Inverse Trigonometric Functions

- The domains and ranges (principal value branches) of inverse trigonometric functions are given in the following table:

Functions	Domain	Range (Principal Value Branches)
$y = sin^{-1}x$	[-1, 1]	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	[-1, 1]	$[\ 0, \pi]$
$y = \operatorname{cosec}^{-1} x$	R- [-1, 1]	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - {0}
$y = \sec^{-1} x$	R-[-1, 1]	$[0, \pi] - \{\frac{\pi}{2}\}$
$y = \tan^{-1} x$	R	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cot^{-1} x$	R	$[0, \pi]$

- $sin^{-1}x$ should not be confused with $(\sin x)^{-1}$. In fact $(\sin x)^{-1} = \frac{1}{\sin x}$ And similarly for other trigonometric functions.
- The value of an inverse trigonometric functions which lies in its principal value branch is called the principal value of that inverse trigonometric functions.
- **For suitable values of domain, we have**

- $y = sin^{-1} x \Rightarrow x = sin\, y$
- $x = sin\ y \Rightarrow y = sin^{-1} x$

$\sin(\sin^{-1} x) = x$

- $\sin^{-1}(\sin x) = x$
- $\sin^{-1} \frac{1}{x} = \operatorname{cosec}^{-1} x$
- $\cos^{-1}(-x) = \pi - \cos^{-1} x$
- $\cos^{-1} \frac{1}{x} = \sec^{-1} x$
- $\cot^{-1}(-x) = \pi - \cot^{-1} x$
- $\tan^{-1} \frac{1}{x} = \cot^{-1} x$

$\cot^{-1} \frac{1}{x} = \tan^{-1} x$

$\operatorname{cos}ec^{-1} \frac{1}{x} = \sin^{-1} x$

- $\sec^{-1}(-x) = \pi - \sec^{-1} x$

$\sec^{-1} \frac{1}{x} = \cos^{-1} x$

- $\sin^{-1}(-x) = -\sin^{-1} x$
- $\tan^{-1}(-x) = -\tan^{-1} x$
- $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
- $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$
- $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$$

$$\cos^{-1}x + o\cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

- $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

- $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$

$$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$$

$$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$2\cos^{-1}x = \cos^{-1}\left(2x^2-1\right)$$

- $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-\sqrt{x}}\right)$

$$3\sin^{-1}x = \sin^{-1}\left(3x-4x^3\right)$$

$$3\cos^{-1}x = \cos^{-1}\left(4x^3-3x\right)$$

$$3\tan^{-1}x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

Conversion:

- $\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}} = \cot^{-1}\frac{\sqrt{1-x^2}}{x} = \sec^{-1}\frac{1}{\sqrt{1-x^2}} = \operatorname{cosec}^{-1}\frac{1}{x}$
- $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}} = \sec^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}\frac{1}{\sqrt{1-x^2}}$
- $\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \sec^{-1}\sqrt{1+x^2} =$

$\operatorname{cosec}^{-1}\frac{\sqrt{1+x^2}}{x} = \cot^{-1}\frac{1}{x}$

- $\cot^{-1}x = \sin^{-1}\frac{1}{\sqrt{1+x^2}} = \cos^{-1}\frac{x}{\sqrt{1+x^2}} = \sec^{-1}\frac{1}{x} = \sec^{-1}\frac{\sqrt{1+x^2}}{x} = \operatorname{cosec}^{-1}\sqrt{1+x^2}$
- $\sec^{-1}x = \tan^{-1}\frac{\sqrt{x^2-1}}{1} = \cot^{-1}\frac{1}{\sqrt{x^2-1}} = \sin^{-1}\frac{\sqrt{x^2-1}}{x} = \cos^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}\frac{x}{\sqrt{x^2-1}}$
- $\operatorname{cosec}^{-1}x = \sin^{-1}\frac{1}{x} = \tan^{-1}\frac{1}{\sqrt{x^2-1}} = \cot^{-1}\sqrt{x^2-1} = \sec^{-1}\frac{x}{\sqrt{x^2-1}} = \cos^{-1}\frac{\sqrt{x^2-1}}{x}$

Some other properties of Inverse Trigonometric Function:

- $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$
- $\cot^{-1}\frac{x}{\sqrt{a^2-x^2}} = \cos^{-1}\frac{x}{a}$
- $\tan^{-1}\frac{a}{\sqrt{x^2-a^2}} = \operatorname{cosec}^{-1}\frac{x}{a}$
- $\cot^{-1}\frac{a}{\sqrt{x^2-a^2}} = \sec^{-1}\frac{x}{a}$