

Problem 15: Let  $f$  be a real-valued function with  $\lim_{x \rightarrow a} f(x) = L$  and let  $c \in \mathbb{R}$ . Prove that

$$\lim_{x \rightarrow a} (cf)(x) = cL$$

*Proof.* Let  $\epsilon > 0$ . Since  $\lim_{x \rightarrow a} f(x) = L$ , there exists a  $\delta > 0$  such that

$$0 < |x - a| < \delta \implies |f(x) - L| < \frac{\epsilon}{|c|}$$

We can multiply both sides of the inequality on the right by  $|c|$ .

$$0 < |x - a| < \delta \implies |c| \cdot |f(x) - L| < |c| \cdot \frac{\epsilon}{|c|} \implies |c| \cdot |f(x) - L| < \epsilon$$

Now we can merge the two absolute value operators.

$$0 < |x - a| < \delta \implies |c(f(x) - L)| < \epsilon \implies |cf(x) - cL| < \epsilon$$

Thus

$$\lim_{x \rightarrow a} (cf)(x) = cL$$

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