Problem 10: Find all roots of $g(x) = 2x^3 + 5x^2 + 15x + 18$. (Source: AoPS Precalculus)

Let $g(x) = 2x^3 + 5x^2 + 15x + 18$.

We have

$$\begin{aligned} g(-1) &= 2(-1)^3 + 5(-1)^2 + 15(-1) + 18 \\ &= -2 + 5 + -15 + 18 \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

and

$$\begin{aligned} g(-2) &= 2(-2)^3 + 5(-2)^2 + 15(-2) + 18 \\ &= -16 + 20 + -30 + 18 \\ &= 4 - 12 \\ &= -8 \end{aligned}$$

So there exists a root of the polynomial $g(x)$ between $x = -2$ and $x = -1$.

Suppose this root is rational. Let $-2 < \frac{p}{q} < -1$ be the rational root. By the Rational Root Theorem, $p$ is a divisor of 18 and $q$ is a divisor of 2. The divisors of 18 are $\pm 1, 2, 3, 6, 9, 18$. The divisors of 2 are $\pm 1, 2$. Now the only possible $(p, q)$ pairs are $(-3, 2)$ and $(3, -2)$. So we'll try $g(-3/2)$.

$$\begin{aligned} g\left(-\frac{3}{2}\right) &= 2\left(-\frac{3}{2}\right)^3 + 5\left(-\frac{3}{2}\right)^2 + 15\left(-\frac{3}{2}\right) + 18 \\ &= 2\left(-\frac{27}{8}\right) + 5\left(\frac{9}{4}\right) + 15\left(-\frac{3}{2}\right) + 18 \\ &= -\frac{27}{4} + \frac{45}{4} + -\frac{90}{4} + \frac{72}{4} \\ &= 0 \end{aligned}$$

Thus $2x + 3$ is a divisor of $g(x)$.

Dividing $g(x)$ by $2x + 3$ gives us

$$g(x) = (2x + 3)(x^2 + x + 6)$$

Applying the quadratic formula to $x^2 + x + 6 = 0$ gives us the other roots.

The three roots of $g(x)$ are

$$\boxed{x = -\frac{3}{2}, \qquad -\frac{1}{2} + \frac{\sqrt{23}}{2}i, \qquad \text{and} \qquad -\frac{1}{2} - \frac{\sqrt{23}}{2}i}$$