Continuity and Differentiability

- Continuity of function at a point: Geometrically we say that a funciton $y = f(x)$ is continuous at $x = a$ if the graph of the function $y = f(x)$ is continuous (without any break) at $x = a$.
- A funciton $f(x)$ is said to be continuous ata point $x = a$ if:

(i) $f(a)$ exists i.e., $f(a)$ is finite, definite and real.

(ii) $\lim_{x \to a} f(x)$ exists.

(iii) $\lim_{x \to a} f(x) = f(a)$

- A function $f(x)$ is continuous at $x = a$ if $\lim_{h \to 0} f(a+h) = \lim_{h \to 0} f(a+h) = f(a)$ where $h \to 0$ through positive values.
- **Continuity of a function in a closed interval**: A function $f(x)$ is said to be continuous in the closed interval if it is continuous for every value of x lying between a and b continuous to the right of a and to the left of $x = b$ i.e., $\lim_{x \to a-0} f(x) = f(a)$ and $\lim_{x \to b-0} f(x) = f(b)$
- **Continuity of a function in a open interval**: A function $f(x)$ is said to be continuous in an open interval (a, b) if it is continuous at every point in (a, b).
- **Discontinuity (Discontinuous function)**: A function $f(x)$ is said to be discontinuous in an interval if it is discontinuous even at a single point of the interval.
- $(\sec x)' = \sec x . \tan x$
- $(\operatorname{cosec} x)' = -\operatorname{cosec} x . \cot x.$
- $(a^x)'$
- Suppose f is a real function and c is a point in its domain. The derivative of f at c is defined by $f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ provided this limit exists.
- A real valued function is continuous at a point in its domain if the limit of the function at that point equals the value of the function at that point. A function is continuous if it is continuous on the whole of its domain.
- $\frac{dy}{dx}$ is derivative of first order and is also denoted by y' or y_1 .
- Sum, difference, product and quotient of continuous functions are continuous. i.e., if f and g are continuous functions, then $(f \pm g)(x) = f(x) \pm g(x)$ is continuous. (f . g) (x) = f (x) . g(x) is continuous.
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ (wherever g (x) ≠ 0) is continuous.

- Every differentiable function is continuous, but the converse is not true.
- Chain rule is rule to differentiate composites of functions. If f = v o u, t = u (x) and if both *and if both* $\frac{dt}{dx}$ *and* $\frac{dv}{dt}$ exist then $\frac{df}{dx} = \frac{dv}{dt} = \frac{dt}{dx}$
- Following are some of the standard derivatives (in appropriate domains):
- $(u \pm v)' = u' \pm v'$
- $(uv)' = u'v + uv'$ [Product Rule]
- $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$, wherever $v \neq 0$ [Quotient Rule]
- If $y = f(u); u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ [Chain Rule]
- $x = f(t); y = g(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ [Parametric Form]
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = -\sec^2 x$
- $\frac{d}{dx}(\cot x) = -\operatorname{cos}ec^2 x$

$\frac{d}{dx}$

$\frac{d}{dx}$

$\frac{d}{dx} \qquad = a^x . \log_e a$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{1+x^2}$

- $\frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{x\sqrt{1-x^2}}$
- $\frac{d}{dx}\left(\operatorname{cosec}^{-1}x\right)=\frac{-1}{x\sqrt{1-x^2}}$
- $\frac{d}{dx}\left(e^x\right)=e^x$
- Logarithmic differentiation is a powerful technique to differentiate functions of the form $f(x) = [u(x)]^{v(x)}$ Here both f (x) and u (x) need to be positive for this technique to make sense.
- If we have to differentiate logarithmic funcitons, other than base e, then we use the result $\log_b a = \frac{\log_e a}{\log_e b}$ and then differentiate R.H.S.
- While differentiating inverse trigonometric functions, first represent it in simplest form by using suitable substitution and then differentiate simplified form.
- If we are given implicit functions then differentiate directly w.r.t. suitable variable involved and get the derivative by readusting the terms.
- $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ is derivative of second order and is denoted by y'' or y_2 .
- **Rolle's Theorem:** If f: [a, b] → R is continuous on [a, b] and differentiable on (a, b) such that f (a) = f (b), then there exists some c in (a, b) such that f ′(c) = 0.
- **Lagrange's Mean Value Theorem**: If f: [a, b] → R is continuous on [a, b] and differentiable on (a, b). Then there exists some c in (a, b) such that

  $$f'(c)=\frac{f(b)-f(a)}{b-a}$$