Chapter 1 Section 2

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March 16 2022

Problem 1. *Find all solutions for the equations*

$$\begin{vmatrix} x + y - 2z = 5 \\ 2x + 3y + 4z = 2 \end{vmatrix}$$

Solution. *We can use Gauss-Jordan elimination.*

$$\begin{vmatrix} 1 & 1 & -2 & 5 \\ 2 & 3 & 4 & 2 \end{vmatrix}$$
$$\begin{vmatrix} 1 & 1 & -2 & 5 \\ 0 & 1 & 8 & -8 \end{vmatrix}$$
$$\begin{vmatrix} 1 & 0 & -10 & 13 \\ 0 & 1 & 8 & -8 \end{vmatrix}$$

This gives us the equations $x - 10z = 13$ *and* $y + 8z = -8$.

Thus there are infinitely many solutions. The solutions are

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 + 10t \\ -8 - 8t \\ t \end{pmatrix}$$

for an arbitrary real number t.

Problem 2. *Find all solutions for the equations*

$$\begin{vmatrix} 3x + 4y - z = 8 \\ 6x + 8y - 2z = 3 \end{vmatrix}$$

Solution. *We can use Gauss-Jordan elimination once again (and after this problem we may use Gauss-Jordan elimination without saying it explicitly).*

$$\begin{vmatrix} 3 & 4 & -1 & 8 \\ 6 & 8 & -2 & 3 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 4 & -1 & 8 \\ 0 & 0 & 0 & -13 \end{vmatrix}$$

This gives us the equations $3x + 4y - z = 8$ and $0 = -13$. The second equation is a contradiction.

Why is the second equation a contradiction? It's because each equation represents a line, and the two lines do not intersect.

When we do Gauss-Jordan elimination, we assume there is a solution. When this assumption is wrong, we can get a contradiction like $0 = 13$.

Our method shows us that the system is inconsistent, so there are no solutions.

Problem 3. *Find all solutions for the equation*

$$x + 2y + 3z = 4$$

Solution. *Let $s = y$ and $t = z$ be arbitrary real numbers. Then the solutions to the equation are:*

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 - 2s - 3t \\ s \\ t \end{pmatrix}$$

Problem 4. *Find all solutions to the equations*

$$\begin{vmatrix} x + y = 1 \\ 2x - y = 5 \\ 3x + 4y = 2 \end{vmatrix}$$

Solution.

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 5 \\ 3 & 4 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & -3 & 3 \\ 3 & 4 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 3 & 4 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 3 & 4 & 2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 4 & -4 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{vmatrix}$$

Thus the solution is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$