Straight Lines

1. **Slope of a Line**
2. **Various Forms of the Equation of a Line**
3. **General Equation of a Line and Distance of a Point From a Line**

First Degree Equation

Every first degree equation like $ax + by + c = 0$ would be the equation of a straight line.

Slope of a line

- Slope (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is given by is given by $m = \frac{y_1 - y_2}{x_1 - x_2} = x_1 \neq x_2$.
- If a line makes an angle á with the positive direction of x-axis, then the slope of the line is given by $m = \tan\alpha$, $\alpha \neq 90^o$
- Slope of horizontal line is zero and slope of vertical line is undefined.
- An acute angle (say θ) between lines L_1 and L_2 with slopes m_1 and m_2 is given by $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|, 1 + m_1 m_2 \neq 0$
- Two lines are parallel if and only if their slopes are equal i.e.$m_1 = m_2$
- Two lines are *perpendicular* if and only if product of their slopes is –1, i.e., $m_1 . m_2 = -1$
- Three points A, B and C are collinear, if and only if slope of AB = slope of BC.
- Equation of the horizontal line having distance a from the x-axis is eithery = a or y = – a.
- Equation of the vertical line having distance b from the y-axis is eitherx = b or x = – b.
- The point (x, y) lies on the line with slope m and through the fixed point (x_o, y_0), if and only if its coordinates satisfy the equation.

Various forms of equations of a line:

- **Two points form**: Equation of the line passing through the points (x_1, y_1) and ((x_2, y_2) is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- **Slope-Intercept form**: The point (x, y) on the line with slope m and y-intercept c lies on the line if and only if $y = mx + c$.
- If a line with slope m makes x-intercept d. Then equation of the line is $y = m(x - d)$.
- **Intercept form**: Equation of a line making intercepts a and b on the x-and y-axis, respectively, is $\frac{x}{a} + \frac{y}{b} = 1$.
- **Normal form**: The equation of the line having normal distance from origin p and angle between normal and the positive $x-$axis ω is given by $x \cos\omega + y \sin\omega = p$
- **General Equation of a Line**: Any equation of the form Ax + By + C = 0, with A and B are not zero, simultaneously, is called the general linear equation or general equation of a line.
- **Working Rule for reducing general form into the normal form**:

(i) Shift constant 'C' to the R.H.S. and get $Ax + By = -C$
(ii) If the R.H.S. is not positive, then make it positive by multiplying the whole equation by -1.
(iii) Divide both sides of equation by $\sqrt{A^2 + B^2}$.

The equation so obtained is in the normal form.

- **Parametric Equation (Symmetric Form)** $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$
- **Equation of a line through origin**: $y = mx$ or $y = x\tan\theta$.
- The perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$
- Distance between the parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, is given by $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$

Concurrent Lines

Three of more straight lines are said to be concurrent if they pass through a common point i.e., they meet at a point. Thus, if three lines are concurrent the point of intersection of two lines lies on the third line.

Condition of concurrency of three lines:

$$a_1\left(b_2 c_3-b_3 c_2\right)+b_1\left(c_2 a_3-c_3 a_2\right)+c_1\left(a_2 b_3-a_3 b_2\right)=0$$

EQUATIONS OF FAMILY OF LINES THROUGH THE INTERSECTION OF TWO LINES

$$\mathrm{A}_1 x+\mathrm{B}_1 y+\mathrm{C}_1+k\left(\mathrm{A}_2 x+\mathrm{B}_2 y+\mathrm{C}_2\right)=0$$

where k is a constant and also called parameter.

This equation is of first degree of x and y, therefore, it represents a family of lines.

DISTANCE BETWEEN TWO PARALLEL LINES

Working Rule to find the distance between two parallel lines:

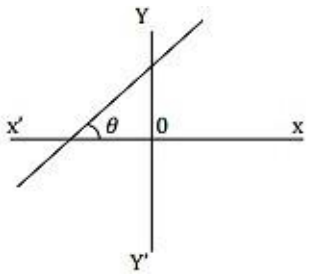
(i) Find the co-ordinates of any point on one of ht egiven line, preferably by putting $x=0$ and $y=0$.

(ii) The perpendicular distance of this point from the other line is the required distance between the lines.

STRAIGHT LINE

Definition: A straight line is a curve such that every point on the line segment joining any two points on It lies on it. (No turning point b/w two points called a straight line)

Slope of Line (Gradient): A line makes with the +ve direction of the x – axis in anticlockwise sense is Called the slope or gradient of the line.

