PHYSICS

MOTION IN A PLANE

1. **Scalars and vectors**
2. **Multiplication of vectors by real numbers**
3. **Addition and subtraction of vectors**
4. **Resolution of vectors**
5. **Motion in a plane with constant acceleration**
6. **Projectile motion and Uniform circular motion**

SUMMARY

1. Scalar quantities have only magnitude. Examples are distance, speed, mass and temperature.

2. Vector quantities have both magnitude and direction. Examples are displacement, velocity and acceleration. They obey special rules of vector algebra.

3. A vector 'A' multiplied by a real number 'λ' is also a vector, whose magnitude is 'λ' times the magnitude of the vector 'A' and whose direction is the same or opposite depending upon whether 'λ' is positive or negative.

4. Two vectors A and B can be added graphically by using head-to-tail method or parallelogram method.

5. Vector addition obeys commutative law i.e, A + B = B + A

It also obeys the associative law i.e, (A + B) + C = A + (B + C)

6. A null or zero vector is a vector with zero magnitude. Since the magnitude is zero, we don't have to specify its direction.

It has the properties:
A+0 =A

λ.0 =0

0.A =0

7. The subtraction of vector B from A is defined as the sum of 'A' and 'negative B'
i.e A-B=A + (-B)

8. A vector A can be resolved into component along two given vectors 'a' and 'b' lying in the same plane i.e, A=λa+μb where λ and μ are real numbers.

9. A unit vector associated with a vector 'A' has magnitude '1' and is along the vector A:
$\widehat{n} = \frac{A}{|A|}$

The unit vectors $\hat{i}$,$\hat{j}$,$\hat{k}$ are vectors of unit magnitude and points in the direction of the x-, y-, and z-axes respectively in a right-handed coordinate system.

10.A vector $\mathbf{A}$ can be expressed as $\mathrm{A} = \mathrm{A}_x\hat{i} = \mathrm{A}_y\hat{j}$

Where A_x A_y are its components along x-, and y -axes. If vector $\mathbf{A}$ makes an angle θ with the x-axis, then A_x=A cosθ, A_y=A sinθ and $|A| = \sqrt{A_x^2 + A_y^2}$ and direction is given by $\tan = \frac{A_y}{A_x}$

11. Vectors can be conveniently added using analytical method. If sum of two vectors A and B, that lie in x-y plane, is R, then: R=A+B

12. The position vector of an object in x-y plane is given by r=x$\hat{i}$+y$\hat{j}$ and the displacement from position r to position r i' is given by $\triangle r = r' - r = (x' - x)\hat{i} + (y' - y)\hat{j}$
$= \triangle x\hat{i} + \triangle y\hat{j}$

13. If an object undergoes a displacement Δr in time Δt, its average velocity is given by $v = \frac{\Delta r}{\Delta t}$. The velocity of an object at time t is the limiting value of the average velocity

As Δt tends to zero: $v = \lim_{\Delta t \to o} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$

$\nu_x = \frac{dx}{dt}, \nu_Z = \frac{dz}{dt}$

It can be written in unit vector notation as $v = \nu_x \hat{i} + \nu y \hat{j} + \nu_z \hat{k}$

When position of an object is plotted on a coordinate system, v is always tangent to the curve representing the path of the object.

14. If the velocity of an object changes from v to v′ in time Δt, then its average acceleration is given by: $\overline{a} = \frac{v - v^1}{\Delta t} = \frac{\Delta v}{\Delta t}$

The acceleration a at any time t is the limiting value $\overline{a} \ \ as \ \Delta t \to 0$

$a = \lim_{x \to \infty} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$ In component form, we have

15. If an object is moving in a plane with constant acceleration $a \ |a| \sqrt{a_x^2 \ \ a_y^2}$ and its position vector at time t $= 0 \text{is} \ \ r_o$ then at any other time t, it will be at a point given by $r - \ \ r_o = \ \ v_o t + \ \ \frac{1}{2} a t^2$ and its velocity is given by $v = v_o + a \ t$

When v_o is the velocity at time t=0

In component form

$x - \ \ x_o = \ \ v_{ox} t \ + \ \ \frac{1}{2} a_x t^2$

$y - \ \ y_o = \ \ v_{ox} t + \ \ \frac{1}{2} a_y t^2$

$\nu_x = \nu_{ax} + a_x t$

$\nu_y = \nu_{ay} + a_y t$

Motion in a plane can be treated as superposition of two separate simultaneous one-dimensional motions along two perpendicular directions

16. An object that is in flight after being projected is called a projectile. If an object is projected with initial velocity v_o making an angle θ_o with x-axis and if we assume its initial position to coincide with the origin of the coordinate system, then the position and velocity of the projectile at time t are given by:

$$\mathrm{x} = (\mathrm{v}_o \cos\theta_o)\,\mathrm{t}$$
$$\mathrm{y} = (\mathrm{v}_o \sin\theta_o)\mathrm{t} - (1/2)\,\mathrm{gt}^2$$
$$\mathrm{vx} = \mathrm{v}_{ox} = \mathrm{v}_o \cos\theta_o$$
$$\mathrm{v}_y = \mathrm{v}_o \sin\theta_o - \mathrm{gt}$$

The path of a projectile is parabolic and is given by $y \quad \tan_o\ x - \frac{gx^2}{2\ v_o \cos_o{}^2}$

The maximum height that a projectile attains is $h_m = \frac{(\nu_o \sin q_o}{2g}$

The time taken to reach this height is: $t_m = \frac{\nu_o \sin\theta_o}{g}$

The horizontal distance travelled by a projectile from its initial position to the position it passes y = 0 during its fall is called the range, R of the projectile. It is: $R = \frac{\nu_0^2}{g} \sin 2.$

17. When an object follows a circular path at constant speed, the motion of the object is called uniform circular motion. The magnitude of its acceleration is $\mathrm{a}_2 = \mathrm{v}^2/\mathrm{R}$. The direction of a_c is always onwards the centre of the circle.

The angular speed ω, is the rate of change of angular distance. It is related to velocity v by v = ωR. The acceleration is $\mathrm{a}_c = \omega^2\mathrm{R}$. If T is the time period of revolution of the object in circular motion and v is its frequency, we have

$$\omega = 2\pi\nu, \mathrm{v} = 2\pi\nu\mathrm{R},\ \mathrm{ac} = 4\pi^2\nu^2\mathrm{R}$$