Fluids

Fluids are those substances which can flow when an external force is applied on it.

Liquids and gases are fluids.

Fluids do not have finite shape but takes the shape of the containing vessel,

The total normal force exerted by liquid at rest on a given surface is called thrust of liquid.

The SI unit of thrust is newton.

In fluid mechanics the following properties of fluid would be considered

(i) When the fluid is at rest **- hydrostatics**

(ii) When the fluid is in motion **- hydrodynamics**

Pressure Exerted by the Liquid

The normal force exerted by a liquid per unit area of the surface in contact is called **pressure of liquid** or**hydrostatic pressure**.

Pressure exerted by a liquid column

$p = h\rho g$

Where, h = height of liquid column, ρ = density of liquid

and g = acceleration due to gravity

Mean pressure on the walls of a vessel containing liquid upto height h is ($h\rho g$ / 2).

Pascal's Law

The increase in pressure at a point in the enclosed liquid in equilibrium is transmitted equally in all directions in liquid and to the Walls of the container.

The working of hydraulic lift, hydraulic press and hydraulic brakes are based on Pascal's law.

Atmospheric Pressure

The pressure exerted by the atmosphere on earth is **atmospheric pressure**.

It is about 100000 N/m^2.

It is equivalent to a weight of 10 tones on 1 m^2.

At sea level, atmospheric pressure is equal to 76 cm of mercury column. Then, atmospheric pressure

= hdg = 76 x 13.6 x 980 $dyne/cm^2$

[The atmospheric pressure does not crush our body because the pressure of the blood flowing through our circulatory system] balanced this pressure.]

Atmospheric pressure is also measured in torr and bar.

1 torr = 1 mm of mercury column

1 bar = $l0^5$ Pa

Aneroid barometer is used to measure atmospheric pressure.

Buoyancy

When a body is partially or fully immersed in a fluid an upward force acts on it, which is called buoyant force or simply buoyancy.

The buoyant force acts at the centre of gravity of the liquid displaced] by the immersed part of the body and this point is called the centre buoyancy.

Archimedes' Principle

When a body is partially or fully immersed in a liquid, it loses some of its weight. and it is equal to the weight of the liquid displaced by the immersed part of the body.

If T is the observed weight of a body of density σ when it is fully immersed in a liquid of density p, then real weight of the body

$w = T / (1 – p / \sigma)$

Laws of Floatation

A body will float in a liquid, if the weight of the body is equal to the weight of the liquid displaced by the immersed part of the body.

If W is the weight of the body and w is the buoyant force, then

(a) If W > w, then body will sink to the bottom of the liquid.

(b) IfW < w, then body will float partially submerged in the liquid.
(c) If W = w, then body will float in liquid if its whole volume is just immersed in the liquid,

The floating body will be in stable equilibrium if meta-centre (centre of buoyancy) lies vertically above the centre of gravity of the body.

The floating body will be in unstable equilibrium if meta-centre (centre of buoyancy) lies vertically below the centre of gravity of the body.

The floating body will be in neutral equilibrium if meta-centre (centre of buoyancy) coincides with the centre of gravity of the body.

Density and Relative Density

Density of a substance is defined as the ratio of its mass to its volume.

Density of a liquid = Mass / Volume

Density of water = 1 g/cm^3 or $l0^3$ kg/m^3

It is scalar quantity and its dimensional formula is $[ML^{-3}]$.

Relative density of a substance is defined as the ratio of its density to the density of water at 4°C,

Relative density = Density of substance / Density of water at 4°C

= Weight of substance in air / Loss of weight in water

Relative density also known as specific gravity has no unit, no dimensions.

For a solid body, density of body = density of substance

While for a hollow body, density of body is lesser than that of Substance.

When immiscible liquids of different densities are poured in a container, the liquid of highest density will be at the bottom while, that of lowest density at the top and interfaces will be plane.