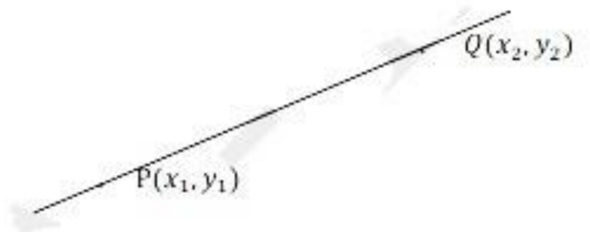


The slope of a line is generally denoted by m. Thus, m = $\tan\theta$.

1. Since a line parallel to x –axis makes an angle of 00 with x – axis, therefore its slope is tan 0° = 0.
2. A line parallel to y – axis makes an angle of 90° with x – axis, so its slope is $\tan\frac{}{2} = \infty$.

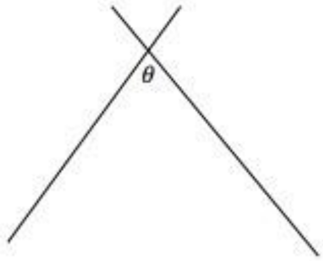
Slope of Line when Passing from two given points:

If P(x_1, y_1) & (x_2, y_2)So, $m = \frac{y_2-y_1}{x_2-x_1}$



Angle between two Lines:

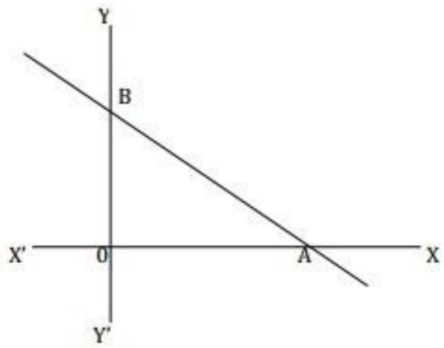
$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$ here m1:m2 are slope of lines and θ is angle $\frac{b}{w}$ two lines.



NOTE: 1. If two lines are parallel to each other ⇒ $m_1 = m_2$ because $\theta = 0$

1. if two line are perpendicular to each other ⇒ $m_1 m_2 = -1$ because $\theta = 90$
2. if line parallel to x - axis ⇒ equation of line y = k
3. if line parallel to y - axis ⇒ equation of line x = k
4. every linear equation of two variable represent a line e.g. ax + by c = 0

Intercepts of line on the Axes:



B Here OA = X axis intercepts

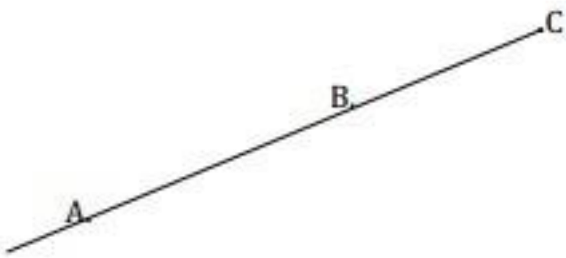
And OB = Y axis intercepts

Let OA = a and OB = b

So, A(a, 0) and B(0, b)

NOTE: If three point are collinear than slope are equal b/w any two point of line

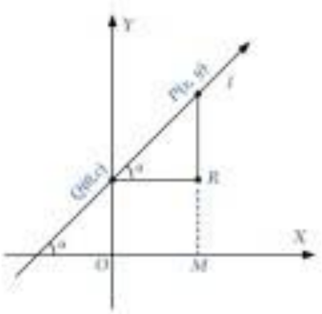
let $A(x_1,y_1)$: $B(x_2,y_2)$ & (x_3,y_3) ⇒ slope of BC = slope of AC

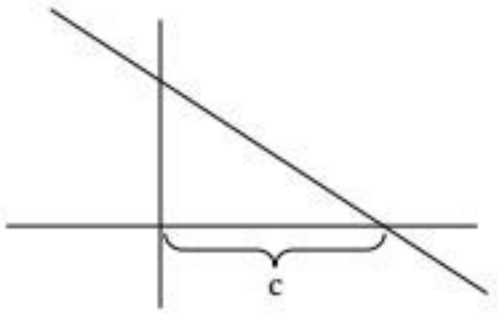


Different forms of the equation of a straight line:

1. **Slope intercept form of a line:**

   The equation of a line with slope m and making an intercept c on y – axis is y = mx + c





   The equation of a line with slope m and making an intercept c on x – axis is y = m(x - c)

2. **Point - slope form of a line:**

   The equation of a line which passes through the point (given) P(x_1, y_1) and has the slope 'm' is

   y - y_1 = m(x - x_1).

