

# Chapter 1 Section 1

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**Problem 1.** *The yield of one bundle of inferior rice, two bundles of medium-grade rice, and three bundles of superior rice is 39 dou of grain. The yield of one bundle of inferior rice, three bundles of medium-grade rice, and two bundles of superior rice is 34 dou. The yield of three bundles of inferior rice, two bundles of medium-grade rice, and one bundle of superior rice is 26 dou. What is the yield of one bundle of each grade of rice?*

**Solution.** *We can turn this information into an equation involving matrices.*

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 39 \\ 34 \\ 26 \end{pmatrix}$$

*Let's get the determinant of the coefficients matrix.*

$$\begin{aligned} \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} &= 1 \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 3 \begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} \\ &= 1 * (3 - 4) - 2 * (1 - 6) + 3 * (2 - 9) \\ &= -1 + 10 - 21 \\ &= -12 \end{aligned}$$

*The determinant is nonzero, so the equation has a solution.*

*Now we can do some row operations to get the identity matrix on the left.*

$$\begin{aligned}
\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 39 \\ -5 \\ 26 \end{pmatrix} \\
\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & -4 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 39 \\ -5 \\ -91 \end{pmatrix} \\
\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -12 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 39 \\ -5 \\ -111 \end{pmatrix} \\
\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 39 \\ -5 \\ 9.25 \end{pmatrix} \\
\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 39 \\ 4.25 \\ 9.25 \end{pmatrix} \\
\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 30.5 \\ 4.25 \\ 9.25 \end{pmatrix} \\
\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 2.75 \\ 4.25 \\ 9.25 \end{pmatrix}
\end{aligned}$$

The solution is  $(x, y, z) = (2.75, 4.25, 9.25)$ .

Thus the yield for one bundle of inferior rice is 2.75 dou. The yield for one bundle of medium-grade rice is 4.25 dou. The yield for one bundle of superior rice is 9.25 dou.

**Problem 2.**