

Problem 6: Find the value of the product

$$\tan\left(\frac{\pi}{12}\right)\tan\left(\frac{2\pi}{12}\right)\tan\left(\frac{3\pi}{12}\right)\tan\left(\frac{4\pi}{12}\right)\tan\left(\frac{5\pi}{12}\right).$$

(Source: AoPS Precalculus Ex. 3.1.5)

Solution:

$$\begin{aligned} & \tan\left(\frac{\pi}{12}\right)\tan\left(\frac{2\pi}{12}\right)\tan\left(\frac{3\pi}{12}\right)\tan\left(\frac{4\pi}{12}\right)\tan\left(\frac{5\pi}{12}\right) \\ &= \tan\left(\frac{\pi}{12}\right)\tan\left(\frac{2\pi}{12}\right)\tan\left(\frac{3\pi}{12}\right)\cot\left(\frac{2\pi}{12}\right)\cot\left(\frac{\pi}{12}\right) \\ &= \tan\left(\frac{3\pi}{12}\right) \\ &= \tan\left(\frac{\pi}{4}\right) \\ &= \boxed{1} \end{aligned}$$