Problem 13: Find $\lim_{x\to 0} \frac{x}{x}$

Proof. Let $\epsilon > 0$. We want to find a $\delta > 0$ such that

$$0 < |x| < \delta \quad \Longrightarrow \quad \left| \frac{x}{x} - 1 \right| < \epsilon$$

Any $\delta > 0$ suffices. Let $\delta = \epsilon$. Then

$$0 < |x| < \delta \quad \Longrightarrow \quad \left| \frac{x}{x} - 1 \right| = \left| 1 - 1 \right| = 0 < \epsilon$$

Thus

$$\lim_{x\to 0} \frac{x}{x} = 1$$

□