

Problem 19: Use the fact that  $e^{-i\theta} = \frac{1}{e^{i\theta}}$  to reproduce the identities for  $\sin(-\theta)$  and  $\cos(-\theta)$ .

(Source: AoPS Precalculus)

We have

$$\cos(-\theta) + i \sin(-\theta) = \frac{1}{\cos \theta + i \sin \theta} = \cos \theta - i \sin \theta$$

Equating the real and imaginary parts, we get

$$\boxed{\cos(-\theta) = \cos \theta} \text{ and } \boxed{\sin(-\theta) = -\sin \theta}.$$