Problem 20: Express $\cos 5\theta$ in terms of $\cos\theta$. (Source: AoPS Precalculus)

We can use De Moivre's theorem and the Binomial Theorem. We can also use the identity $\sin^2\theta = 1 - \cos^2\theta$.

$$\begin{aligned}
\cos 5\theta + i\sin 5\theta &= (\cos\theta + i\sin\theta)^5 \\
&= (\cos\theta)^5 + 5(\cos\theta)^4(i\sin\theta) + 10(\cos\theta)^3(i\sin\theta)^2 + 10(\cos\theta)^2(i\sin\theta)^3 + 5(\cos\theta)(i\sin\theta)^4 + (i\sin\theta)^5 \\
&= \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta(1-\cos^2\theta) - 10i\cos^2\theta\sin^3\theta + 5\cos\theta(1-\cos^2\theta)^2 + i\sin^5\theta
\end{aligned}$$

Now we can equate the real parts.

$$\begin{aligned}
\cos 5\theta &= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2 \\
&= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta) \\
&= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta \\
&= \boxed{16\cos^5\theta - 20\cos^3\theta + 5\cos\theta}
\end{aligned}$$