

Problem 9: Write parametric equations to describe the curve traced by the following motion: A particle tracing a circle with center  $(0,0)$  and radius 2, starting at  $(2,0)$  at time  $t = 0$ , moving clockwise with speed  $\sqrt{t}$ . (Source: AoPS Calculus)

We can figure out how much distance the particle travels in  $t$  seconds.

$$\begin{aligned} d &= \int_0^t s(t) dt \\ &= \int_0^t \sqrt{t} dt \\ &= \frac{2}{3} t^{3/2} \Big|_0^t \\ &= \frac{2}{3} t^{3/2} \end{aligned}$$

The particle travels  $\frac{2}{3}t^{3/2}$  distance in  $t$  time.

Thus it makes  $\frac{\frac{2}{3}t^{3/2}}{4\pi}$  revolutions in  $t$  time, and travels  $\frac{\frac{2}{3}t^{3/2}}{4\pi} \cdot 2\pi = \frac{\frac{2}{3}t^{3/2}}{2} = \frac{1}{3}t^{3/2}$  radians in  $t$  time.

Using this, we can write down the parametric equations.

$$\begin{aligned} x(t) &= 2 \cos \left( \frac{1}{3} t^{3/2} \right) \\ y(t) &= -2 \sin \left( \frac{1}{3} t^{3/2} \right) \end{aligned}$$

This gives a parameterization of

$$(x, y) = \left( 2 \cos \left( \frac{1}{3} t^{3/2} \right), -2 \sin \left( \frac{1}{3} t^{3/2} \right) \right)$$