

Problem 1: If the six solutions of  $x^6 = -64$  are written in the form  $a + bi$ , where  $a$  and  $b$  are real, then find the product of the solutions with  $a > 0$ . (Source: AHSME)

The sixth roots of unity are

$$1, e^{2\pi i/6}, e^{4\pi i/6}, e^{6\pi i/6}, e^{8\pi i/6}, e^{10\pi i/6}$$

Now let  $x^6 = -64$ . We have  $x = \sqrt[6]{-64} = 2\sqrt[6]{-1} = 2i$ .

We can get all six solutions if we multiply  $2i$  by each of the sixth roots of unity.

$$x = 2i, 2ie^{2\pi i/6}, 2ie^{4\pi i/6}, 2ie^{6\pi i/6}, 2ie^{8\pi i/6}, 2ie^{10\pi i/6}$$

Converting these solutions to rectangular form, we get

$$x_1 = 2i$$

$$x_2 = 2i\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -\sqrt{3} + i$$

$$x_3 = 2i\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -\sqrt{3} - i$$

$$x_4 = 2i(-1 + 0) = -2i$$

$$x_5 = 2i\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \sqrt{3} - i$$

$$x_6 = 2i\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \sqrt{3} + i$$

There are only two solutions with  $a > 0$ , where  $a$  is the real part of the complex number. These solutions are  $x_5 = \sqrt{3} - i$  and  $x_6 = \sqrt{3} + i$ . Their product is  $\boxed{(\sqrt{3} - i)(\sqrt{3} + i) = 4}$ .