Problem 15: Find the equation, in polar coordinates, of the line passing through (x_1, y_1) with slope m.

(Source: AoPS Calculus)

We can start with the equation of this line in point-slope form.

$$y - y_1 = m(x - x_1)$$

Now we can substitute $r\sin\theta$ for y and $r\cos\theta$ for x.

$$r\sin\theta - y_1 = m(r\cos\theta - x_1)$$

We can manipulate this equation to get r as a function of θ.

$$\begin{aligned} r\sin\theta - y_1 &= m(r\cos\theta - x_1) \\ r\sin\theta - y_1 &= mr\cos\theta - mx_1 \\ r\sin\theta - mr\cos\theta &= y_1 - mx_1 \\ r\left(\sin\theta - m\cos\theta\right) &= y_1 - mx_1 \\ r &= \frac{y_1 - mx_1}{\sin\theta - m\cos\theta} \end{aligned}$$

Thus the equation of the line, in polar coordinates, is

$$\boxed{r = \frac{y_1 - mx_1}{\sin\theta - m\cos\theta}}$$