Density of a Mixture of Substances

When two liquids of mass m_1 and m_2 having density p_1 and p_2 are mixed together then density of mixture is

$p = m_1 + m_2 / (m_1 / p_1) + (m_2 + p_2)$

$= p_1p_2 (m_1 + m_2) / (m_1p_2 + m_2p_1)$

When two liquids of same mass m but of different densities p_1 and p_2 are mixed together then density of mixture is

$p = 2p_1p_2 / p_1 + p_2$

When two liquids of same volume V but of different densities p_1 and p_2 are mixed together then density of mixture is

$p = p_1 + p_2 / 2$

Density of a liquid varies with

pressure $p = p_o [1 + \Delta p / K]$

where, p_o = initial density of the liquid, K = bulk modulus of elasticity of the liquid and Δp = change in pressure

Viscosity

The property of a fluid by virtue of which an internal frictional force acts between its different layers which opposes their relative motion is called viscosity.

These internal frictional force is called viscous force.

Viscous forces are intermolecular forces acting between the molecules of different layers of liquid moving with different velocities.

$$\text{Viscous force } (F) = -\eta A \frac{dv}{dx}$$

$$\eta = -\frac{F}{A\left(\frac{dv}{dx}\right)}$$

where, (dv/dx) = rate of change of velocity with distance called velocity gradient, A = area of cross-section and = coefficient of viscosity.

SI unit of η is Nsm^{-2} or pascal-second or decapoise. Its dimensional formula is $[ML^{-1}T^{-1}]$.

The knowledge of the coefficient of viscosity of different oils and its variation with temperature helps us to select a suitable lubricant for a given machine.

Viscosity is due to transport of momentum. The value of viscosity (and compressibility) for ideal liquid is zero.

The viscosity of air and of some liquids is utilised for damping the n.ving parts of some instruments.

The knowledge of viscosity of some organic liquids is used in determining the molecular weight and shape of large organic moleculars like proteins and cellulose.

Variation of Viscosity

The viscosity of liquids decreases with increase in temperature

$$\eta_t = \frac{\eta_0}{(1 + \alpha t + \beta t^2)}$$

where, η_0 and η_t is are coefficient of viscosities at 0°C t°C, α and β are constants.

The viscosity of gases increases with increase in temperatures as

$\eta \propto \sqrt{T}$

The viscosity of liquids increases with increase in pressure but the viscosity of water decreases with increase in pressure.

The viscosity of gases do not changes with pressure.

Poiseuille's Formula

The rate of flow (v) of liquid through a horizontal pipe for steady flow is given by

$$v = \frac{\pi}{8} \frac{pr^4}{\eta l}$$

where, p = pressure difference across the two ends of the tube. r = radius of the tube, n = coefficient of viscosity and 1 = length of th tube.

The Rate of Flow of Liquid

Rate of flow of liquid through a tube is given by

v = (P/R)

where, $R = (8\,\eta l/\pi r^4)$, called liquid resistance and p = liquid pressure.

(i) When two tubes are connected in series

- Resultant pressure difference $p = p_1 + p_2$
- Rate of flow of liquid (v) is same through both tubes.
- Equivalent liquid resistance, $R = R_1 + R_2$

(ii) When two tubes are connected in parallel

1. Pressure difference (p) is same across both tubes.
2. Rate of flow of liquid $v = v_1 + v_2$
3. Equivalent liquid resistance $(1/R) = (1/R_1) + (1/R_2)$

Stoke's Law

When a small spherical body falls in a long liquid column, then after sometime it falls with a constant velocity, called terminal velocity. When a small spherical body falls in a liquid column with terminal velocity then viscous force acting on it is

$F = 6\pi\eta rv$

where, r = radius of the body, V = terminal velocity and η = coefficient of viscosity.

This is called **Stoke's law.**

Terminal velocity $v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

where,

- ρ = density of body,
- σ = density of liquid,
- η = coefficient of viscosity of liquid and,
- g = acceleration due to gravity

1. If $\rho > \rho_0$, the body falls downwards.
2. If $\rho < \rho_0$, the body moves upwards with the constant velocity.
3. If po << ρ, $v = (2r^2\rho g/9\eta)$

Importance of Stoke's Law

1. This law is used in the determination of electronic charge by Millikan in his oil drop experiment.
2. This law helps a man coming down with the help of parachute.
3. This law account for the formation of clouds.

