

Problem 11: Let  $\omega$  be a primitive 13<sup>th</sup> root of unity. Evaluate  $(1 - \omega)(1 - \omega^2) \cdots (1 - \omega^{12})$ .

(Source: AoPS Precalculus)

We have  $x^{13} - 1 = (x - 1)(x^{12} + x^{11} + \cdots + x + 1)$ .

The roots of  $x^{13} - 1$  are the 13th roots of unity,  $1, \omega, \omega^2, \dots, \omega^{12}$ .

Thus the roots of  $x^{12} + x^{11} + \cdots + x + 1$  are  $\omega, \omega^2, \dots, \omega^{12}$ .

This means that  $(x - \omega)(x - \omega^2) \cdots (x - \omega^{12}) = x^{12} + x^{11} + \cdots + x + 1$ .

Plugging in 1 for  $x$ , we get

$$(1 - \omega)(1 - \omega^2) \cdots (1 - \omega^{12}) = \boxed{13}$$