Chapter 2 Section 3

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Problem 1. *Calculate the matrix product*

$$\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

Solution.

$$\begin{aligned} \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} &= \begin{bmatrix} 6*1+7*3 & 6*2+7*5 \\ 8*1+9*3 & 8*2+9*5 \end{bmatrix} \\ &= \begin{bmatrix} 27 & 47 \\ 35 & 61 \end{bmatrix} \end{aligned}$$

Problem 2. *Compute the products BA and AB for*

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Interpret your answers geometrically, as composites of linear transformation.

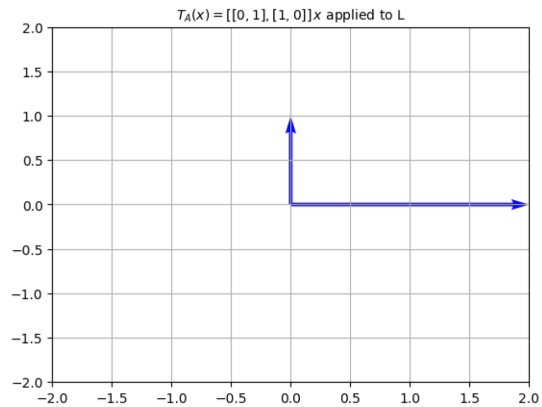
Solution. *Let $T_A(\vec{x}) = A\vec{x}$ and $T_B(\vec{y}) = B\vec{y}$. We can write*

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} T_A(e_1) & T_A(e_2) \end{bmatrix} \\ B &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} T_B(e_1) & T_B(e_2) \end{bmatrix} \end{aligned}$$

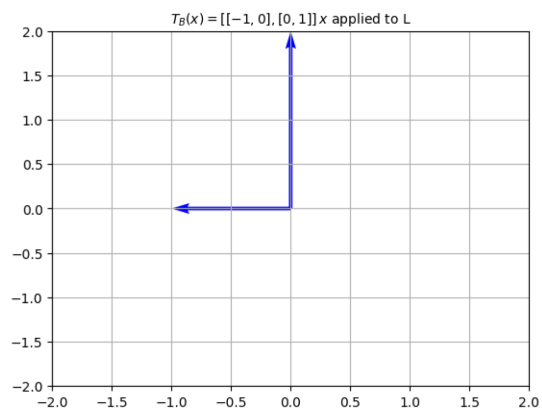
Thus

$$\begin{aligned} T_A(\vec{e_1}) &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, T_A(\vec{e_2}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ T_B(\vec{e_1}) &= \begin{bmatrix} -1 \\ 0 \end{bmatrix}, T_B(\vec{e_2}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned}$$

We see that T_A is a reflection about the line $y = x$. In other words, T_A is a reflection about the line spanned by the vector $\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.



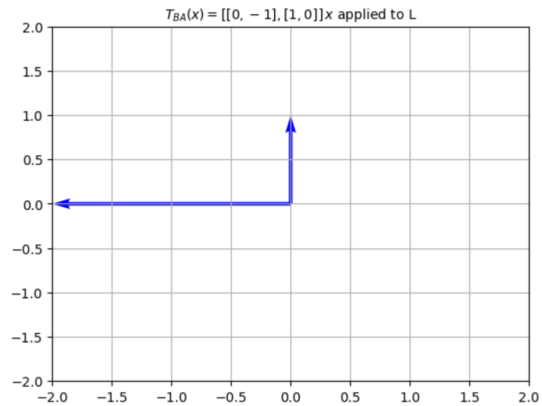
We see that T_B is a reflection about the line $x = 0$. In other words, T_B is a reflection about the line spanned by the vector $\vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.



Now let's compute the products BA and AB.

$$\begin{aligned} BA &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

The product BA is a rotation matrix that rotates a vector ninety degrees counterclockwise.



$$AB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The product AB is a rotation matrix that rotates a vector ninety degrees clockwise.