Problem 10: The Archimedes spiral is the graph of $r = \theta$ for $\theta \geq 0$. Find the slope of the tangent line to the spiral at θ. (Source: AoPS Calculus)

We have

$$\begin{aligned} y &= r\sin\theta = \theta\sin\theta \\ x &= r\cos\theta = \theta\cos\theta \end{aligned}$$

Taking the derivative with respect to θ, we get

$$\begin{aligned} \frac{dy}{d\theta} &= \sin\theta + \theta\cos\theta \\ \frac{dx}{d\theta} &= \cos\theta - \theta\sin\theta \end{aligned}$$

Now we can write the slope of the tangent line as a function of θ.

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \\ &= \boxed{\frac{\sin\theta + \theta\cos\theta}{\cos\theta - \theta\sin\theta}} \end{aligned}$$