Problem 30: Given that z is a complex number such that $z + \frac{1}{z} = 2\cos 3^\circ$, find the least integer that is greater than $z^{2000} + \frac{1}{z^{2000}}$. (Source: AIME)

Solving for z, we get

$$\begin{aligned}
z + \frac{1}{z} &= 2\cos 3^\circ \\
z^2 + 1 &= 2\cos 3^\circ z \\
z^2 - 2\cos 3^\circ z + 1 &= 0 \\
z &= \frac{2\cos 3^\circ \pm \sqrt{4\cos^2 3^\circ - 4}}{2} \\
&= \frac{2\cos 3^\circ \pm 2\sqrt{\cos^2 3^\circ - 1}}{2} \\
&= \cos 3^\circ \pm \sqrt{\cos^2 3^\circ - 1} \\
&= \cos 3^\circ \pm \sqrt{-\sin^2 3^\circ} \\
&= \cos 3^\circ \pm i\sin 3^\circ \\
&= \{e^{\pi i/60}, e^{-\pi i/60}\}
\end{aligned}$$

Now we can plug these values into the expression $z^{2000} + \frac{1}{z^{2000}}$.

$$\begin{aligned}
z^{2000} + \frac{1}{z^{2000}} &= (e^{\pi i/60})^{2000} + \frac{1}{(e^{\pi i/60})^{2000}} \\
&= e^{2000\pi i/60} + \frac{1}{e^{2000\pi i/60}} \\
&= e^{200\pi i/6} + \frac{1}{e^{200\pi i/6}} \\
&= e^{100\pi i/3} + \frac{1}{e^{100\pi i/3}} \\
&= e^{4\pi i/3} + \frac{1}{e^{4\pi i/3}} \\
&= \frac{e^{8\pi i/3}}{e^{4\pi i/3}} + \frac{1}{e^{4\pi i/3}} \\
&= \frac{e^{8\pi i/3} + 1}{e^{4\pi i/3}} \\
&= \frac{e^{2\pi i/3} + 1}{e^{4\pi i/3}} \\
&= \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i + 1}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i} \\
&= \frac{\frac{1}{2} + \frac{\sqrt{3}}{2}i}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i} \\
&= -1
\end{aligned}$$

Plugging in $z = e^{-\pi i/60}$, we get the same value.

The least integer greater than -1 is 0.

Thus the answer is $\boxed{0}$.