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CS 22 — Spring 2012

Sample Proofs

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Prove that for all integers $n \geq 0$, $3n^2 + 15n + 12$ is even.

Proof by induction:
Let $P(n)$ be the property that $3n^2 + 15n + 12$ is even.

Base Case:
$P(0)$ holds, since $3(0)^2 + 15(0) + 12 = 12$, which is even.
Inductive Hypothesis:
Assume $P(k)$ holds for integer $k \geq 0$. That is, $3k^2 + 15k + 12$ is even.
Inductive Step:
We now prove that $P(k+1)$ holds. By the inductive hypothesis and definition of even, $3k^2 + 15k + 12 = 2m$ for some integer m. Then we have:

$$\begin{aligned}
3(k+1)^2 + 15(k+1) + 12 &= 3k^2 + 6k + 3 + 15k + 15 + 12 \\
&= (3k^2 + 15k + 12) + (6k + 18) \\
&= (2m) + 2(3k + 9) \text{ by the inductive hypothesis.} \\
&= 2(m + 3k + 9) = 2q \text{ for } q = m + 3k + 9.
\end{aligned}$$

Since m, $3k$, and 9 are all integers, q is an integer.
Then since $3(k+1)^2 + 15(k+1) + 12 = 2q$, where q is an integer, $3(k+1)^2 + 15(k+1) + 12$ is even and so $P(k+1)$ holds.

Thus, since we have shown that the base case $P(0)$ holds and $P(k) \rightarrow P(k+1)$ for integers $k \geq 0$, $P(n)$ is true by induction for all integers $n \geq 0$.