Problem 3: Find the area of the region that is outside the curve $r = 1 - \cos\theta$ but inside the curve $r = 1$. (Source: AoPS Calculus)

The curve $r = 1 - \cos\theta$ is a cardioid, and the curve $r = 1$ is a circle. We can find the area of the region that is outside the cardioid but inside the circle by subtracting twice the area of a specific region from the area of the semicircle. The specific region we refer to is the region of the cardioid from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

$$\begin{aligned}
A &= \frac{1}{2}\pi - 2\cdot\frac{1}{2}\int_0^{\frac{\pi}{2}} (1-\cos\theta)^2\,d\theta \\
&= \frac{1}{2}\pi - \int_0^{\frac{\pi}{2}} (1-\cos\theta)^2\,d\theta \\
&= \frac{1}{2}\pi - \int_0^{\frac{\pi}{2}} (1 - 2\cos\theta + \cos^2\theta)\,d\theta \\
&= \frac{1}{2}\pi - \int_0^{\frac{\pi}{2}} (1 - 2\cos\theta + \frac{1}{2}(1-\cos 2\theta))\,d\theta \\
&= \frac{1}{2}\pi - \int_0^{\frac{\pi}{2}} (1 - 2\cos\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta)\,d\theta \\
&= \frac{1}{2}\pi - (\theta - 2\sin\theta + \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta)\Big|_0^{\frac{\pi}{2}} \\
&= \frac{1}{2}\pi - \left(\frac{\pi}{2} - 2 + \frac{1}{2}\cdot\frac{\pi}{2}\right) \\
&= \frac{1}{2}\pi - \left(\frac{3\pi}{4} - 2\right) \\
&= \boxed{2 - \frac{\pi}{4}}
\end{aligned}$$