OSCILLATIONS

1. **Periodic and oscillatory motions**
2. **Simple harmonic motion and its equations**
3. **Free, forced and damped oscillations, Resonance**

- **Periodic Motion:** A motion which repeats itself over and over again after a regular interval of time.

- **Oscillatory Motion:** A motion in which a body moves back and forth repeatedly about a fixed point.
- **Periodic function:** A function that repeats its value at regular intervals of its argument is called periodic function. The following sine and cosine functions are periodic with period T.
  $f(t) = \sin \frac{2\pi t}{T}$ and $g(t) = \cos \frac{2\pi t}{T}$
  These are called Harmonic Functions
  $f(t) = A \cos [\varpi\tau = 2\pi/\mathrm{T}]$

Note :- All Harmonic functions are periodic but all periodic functions are not harmonic.

One of the simplest periodic functions is given by
f(t) = A cos ωt [ω = 2π/T]

If the argument of this function ωt is increased by an integral multiple of 2π radians, the value of the function remains the same. The function f(t) is then periodic and its period, T is given by

$$T = \frac{2\pi}{\omega}$$

Thus the function f(t) is periodic with period T
F(t) = f (t + T)

Thus the function f(t) is periodic with period T

F(t) = f (t + T)

Linear combination of sine and cosine functions

$$f(t) = A \sin \omega t + B \cos \omega t$$

A periodic function with same period T is given as

$$A = D \cos o \text{ and } B = D \sin o$$

$$\therefore f(t) = D \sin (\omega t + o)$$

$$\therefore D = \sqrt{A^2 + B^2} \text{ and } o = \tan^{-1} \frac{x}{a}$$

- **Simple Harmonic Motion (SHM):** A particle is said to execute SHM if it moves to and fro about a mean position under the action of a restoring force which is directly proportional to its displacement from mean position and is always directed towards mean position.

Restoring Force Displacement

F a x

F= - k x

Where 'k' is force constant.

- **Amplitude**: Maximum displacement of oscillating particle from its mean position

$$X_{Max} = \pm A$$

- **Time Period**: Time taken to complete one oscillation.
- **Frequency =** $\frac{1}{T}$ Unit of frequency is Hertz (Hz).

$$1 \text{ Hz} = 1 \ s^{-1}$$

  - **Angular Frequency:**

$$\text{S.I unit } \omega = \text{rad } s^{-1}$$

- **Phase:**

1. The Phase of Vibrating particle at any instant gives the state of the particle as regards its position and the direction of motion at that instant. It is denoted by ø.

2. **Initial phase or epoch**: The phase of particle corresponding to time t = 0. It is denoted by ø.

- Displacement in SHM :

  $x = A\ \cos\left(\omega t + \phi_o\right)$

  Where, = Displacement,

  A = Amplitude

  ωt = Angular Frequency

  ø0 = Initial Phase.

Case 1: When Particle is at mean position

$v = -\omega\sqrt{A^2 - 0^2} = -\omega A$

$V_{\max} = \omega A = \frac{2\pi}{\mathrm{T}} A$

Case 2: When Particle is at extreme position $x = \pm A$

$\mathrm{v}\ = -\omega\sqrt{A^2 - A^2}\ \ = \ 0$

Acceleration

Case 3: When particle is at mean position x = 0,

acceleration = $\text{acceleration}\ \ = -\omega^2\ (o)\ \ = 0$

Case 4: When particle is at extreme position then

$\mathrm{x} = \mathrm{A}$ acceleration = - $\omega^2\ \mathrm{A}$