

Problem 7: Write parametric equations to describe the curve traced by the following motion: A point starting at the bottom edge of a bicycle wheel (with radius 30cm) that is rotating at 1 revolution per second, where the bicycle is moving forward at the rate implied by the rotation of the wheel. (Source: AoPS Calculus)

The position of the center of the wheel is given by

$$\begin{aligned}x_c(t) &= 0.6\pi t \\ y_c(t) &= 0.3\end{aligned}$$

The position of the point at the bottom edge is given by

$$\begin{aligned}x_p(t) &= .6\pi t + 0.3 \cos\left(2\pi t + \frac{\pi}{2}\right) \\ &= .6\pi t - 0.3 \sin(2\pi t) \\ y_p(t) &= 0.3 - 0.3 \sin\left(2\pi t + \frac{\pi}{2}\right) \\ &= 0.3 - 0.3 \cos(2\pi t)\end{aligned}$$

The position of the point at the bottom edge is given by the parameterization

$$\boxed{(.6\pi t - 0.3 \sin(2\pi t), 0.3 - 0.3 \cos(2\pi t))}$$