3. **Two point form of a line:**

   The equation of a line passing through two points P(x_1, y_2) and Q(x_2, y_2) is

   $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

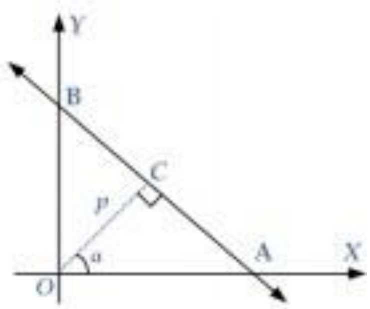
4. **Intercept form of a line:**

   The equation of a line which cuts off intercepts 'a' and 'b' respectively from the x – axis and y – axis is $\frac{x}{a} + \frac{y}{b} = 1$.

5. **Normal form or Perpendicular form of a line:**

   The equation of the straight line upon which the length of the perpendicular from the origin is p and this Perpendicular makes an angle α with x – axis is

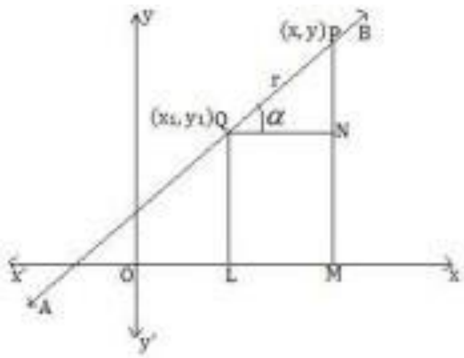
   $x \cos\alpha + y \sin\alpha = p$



6. **Distance form of a line:**

The equation of the straight line passing through (x_1, y_1) and making an angle θ with the +ve direction of x – axis is $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

Where r is the distance of the point (x, y) on the line from the point (x_1, y_1)



Transformation of general equation in different standard forms:

1. Transformation of Ax + By + C = 0 in the slope intercept form y = m x +c

$y = \left(-\frac{A}{B}\right)x + \left(-\frac{C}{B}\right)$

This is of the form y = m x +c, where

$m = -\frac{A}{B} = -\frac{\text{cof } ficient\ of\ x}{cof\ ficient\ of\ y}$, and intercept on y – axis $= -\frac{C}{B} = -\frac{constent}{cof\ ficient\ of\ y}$

2. Transformation of Ax + By + C = 0 in intercept form $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{\left(-\frac{C}{A}\right)} + \frac{y}{\left(-\frac{C}{B}\right)} = 1$

Intercept on x – axis $= -\frac{C}{A} = -\frac{cons\tan t\ term}{cof\ ficient\ of\ x}$, Intercept on y - axis $= -\frac{C}{B} = -\frac{cons\tan t\ term}{cof\ ficient\ of\ y}$

3. Transformation of Ax + By + C = 0 in intercept form $x\cos\alpha + y\sin\alpha\ = p$

$-\frac{A}{\sqrt{A^2+B^2}}x - \frac{B}{\sqrt{A^2+B^2}}y = \frac{C}{\sqrt{A^2+B^2}}$

Here $\cos\alpha = -\frac{A}{\sqrt{A^2+B^2}}$ and $\sin\alpha = -\frac{B}{\sqrt{A^2+B^2}}$; $p = \pm\frac{C}{\sqrt{A^2+B^2}}$

NOTE: Three lines are said to be concurrent if they pass through a common point OR they meet at a point.

Lines parallel and Perpendicular to a given line:

1. **Line parallel to a guven line**

   To write a line parallel to a given line we keep the expression containing x and y same and simply replace The given constant by an unknown constant k. the value of k can be determined by some given condition.
2. **Line perpendicular to a guven line**

   The equation of a line perpendicular to a given line ax + by + c = 0 is bx – ay + k = 0.

Distance of a point from a line:

The length of the perpendicular from a point (α, β) to a line ax + by + c = 0 is

Distance B/W Parallel lines:

The distance between two parallel lines ax + by +c_1 = 0 and ax + by + c_2 = 0 is

$$\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$$