

Problem 5: The yield of one bundle of inferior rice, two bundles of medium-grade rice, and three bundles of superior rice is 39 dou of grain (1 dou = 2 liters). The yield of one bundle of inferior rice, three bundles of medium-grade rice, and two bundles of superior rice, is 34 dou. The yield of three bundles of inferior rice, two bundles of medium-grade rice, and one bundle of superior rice, is 26 dou. What is the yield of one bundle of each grade of rice?

(Source: Linear Algebra with Applications, chapter 1 section 1)

We can turn this word problem into a system of equations.

$$x + 2y + 3z = 39$$

$$x + 3y + 2z = 34$$

$$3x + 2y + z = 26$$

We have three equations and three variables. We can use Gaussian elimination to solve for x , y , and z .

$$\begin{aligned}
& \text{rref} \begin{pmatrix} 1 & 2 & 3 & 39 \\ 1 & 3 & 2 & 34 \\ 3 & 2 & 1 & 26 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 2 & 3 & 39 \\ 0 & 1 & -1 & -5 \\ 3 & 2 & 1 & 26 \end{pmatrix} & \text{Row2} = \text{Row2} - \text{Row1} \\
&= \begin{pmatrix} 1 & 2 & 3 & 39 \\ 0 & 1 & -1 & -5 \\ 0 & -4 & -8 & -91 \end{pmatrix} & \text{Row3} = \text{Row3} - 3 * \text{Row1} \\
&= \begin{pmatrix} 1 & 2 & 3 & 39 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & -12 & -111 \end{pmatrix} & \text{Row3} = \text{Row3} + 4 * \text{Row2} \\
&= \begin{pmatrix} 1 & 0 & 5 & 49 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & -12 & -111 \end{pmatrix} & \text{Row1} = \text{Row1} - 2 * \text{Row2} \\
&= \begin{pmatrix} 1 & 0 & 5 & 49 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & \frac{111}{12} \end{pmatrix} & \text{Row3} = \text{Row3} / -12 \\
&= \begin{pmatrix} 1 & 0 & 5 & 49 \\ 0 & 1 & 0 & -5 + \frac{111}{12} \\ 0 & 0 & 1 & \frac{111}{12} \end{pmatrix} & \text{Row2} = \text{Row2} + \text{Row3} \\
&= \begin{pmatrix} 1 & 0 & 0 & 49 - \frac{555}{12} \\ 0 & 1 & 0 & -5 + \frac{111}{12} \\ 0 & 0 & 1 & \frac{111}{12} \end{pmatrix} & \text{Row1} = \text{Row1} - 5 * \text{Row3}
\end{aligned}$$

We get $x = 49 - \frac{555}{12}$, $y = -5 + \frac{111}{12}$, and $z = \frac{111}{12}$.

Let's check our solution by plugging these values into the first equation.

$$\begin{aligned}
 x + 2y + 3z &= 39 \\
 49 - \frac{555}{12} + 2(-5 + \frac{111}{12}) + 3\frac{111}{12} &= 39 \\
 49 - \frac{555}{12} + -10 + \frac{222}{12} + \frac{333}{12} &= 39 \\
 49 + -10 + -\frac{555}{12} + \frac{555}{12} &= 39 \\
 39 &= 39
 \end{aligned}$$

It works for the first equation, which gives me enough confidence to proceed.

The yield of one bundle of inferior rice is $49 - \frac{555}{12} = 2.75$ dou. The yield
of one bundle of medium-grade rice is $-5 + \frac{111}{12} = 4.25$ dou. The yield of one
bundle of superior rice is $\frac{111}{12} = 9.25$ dou.