

Problem 9: Write parametric equations to describe the curve traced by the following motion: A particle tracing a circle with center $(0,0)$ and radius 2, starting at $(2,0)$ at time $t = 0$, moving clockwise with speed \sqrt{t} . (Source: AoPS Calculus)

We can figure out how much distance the particle travels in t seconds.

$$\begin{aligned} d &= \int_0^t s(t) dt \\ &= \int_0^t \sqrt{t} dt \\ &= \frac{2}{3} t^{3/2} \Big|_0^t \\ &= \frac{2}{3} t^{3/2} \end{aligned}$$

The particle travels $\frac{2}{3}t^{3/2}$ distance in t time.

Thus it makes $\frac{\frac{2}{3}t^{3/2}}{4\pi}$ revolutions in t time, and travels $\frac{\frac{2}{3}t^{3/2}}{4\pi} \cdot 2\pi = \frac{\frac{2}{3}t^{3/2}}{2} = \frac{1}{3}t^{3/2}$ radians in t time.

Using this, we can write down the parametric equations.

$$\begin{aligned} x(t) &= 2 \cos \left(\frac{1}{3} t^{3/2} \right) \\ y(t) &= -2 \sin \left(\frac{1}{3} t^{3/2} \right) \end{aligned}$$

This gives a parameterization of

$$\boxed{\left(2 \cos \left(\frac{1}{3} t^{3/2} \right), -2 \sin \left(\frac{1}{3} t^{3/2} \right) \right)}$$