Problem 20: Describe the graph of $r = a + b\sin\theta$, where a and b are positive real numbers, and find the slope of the tangent line at the point where $\theta = \frac{\pi}{2}$. (Source: AoPS Calculus)

The graph of $r = a + b\sin\theta$ is a limacon, which resembles a circle but has a loop around the origin.

When $0 \le \theta \le \pi$, the magnitude $r(\theta)$ is always positive. But when $\pi < \theta < 2\pi$, the magnitude $r(\theta)$ can be positive or negative.. The loop around the origin is caused by a sub-interval of $(\pi, 2\pi)$ where $r(\theta) < 0$.

We would like to know the slope of the tangent line at the point where $\theta = \frac{\pi}{2}$. To find this slope we will differentiate the parametric equations $x = r\cos\theta = a\cos\theta + b\cos\theta\sin\theta$ and $y = r\sin\theta = a\sin\theta + b\sin^2\theta$.

$$\frac{dy}{d\theta} = a\cos\theta + 2b\sin\theta\cos\theta$$
$$\frac{dx}{d\theta} = -a\sin\theta + -b\sin^2\theta + b\cos^2\theta$$

Now we can get the slope of the tangent line, since $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy/d\theta}{dx/d\theta} \\ &= \frac{a\cos\theta + 2b\sin\theta\cos\theta}{-a\sin\theta + -b\sin^2\theta + b\cos^2\theta}\end{aligned}$$

Plugging in $\theta = \frac{\pi}{2}$, we get

$$\frac{dy}{dx}\left(\frac{\pi}{2}\right) = \frac{0}{-a + -b} = \boxed{0}$$

The slope of the tangent line at the point where $\theta = \frac{\pi}{2}$ is $\boxed{0}$.