LIMITS AND DERIVATIVES

---

1. **Limits**
2. **Derivatives**
3. **Miscellaneous Questions**

Meaning of $x \to a$ or "x tends to a" or "x approaches a", x is a variable. The expected value of the function as dictated by the points to the left of a point defines the left hand limit of the function at that point. Similarly the right hand limit. It can be changed so that its value comes nearer and nearer to a. $0 < |x - a| < \delta$

(i) $x \neq a$, (ii) $|x - a|$ becomes smaller and smaller as we please.

Neighbourhood: The set of all real numbers lying between $a - \delta$ and $a + \delta$ is called the neighbourhood of a. Neighbourhood of a = $(a - \delta, a + \delta)$, $x \in (a - \delta, a + \delta)$

- **Limit** of a function at a point is the common value of the left and right hand limits, if they coincide

Left hand limit of f at $x = a$. When x approches a from left hand side of a, the function $f(x)$ tends to l "a definite number". This definite number l is said to be the left hand limit of f at $x = a$.

Right hand limit of f at $x = a$. When x approches a from right hand side of a, the function $f(x)$ tends to l "a definite number". This definite number l is said to be the right hand limit of f at $x = a$.

Therefore, if Left hand limit of f at $x = a$ = Right hand limit of f at $x = a$, then the limit of $f(x)$ at $x = a$ exists.

- For function f and a real number a, $\lim_{x \to \infty} f(x)$ and f (a) may not be same (Infact, one may be defined and not the other one).
- For functions f and g the following holds:.

$$\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

$$\lim_{x \to a} [f(x). g(x)] = \lim_{x \to a} f(x). \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to \infty} g(x)}$$

Following are some of the standard limits

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1, \lim_{x \to a} \frac{\sin(x-a)}{x-a} = 1$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1, \lim_{x \to a} \frac{\tan(x-a)}{x-a} = 1$$

$$\lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1, \lim_{x \to 0} \frac{\tan^{-1} x}{x} = 1$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a, a > 0, a \neq 1$$

Derivatives

The derivative of a function f at a is defined by

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

Derivative of a function f at any point x is defined by

$$f'(x) = \frac{df(x)}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

For functions u and v the following holds:

$$(u \pm v)' = u' \pm v'$$

$$(uv)' = u'v + uv' \qquad \Rightarrow \qquad \frac{d}{dx}(uv) = u.\frac{dv}{dx} + v.\frac{du}{dx}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2} \qquad \Rightarrow \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx}-u.\frac{dv}{dx}}{v^2}$$

provided all are defind.

Following are some of the standard derivatives

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\cos ec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x.\tan x$$

$$\frac{d}{dx}(\cos ecx) = -\cos ecx.\cot x$$