

Problem 7: The sums of any two of three real numbers are 24, 28, and 30. Find these three numbers.

(Source: Linear Algebra with Applications, Exercise 1.1.21)

Let  $x$ ,  $y$ , and  $z$  stand for the three real numbers. Using what we're given, we know that

$$x + y = 24$$

$$x + z = 28$$

$$y + z = 30$$

We can solve for  $x$ ,  $y$ , and  $z$  by Gaussian elimination.

$$\begin{aligned}
 & \text{rref} \begin{pmatrix} 1 & 1 & 0 & 24 \\ 1 & 0 & 1 & 28 \\ 0 & 1 & 1 & 30 \end{pmatrix} \\
 &= \text{rref} \begin{pmatrix} 2 & 1 & 1 & 52 \\ 1 & 0 & 1 & 28 \\ 0 & 1 & 1 & 30 \end{pmatrix} && \text{Row1} = \text{Row1} + \text{Row2} \\
 &= \text{rref} \begin{pmatrix} 2 & 0 & 0 & 22 \\ 1 & 0 & 1 & 28 \\ 0 & 1 & 1 & 30 \end{pmatrix} && \text{Row1} = \text{Row1} - \text{Row3} \\
 &= \text{rref} \begin{pmatrix} 1 & 0 & 0 & 11 \\ 1 & 0 & 1 & 28 \\ 0 & 1 & 1 & 30 \end{pmatrix} && \text{Row1} = \text{Row1} / 2 \\
 &= \text{rref} \begin{pmatrix} 1 & 0 & 0 & 11 \\ 1 & 0 & 1 & 28 \\ -1 & 1 & 0 & -2 \end{pmatrix} && \text{Row3} = \text{Row3} - \text{Row2} \\
 &= \text{rref} \begin{pmatrix} 1 & 0 & 0 & 11 \\ 1 & 0 & 1 & 28 \\ 0 & 1 & 0 & 9 \end{pmatrix} && \text{Row3} = \text{Row3} + \text{Row1} \\
 &= \text{rref} \begin{pmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 9 \\ 1 & 0 & 1 & 28 \end{pmatrix} && \text{Swap Row2 and Row3} \\
 &= \begin{pmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 17 \end{pmatrix} && \text{Row3} = \text{Row3} - \text{Row1}
 \end{aligned}$$

We arrive at the solutions  $x = 11$ ,  $y = 9$ , and  $z = 17$ .

The three numbers that we seek are 9, 11, and 17.