Problem 3: Find the roots of the polynomial $p(x) = x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$. (Source: AoPS Precalculus)

Multiplying $p(x)$ by x we get

$$xp(x) = x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x$$

Subtracting $p(x)$ from $xp(x)$, we get

$$\begin{aligned} xp(x) - p(x) &= x^9 - 1 \\ p(x)(x-1) &= x^9 - 1 \\ p(x) &= \frac{x^9 - 1}{x - 1} && \text{provided } x \neq 1 \end{aligned}$$

Now we know that $x = 1$ is not a root of $p(x)$ since $p(1) = 9$. But from our equations above, we see that all of the ninth roots of unity except for $x = 1$ are roots of $p(x)$. Thus the roots of $p(x)$ are

$$\boxed{e^{2\pi i/9}, e^{4\pi i/9}, e^{6\pi i/9}, e^{8\pi i/9}, e^{10\pi i/9}, e^{12\pi i/9}, e^{14\pi i/9}, e^{16\pi i/9}}$$