Problem 17: Determine all integer values of θ with $0° \leq \theta \leq 90°$ for which $(\cos\theta + i\sin\theta)^{75}$ is a real number. (Source: ARML)

Let $z = (\cos\theta + i\sin\theta)^{75}$. By de Moivre's theorem, we have

$$z = (\cos\theta + i\sin\theta)^{75} = \cos(75\theta) + i\sin(75\theta)$$

Now z is a real number when 75θ is an integer multiple of 180, that is, for all values

$$\theta = \frac{180n}{75} = \frac{12n}{5} \text{ where } n \text{ is an integer and } 0 \leq \theta \leq 90$$

For θ to be an integer, n has to be an integer multiple of 5. Thus $n = 0, 5, 10, ..., 35$. This gives us

$$\boxed{\theta = 0°, 12°, 24°, 36°, 48°, 60°, 72°, 84°}$$