Problem 19: A function f is defined on the complex numbers by $f(z) = (a + bi)z$, where a and b are positive numbers. This function has the property that the image of each point in the complex plane is equidistant from that point and the origin. Given that $|a + bi| = 8$, find the value of b^2. (Source: AIME)

We are given that $|f(z) - z| = |f(z)|$ for all complex numbers z. Let's plug in 1 for z.

$$\begin{aligned}
|f(1) - 1| &= |f(1)| \\
|a + bi - 1| &= |a + bi| \\
(a - 1)^2 + b^2 &= a^2 + b^2 \\
a^2 - 2a + 1 + b^2 &= a^2 + b^2 \\
-2a + 1 &= 0 \\
-2a &= -1 \\
a &= \frac{1}{2}
\end{aligned}$$

We are also given that $|a + bi| = 8$. Knowing a, we can solve for b.

$$\begin{aligned}
|a + bi| &= 8 \\
a^2 + b^2 &= 64 \\
\left(\frac{1}{2}\right)^2 + b^2 &= 64 \\
\frac{1}{4} + b^2 &= \frac{256}{4} \\
b^2 &= \boxed{\frac{255}{4}}
\end{aligned}$$