Problem 24: The equation $z^6 + z^3 + 1 = 0$ has one complex root with argument (angle) θ between $90°$ and $180°$ in the complex plane. Determine the degree measure of θ. (Source: AIME)

Let $u = z^3$. Then $u^2 + u + 1 = 0$.

$$\begin{aligned} u &= \frac{-1 \pm \sqrt{1-4}}{2} \\ &= \frac{-1 \pm \sqrt{-3}}{2} \\ &= -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i \end{aligned}$$

Putting u in exponential form, we get

$$u = \{e^{2\pi i/3}, e^{4\pi i/3}\}$$

Now we can solve for z.

$$\begin{aligned} z^3 = e^{2\pi i/3} &\implies z = \{e^{2\pi i/9}, e^{8\pi i/9}, e^{14\pi i/9}\} \\ z^3 = e^{4\pi i/3} &\implies z = \{e^{4\pi i/9}, e^{10\pi i/9}, e^{16\pi i/9}\} \end{aligned}$$

Combining all six solutions, we have

$$z = \{e^{2\pi i/9}, e^{4\pi i/9}, e^{8\pi i/9}, e^{10\pi i/9}, e^{14\pi i/9}, e^{16\pi i/9}\}$$

Only one of these roots is between $90°$ and $180°$, and that root is $z = e^{8\pi i/9}$.

The angle measure of $\dfrac{8\pi}{9}$ radians is equivalent to $160°$.

Thus $\boxed{\theta = 160°}$.