t^{pv\to g}+q_t^{b\to g})\,c_t^{\text{sell}}+(q_t^{b\to c}+q_t^{b\to g})\,c_t^{\text{LCOS}}\Big]$$

$$\begin{aligned}
\text{s.t.}\quad & D_t = q_t^{pv\to c}+q_t^{g\to c}+q_t^{b\to c} && \forall t\in T,\\
& \text{PV}_t = q_t^{pv\to c}+q_t^{pv\to b}+q_t^{pv\to g} && \forall t\in T,\\
& s_0=0,\quad s_t = s_{t-1}+\eta\big(q_{t-1}^{pv\to b}+q_{t-1}^{g\to b}\big)-\big(q_{t-1}^{b\to g}+q_{t-1}^{b\to c}\big) && \forall t\in T\setminus\{0\},\\
& 0\le s_t\le B && \forall t\in T,\\
& q_t^{pv\to b}+q_t^{g\to b}\le R && \forall t\in T,\\
& q_t^{b\to g}+q_t^{b\to c}\le R && \forall t\in T,\\
& q_t^{pv\to g}+q_t^{b\to g}\le G && \forall t\in T,\\
& q_t^{g\to c}+q_t^{g\to b}\le G && \forall t\in T.
\end{aligned}$$

1.3.2 part B

$$\min_{q,s,b} \quad \sum_{t\in T}\Big[(q_t^{g\to c}+q_t^{g\to b})\,c_t^{\text{buy}}-(q_t^{pv\to g}+q_t^{b\to g})\,c_t^{\text{sell}}+(q_t^{b\to c}+q_t^{b\to g})\,c_t^{\text{LCOS}}\Big]$$

$$\begin{aligned}
\text{s.t.}\quad & D_t = q_t^{pv\to c}+q_t^{g\to c}+q_t^{b\to c} && \forall t\in T,\\
& \text{PV}_t = q_t^{pv\to c}+q_t^{pv\to b}+q_t^{pv\to g} && \forall t\in T,\\
& s_0=0,\quad s_t = s_{t-1}+\eta\big(q_{t-1}^{pv\to b}+q_{t-1}^{g\to b}\big)-\big(q_{t-1}^{b\to g}+q_{t-1}^{b\to c}\big) && \forall t\in T\setminus\{0\},\\
& 0\le s_t\le B && \forall t\in T,\\
& q_t^{pv\to b}+q_t^{g\to b}\le R && \forall t\in T,\\
& q_t^{b\to g}+q_t^{b\to c}\le R && \forall t\in T,\\
& q_t^{pv\to g}+q_t^{b\to g}\le G && \forall t\in T,\\
& q_t^{g\to c}+q_t^{g\to b}\le G && \forall t\in T,\\
& y_t+z_t\le 1 && \forall t\in T,\\
& q_t^{g\to c}+q_t^{g\to b}\le G\,y_t && \forall t\in T,\\
& q_t^{pv\to g}+q_t^{b\to g}\le G\,z_t && \forall t\in T.
\end{aligned}$$