

Electromagnetic Induction

- **Magnetic Flux:** Magnetic flux through a plane of area dA placed in a uniform magnetic field B $\phi = \int \vec{B} \cdot d\vec{A} = BA \cos \theta$ where θ is the angle between magnetic field lines and area vector of the surface.
- **Dimensions of magnetic flux:** $[ML^2T^{-2}A^{-1}]$
- **SI unit:-** Weber (Wb)
- **Faraday's Law:**
 - a) **First Law:** whenever there is a change in the magnetic flux linked with a circuit with time, an induced emf is produced in the circuit which lasts as long as the change in magnetic flux continues.
 - b) **Second Law:** The magnitude of the induced emf is directly proportional to the rate of change of magnetic flux linked with the closed circuit. $|\varepsilon| \propto \frac{d\phi}{dt}$
- **Lenz's Law:** The direction of the induced emf or current in the circuit is such that it opposes the cause due to which it is produced i.e. it opposes the change in magnetic flux, so that-

$$E = -N \left(\frac{d\phi}{dt} \right)$$
 Where N is the number of turns in the coil
 Lenz's law is based on energy conservation.
- **Induced EMF and Induced Current:** $E = -N \frac{d\phi}{dt} = -\frac{N(\phi_2 - \phi_1)}{t}$
 Charge depends only on net change in flux does not depends on time.

1. Induced EMF,

$$I = \frac{E}{R} = -\frac{N}{R} \left(\frac{d\phi}{dt} \right)$$

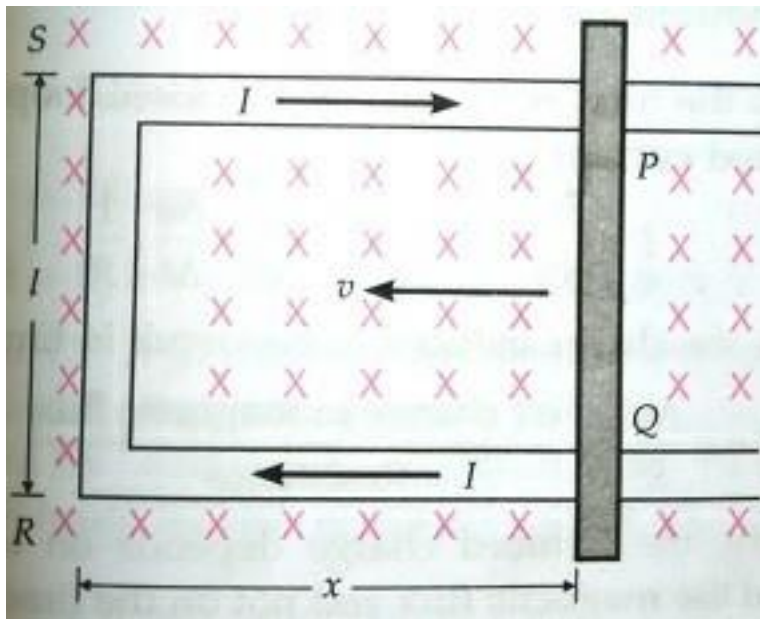
2. Induced current,

$$= -\frac{N}{R} \frac{(\phi_2 - \phi_1)}{t}$$

- **Induced Emf due to Linear Motion of a Conducting Rod in a Uniform Magnetic Field**

The induced emf, $E = \vec{l} \cdot (\vec{v} \times \vec{B})$

If \vec{l} , \vec{v} and \vec{B} are perpendicular to each other, then $E = Bvl$



- Fleming's Right Hand Rule is used to find the direction of induced current set up in the conductor.
- Induced EMF due to Rotation of a Conducting Rod in a Uniform Magnetic Field:** The induced emf, $E = \frac{1}{2} B \omega l^2 = B \pi n l^2 = B A n$ Where n is the frequency of rotation of the conducting rod.
- Induced EMF due to Rotation of a Metallic Disc in a Uniform Magnetic Field:**
 $E_{OA} = \frac{1}{2} B \omega R^2 = B \pi R^2 n = B A n$
- Induced EMF, Current and Energy Conservation in a Rectangular Loop Moving in a Non – Uniform Magnetic Field with a Constant Velocity:**
- Energy supplied in this process appears in the form of heat energy in the circuit.

