MECHANICAL PROPERTIES OF SOLIDS

- **Elastic behaviour of solids**
- **Stress and strain**
- **Hooke's law**
- **Young's modulus, bulk modulus, shear modulus of rigidity**
- **Poisson's Ratio, Elastic Energy**

Deforming Force : A force which produces a change in configuration of the object on applying it, is called a deforming force.

Elasticity: Elasticity is that property of the object by virtue of which it regain its original configuration after the removal of the deforming force.

Elastic Limit : Elastic limit is the upper limit of deforming force upto which, if deforming force is removed, the body regains its original form completely and beyond which if deforming force is increased the body loses its property of elasticity and get permanently deformed.

Perfectly Elastic Bodies: Those bodies which regain its original configuration immediately and completely after the removal of deforming force are called perfectly elastic bodies. e.g., quartz and phosphor bronze etc.

Perfectly Plastic Bodies: Those bodies which does not regain its original configuration at all on the removal of deforming force are called perfectly plastic bodies, e.g., putty, paraffin, wax etc.

Stress : The internal restoring force acting per unit area of a deformed body is called stress.

Stress = $\frac{Restoring\ force}{Area}$

Its unit is N/m^2 or Pascal and dimensional formula is $[ML^{-11}T^{-2}]$.

Stress is a tensor quantity.

Stress is of Two Types

(i) Normal Stress If deforming force is applied normal to the area, then the stress is called normal stress.

If there is an increase in length, then stress is called **tensile stress**.

If there is a decrease in length, then stress is **called compression stress**.

(ii) **Tangential Stress** If deforming force is applied tangentially, then the stress is called tangential stress.

Strain : The fractional change in configuration is called strain.

Strain = $\frac{Change\ in\ the\ configuration}{Original\ configuration.}$

It has no unit and it is a dimensionless quantity. According to the change in configuration, the strain is of three types

(1) Longitudinal strain= $\frac{Change\ in\ length}{Original\ length}$

(2) Volumetric strain = $\frac{Change\ in\ volume}{Original\ volume}$

(iii) Shearing strain = Angular displacement of the plane perpendicular to the fixed surface.

Hooke's Law

Within the limit of elasticity, the stress is proportional to the strain.

Stress &infi; Strain

or Stress = E * Strain

where, E is the modulus of elasticity of the material of the body.

Types of Modulus of Elasticity

1. **Young's Modulus ofElasticity**

It is defined as the ratio of normal stress to the longitudinal strain Within the elastic limit.

y = Normal stress / Longitudinal strain

$$y = \frac{F\Delta}{Al} = \frac{Mg\Delta l}{\pi r^2 l}$$

Its unit is N/m^2 or Pascal and its dimensional formula is $[ML^{-2}T^{-2}]$.

2. Bulk Modulus ofElasticity

It is defined as the ratio of normal stress to the volumetric strain within the elastic limit.

$K = \frac{Normal\ stress}{Volumetric\ strain}$

$K = \frac{FV}{A\Delta V} = \Delta p\ V / \Delta V$

where, $\Delta p = \frac{F}{A}$ = Change in pressure.

Its unit is N/m^2 or Pascal and its dimensional formula is $[M^{-1}T^{-2}]$.

3. **Modulus of Rigidity** (η)

It is defined as the ratio of tangential stress to the shearing strain, within the elastic limit.

η = Tangential stress / Shearing strain

Its urut is N/m^2 or Pascal and its dimensional formula is $[ML^{-1}T^{-2}]$.

Compressibility

Compressibility of a material is the reciprocal of its bulk modulus of elasticity.
Compressibility (C) = 1 / k

Its SI unit is $N^{-1}m^2$ and CGS unit is $dyne^{-1}\ cm^2$. Steel is more elastic than rubber. Solids are more elastic and gases are least elastic. For liquids. modulus of rigidity is zero. Young's modulus (Y) and modulus of rigidity (η) are possessed by solid materials only.

Limit of Elasticity

The maximum value of deforming force for which elasticity is present in the body is called itslimit of elasticity.

Breaking Stress

The minimum value of stress required to break a wire, is called breaking stress. Breaking stress is fixed for a material but breaking force varies with area of cross-section of the wire.
Safety factor $= \frac{Breaking\ stress}{Working\ stress}$

Elastic Relaxation Time

The time delay in restoring the original configuration after removal of deforming force is called elastic relaxation time.

For quartz and phosphor bronze this time is negligible.

Elastic After Effect

The temporary delay in regaining the original configuration by the elastic body after the removal of deforming force, is called elastic after effect.

Elastic Fatigue

The property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming force is called elastic fatigue.

