Problem 5: The yield of one bundle of inferior rice, two bundles of medium-grade rice, and three bundles of superior rice is 39 dou of grain (1 dou 2 liters). The yield of one bundle of inferior rice, three bundles of medium-grade rice, and two bundles of superior rice, is 34 dou. The yield of three bundles of inferior rice, two bundles of medium-grade rice, and one bundle of superior rice, is 26 dou. What is the yield of one bundle of each grade of rice?

(Source: Linear Algebra with Applications, chapter 1 section 1)

We can turn this word problem into a system of equations.

$$\begin{aligned} x + 2y + 3z &= 39 \\ x + 3y + 2z &= 34 \\ 3x + 2y + z &= 26 \end{aligned}$$

We have three equations and three variables. We can use Gaussian elimination to solve for x, y, and z.

$$\text{rref}\begin{pmatrix}1 & 2 & 3 & 39\\ 1 & 3 & 2 & 34\\ 3 & 2 & 1 & 26\end{pmatrix}$$

$$= \text{rref}\begin{pmatrix}1 & 2 & 3 & 39\\ 0 & 1 & -1 & -5\\ 3 & 2 & 1 & 26\end{pmatrix} \qquad \text{Row2 = Row2 - Row1}$$

$$= \text{rref}\begin{pmatrix}1 & 2 & 3 & 39\\ 0 & 1 & -1 & -5\\ 0 & -4 & -8 & -91\end{pmatrix} \qquad \text{Row3 = Row3 - 3 * Row1}$$

$$= \text{rref}\begin{pmatrix}1 & 2 & 3 & 39\\ 0 & 1 & -1 & -5\\ 0 & 0 & -12 & -111\end{pmatrix} \qquad \text{Row3 = Row3 + 4 * Row2}$$

$$= \text{rref}\begin{pmatrix}1 & 0 & 5 & 49\\ 0 & 1 & -1 & -5\\ 0 & 0 & -12 & -111\end{pmatrix} \qquad \text{Row1 = Row1 - 2 * Row2}$$

$$= \text{rref}\begin{pmatrix}1 & 0 & 5 & 49\\ 0 & 1 & -1 & -5\\ 0 & 0 & 1 & \frac{111}{12}\end{pmatrix} \qquad \text{Row3 = Row3 / -12}$$

$$= \text{rref}\begin{pmatrix}1 & 0 & 5 & 49\\ 0 & 1 & 0 & -5+\frac{111}{12}\\ 0 & 0 & 1 & \frac{111}{12}\end{pmatrix} \qquad \text{Row2 = Row2 + Row3}$$

$$= \begin{pmatrix}1 & 0 & 0 & 49-\frac{555}{12}\\ 0 & 1 & 0 & -5+\frac{111}{12}\\ 0 & 0 & 1 & \frac{111}{12}\end{pmatrix} \qquad \text{Row1 = Row1 - 5 * Row3}$$

We get $x = 49 - \frac{555}{12}$, $y = -5 + \frac{111}{12}$, and $z = \frac{111}{12}$.

Let's check our solution by plugging these values into the first equation.

$$\begin{aligned} x + 2y + 3z &= 39 \\ 49 - \frac{555}{12} + 2(-5 + \frac{111}{12}) + 3\frac{111}{12} &= 39 \\ 49 - \frac{555}{12} + -10 + \frac{222}{12} + \frac{333}{12} &= 39 \\ 49 + -10 + -\frac{555}{12} + \frac{555}{12} &= 39 \\ 39 &= 39 \end{aligned}$$

It works for the first equation, which gives me enough confidence to proceed.

The yield of one bundle of inferior rice is $\boxed{49 - \frac{555}{12} = 2.75 \text{ dou.}}$ The yield of one bundle of medium-grade rice is $\boxed{-5 + \frac{111}{12} = 4.25 \text{ dou.}}$ The yield of one bundle of superior rice is $\boxed{\frac{111}{12} = 9.25 \text{ dou.}}$