Problem 7: The sums of any two of three real numbers are 24, 28, and 30. Find these three numbers.

(Source: Linear Algebra with Applications, Exercise 1.1.21)

Let x, y, and z stand for the three real numbers. Using what we're given, we know that

$$\begin{aligned} x + y &= 24 \\ x + z &= 28 \\ y + z &= 30 \end{aligned}$$

We can solve for x, y, and z by Gaussian elimination.

$$\begin{aligned}
&\operatorname{rref}\begin{pmatrix} 1 & 1 & 0 & 24 \\ 1 & 0 & 1 & 28 \\ 0 & 1 & 1 & 30 \end{pmatrix} & \\
=&\operatorname{rref}\begin{pmatrix} 2 & 1 & 1 & 52 \\ 1 & 0 & 1 & 28 \\ 0 & 1 & 1 & 30 \end{pmatrix} & \text{Row1 = Row1 + Row2} \\
=&\operatorname{rref}\begin{pmatrix} 2 & 0 & 0 & 22 \\ 1 & 0 & 1 & 28 \\ 0 & 1 & 1 & 30 \end{pmatrix} & \text{Row1 = Row1 - Row3} \\
=&\operatorname{rref}\begin{pmatrix} 1 & 0 & 0 & 11 \\ 1 & 0 & 1 & 28 \\ 0 & 1 & 1 & 30 \end{pmatrix} & \text{Row1 = Row1 / 2} \\
=&\operatorname{rref}\begin{pmatrix} 1 & 0 & 0 & 11 \\ 1 & 0 & 1 & 28 \\ -1 & 1 & 0 & -2 \end{pmatrix} & \text{Row3 = Row3 - Row2} \\
=&\operatorname{rref}\begin{pmatrix} 1 & 0 & 0 & 11 \\ 1 & 0 & 1 & 28 \\ 0 & 1 & 0 & 9 \end{pmatrix} & \text{Row3 = Row3 + Row1} \\
=&\operatorname{rref}\begin{pmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 9 \\ 1 & 0 & 1 & 28 \end{pmatrix} & \text{Swap Row2 and Row3} \\
=&\operatorname{rref}\begin{pmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & 17 \end{pmatrix} & \text{Row3 = Row3 - Row1}
\end{aligned}$$

We arrive at the solutions $x = 11$, $y = 9$, and $z = 17$.

The three numbers that we seek are 9, 11, and 17.