Problem 1: Define a sequence $\{a_n\}_{n=1}^{\infty}$ by $a_1 = \sqrt{2}$ and $a_n = \sqrt{2a_{n-1}}$ for all $n > 1$. Determine $\lim\limits_{n\to\infty} a_n$.

Let's write out some of the initial terms, so that we can visualize the sequence.

$$\begin{aligned} a_1 &= \sqrt{2} \\ a_2 &= \sqrt{2\sqrt{2}} \\ a_3 &= \sqrt{2\sqrt{2\sqrt{2}}} \end{aligned}$$

Let $L = \lim_{n\to\infty} a_n$.

We can write the equation $L = \sqrt{2L}$.

Solving for L we get,

$$L^2 = 2L \implies L(L-2) = 0 \implies L = \{0, 2\}$$

There are only two possible solutions to the equation $L = \sqrt{2L}$, so $L$ must be either 0 or 2.

We can rule out 0 as a possibility, since $\frac{a_{n+1}}{a_n} > 1$ for all positive integers n.

Therefore $\boxed{\lim\limits_{n\to\infty} a_n = 2}$.