Problem 19: Describe the graph of $r = a\cos\theta + b\sin\theta$, where a and b are positive real numbers, and find the slope of the tangent line at the point where $\theta = 0$. (Source: AoPS Calculus)

The graph of $r = a\cos\theta + b\sin\theta$ is the graph of a circle centered at $\left(\frac{a}{2}, \frac{b}{2}\right)$ with a radius of $\dfrac{\sqrt{a^2+b^2}}{2}$.

We will compute the slope of the tangent line in terms of θ to get the slope at $\theta = 0$. To do so, we will differentiate the parametric equations $x = r\cos\theta = a\cos^2\theta + b\cos\theta\sin\theta$ and $y = r\sin\theta = a\cos\theta\sin\theta + b\sin^2\theta$.

$$\begin{aligned}\frac{dy}{d\theta} &= -2b\cos\theta\sin\theta - a\sin^2\theta + a\cos^2\theta \\ \frac{dx}{d\theta} &= -2a\cos\theta\sin\theta - b\sin^2\theta + b\cos^2\theta\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \\ &= \frac{-2b\cos\theta\sin\theta - a\sin^2\theta + a\cos^2\theta}{-2a\cos\theta\sin\theta - b\sin^2\theta + b\cos^2\theta}\end{aligned}$$

Plugging in $\theta = 0$, we get $\frac{dy}{dx}(0) = \frac{a}{b}$. Thus the slope of the tangent line at the point where $\theta = 0$ is $\boxed{\dfrac{a}{b}}$.