Problem 2: Compute the length of the astroid given by the parameterization $(\cos^3(t), \sin^3(t))$.

(Source: AoPS Calculus)

The length of the astroid is the integral of its speed with respect to t from $t = 0$ to $t = 2\pi$.

We are given the parametric functions $x(t) = \cos^3(t)$ and $y(t) = \sin^3(t)$.

We can use the chain rule to compute the derivatives $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

$$\frac{dx}{dt} = -3\sin(t)\cos^2(t) \qquad \text{and} \qquad \frac{dy}{dt} = 3\sin^2(t)\cos(t)$$

Now we can substitute these derivatives into the equation for length.

$$\begin{aligned}
\int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt &= 4\int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \\
&= 4\int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \\
&= 4\int_0^{\frac{\pi}{2}} \sqrt{\left(-3\sin(t)\cos^2(t)\right)^2 + \left(3\sin^2(t)\cos(t)\right)^2}\, dt \\
&= 4\int_0^{\frac{\pi}{2}} \sqrt{9\sin^2(t)\cos^4(t) + 9\sin^4(t)\cos^2(t)}\, dt \\
&= 4\int_0^{\frac{\pi}{2}} \sqrt{9\sin^2(t)\cos^2(t)(\sin^2(t) + \cos^2(t))}\, dt \\
&= 4\int_0^{\frac{\pi}{2}} \sqrt{9\sin^2(t)\cos^2(t)}\, dt \\
&= 4\int_0^{\frac{\pi}{2}} 3\sin(t)\cos(t)\, dt \\
&= 4\int_0^{\frac{\pi}{2}} 3 \cdot \frac{1}{2} \cdot \sin(2t)\, dt \\
&= 6\int_0^{\frac{\pi}{2}} \sin(2t)\, dt \\
&= 6 \cdot -\frac{1}{2}\cos(2t)\Big|_0^{\frac{\pi}{2}} \\
&= -3\cos(2t)\Big|_0^{\frac{\pi}{2}} \\
&= -3\left(\cos(\pi) - \cos(0)\right) \\
&= -3(-1-1) \\
&= -3(-2) \\
&= \boxed{6}
\end{aligned}$$

Thus the length of the astroid is $\boxed{6}$.