CS157 Homework 8

1. The variables we define are as follows:

$$\begin{aligned} w_i &= \text{number of workers for month } i \\ t_i &= \text{number of experienced workers participate in the training for month } i \\ c_i &= \text{the cost for employing in month } i \end{aligned}$$

The constraints are as follows:

$$\begin{align} 150 \cdot (w_{May} - t_{May}) + 50 \cdot t_{May} &\geqslant 8000 \tag{1} \\ 150 \cdot (w_{Jun} - t_{Jun}) + 50 \cdot t_{Jun} &\geqslant 9000 \tag{2} \\ 150 \cdot (w_{Jul} - t_{Jul}) + 50 \cdot t_{Jul} &\geqslant 7000 \tag{3} \\ 150 \cdot \left(w_{Aug} - t_{Aug}\right) + 50 \cdot t_{Aug} &\geqslant 10000 \tag{4} \\ 150 \cdot \left(w_{Sep} - t_{Sep}\right) + 50 \cdot t_{Sep} &\geqslant 9000 \tag{5} \\ 150 \cdot (w_{Oct} - t_{Oct}) + 50 \cdot t_{Oct} &\geqslant 11000 \tag{6} \\ w_{May} &= 60 \tag{7} \\ w_{Jun} &= 0.9 \cdot w_{May} + t_{May} \tag{8} \\ w_{Jul} &= 0.9 \cdot w_{Jun} + t_{Jun} \tag{9} \\ w_{Aug} &= 0.9 \cdot w_{Jul} + t_{Jul} \tag{10} \\ w_{Sep} &= 0.9 \cdot w_{Aug} + t_{Aug} \tag{11} \\ w_{Oct} &= 0.9 \cdot w_{Sep} + t_{Sep} \tag{12} \end{align}$$

For each c_i, it could be interpretted as

$$c_i = 3400 \cdot w_i - 1600 \cdot t_i$$

Our object is to minimize the toal cost:

$$c_{May} + c_{Jun} + c_{Jul} + c_{Aug} + c_{Sep} + c_{Oct}$$

and plugging the *linear equality constraints* (7) to (12) into it we can get the objective function:

$$\min \left(3400 \times \left(60 \times \sum_{i=0}^{5} 0.9^i + t_{May} \cdot \sum_{j=0}^{4} 0.9^j + t_{Jun} \cdot \sum_{k=0}^{3} 0.9^k + t_{Jul} \cdot \sum_{p=0}^{2} 0.9^p + t_{Aug} \cdot \sum_{q=0}^{1} 0.9^q + t_{Sep} \cdot 0.9 \right) - 1600(t_{May} + t_{Jun} + t_{Jul} + t_{Aug} + t_{Sep} + t_{Oct}) \right)$$

Plugging the *linear equality constraints* (7) to (12) into *linear inequality constraints* (1) to (6), and negating both sides of them, we can get new *linear inequalitiy constraints* containing up to 6 variables shown as follows:

$$\begin{aligned} -150 \times 60 + 100 \cdot t_{May} &\leqslant -8000 \\ -150 \cdot (0.9 \times 60 + t_{May}) + 100 \cdot t_{Jun} &\leqslant -9000 \\ -150 \cdot (0.9^2 \times 60 + 0.9 t_{May} + t_{Jun}) + 100 \cdot t_{Jul} &\leqslant -7000 \\ -150 \cdot (0.9^3 \times 60 + 0.9^2 t_{May} + 0.9 t_{Jun} + t_{Jul}) + 100 \cdot t_{Aug} &\leqslant -10000 \\ -150 \cdot (0.9^4 \times 60 + 0.9^3 t_{May} + 0.9^2 t_{Jun} + 0.9 t_{Jul} + t_{Aug}) + 100 \cdot t_{Sep} &\leqslant -9000 \\ -150 \cdot (0.9^5 \times 60 + 0.9^4 t_{May} + 0.9^3 t_{Jun} + 0.9^2 t_{Jul} + 0.9 t_{Aug} + t_{Sep}) + 100 \cdot t_{Oct} &\leqslant -11000 \end{aligned}$$

$\Rightarrow$

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\begin{aligned}
100 \cdot t_{May} &\leqslant -8000 + 150 \times 60 \times 0.9^0 && (1') \\
-150 \cdot t_{May} + 100 \cdot t_{Jun} &\leqslant -9000 + 150 \times 60 \times 0.9^1 && (2') \\
-150 \cdot (0.9t_{May} + t_{Jun}) + 100 \cdot t_{Jul} &\leqslant -7000 + 150 \times 60 \times 0.9^2 && (3') \\
-150 \cdot (0.9^2 t_{May} + 0.9t_{Jun} + t_{Jul}) + 100 \cdot t_{Aug} &\leqslant -10000 + 150 \times 60 \times 0.9^3 && (4') \\
-150 \cdot (0.9^3 t_{May} + 0.9^2 t_{Jun} + 0.9t_{Jul} + t_{Aug}) + 100 \cdot t_{Sep} &\leqslant -9000 + 150 \times 60 \times 0.9^4 && (5') \\
-150 \cdot (0.9^4 t_{May} + 0.9^3 t_{Jun} + 0.9^2 t_{Jul} + 0.9t_{Aug} + t_{Sep}) + 100 \cdot t_{Oct} &\leqslant -11000 + 150 \times 60 \times 0.9^5 && (6')
\end{aligned}
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