Ductile Materials

The materials which show large plastic range beyond elastic limit are called ductile materials, e.g., copper, silver, iron, aluminum, etc. Ductile materials are used for making springs and sheets. Brittle Materials The materials which show very small plastic range beyond elastic limit are called brittle materials, e.g., glass, cast iron, etc.

Elastomers

The materials for which strain produced is much larger than the stress applied, with in the limit of elasticity are called elastomers, e.g., rubber, the elastic tissue of aorta, the large vessel carrying blood from heart. etc. Elastomers have no plastic range.

Elastic Potential Energy in a Stretched Wire

The work done in stretching a wire is stored in form of potential energy of the wire.

Potential energy U = Average force * Increase in length

$= \frac{1}{2} F\Delta l$

$= \frac{1}{2}$ Stress * Strain * Volume of the wire

Elastic potential energy per unit volume

$U = \frac{1}{2}$ * Stress * Strain

$= \frac{1}{2}$ (Young's modulus) * $(\text{Strain})^2$

Elastic potential energy of a stretched spring = $\frac{1}{2} kx^2$

where, k = Force constant of spring and x = Change in length.

Thermal Stress When temperature of a rod fixed at its both ends is changed, then the produced stress is called thermal stress.

Thermal stress = $\frac{F}{A} = y\alpha\Delta\theta$

where, α = coefficient of linear expansion of the material of the rod.

When temperature of a gas enclosed in a vessel is changed, then the thermal stress produced is equal to change in pressure (Δp) of the gas.

Thermal stress = $\Delta p = K\gamma \Delta \theta$

where, K = bulk modulus of elasticity and

γ = coefficient of cubical expansion of the gas.

Interatomic force constant

$K = Yr_o$

where, r_o = interatomic distance.

Poisson's Ratio

When a deforming force is applied at the free end of a suspended wire of length 1 and radius R, then its length increases by dl but its radius decreases by dR. Now two types of strains are produced by a single force.

1. Longitudinal strain = ΔUl
2. Lateral strain = $\frac{-\Delta R}{R}$
3. ∴ Poisson's Ratio (σ) = $\frac{Lateral\ strain}{Longitudinal\ strain}$ = $\frac{-\Delta R}{R \times \Delta Ul}$ The theoretical value of Poisson's ratio lies between – 1 and 0.5. Its practical value lies between 0 and 0.5

Relation Between Y, K, η and σ

(i) Y = 3K (1 – 2σ)

(ii) Y = 2 η (1 + σ)

(iii) σ = $\frac{3K-2\eta}{2\eta+6K}$

Coefficient of elasticity depends upon the material, its temperature and purity but not on stress or strain.

For the same material, the three coefficients of elasticity γ, η and K have different magnitudes.

Isothermal elasticity of a gas ET = ρ where, ρ = pressure of the gas.

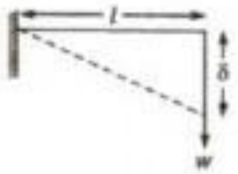
Adiabatic elasticity of a gas Es = γρ where, γ = $\frac{C_p}{C_v}$; ratio of specific heats at constant pressure and at constant volume

Ratio between isothermal elasticity and adiabatic elasticity; $\frac{E_s}{E_T} = \gamma = \frac{C_p}{C_v}$

Cantilever

A beam clamped at one end and loaded at free end is called a cantilever. Depression at the free end of a cantilever is given by

$$\delta = \frac{wl^3}{3YI^G}$$



where, w = load, 1 = length of the cantilever,

y = Young's modulus of elasticity, and I^G = geometrical moment of inertia.

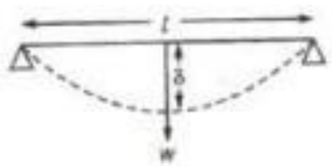
For a beam of rectangular cross-section having breadth b and thickness d.

$$I^G = \frac{bd^3}{11}$$

For a beam of circular cross-section area having radius r

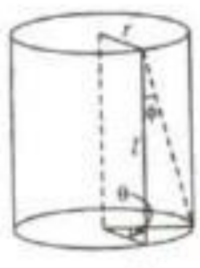
$$I^G = \frac{\pi r^4}{4}$$

Beam Supported at Two Ends and Loaded at the Middle



Depression at middle $\delta = \frac{wl^3}{48YI^G}$

Torsion of a Cylinder



Couple per unit twist $C = \frac{\pi\eta r^4}{2l}$

where, η = modulus of rigidity of the material of cylinder, r = radius of cylinder,

and 1 = length of cylinder,

Work done in twisting the cylinder through an angle θ

W = 1 / 2 Cθ2

Relation between angle of twist (θ) and angle of shear (φ) rθ = lφ or φ = r / l = θ