Problem 21: Describe the graph of $r = 1 - \sin 2\theta$, and find the slope of the tangent line at the origin. (Source: AoPS Calculus)

The polar equation $r = 1 - \sin 2\theta$ is the graph of a figure eight rotated forty five degrees counter-clockwise. We can find the polar coordinates of the origin by setting $r = 0$ and solving for θ.

$$\begin{aligned}
r &= 1 - \sin 2\theta \\
0 &= 1 - \sin 2\theta \\
1 &= \sin 2\theta \\
\arcsin 1 &= 2\theta \\
\frac{\pi}{2} &= 2\theta \\
\theta &= \frac{\pi}{4}
\end{aligned}$$

Thus the polar coordinates of the origin are $\left(0, \frac{\pi}{4}\right)$.

Now let's get an equation for the slope of the tangent line in terms of θ.

$$\begin{aligned}
y &= r\sin\theta \\
&= (1 - \sin 2\theta)\sin\theta \\
&= \sin\theta - \sin\theta\sin 2\theta \\
\frac{dy}{d\theta} &= \cos\theta - \cos\theta\sin 2\theta - 2\sin\theta\cos 2\theta \\
x &= r\cos\theta \\
&= (1 - \sin 2\theta)\cos\theta \\
&= \cos\theta - \cos\theta\sin 2\theta \\
\frac{dx}{d\theta} &= -\sin\theta + \sin\theta\sin 2\theta - 2\cos\theta\cos 2\theta
\end{aligned}$$

Since $\frac{dy}{d\theta} = \frac{dx}{d\theta} = 0$ at $\theta = \frac{\pi}{4}$, we have to take the limit of $\frac{dy/d\theta}{dx/d\theta}$ as θ approaches $\frac{\pi}{4}$.

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{\theta\to\frac{\pi}{4}} \frac{dy/d\theta}{dx/d\theta} \\
&= \lim_{\theta\to\frac{\pi}{4}} \frac{-\sin\theta - (-\sin\theta\sin 2\theta + 2\cos\theta\cos 2\theta) - (2\cos\theta\cos 2\theta - 4\sin\theta\sin 2\theta)}{-\cos\theta + (\cos\theta\sin 2\theta + 2\sin\theta\cos 2\theta) - (-2\sin\theta\cos 2\theta - 4\cos\theta\sin 2\theta)} \\
&= \lim_{\theta\to\frac{\pi}{4}} \frac{-\sin\theta - (-\sin\theta\sin 2\theta) - (-4\sin\theta\sin 2\theta)}{-\cos\theta + (\cos\theta\sin 2\theta) - (-4\cos\theta\sin 2\theta)} \\
&= \lim_{\theta\to\frac{\pi}{4}} \frac{-\sin\theta + \sin\theta\sin 2\theta + 4\sin\theta\sin 2\theta}{-\cos\theta + \cos\theta\sin 2\theta + 4\cos\theta\sin 2\theta} \\
&= \lim_{\theta\to\frac{\pi}{4}} \frac{-\sin\theta + \sin\theta + 4\sin\theta}{-\cos\theta + \cos\theta + 4\cos\theta} \\
&= \lim_{\theta\to\frac{\pi}{4}} \frac{\sin\theta}{\cos\theta} \\
&= \boxed{1}
\end{aligned}$$

Thus the slope of the tangent line at the origin is $\boxed{1}$.