Problem 2: Compute $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n-1}}{n!}$

The Taylor series for e^{2x} at $x = 0$ is

$$e^{2x} = 1 + 2x + \frac{4}{2!}x^2 + \frac{8}{3!}x^3 + \frac{16}{4!}x^4 + \cdots$$

Plugging in $x = 1$ we get

$$e^2 = 1 + 2 + \frac{4}{2!} + \frac{8}{3!} + \frac{16}{4!} + \cdots = 2\sum_{n=0}^{\infty} \frac{2^{n-1}}{n!}$$

Thus

$$\boxed{\sum_{n=0}^{\infty} \frac{2^{n-1}}{n!} = \frac{e^2}{2}}$$