Chapter 1 Section 3

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Problem 1. *The reduced row echelon forms of augmented matrices A, B, and C are given below. The augmented matrices A, B, and C each represent a linear system. How many solutions does each system have?*

$$rref(A) = \begin{vmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$
$$rref(B) = \begin{vmatrix} 1 & 0 & 5 \\ 0 & 1 & 6 \end{vmatrix}$$
$$rref(C) = \begin{vmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{vmatrix}$$

Solution. *The first linear system is inconsistent, as the third row of $rref(A)$ gives the equation 0 = 1. Thus the first linear system has no solutions. The second linear system is consistent, has two unknowns, and rank two, which means there is exactly one solution. The solution to the second linear system is*

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

The third linear system has infinitely many solutions. We know this because $rref(C)$ proves that C is consistent, yet $rank(C) = 2$, which is less than the number of unknowns. The solutions for the third system are

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r \\ 2 \\ 3 \end{pmatrix}$$

where r is any arbitrary real number.

Find the rank of the matrices in problems 2, 3, and 4.

Problem 2. $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix}$

Solution.

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Thus the matrix has rank 3.