

Problem 25: Let  $z$  be a root of  $x^5 - 1 = 0$ , with  $z \neq 1$ . Compute the value of

$$z^{15} + z^{16} + z^{17} + \cdots + z^{50}.$$

(Source: ARML)

$z$  is a nonreal fifth root of unity, and since every nonreal fifth root of unity is primitive,  $z$  is a primitive fifth root of unity.

$$\begin{aligned} z^{15} + z^{16} + z^{17} + \cdots + z^{50} &= 7(1 + z + z^2 + z^3 + z^4) + 1 \\ &= \boxed{1} \end{aligned}$$