COMPLEX NUMBERS AND QUADRATIC EQUATIONS

1. **Algebra, Modulus and Conjugate of Complex Numbers**
2. **Argand Plane and Polar Representation**
3. **Quadratic Equations**

- $\Rightarrow \quad i^2 = -1$
- **Imaginary Number:** Square root of a negative number is called an Imaginary number. For example, $\sqrt{-5}$, $\sqrt{-16}$, etc. are imaginary numbers.
- **Integral power of Iota (i)** : $i^p\ (p > 4) = i^{4q+r} = \left(i^4\right)^q . i^r = i^r$, where $\sqrt{-1} = i$ and $i^4 = 1$
- **Complex Number**: A number of the form $\mathrm{a} + \mathrm{ib}$, where a and b are real numbers, is called a complex number, a is called the real part and b is called the imaginary part of the complex number. It is denoted by z.
- **Real part of** $z = a + ib$ is a and is denoted by $Re(z) = a$.
- **Imaginary part of** $z = a + ib$ is b and is written as $Im(z) = b$.
- **Equality of complex numbers**: Two complex numbers $z_1 = a + ib$ and $z_2 = c + id$ are said to be equal, if $a = c$ and $b = d$.
- **Conjugate of a complex number**: Two complex numbers are said to be conjugate of each other, if their sum is real and their product is also real. Conjugate of a complex number $z = a + ib$ is $\overline{z} = a - ib$ i.e., conjugate of a complex number is obtained by changing the sign of imaginary part of z.
- **Modulus of a complex number**: Modulus of a complex number $z = x + iy$ is denoted by $|z| = \sqrt{x^2 + y^2}$.
- **Argument of a complex number** $x + iy$: Arg$(x + iy) = \tan^{-1}\frac{y}{x}$.
- **Representation of complex number as ordered pair**: Any complex number $a + ib$ can be written in ordered pair as (a, b), where a is the real past and b is the imaginary part of a complex number.

- Let $z_1 = a + ib$ and $z_2 = c + id$. Then

(i) $z_1 + z_2 = (a + c) + i\,(b + d)$

(ii) $z_1 z_2 = (ac - bd) + i\,(ad + bc)$

- Division of a complex number: If $z_1 = a + ib$ and $z_2 = c + id$, then,

$$\frac{z_1}{z_2} = \frac{a+ib}{c+id} = \frac{(a+ib)(c-id)}{(c+id)(c-id)} = \frac{ac+bd}{c^2+d^2} + i\,\frac{bc-ad}{c^2+d^2}$$

- For any non-zero complex number $z = a + ib\ (a \neq 0,\ b \neq 0)$ there exists the complex number $\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}$ denoted by $\frac{1}{z}$ or z^{-1}, called the multiplicative inverse of z such that $(a + ib)\left(\frac{a^2}{a^2+b^2} + i\frac{-b}{a^2+b^2}\right) = 1 + io = 1$

- **Polar form of a complex number**: The polar form of the complex number $z = x + iy$ is $r\,(\cos\theta + i\sin\theta)$, where $r = \sqrt{x^2+y^2}$ (the modulus of z) and $\cos\theta = \frac{x}{r},\ \sin\theta = \frac{y}{r}$, (θ is known as the argument of z. The value of θ, such that is called the principal argument of z.
- **Important properties**: (i) $|z_1| + |z_2| \geq |z_1 + z_2|$, (ii) $|z_1| - |z_2| \leq |z_1 + z_2|$
- **Fundamental Theorem of algebra**: A polynomial equation of n degree has n roots.

Quadratic Equation:

- **Quadratic Equation**: Any equation containing a varibale of highest degree 2 is known as quadratic equation. e.g., $ax^2 + bx + c = 0$.
- **Roots of an equation**: The values of variable satisfying a given equation are called its roots. Thus, $x = \alpha$ is a root of the equation $p(x) = 0$ if $p(\alpha) = 0$.
- **Solution of quadratic equation**: The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in \mathrm{R},\ a \neq 0,\ b^2 - 4ac < 0$, are given by $x = \frac{-b \pm i\sqrt{4ac-b^2}}{2a}$.