Problem 17: Find all complex numbers z such that $|z-1| = |z+3| = |z-i|$. (Source: ARML)

Let $z = a + bi$.

$$\begin{aligned}
(a-1)^2 + b^2 &= (a+3)^2 + b^2 \\
(a-1)^2 &= (a+3)^2 \\
a - 1 &= \pm(a+3) \\
a - 1 &= -a - 3 \\
2a &= -2 \\
a &= -1
\end{aligned}$$

$$\begin{aligned}
(a-1)^2 + b^2 &= a^2 + (b-1)^2 \\
a^2 - 2a + 1 + b^2 &= a^2 + b^2 - 2b + 1 \\
-2a + 1 &= -2b + 1 \\
-2a &= -2b \\
a &= b
\end{aligned}$$

Thus $\boxed{z = -1 - i}$.

Now let's think about this geometrically. The points $1, -3,$ and $-i$ form a triangle in the complex plane. There is only one complex number that is equidistant from the vertices of this triangle. z is the point inside the triangle that is equidistant from the vertices.