

Since the task is to prove the claim for any $n \geq 0$, induction seems like a good way to go. (But you don't have to explain that in your proof)

We proceed by induction. Let $P(n)$ be a statement on $n \in \mathbb{Z}$ which means $n^2 + 15n + 12$ is even.

Base case: We will show $P(0)$ is true: $3(0^2) + 15(0) + 12 = 12$ and 12 is even. So $P(0)$ is true.

Inductive Step: Assume $P(k)$ is true for some $k \geq 0$. Then $3(k^2) + 15(k) + 12$ is even. We will now prove $P(k+1)$ is true. That is, we will show $3(k+1)^2 + 15(k+1) + 12$ is even. We can expand the expression:

$$\begin{aligned} 3(k+1)^2 + 15(k+1) + 12 &= 3(k^2 + 2k + 1) + 15k + 15 + 12 \\ &= 3k^2 + 6k + 3 + 15k + 17 \text{ (see note*.)} \\ &= (3k^2 + 15k + 12) + 6k + 8 \end{aligned}$$

note*: At this point the goal is to try to use what we know about the k case to figure out something about the $k+1$ case.) So we want **something** of the form $3k^2 + 15k + 12$. You don't have to explain that in your proof either.

By the inductive hypothesis, $3k^2 + 15k + 12$ is even.

Furthermore, the quantity $6k + 8$ can be written $2(3k + 4)$ which is even by definition of even. The entire expression is thus the sum of two even numbers. Since the sum of even numbers is even, $3(k+1)^2 + 15(k+1) + 12$ is even, which is what we were trying to show. The claim follows by induction.