Homework 8

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Problem 2 (write-up)

1. The constraints for every point should be

$$-Z_i \leqslant px_i + q - y_i \leqslant Z_i, Z_i \geqslant 0 \tag{1}$$

(1) could be expanded into two related constraints for each point (x_i, y_i)

$$
\begin{aligned}
px_i + q - y_i &\leqslant Z_i \\
-px_i - q + y_i &\leqslant Z_i
\end{aligned} \tag{2}
$$

Extracting the variables x_i, y_i, Z_i, our constraint inequality functions (2) become

$$
\begin{aligned}
px_i + q - Z_i &\leqslant y_i \\
-px_i - q - Z_i &\leqslant -y_i
\end{aligned} \tag{3}
$$

Our objective is to minimize $\sum_{i=1}^{n} |px_i + q - y_i|$ and taking the variables p and q into account, the objective function could be interpretted as follows:

$$0p + 0q + \sum_{i=1}^{n} Z_i \tag{4}$$

2. The constraints for every point should be

$$-Z \leqslant px_i + q - y_i \leqslant Z, Z \geqslant 0 \tag{5}$$

And now the Z is the overall constraint for every point, the constraints inequality function could be represented as follows:

$$
\begin{aligned}
px_i + q - Z &\leqslant y_i \\
-px_i - q - Z &\leqslant -y_i
\end{aligned} \tag{6}
$$

Our objective is to minimize $\max_i |px_i + q - y_i|$ and taking the variables p and q into account, the objective function could be interpretted as follows:

$$0p + 0q + Z \tag{7}$$

3. In Figure 1, we plot 10 points in the coordinates and arbitrarily setting the first point be the outlier. The legend is that gray line represents the built-in least-squares regression routine polyfit; red line represents our L1Regression and green line denotes L1MaxRegression.

   From the image, we know that the built-in least squares regression polyfit looks *worse* than L1Regression but still better than L1MaxRegression.
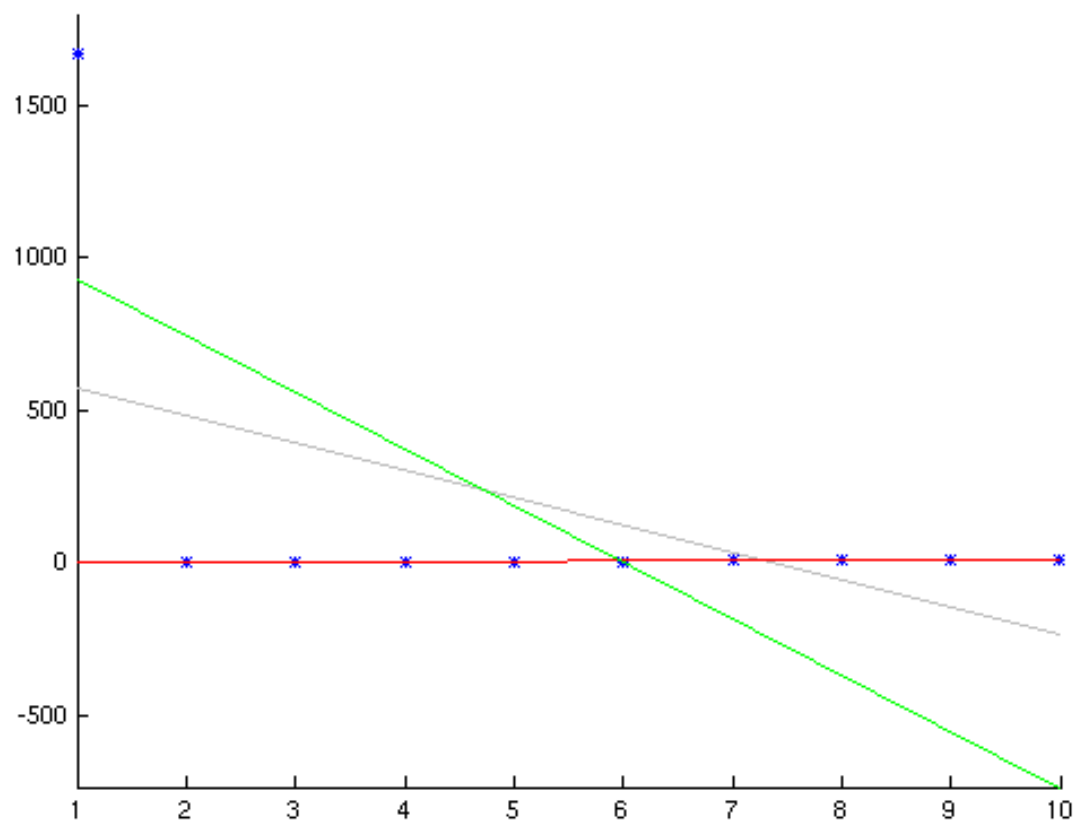
Figure 1.