- The net increase in flux crossing through the coil in time Δt is, $\Delta \phi = (B_2 - B_1) l v \Delta t$
- Induced emf in the coil is, $E = (B_1 - B_2) l v$
- If the resistance of the coil is R, then the induced current in the coil is,

$$I = \frac{E}{R} = \frac{(B_1 - B_2)}{R} l v$$

- Resultant force acting on the coil is $F = Il(B_1 - B_2)$ (towards left)
- The work done against the resultant force $W = (B_1 - B_2)^2 \frac{l^2 v^2}{R} \Delta t$ joule
- Energy supplied due to flow of current I in time Δt is, $H = I^2 R \Delta t$

$$\text{Or } H = (B_1 - B_2)^2 \frac{l^2 v^2}{R} \Delta t \text{ joule}$$

$$\text{Or } H = W$$

● **Rotation of Rectangular Coil in a Uniform Magnetic Field:**

1. Magnetic flux linked with coil $\phi = NBA \cos \theta = NBA \cos \omega t$
2. Induced emf in the coil $E = \frac{d\phi}{dt} = BAN\omega \sin \omega t = E_0 \sin \omega t$
3. Induced current in the coil. $I = \frac{E}{R} = \frac{NBA\omega}{R} \sin \omega t = \frac{E_0}{R} \sin \omega t$
4. Both Emf and current induced in the coil are alternating.

● **Self-Induction and Self Inductance:**

1. The phenomenon in which an induced emf is produced by changing the current in a coil is called self induction.

$$\phi \propto I \text{ or } \phi = LI$$

$$\text{or } L = \frac{\phi}{I}$$

$$E = -L \frac{dI}{dt}$$

$$L = \frac{E}{-(dI/dt)}$$

where L is a constant, called self inductance or coefficient of self – induction.

2. S.I. Unit- Henry (H)
3. Dimension- $[ML^2T^{-2}A^{-2}]$
4. Self inductance of a circular coil $L = \frac{\mu_0 N^2 \pi R}{2} = \frac{\mu_0 N^2 A}{2R}$
5. Self inductance of a solenoid $L = \frac{\mu_0 N^2 A}{l}$
6. Two coils of self – inductances L_1 and L_2 , placed far away (i.e., without coupling) from each other.
7. For series combination: $L = L_1 + L_2 + \dots + L_n$
8. For parallel combination: $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}$

● **Mutual Induction and Mutual Inductance:**

1. On changing the current in one coil, if the magnetic flux linked with a second coil changes and induced emf is produced in that coil, then this phenomenon is called mutual induction.

$$\phi_2 \propto I_1 \text{ or } \phi_2 = MI_1$$

Or $M = \frac{\phi_2}{I_1}$

$$E_2 = -\frac{d\phi_2}{dt} = -M \frac{dI_1}{dt}$$

$$M = \frac{E_2}{-(dI_1/dt)} \text{ Therefore, } M_{12} = M_{21} = M$$

2. Mutual inductance two coaxial solenoids $M = \frac{\mu_0 N_1 N_2 A}{l}$

3. If two coils of self-inductance L_1 and L_2 are wound over each other, the mutual inductance is, $M = K \sqrt{L_1 L_2}$ Where K is called coupling constant.

4. Mutual inductance for two coils wound in same direction and connected in series
 $L = L_1 + L_2 + 2M$

5. Mutual inductance for two coils wound in opposite direction and connected in series
 $L = L_1 + L_2 - 2M$

6. Mutual inductance for two coils in parallel $L = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm M}$

- **Energy Stored in an Inductor:** $U_B = \frac{1}{2} L I_{\max}^2$

- **Magnetic Energy Density:** $U_B = \frac{B^2}{2\mu_0}$

- **Eddy Current:** When a conductor is moved in a magnetic field, induced currents are generated in the whole volume of the conductor. These currents are called eddy currents.

- **Transformer:**

1. It is a device which changes the magnitude of alternating voltage or current.

$$\frac{E_s}{E_p} = \frac{n_s}{n_p} = K$$

2. For ideal transformer: $\frac{I_p}{I_s} = \frac{n_s}{n_p}$

3. In an ideal transformer: $E_p I_p = E_s I_s$

4. In step-up transformer: $n_s > n_p$ or $K > 1$ $E_s > E_p$ and $I_s < I_p$

5. In step-down transformer: $n_s < n_p$ or $K < 1$ $E_s < E_p$ and $I_s > I_p$

6. Efficiency $\eta = \frac{E_s I_s}{E_p I_p} \times 100\%$

- **Generator or Dynamo:** It is a device by which mechanical energy is converted into electrical energy. It is based on the principle of electromagnetic induction.

- **Different Types of Generator:**

1. AC Generator- It consists of field magnet, armature, slip rings and brushes.
2. DC Generator- It consists of field magnet, armature, commutator and brushes.

- **Motor:** It is a device which converts electrical energy into mechanical energy.

Back emf $e \propto \omega$

Current flowing in the coil, $i_a = \frac{E - e_b}{R}$

$$E = e_b + i_a R$$

Where R is the resistance of the coil.

Out put Power = $i_a e_b$

Efficiency, $\eta = \frac{e_b}{E} \times 100$