TRIGONOMETRIC FUNCTIONS

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1. **Angles**
2. **Trigonometric Functions**
3. **Sum and Difference of Two Angles**
4. **Trigonometric Equations**

- **Measurement of an angle**: The measure of an angle is the amount of rotation from the initial side to the terminal side.
- **Right angle**: If the rotating ray starting from its initial position to final position, describes one quarter of a circle, then we say that the measure of the angle formed is a right angle.
- If in a circle of radius r, an arc of length l subtends an angle of θ radians, then l = rθ
- Radian measure = $\frac{\pi}{180} \times \text{ Degree measure}$
- Degree measure = $\frac{180}{\pi} \times \text{ }Radian\text{ measure}$

Trigonometric Functions

- **Quadrant**:

$t-$ ratios	I	II	III	IV
$\sin\theta = y$	+	+	-	-
$\cos\theta = x$	+	-	-	+
$\tan\theta = \frac{y}{x}$	+	-	+	-

- $\cos ec\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

- **Trigonometric values of some angles**:

	0°	30°	45°	60°	90°	180°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0

Trigonometric Identities:

- $\cos^2 \mathrm{x} + \sin^2\mathrm{x} = 1$
- $1 + \tan^2\mathrm{x} = \sec^2\mathrm{x}$
- $1 + \cot^2\mathrm{x} = \mathrm{cosec}^2\mathrm{x}$

Trigonometric ratio of $(90^\circ + x)$ **in terms of** x:

- $\cos(\frac{\pi}{2} + x) =$ -sin x
- $\sin(\frac{\pi}{2} + x) =$ cos x
- $\tan\left(\frac{\pi}{2} + x\right) = -\cot x$

Trigonometric ratio of $(90^\circ - x)$ in terms of x:

- $\cos(\frac{\pi}{2} - (x) =$ sin x
- $\sin(\frac{\pi}{2} - x) =$ cos x
- $\tan\left(\frac{\pi}{2} - x\right) = \cot x$

Trigonometric ratio of $(180^\circ - x)$ **in terms of** x:

- $\cos(\pi - x) =$ -cos x
- $\sin(\pi - x) =$ sin x
- $\tan(\pi - x) = -\tan x$

Trigonometric ratio of $(270^\circ - x)$ **in terms of** x:

- $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$
- $\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$
- $\tan\left(\frac{3\pi}{2} - x\right) = \cot x$

Trigonometric ratio of $(270^\circ + x)$ in terms of x:

- $\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$
- $\cos\left(\frac{3\pi}{2} + x\right) = \sin x$
- $\tan\left(\frac{3\pi}{2} + x\right) = -\cot x$

Trigonometric ratio of $(360^\circ - x)$ in terms of x:

- $\cos(2\pi - x) = \cos x$
- $\sin(2\pi - x) = -\sin x$
- $\tan(2\pi - x) = -\tan x$

Trigonometric ratio of $(360^\circ + x)$ in terms of x:

- $\cos(2\pi + x) = \cos x$
- $\sin(2\pi + x) = \sin x$
- $\tan(2\pi + x) = \tan x$
- $\cos\ (2\mathrm{n}\pi +\ \mathrm{x})\ =\ \cos \mathrm{x}$
- $\sin\ (2\mathrm{n}\pi +\ \mathrm{x})\ =\ \sin \mathrm{x}$

Trigonometric Ratios of Compound Angles:

Sum Formulae:

- $\sin\ (\mathrm{x} + \mathrm{y}) = \sin \mathrm{x} \cos \mathrm{y} + \cos \mathrm{x} \sin \mathrm{y}$
- $\cos\ (\mathrm{x} + \mathrm{y}) = \cos \mathrm{x} \cos \mathrm{y}$ - $\sin \mathrm{x} \sin \mathrm{y}$
- $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$

Difference Formulae:

- $\sin\ (\mathrm{x}$ - $\mathrm{y}) = \sin \mathrm{x} \cos \mathrm{y}$ - $\cos \mathrm{x} \sin \mathrm{y}$
- $\cos\ (\mathrm{x}$ - $\mathrm{y}) = \cos \mathrm{x} \cos \mathrm{y} + \sin \mathrm{x} \sin \mathrm{y}$
- $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$
- $\cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$

Some Useful Results:

- $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$
- $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x$
- $\tan(x+y+z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan z \tan x}$

<u>Transformation Formulae</u>:

Product Formulae (on the basis of L.H.S.) or A-B formulae:

- $2\sin x \cos y = \sin(x+y) + \sin(x-y)$
- $2\cos x \sin y = \sin(x+y) - \sin(x-y)$
- $2\cos x \cos y = \cos(x+y) + \cos(x-y)$
- $2\sin x \sin y = \cos(x-y) - \cos(x+y)$

Sum and Difference Formulae (on the basis of L.H.S.) or C-D formulae:

- $\sin \mathrm{C} + \sin \mathrm{D} = 2\sin \frac{\mathrm{C+D}}{2} \cos \frac{\mathrm{C-D}}{2}$
- $\sin \mathrm{C} - \sin \mathrm{D} = 2\cos \frac{\mathrm{C+D}}{2} \sin \frac{\mathrm{C-D}}{2}$
- $\cos \mathrm{C} + \cos \mathrm{D} = 2\cos \frac{\mathrm{C+D}}{2} \cos \frac{\mathrm{C-D}}{2}$
- $\cos \mathrm{C} - \cos \mathrm{D} = 2\sin \frac{\mathrm{C+D}}{2} \sin \frac{\mathrm{D-C}}{2}$

<u>Trigonometric Functions of Multiple and Sub-multiples of Angles</u>:

- $\sin 2x = 2\sin x \cos x = \frac{2\tan x}{1+\tan^2 x}$
- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1+\tan^2 x}{1+\tan^2 x}$
- $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$
- $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$
- $\sin 3\mathrm{x} = 3\sin \mathrm{x} - 4\sin^3 \mathrm{x}$
- $\sin \frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$
- $\cos 3\mathrm{x} = 4\cos^3 \mathrm{x} - 3\cos \mathrm{x}$
- $\cos \frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$
- $\tan \frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}}$
- $\sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5}-1}{4}$
- $\cos 18^\circ = \sin 72^\circ = \frac{1}{4}\sqrt{10+2\sqrt{5}}$
- $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$
- $\sin 36^\circ = \frac{1}{4}\sqrt{10-2\sqrt{5}}$

Trigonometric Equations:

- **Principle Solutions**: The solutions of a trigonometric equation, for which $0 \leq 2\pi$ are called the principle solutions.
- **General Solutions**: The solution, consisting of all possible solutions of a trigonometric equation is called its general solutions>
- **Some General Solutions**:
- $\sin x = 0$ gives $x = n\pi$, where $n \in Z$.
- $\cos x = 0$ gives $x = (2n + 1)\frac{\pi}{2}$, where $n \in Z$.
- $\tan x = 0$ gives $x = n\pi$
- $\cot x = 0$ gives $x = (2n + 1)\frac{\pi}{2}$
- $\sec x = 0$ gives no solution
- $\cos ecx = 0$ gives no solution
- $\sin x = \sin y$ gives $x = n\pi + (-1)^n y$
- $\cos x = \cos y$, implies $x = 2n\pi \pm y$, where $n \in Z$.
- $\tan x = \tan y$ implies $x = n\pi + y$, where $n \in Z$.
- $\sin^2 x = \sin^2 y$ gives $x = n\pi \pm y$
- $\cos^2 x = \cos^2 y$ gives $x = n\pi \pm y$
- $\tan^2 x = \tan^2 y$ gives $x = n\pi \pm y$