

Problem 4: Find all x with $0 \leq x < 2\pi$ such that $\tan x + \sec x = \sqrt{3}$. (Source: AoPS Precalculus Ex. 3.1.3)

Solution:

$$\begin{aligned}\tan x + \sec x &= \sqrt{3} \\ \sec x &= \sqrt{3} - \tan x \\ \tan^2 x + 1 &= \sec^2 x \\ \tan^2 x + 1 &= (\sqrt{3} - \tan x)^2 \\ \tan^2 x + 1 &= 3 - 2\sqrt{3}\tan x + \tan^2 x \\ 1 &= 3 - 2\sqrt{3}\tan x \\ 2\sqrt{3}\tan x &= 2 \\ \sqrt{3}\tan x &= 1 \\ \tan x &= \frac{1}{\sqrt{3}} \\ x &= \left\{ \frac{\pi}{6}, \frac{7\pi}{6} \right\}\end{aligned}$$

Checking these values, only $x = \frac{\pi}{6}$ satisfies the original equation.

So there is only one solution to the original equation, $\boxed{x = \frac{\pi}{6}}$.