

Chapter 1 Section 2

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Problem 1. Find all solutions for the equations

$$\begin{cases} x + y - 2z = 5 \\ 2x + 3y + 4z = 2 \end{cases}$$

Solution. We can use Gauss-Jordan elimination.

$$\begin{array}{l} \left| \begin{array}{cccc} 1 & 1 & -2 & 5 \\ 2 & 3 & 4 & 2 \end{array} \right| \\ \left| \begin{array}{cccc} 1 & 1 & -2 & 5 \\ 0 & 1 & 8 & -8 \end{array} \right| \\ \left| \begin{array}{cccc} 1 & 0 & -10 & 13 \\ 0 & 1 & 8 & -8 \end{array} \right| \end{array}$$

This gives us the equations $x - 10z = 13$ and $y + 8z = -8$.

Thus there are infinitely many solutions. The solutions are

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 + 10t \\ -8 - 8t \\ t \end{pmatrix}$$

for an arbitrary real number t .

Problem 2. Find all solutions for the equations

$$\begin{cases} 3x + 4y - z = 8 \\ 6x + 8y - 2z = 3 \end{cases}$$

Solution. We can use Gauss-Jordan elimination once again (and after this problem we may use Gauss-Jordan elimination without saying it explicitly).

$$\begin{array}{l} \left| \begin{array}{cccc} 3 & 4 & -1 & 8 \\ 6 & 8 & -2 & 3 \end{array} \right| \\ \left| \begin{array}{cccc} 3 & 4 & -1 & 8 \\ 0 & 0 & 0 & -13 \end{array} \right| \end{array}$$

This gives us the equations $3x + 4y - z = 8$ and $0 = -13$. The second equation is a contradiction.

Why is the second equation a contradiction? It's because each equation represents a line, and the two lines do not intersect.

When we do Gauss-Jordan elimination, we assume there is a solution. When this assumption is wrong, we can get a contradiction like $0 = 13$.

Our method shows us that the system is inconsistent, so there are no solutions.