Set Equivalence

The following are two sample proofs of the equivalence

$$(A \cap B) \cup (A - B) = A \cap (B \cup (A - B)).$$

One uses the element method. The other uses set algebra.

NOTE: Drawing a Venn diagram of each does not constitute a proof and will not be graded as such.

Element Method

Requirements

1. Show the left side of the of the equation is a subset of the right.
2. Show the right side of the of the equation is a subset of the left.
3. Conclude they are equal.

Proof

Suppose $x \in (A \cap B) \cup (A - B)$. Then either x is in both A and B or x is in A but not B. Either way, x must be in A. Therefore, x must either be in the portion of A that does not overlap with B or the portion that does. So either $x \in A - B$ or $x \in B$. But we also know x is definitely in A, so $x \in A \cap (B \cup (A - B))$. And since x was arbitrary, this is true for all elements in the set, and therefore the set as a whole. This proves the first direction. For the second, suppose $x \in A \cap (B \cup (A - B))$. Then $x \in A$ and $x \in (B \cup (A - B))$. Therefore either x is in B or x is in A but not B. Since x is also in A, x is either in $(A \cap B)$ or $(A - B)$. So $x \in (A \cap B) \cup (A - B)$. This proves the second direction, and as both directions hold, the equality is proven.

Set Algebra

Requirements

1. Conversion of one side of the equation to the other (or conversion of both sides to an identical expression) using *stated* laws of set algebra
2. Conclusion based on the biconditionality of the steps taken

Set Equivalence

Proof

$$
\begin{aligned}
&(A \cap B) \cup (A - B) && \\
&(A \cap B) \cup (A \cap B^c) && \text{(Set Difference Law)} \\
&A \cup (B \cap B^c) && \text{(Distribution)} \\
&A \cup \emptyset && \text{(Complement Law)} \\
&A && \text{(Identity Law)} \\
&A \cap (A \cup B) && \text{(Absorbtion)} \\
&A \cap (B \cup A) && \text{(Commutivity)} \\
&A \cap ((B \cup A) \cap U) && \text{(Identity Law)} \\
&A \cap ((B \cup A) \cap (B \cup B^c)) && \text{(Complement Law)} \\
&A \cap (B \cup (A \cap B^c)) && \text{(Distribution)}
\end{aligned}
$$

All these steps are biconditionally true, therefore the equality holds.