

Problem 3: Let  $p$  and  $q$  be positive real numbers. Compute  $\lim_{n \rightarrow \infty} \sqrt[n]{p^n + q^n}$ .

Suppose  $p > q$ . Then

$$\begin{aligned} \lim_{n \rightarrow \infty} \sqrt[n]{p^n + q^n} &= \lim_{n \rightarrow \infty} \sqrt[n]{p^n \left(1 + \left(\frac{q}{p}\right)^n\right)} \\ &= \lim_{n \rightarrow \infty} p \sqrt[n]{1 + \left(\frac{q}{p}\right)^n} \\ &= p \lim_{n \rightarrow \infty} \sqrt[n]{1 + \left(\frac{q}{p}\right)^n} \\ &= p \end{aligned}$$

Similarly, if  $p < q$ , then

$$\lim_{n \rightarrow \infty} \sqrt[n]{p^n + q^n} = q$$

If  $p = q$ , then

$$\begin{aligned} \lim_{n \rightarrow \infty} \sqrt[n]{p^n + q^n} &= \lim_{n \rightarrow \infty} \sqrt[n]{2p^n} \\ &= \lim_{n \rightarrow \infty} p \sqrt[n]{2} \\ &= p \lim_{n \rightarrow \infty} \sqrt[n]{2} \\ &= p \end{aligned}$$

Thus

$$\boxed{\lim_{n \rightarrow \infty} \sqrt[n]{p^n + q^n} = \max(p, q)}$$