Problem: Find a formula for the sum of the first n squares.

My family taught me that, if we know the key to a problem, we can unlock the problem. There are so many problems that we can unlock if we just know the key. Law is a problem. We need to write correct law and prove that it is correct. What is the key to law? The key to law is mathematics. Mathematics is the key that unlocks law.

The problem on hand is more specific than law. It's actually an arithmetic problem. We are using two operations, addition and exponentiation. How do we unlock this problem? Well, the key to solving this problem is starting with the right equation. It is so important to give this emphasis that I will say it again. Maybe I will say it many times.

The key to solving this problem is starting with the right equation.

What is the right equation? The right equation is this:

$(n+1)^3 - 1^3 = \sum_1^n ((k+1)^3 - k^3)$

We call this equation a telescope, because it can expand and contract. The left hand side is the contracted form, and the right hand side is the expanded form.

If we start with the above equation, we can work our way down and figure out a formula.

$$(n+1)^3 - 1^3 = \sum_1^n ((k+1)^3 - k^3)$$

$$n^3 + 3n^2 + 3n + 1 - 1^3 = \sum_1^n ((k+1)^3 - k^3)$$

$$n^3 + 3n^2 + 3n = \sum_1^n ((k+1)^3 - k^3)$$

$$n^3 + 3n^2 + 3n = \sum_1^n (k^3 + 3k^2 + 3k + 1 - k^3)$$

$$n^3 + 3n^2 + 3n = \sum_1^n (3k^2 + 3k + 1)$$

$$n^3 + 3n^2 + 3n = \sum_1^n 3k^2 + \sum_1^n 3k + \sum_1^n 1$$

$$n^3 + 3n^2 + 3n = 3\sum_1^n k^2 + 3\sum_1^n k + \sum_1^n 1$$

$$n^3 + 3n^2 + 3n = 3\sum_1^n k^2 + 3\sum_1^n k + n$$

$$n^3 + 3n^2 + 3n = 3\sum_1^n k^2 + 3n(n+1)/2 + n$$

$$n^3 + 3n^2 + 2n = 3\sum_1^n k^2 + 3n(n+1)/2$$

$$2n^3 + 6n^2 + 4n = 6\sum_1^n k^2 + 3n(n+1)$$

$$2n^3 + 6n^2 + 4n = 6\sum_1^n k^2 + 3n^2 + 3n$$

$$2n^3 + 3n^2 + n = 6\sum_1^n k^2$$

$$n(2n^2 + 3n + 1) = 6\sum_1^n k^2$$

$$n(n+1)(2n+1) = 6\sum_1^n k^2$$

$$\sum_1^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

Finally, we arrive at a formula for the sum of the first n squares. The trick to solving this problem, the key, is starting with the right equation. If we know the key to a problem, we can unlock the problem.

Before solving this problem, I got a little philosophical, and I said, "Mathematics is the key to law. Mathematics is the key that unlocks law."

It really is. Mathematics is one of the keys to law. There is more than one key, but mathematics is one of them.

In my family we say, "Mathematics is the key that unlocks everything."