Homework 0

Due: Jan 28, 2013 11:59 PM

The written portion of the homework must be handed in to the CS157 hand-in bin located on the CIT 2nd floor between the Fishbowl and the locker. The programming portion of homeworks (when applicable) must be handed in electronically via the hand-in script.

Each problem must be handed in separately with your name on the top and the time of hand-in. You may turn in different problems for different deadlines. For more information, please review the assignment and due date policy in the course missive on the course webpage.

Please ensure that your solutions are complete and communicated clearly: use full sentences and plan your presentation before you write. Except in the rare cases where it is indicated otherwise, consider every problem as asking you to prove your result.

You are allowed, and often encouraged, to use Wikipedia to help with your homework. See the collaboration policy on the course webpage for more details.

Important: The early and late hand-in policy does *not* apply to this homework. This homework must be handed in before the ultimate deadline indicated above.

This homework must be handed in with a signed COLLABORATION POLICY from the course website.

Problem 1

Let the function $f(n)$ be defined by the following sum

$$f(n) = \sum_{k=1}^{n} 2^k (k+1)$$

Show that the function $f(n)$ has the closed form $n \cdot 2^{n+1}$ by using mathematical induction.

Problem 2

Sort the following functions by order of growth from slowest to fastest. For each pair of adjacent functions in your list, please write one sentence describing (informally is fine) why you ordered them as you did.

a. $7n^3 - 10n$	b. $4n^2$	c. n
d. $n^{8621909}$	e. 3^n	f. $e^{\log \log n}$
g. $n^{\log n}$	h. $6n \log n$	i. $n!$

Problem 3

1. Show that $n! \leq n^n$ for $n \geq 1$.
2. Show that $n! \geq (\lceil n/2 \rceil)^{\lceil n/2 \rceil}$ for $n \geq 1$.

3. Use the fact above to prove that $\log(n!) = \Theta(n \log n)$.
4. Prove that "$n! = \Theta(n^n)$" is false.

Problem 4

Without appealing to the Master Theorem, find (and prove) for each recurrence relation below, a simple closed-form function $f(n)$ which satisfies $T(n) = \Theta(f(n))$. If your calculation involves a noninteger value (e.g. $n/2$ for n odd), you are welcome to simplify your calculation by rounding to the nearest whole number.

1. $T(n) = 2 \cdot T(n-1) + 1, \qquad \text{where } T(0) = 0$
2. $T(n) = \begin{cases} 2 \cdot T(\frac{n}{2}) + n & \text{if } n > 1 \\ 1 & \text{if } n \leq 1 \end{cases}$

Problem 5

1. Find the following sum of matrices
$$\begin{bmatrix} 1 & 0 & 5 \\ 2 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 6 & 2 \\ 1 & 7 & 4 \\ 0 & 2 & 1 \end{bmatrix}$$
2. Find the following products of matrices
   (a) $\begin{bmatrix} 2 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 1 & 3 \\ 2 & 3 \end{bmatrix}$
   (b) $\begin{bmatrix} 1 & 7 & 4 & 2 & 6 \\ 2 & 1 & 1 & 1 & 9 \\ 100 & 99 & 98 & 97 & 96 \\ 0 & 10 & 5 & 2 & 1 \\ 10 & 1 & 19 & 28 & 33 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$
3. Find the transpose of the following matrix
$$\begin{bmatrix} 1 & 2 \\ 0 & 6 \\ 3 & 2 \end{bmatrix}^T$$
4. Find the inverse of the following matrices where possible. If it is impossible, say why it is impossible
   (a) $\begin{bmatrix} 1 & 3 & 5 \\ 8 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$
   (b) $\begin{bmatrix} 0 & 5 & 0 \\ 0 & 4 & 1 \\ 0 & 1 & 3 \end{bmatrix}$

5. (a) Express the vector $\begin{bmatrix} 7 & 0 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 3 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \end{bmatrix}$.
   (b) Describe part (a) in a sentence using the technical term "basis".
   (c) Describe part (a) in a sentence involving the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}$.

Problem 6

(Note: this problem is warmup for the lecture in a couple weeks on Fourier transforms. If you are not familiar with complex numbers, brush up on them now!)

In this problem there is no need to rigorously prove your statements, but remember to use full sentences to describe anything that will not be obvious to the reader. All of your answers should involve exact expressions, not decimals.

1. Plot the number $1 + \sqrt{3}\,i$ on the complex plane. Express $1 + \sqrt{3}\,i$ in polar coordinates: label the angle and magnitude on your plot.
2. Find the complex conjugate of $3 + 2i$.
3. Find the reciprocal of $3 + 2i$ and plot it, the conjugate of $3 + 2i$, and $3 + 2i$ on the same plot.
4. Given two complex numbers in polar coordinates, how do you multiply them?
5. Given an integer k and a complex number with angle θ and magnitude r, what is the expression for the kth power of the complex number?
6. What are *all* the complex numbers whose 6th powers are 1? (Hint: use the first part and the previous part. There are 6 answers.)