

## Chapter 2 Section 3

Andrew Taylor

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**Problem 1.** *Calculate the matrix product*

$$\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

**Solution.**

$$\begin{aligned} \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} &= \begin{bmatrix} 6*1 + 7*3 & 6*2 + 7*5 \\ 8*1 + 9*3 & 8*2 + 9*5 \end{bmatrix} \\ &= \begin{bmatrix} 27 & 47 \\ 35 & 61 \end{bmatrix} \end{aligned}$$

**Problem 2.** *Compute the products  $BA$  and  $AB$  for*

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

*Interpret your answers geometrically, as composites of linear transformation.*

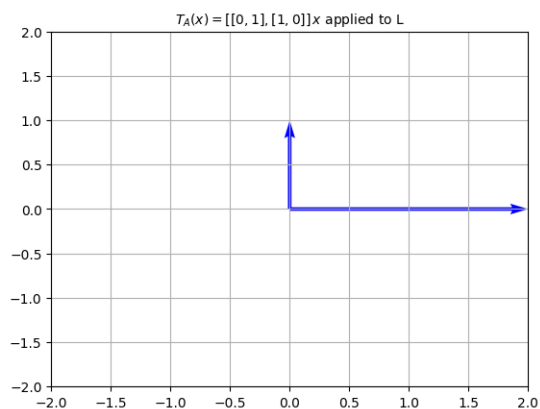
**Solution.** Let  $T_A(\vec{x}) = A\vec{x}$  and  $T_B(\vec{y}) = B\vec{y}$ . We can write

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = [T_A(e_1) \quad T_A(e_2)] \\ B &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = [T_B(e_1) \quad T_B(e_2)] \end{aligned}$$

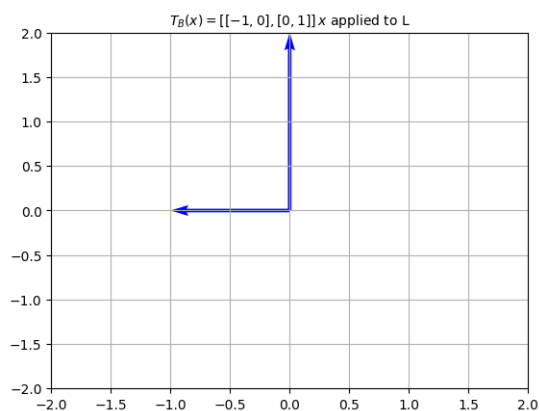
*Thus*

$$\begin{aligned} T_A(\vec{e}_1) &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, T_A(\vec{e}_2) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ T_B(\vec{e}_1) &= \begin{bmatrix} -1 \\ 0 \end{bmatrix}, T_B(\vec{e}_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned}$$

We see that  $T_A$  is a reflection about the line  $y = x$ . In other words,  $T_A$  is a reflection about the line spanned by the vector  $\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ .



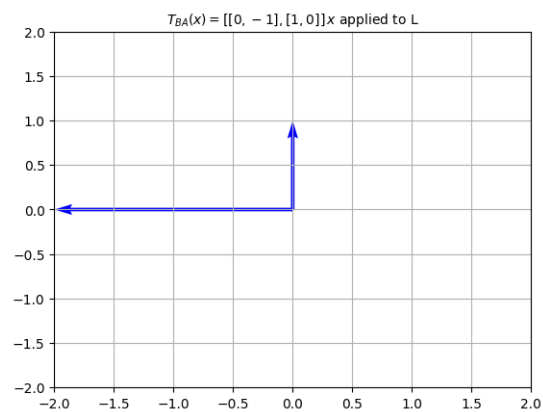
We see that  $T_B$  is a reflection about the line  $x = 0$ . In other words,  $T_B$  is a reflection about the line spanned by the vector  $\vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ .



Now let's compute the products  $BA$  and  $AB$ .

$$\begin{aligned} BA &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

The product  $BA$  is a rotation matrix that rotates a vector ninety degrees counterclockwise.



$$\begin{aligned}
 AB &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}
 \end{aligned}$$

*The product  $AB$  is a rotation matrix that rotates a vector ninety degrees clockwise.*

