Problem 11: Find all roots of $f(t) = 2t^4 - 23t^2 + 27t - 36$. (Source: AoPS Precalculus)

Let $f(t) = 2t^4 - 23t^2 + 27t - 36$.

We have

$$\begin{aligned}
f(1) &= 2 - 23 + 27 - 36 \\
f(1) &= 2 + 4 - 36 \\
f(1) &= -30 \\
f(2) &= 2(16) - 23(4) + 27(2) - 36 \\
f(2) &= 32 - 92 + 54 - 36 \\
f(2) &= -60 + 54 - 36 \\
f(2) &= -42 \\
f(3) &= 2(81) - 23(9) + 27(3) - 36 \\
f(3) &= 162 - 207 + 81 - 36 \\
f(3) &= 162 - 126 - 36 \\
f(3) &= 0
\end{aligned}$$

Thus $t - 3$ is a factor of $f(t) = 2t^4 - 23t^2 + 27t - 36$.

Dividing $f(t)$ by $t - 3$, we get

$$f(t) = (t-3)(2t^3 + 6t^2 - 5t + 12)$$

Now we can experiment with some values of t and plug in $t = -1, -2, -3, -4$.

After experimenting, we find that $f(-4) = 0$. So $t + 4$ is a factor of $f(t)$.

Dividing $f(t)$ by $t + 4$, we get

$$f(t) = (t-3)(t+4)(2t^2 - 2t + 3)$$

Applying the quadratic formula, we get the other two roots.

$$\boxed{t = 3, \quad -4, \quad \frac{1 + i\sqrt{5}}{2}, \quad \text{and} \quad \frac{1 - i\sqrt{5}}{2}}$$