Permutations and Combinations

1. **Fundamental Principle of Counting**
2. **Permutations**
3. **Combinations**

Fundamental Principle of Counting

- **Addition Law**: If there are two operations such that such that they can be performed independently in m and n ways respectively, then either of the two operations can be performed in $(m+n)$ ways.
- **Multiplication**: If one operation can be performed in m ways and if corresponding to each of the m ways of performing this operation, there are n ways of performing a second operation, then the number of ways of performing two operations together in $m \times n$.
- **Factorial Notation**: The continued product of first n natural numbers is called the 'n factorial' and is denoted by $n!$.
- $0! = 1$
- **Permutations**: The number of permutations of n different things taken r at a time, where repetition is not allowed, is denoted by ${}^nP_r$ and is given by ${}^nP_r = \frac{n!}{(n-r)!}$

  where $0 \leq r \leq n$.

  $\mathrm{n}! = 1 \times 2 \times 3 \times \ldots \times \mathrm{n}$

  $\mathrm{n}! = \mathrm{n} \times (\mathrm{n} - 1)!$

- The number of permutations of n different things, taken r at a time, where repeatition is allowed, is n^r.

- The number of permutations of n objects taken all at a time, where p_1 objectare of first kind, p_2 objects are of the second kind, ...,p_k objects are of the k^{th} kind and rest, if any, are all different is $\frac{n!}{p_1!p_2!..p_k!}$.

Combinations:

- The number of combinations of n different things taken r at a time, denoted by ${}^nC_r$ is given by ${}^nC_r = \frac{n!}{r!(n-r)!}, o \leq r \leq n.$
- ${}^n\mathrm{C}_0 = 1$
- ${}^n\mathrm{C}_n = 1$
- ${}^n\mathrm{C}_r = {}^n\mathrm{C}_{n-r}$
- ${}^n\mathrm{C}_r + {}^n\mathrm{C}_{r-1} = {}^{n+1}\mathrm{C}_r$
- ${}^n\mathrm{C}_r = \frac{n}{r} . {}^{n-1}\mathrm{C}_{r-1}$
- $n . {}^{n-1}\mathrm{C}_{r-1} = (n-r+1) {}^n\mathrm{C}_{r-1}$
- Division into Groups: The number of ways $m+n$ things can be divided into two groups containing m and n things respectively = ${}^{m+n}\mathrm{C}_m = \frac{(m+n)!}{m!.n!}$