Problem 1: Prove the identity $\cot^2\theta + 1 = \csc^2\theta$. (Source: AoPS Precalculus Ex. 3.1.1)

Proof:

$$\begin{aligned}
\cot^2\theta + 1 &= \frac{\cos^2\theta}{\sin^2\theta} + 1 \\
&= \frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} \\
&= \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta} \\
&= \frac{1}{\sin^2\theta} \\
&= \boxed{\csc^2\theta}
\end{aligned}$$