

Chapter 2 Section 3

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Problem 1. *Calculate the matrix product*

$$\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

Solution.

$$\begin{aligned} \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} &= \begin{bmatrix} 6 * 1 + 7 * 3 & 6 * 2 + 7 * 5 \\ 8 * 1 + 9 * 3 & 8 * 2 + 9 * 5 \end{bmatrix} \\ &= \begin{bmatrix} 27 & 47 \\ 35 & 61 \end{bmatrix} \end{aligned}$$

Problem 2. *Compute the products BA and AB for*

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Interpret your answers geometrically, as composites of linear transformation.

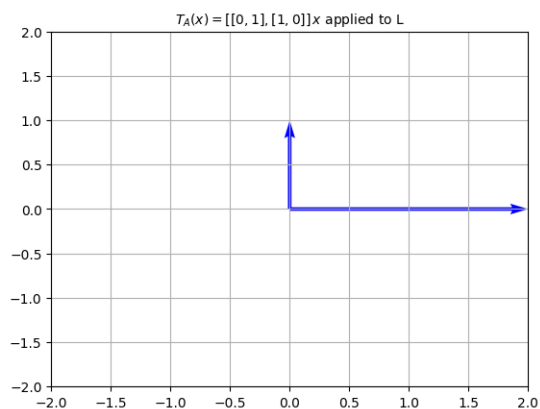
Solution. Let $T_A(\vec{x}) = A\vec{x}$ and $T_B(\vec{y}) = B\vec{y}$. We can write

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = [T_A(e_1) \quad T_A(e_2)] \\ B &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = [T_B(e_1) \quad T_B(e_2)] \end{aligned}$$

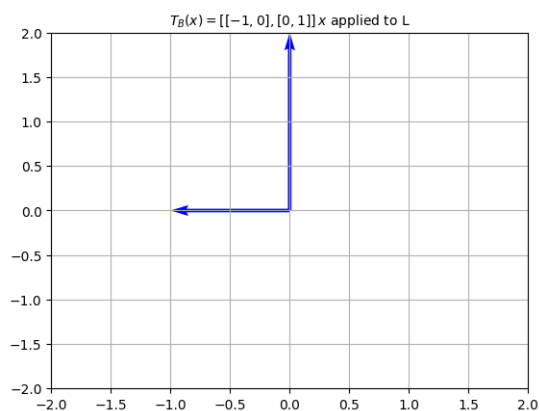
Thus

$$\begin{aligned} T_A(\vec{e}_1) &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, T_A(\vec{e}_2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ T_B(\vec{e}_1) &= \begin{bmatrix} -1 \\ 0 \end{bmatrix}, T_B(\vec{e}_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned}$$

We see that T_A is a reflection about the line $y = x$. In other words, T_A is a reflection about the line spanned by the vector $\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.



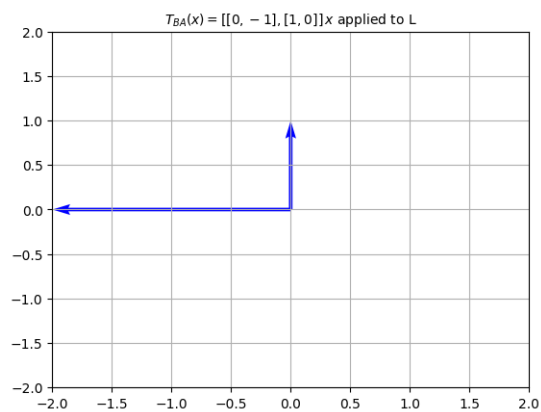
We see that T_B is a reflection about the line $x = 0$. In other words, T_B is a reflection about the line spanned by the vector $\vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.



Now let's compute the products BA and AB .

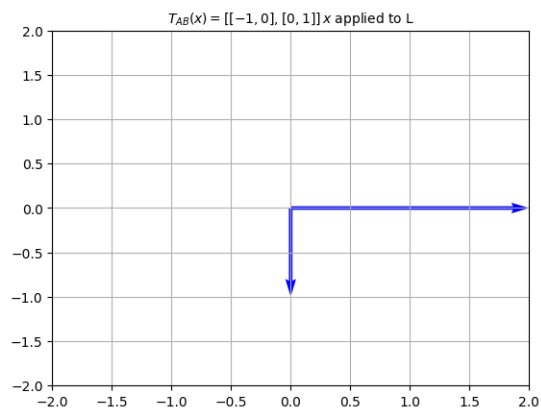
$$\begin{aligned} BA &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

The product BA is a rotation matrix that rotates a vector ninety degrees counterclockwise.



$$\begin{aligned}
 AB &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}
 \end{aligned}$$

The product AB is a rotation matrix that rotates a vector ninety degrees clockwise.



Problem 3. Multiply the block matrices

$$\left(\begin{array}{cc|c} 0 & 1 & -1 \\ 1 & 0 & 1 \end{array} \right) \left(\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \\ \hline 7 & 8 & 9 \end{array} \right)$$

Afterwards, compute the product without a partition and see if you get the same result.

Solution.

$$\begin{aligned}
& \left(\begin{array}{cc|c} 0 & 1 & -1 \\ 1 & 0 & 1 \end{array} \right) \left(\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \\ \hline 7 & 8 & 9 \end{array} \right) \\
&= \left(\begin{array}{cc} [0 & 1] \\ [1 & 0] \end{array} \right) \left(\begin{array}{cc} [1 & 2] \\ [4 & 5] \end{array} \right) + \begin{bmatrix} -1 \\ 1 \end{bmatrix} \begin{bmatrix} 7 & 8 \end{bmatrix} \mid \begin{array}{cc} [0 & 1] \\ [1 & 0] \end{array} \left(\begin{array}{c} [3] \\ [6] \end{array} \right) + \begin{bmatrix} -1 \\ 1 \end{bmatrix} \begin{bmatrix} 9 \end{bmatrix} \right) \\
&= \left(\begin{array}{cc} [4 & 5] \\ [1 & 2] \end{array} \right) + \begin{bmatrix} -7 & -8 \\ 7 & 8 \end{bmatrix} \mid \begin{array}{c} [6] \\ [3] \end{array} + \begin{array}{c} [-9] \\ [9] \end{array} \right) \\
&= \left(\begin{array}{cc} [-3 & -3] \\ [8 & 10] \end{array} \mid \begin{array}{c} [-3] \\ [12] \end{array} \right) \\
&= \left(\begin{array}{cc|c} -3 & -3 & -3 \\ 8 & 10 & 12 \end{array} \right)
\end{aligned}$$

Now let's compute the product without a partition.

$$\begin{aligned}
& \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \\
&= \begin{bmatrix} 4-7 & 5-8 & 6-9 \\ 1+7 & 2+8 & 3+9 \end{bmatrix} \\
&= \begin{bmatrix} -3 & -3 & -3 \\ 8 & 10 & 12 \end{bmatrix}
\end{aligned}$$

We get the same result.