Vector Algebra

- **Vector**: A quantity that has magnitude as well as direction is called vector.
- **Zero Vector**: A vector whose intial and terminal point coincide is called a zero vector or a null vector. It is denoted as $\vec{O}$.
- **Co-initial vectors**: Two or more vectors having the same initial points are called co-initial vectors.
- **Collinear vectors**: Two or more vectors are said to be collinear if they are parallel to the same line, irrespective of their magnitudes and directions.
- **Equal vectors**: Two vectors are said to be equal, if they have the same magnitude and direction regardless of the position of their initial points.
- **Negative of a vector**: A vector whose magnitude is the same as that of a given vector, but direction is opposite to that of it, is called negative of the given vector.
- Position vector of a point P (x, y) is given as $\overrightarrow{OP}\left(=\vec{r}\right) = x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude by $\sqrt{x^2+y^2+z^2}$
- The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.
- The magnitude (r), direction ratios (a, b, c) and direction cosines (l, m, n) of any vector are related as: $l = \frac{a}{r}$, $m = \frac{b}{r}$, $n = \frac{c}{r}$
- The vector sum of the three sides of a triangle taken in order is $\vec{O}$
- The vector sum of two conidial vectors is given by the diagonal of the parallelogram whose adjacent sides are the given vectors.
- The multiplication of a given vector by a scalar λ, changes the magnitude of the vector by the multiple $|\lambda|$, and keeps the direction same (or makes it opposite) according as the value of λ is positive (or negative).
- For a given vector $\vec{a}$, the vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$ gives the unit vector in the direction of $\vec{a}$
- The position vector of a point R dividing a line segment joining the points P and Q

- whose position vectors are $\vec{a}$ and $\vec{b}$ respectively, in the ratio $m : n$

  (i) internally, is given by $\frac{n\vec{a}+m\vec{b}}{m+n}$

  (ii) externally, is given by $\frac{m\vec{b}-n\vec{a}}{m-n}$

- The scalar product of two given vectors $\vec{a}$ and $\vec{b}$ having angle θ between them is defined as $\vec{a}\,.\,\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$

  Also, when $\vec{a}\,.\,\vec{b}$ is given, the angle 'θ' between the vectors $\vec{a}$ and $\vec{b}$ may be determined by $\cos\theta = \frac{\vec{a}\,.\,\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$

- If θ is the angle between two vector $\vec{a}$ and $\vec{b}$, then their cross product is given as $\vec{a}\times\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta.\hat{n}$ where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$. Such that $\vec{a}$, $\vec{b}$, $\hat{n}$ form right handed system of coordinate axes.

- If we have two vectors $\vec{a}$ and $\vec{b}$ given in component form as
  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and λ be any scalar, then,

  $$\vec{a} + \vec{b} = (a_1 + b_1)\,\hat{i} + (a_2 + b_2)\,\hat{j} + (a_3 + b_3)\,\hat{k}$$

  $$\lambda\vec{a} = (\lambda a_1)\,\hat{i} + (\lambda a_2)\,\hat{j} + (\lambda a_3)\,\hat{k}$$

  $$\vec{a}\,.\,\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

  and $\overline{a}\times\overline{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

Parallelogram Law of vector addition: If two vectors $\vec{a}$ and $\vec{b}$ are represented by adjacent sides of a parallelogram in magnitude and direction, then their sum $\vec{a} + \vec{b}$ is represented in magnitude and direction by the diagonal of the parallelogram through their common initial point. This is known as Parallelogram Law of vector addition.