SEQUENCES AND SERIES

1. Sequences and series
2. Arithmetic Progression (A.P.)
3. Geometric Progression (G.P.), relation in A.M. and G.M.
4. Sum to n terms of Special Series

- **Sequence**: By a sequence, we mean an arrangement of number in definite order according to some rules. Also, we define a sequence as a function whose domain is the set of natural numbers or some subsets of the type {1, 2, 3, ...k}. A sequence containing a finite number of terms is called a finite sequence. A sequence is called infinite if it is not a finite sequence.
- Let a_1, a_2, a_3, ... be the sequence, then the sum expressed as $a_1 + a_2 + a_3 + \ldots\ldots$ is called series. A series is called finite series if it has got finite number of terms.

ARITHMETIC PROGRESSION

- An arithmetic progression (A.P .) is a sequence in which terms increase or decrease regularly by the same constant. This constant is called common difference of the A.P. Usually, we denote the first term of A.P . by a, the common difference by d and the last term by . $l$he general term or the n^{th} term of the A.P. is given by $a_n = a + (n - 1) d$.
- Single Arithmetic mean between any two given numbers a and b: A.M. = $\frac{a+b}{2}$
- n Arithmetic mean between two given numbers a and b: $a, A_1, A_2, A_3, \ldots\ldots, b$ form an A.P.
- If a constant is added to each term of an A.P., then the resulting sequence is also an A.P.
- If a constant is subtracted to each term of an A.P., then the resulting sequence is also an A.P.
- If each term of an A.P. is multiplied by a constant, then the resuting sequence is also an A.P.
- If each term of an A.P. is divided by a constant, then the resuting sequence is also an A.P.
- Sum of first n terms of an A.P.: $S_n = \frac{n}{2}[2a + (n-1)d]$ and $S_n = \frac{n}{2}[a + l]$, where l is the last term, i.e., $a + (n-1)d$.

GEOMETRIC PROGRESSION

- A sequence of non-zero numbers is said to be a geometric progression, if the ratio of each term, except the first one, by its preceding term is always the same. $a, ar, ar^2, \ldots\ldots, ar^{n-1}, \ldots$, where a is the first term and r is the common ratio.
- n^{th} term of a G.P.: $a_n = ar^{n-1}$
- Sum of n^{th} terms of a G.P.: $S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$.
- Sum to infinity of a G.P.: $S_\infty = \frac{a}{1-r}$
- Geomtric mean between a and b: $\sqrt{ab}$
- n Geometric means between a and b: $a.\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, a.\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, \ldots. a.\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$
- If all the terms of a G.P. be multiplied or divided by the same quantity the resulting sequence is also a G.P.
- The reciprocal of the terms of a given G.P. form a G.P.
- If each term of a G.P. be raised to the same power, the resulting sequence is also a G.P.

ARITHMETIC - GEOMETRIC SERIES

- A sequence of non-zero numbers is said to be a arithmetic-geometric series, if its terms are obtained on multiplying the terms of an A.P. by the corresponding terms of a G.P. For example: $1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \frac{5}{3^4} + \ldots\ldots\ldots$
- The general form of an arithmetic-geometric series: $a, (a+d)\,r, (a+2d)\,r^2, \ldots\ldots\ldots, [a+(n-1)\,d]\,r^{n-1}$
- n^{th} term of an arithmetic-geometric series: a_n of A.P. x a_n of G.P.
- Sum of n terms of some special series : $\sum n = \frac{n(n+1)}{2}$
- Sum of squares of first n natural numbers = $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of fist n natural numebrs = $\sum n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\sum n\right)^2$