Problem 2: Find the area between the curves $r = \theta$ and $r = 2\theta$ for $0 \leq \theta \leq \pi$. (Source: AoPS Calculus)

$$A = A_1 - A_2$$

$$= \frac{1}{2} \int_0^\pi (2\theta)^2 \, d\theta - \frac{1}{2} \int_0^\pi \theta^2 \, d\theta$$

$$= \frac{1}{2} \int_0^\pi 4\theta^2 \, d\theta - \frac{1}{2} \int_0^\pi \theta^2 \, d\theta$$

$$= 2 \int_0^\pi \theta^2 \, d\theta - \frac{1}{2} \int_0^\pi \theta^2 \, d\theta$$

$$= \frac{3}{2} \int_0^\pi \theta^2 \, d\theta$$

$$= \frac{3}{2} \left(\frac{1}{3} \theta^3 \right) \Big|_0^\pi$$

$$= \left(\frac{1}{2} \theta^3 \right) \Big|_0^\pi$$

$$= \boxed{\frac{1}{2} \pi^3}$$