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Prove that for any integer $n \geq 0$, $3n^2 + 15n + 12$ is even.

Proof: Prove by induction on n.

Let $P(n)$ be the property "$3n^2 + 15n + 12$ is even."

Base case: let $n = 0$. $3 \times (0^2) + 15(0) + 12 = 12$. $12 = 2 \times 6$, and is thus even. Thus $P(0)$ is true.

Inductive hypothesis: suppose that $P(k)$ is true for some $k \geq 0$; that is, "$3k^2 + 15k + 12$ is even." Now it must be shown that $P(k+1)$ is true: that $3(k+1)^2 + 15(k+1) + 12$ is even.

Inductive step:

$$\begin{aligned} 3(k+1)^2 + 15(k+1) + 12 &= 3(k^2 + 2k + 1) + 15k + 15 + 12 \\ &= 3k^2 + 6k + 3 + 15k + 15 + 12 \\ &= 3k^2 + 15k + 12 + 6k + 18 \end{aligned}$$

Because $P(k)$, $k^2 + 15k + 12 = 2z$ for some $z \in \mathbb{Z}$. Thus,

$$\begin{aligned} 3(k+1)^2 + 15(k+1) + 12 &= 2z + 6k + 18 \\ &= 2(z + 6k + 18) \end{aligned}$$

which is an even integer for all integers $z, k \in \mathbb{Z}$. So $P(k+1)$ is true.

Thus, because the base case $P(0)$ is true and $P(k)$ implies $P(k+1)$ for all $k \geq 0$, $3n^2+15n+12$ is even for all $n \geq 0$.