

Problem 16: Jerry writes down all binary strings of length 10 without any 2 consecutive 1s. How many 1s does Jerry write? (Source: AoPS Community Forums)

Case 1: The string has five 1s.

_ 1 0 _ 1 0 _ 1 0 _ 1 0 _ 1 _

We can add one 0 to six bins. There are $\boxed{6}$ ways to do this.

Case 2: The string has four 1s.

_ 1 0 _ 1 0 _ 1 0 _ 1 _

We can add three 0s to five bins. There are $\binom{5+3-1}{5-1} = \binom{7}{4} = \binom{7}{3} = \boxed{35}$ ways to do this.

Case 3: The string has three 1s.

_ 1 0 _ 1 0 _ 1 _

We can add five 0s to four bins. There are $\binom{4+5-1}{4-1} = \binom{8}{3} = \boxed{56}$ ways to do this.

Case 4: The string has two 1s.

_ 1 0 _ 1 _

We can add seven 0s to three bins. There are $\binom{3+7-1}{3-1} = \binom{9}{2} = \boxed{36}$ ways to do this.

Case 5: The string has one 1.

There are $\boxed{10}$ possible strings that have one 1 (since there are ten digits, and any of them can be a 1).

Wrapping up

Now we can use these numbers to calculate the total number of ones Jerry writes.

$$6(5) + 35(4) + 56(3) + 36(2) + 10(1) = 30 + 140 + 168 + 72 + 10 = 420$$

Jerry writes $\boxed{420}$ ones.

Note:

I used the ball and bin formula (see AoPS wiki) to count the number of possible strings in each case.