Problem 8: Evaluate the sum

$$\cos\frac{2\pi}{m}+\cos\frac{2\cdot 2\pi}{m}+\cos\frac{3\cdot 2\pi}{m}+\cdots+\cos\frac{m\cdot 2\pi}{m},$$

where m is a positive integer. (Source: AoPS Precalculus)

The terms in this series correspond to the real parts of the m^{th} roots of unity. Since the sum of the m^{th} roots of unity is zero, the sum of the real parts of the m^{th} roots of unity also has to be zero.

So

$$\boxed{\cos\frac{2\pi}{m}+\cos\frac{2\cdot 2\pi}{m}+\cos\frac{3\cdot 2\pi}{m}+\cdots+\cos\frac{m\cdot 2\pi}{m}=0}$$

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