

Chapter 3 Section 1

Andrew Taylor

April 23 2022

Problem 1. Find the kernel of the linear transformation

$$T(\vec{x}) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \vec{x}$$

from \mathbb{R}^3 to \mathbb{R}^2 .

Solution. Let's solve the linear system $T(\vec{x}) = 0$ to get the kernel of T .

$$\begin{array}{l} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \end{array} \right) \\ \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right) \\ \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right) \end{array}$$

This tells us that $x_1 = x_3$ and $x_2 = -2x_3$.

Let $t = x_3$ be an arbitrary real number. Then the solutions to the linear system are

$$\ker(T) = \begin{bmatrix} t \\ -2t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

The kernel of T is the line spanned by the vector $\vec{v} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.