Chapter 3 Section 4

Andrew Taylor

May 16 2022

Problem 1. *Let* $\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ *and* $\vec{v_2} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. *Let* $V = \mathrm{span}(v_1, v_2)$. *Is the vector*

$\vec{w} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$ *on the plane V?*

Solution. *If the vector* $\vec{w}$ *is on the plane V, then there exist some* $x_1, x_2 \in \mathbb{R}$ *such that* $\vec{w} = x_1\vec{v_1} + x_2\vec{v_2}$. *This gives us the equations*

$$x_1 + x_2 = 5$$
$$x_1 + 2x_2 = 7$$
$$x_1 + 3x_2 = 9$$

We can solve these equations using a matrix.

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 1 & 2 & 7 \\ 1 & 3 & 9 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

This gives us $x_1 = 3$ *and* $x_2 = 2$.

Thus $\vec{w}$ *is on the plane V because* $\vec{w} = 3\vec{v_1} + 2\vec{v_2}$.