Flow of Liquid

1. **Streamline Flow** The flow of liquid in which each of its particle follows the same path as followed by the proceeding particles, is called streamline flow.
2. **Laminar Flow** The steady flow of liquid over a horizontal surface in the form of layers of different velocities, is called laminar flow.
3. **Turbulent Flow** The flow of liquid with a velocity greater than its critical velocity is disordered and called turbulent flow.

Critical Velocity

The critical velocity is that velocity of liquid flow, below which its fl is streamlined and above which it becomes turbulent.

Critical velocity $v_c = (k_\eta/r\rho)$

where,

- K = Reynold's number,
- η = coefficient of viscosity of liquid
- r = radius of capillary tube and ρ = density of the liquid.

Reynold's Number

Reynold's number is a pure number and it is equal to the ratio of inertial force per unit area to the viscous force per unit area for flowing fluid.

$$\text{Reynold number } K = \frac{v_c \rho r}{\eta}$$

where, p = density of the liquid and v_c = critical velocity.

For pure water flowing in a cylindrical pipe, K is about 1000.

When 0< K< 2000, the flow of liquid is streamlined.

When 2000 < K < 3000, the flow of liquid is variable betw streamlined and turbulent.

When K > 3000, the flow of liquid is turbulent.

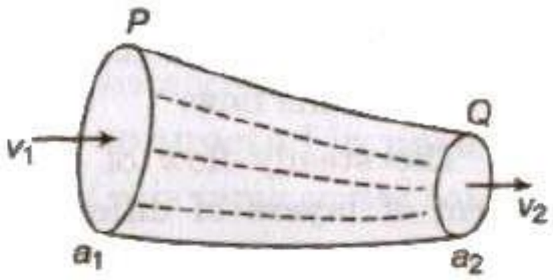
It has no unit and dimension.

Equation of Continuity

If a liquid is flowing in streamline flow in a pipe of non-unif cross-section area, then rate of flow of liquid across any cross-sec remains constant.

$a_1v_1 = a_2v_2$ av = constant

The velocity of liquid is slower where area of cross-section is larger faster where area of cross-section is smaller.



The falling stream of water becomes narrower, as the velocity of f stream of water increases and therefore its area of cross-s decreases.

Energy of a Liquid

A liquid in motion possess three types of energy

(i) Pressure Energy Pressure energy per unit mass = p/ρ

where,

p= pressure of the liquid and p = density of the liquid.
Pressure energy per unit volume = p

(ii) Kinetic Energy

- Kinetic energy per unit mass = $(1/2v^2)$
- Kinetic energy per unit volume = $1/2\rho v^2$

(iii) Potential Energy

- Potential energy per unit mass = gh
- Potential energy per unit volume = ρgh

Bernoulli's Theorem

If an ideal liquid is flowing in streamlined flow then total energy, i.e., sum of pressure energy, kinetic energy and potential energy per unit
volume of the liquid remains constant at every cross-section of the tube.

Mathematically

$$p+\frac{1}{2}\rho v^2+\rho gh=\text{constant}$$

It can be expressed as

$$\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

where, $(p/\rho g)$ = pressure head, $(v^2/2g)$ = velocity head and h = gravitational head.

For horizontal flow of liquid,

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

- where, pis called static pressure and $(1/2\ \rho v^2)$ is called dynamic pressure.
- The erefore in horizontal flow of liquid, if p increases, v decreases and Nce-versa.
- The theorem is applicable to ideal liquid, i.e., a liquid which is non-viscous incompressible and irrotational.

