Application of Derivatives

- If a quantity y varies with another quantity x, satisfying some rule y = f(x), then $\frac{dx}{dy}$(or $f'(x)$ represents the rate of change of y with respect to x and $\left.\frac{dy}{dx}\right]_{x=x_0}$ (or $f'(x_0)$) represents the rate of change of y with respect to x at x = x_0.
- If two variables x and y are varying with respect to another variable t, i.e., if $x = f(t)$ *and* $y = g(t)$ then by Chain Rule

  $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \text{ if } \frac{dx}{dt} \neq 0$$

- A function f is said to be increasing on an interval (a, b) if $x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in (a,b)$. Alternatively, if $f'(x) > 0$ for each x in, then f(x) is an increasing funciton on (a, b).
- A function f is said to be decreasing on an interval (a, b) if $x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in (a,b)$. Alternatively, if $f'(x) > 0$ for each x in, then f(x) is an decreasing funciton on (a, b).
- The equation of the tangent at (x_0, y_0) to the curve y = f (x) is given by

  $$y - y_o = \left.\frac{dy}{dx}\right]_{(x_0, y_0)} (x - x_o)$$

- If $\frac{dy}{dx}$ does not exist at the point (x_0, y_0), then the tangent at this point is parallel to the y-axis and its equation is $x = x_0$.
- If tangent to a curve $y = f(x)$ *at* $x = x_0$ is parallel to x-axis, then $\left.\frac{dy}{dx}\right]_{x=x_0}$ = 0

- **Equation of the normal** to the curve y = f (x) at a point $\left(x_o, y_o\right)$ is given by

$$y - y_o = \frac{-1}{\left.\frac{dy}{dx}\right]_{(x_0, y_0)}}(x - x_0)$$

- If $\frac{dy}{dx}$ at the point $\left(x_o, y_o\right)$ is zero, then equation of the normal is $x = x_0$.
- If $\frac{dy}{dx}$ at the point $\left(x_o, y_o\right)$ does not exist, then the normal is parallel to x-axis and its equation is $y = y_0$.
- Let y = f (x), Δx be a small increment in x and Δy be the increment in y corresponding to the increment in x, i.e., Δy = f (x + Δx) – f (x). Then dy given by $dy = f'(x)\,dx$ or $dy = \left(\frac{dy}{dx}\right)dx$ is a good approximation of Δy when dx x = Δ is relatively small and we denote it by dy ≈ Δy.
- A point c in the domain of a function f at which either f ′(c) = 0 or f is not differentiable is called a critical point of f.
- **First Derivative Test :** Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then,

(i) If f ′(x) changes sign from positive to negative as x increases through c, i.e., if f ′(x) > 0 at every point sufficiently close to and to the left of c, and f ′(x) < 0 at every point sufficiently close to and to the right of c, then c is a point of local maxima.

(ii) If f ′(x) changes sign from negative to positive as x increases through c, i.e., if f ′(x) < 0 at every point sufficiently close to and to the left of c, and f ′(x) > 0 at every point sufficiently close to and to the right of c, then c is a point of local minima.

(iii) If f ′(x) does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. In fact, such a point is called point of inflexion.

- **Second Derivative Test :** Let f be a function defined on an interval I and $c \in I$. Let f be twice differentiable at c. Then,

(i) $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$
The values f (c) is local maximum value of f.

(ii) $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$
In this case, f (c) is local minimum value of f.

(iii) The test fails if $f'(c) = 0$ and $f''(c) = 0$.
In this case, we go back to the first derivative test and find whether c is a point of maxima, minima or a point of inflexion.

- **Working rule for finding absolute maxima and/or absolute minima**

Step 1: Find all critical points of f in the interval, i.e., find points x where either $f'(x) = 0$ or f is not differentiable.

Step 2: Take the end points of the interval.

Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.

Step 4: Identify the maximum and minimum values of f out of the values calculated in Step 3.

This maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f .