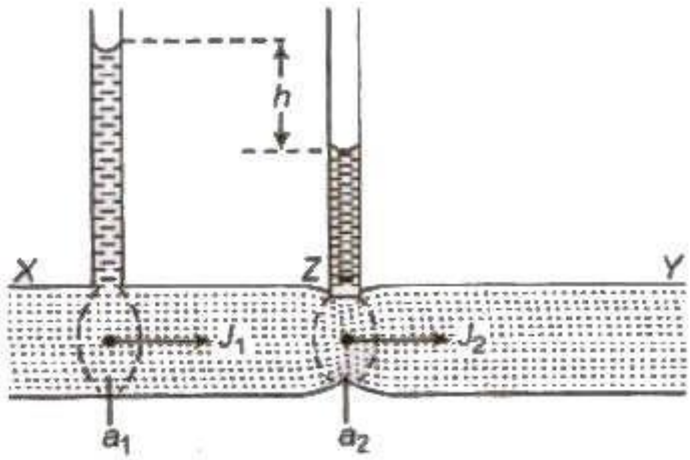
Applications of Bernoulli's Theorem

1. The action of carburetor, paintgun, scent sprayer atomiser insect sprayer is based on Bernoulli's theorem.
2. The action of Bunsen's burner, gas burner, oil stove exhaust pump is also based on Bernoulli's theorem.
3. Motion of a spinning ball (Magnus effect) is based on Bernoulli theorem.
4. Blowing of roofs by wind storms, attraction between two close parallel moving boats, fluttering of a flag etc are also based Bernoulli's theorem.

Venturimeter

It is a device used for measuring the rate of flow of liquid t pipes. Its working is based on Bernoulli's theorem.

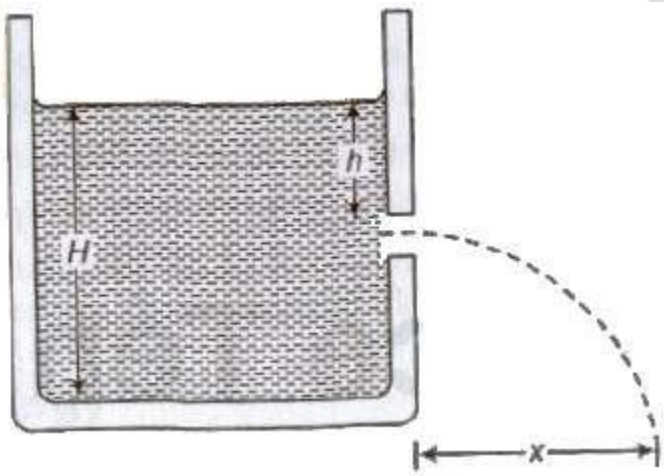
Rate of flow of liquid, $v = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$



where, a_1 and a_2 are area of cross-sections of tube at bra and narrower part and h is difference of liquid columns in ver tubes.

Torricelli's Theorem

Velocity of efflux (the velocity with which the liquid flows out orifice or narrow hole) is equal to the velocity acquired by a falling body through the same vertical distance equal to the dep orifice below the free surface of liquid.



Velocity of efflux, $v = \sqrt{2gh}$

where, h = depth of orifice below the free surface of liquid.

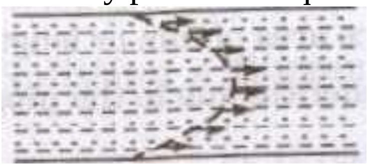
Horizontal range, $S = \sqrt{4h(H - h)}$

where, H = height of liquid column.

Horizontal range is maximum, equal to height of the liquid column H, when orifice is at half of the height of liquid column.

Important Points

- In a pipe the inner layer .central layer) have maximum velocity and the layer in contact with pipe have least velocity.
- Velocity profile of liquid flow in a pipe is parabolic.



- Solid friction is independent of area of surfaces in contact while viscous force depends on area of liquid layers.
- A lubricant is chosen ac:ording to the nature of machinary. In heavy machines lubricating oils of high viscosity are used and in light machines low viscosity oils are used.
- The cause of viscosity 1 liquids is the cohesive forces among their molecules while cause of viscosity in gases is diffusion.