

Electrostatic Potential and Capacitance

- **Electrostatic Potential at a Point:** It is the work done by per unit charge by an external agency, in bringing a charge from infinity to that point.

- **Electrostatic Potential due to a Charge at a Point**

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

- The electrostatic potential at a point with position vector r due to a point dipole of dipole moment p placed at the origin

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{r^2}$$

The result is true also for a dipole (with charges $-q$ and q separated by $2a$) for $r \gg a$.

- For a charge configuration q_1, q_2, \dots, q_n with position vectors r_1, r_2, \dots, r_n , the potential at a point P is given by the superposition principle,

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{1p}} + \frac{q_2}{r_{2p}} + \dots + \frac{q_n}{r_{np}} \right)$$

where r_{1p} is the distance between q_1 and P , as and so on.

- **Electrostatics Potential Energy Stored in a System of Charges:** It is the work done (by an external agency) in assembling the charges at their locations.
- **Electrostatic Potential Energy of Two Charges q_1, q_2 , at r_1, r_2 :**
- $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$ where r_{12} is distance between q_1 and q_2
- **Potential Energy of a Charge q in an External Potential:** $V(r) = qV(r)$
- **Potential Energy of a Dipole of Dipole Moment p in a Uniform Electric Field:** $E = -p \cdot E$.
- **Equipotential Surface:** An equipotential surface is a surface over which potential has a constant value.

1. For a point charge, concentric spheres centered at a location of the charge are equipotential surfaces.
2. The electric field E at a point is perpendicular to the equipotential surface through the point.

3. E is in the direction of the steepest decrease of potential.

- **Capacitance C of a System of Two Conductors Separated by an Insulator:** It is defined as, $C = \frac{Q}{V}$ where Q and $-Q$ are the charges on the two conductors V is the potential difference between them.
- Capacitance is determined purely geometrically, by the shapes, sizes, and relative positions of the two conductors.
- **Capacitance C of a parallel plate capacitor (with the vacuum between the plates):** $C = \epsilon_0 \frac{A}{d}$ where A is the area of each plate and d the separation between them.
- **For capacitors in the series combination:** The total capacitance C is $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$
- **For capacitors in the parallel combination:** The total capacitance C is $C = C_1 + C_2 + C_3 + \dots$ where $C_1, C_2, C_3 \dots$ are individual capacitances.
- **The energy U stored in a capacitor of capacitance C , with charge Q and voltage V :** $U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$
- **The electric energy density (energy per unit volume) in a region with electric field:** $(1/2)\epsilon_0 E^2$
- **The potential difference between the conductor (radius r_0) inside & outside spherical shell (radius R):** $\phi(r_0) - \phi(R) = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_0} - \frac{1}{R} \right)$ which is always positive.
- **When the medium between the plates of a capacitor filled with an insulating substance:** Changes observed are as follows: Polarization of the medium gives rise to a field in the opposite direction.

1. The net electric field inside the insulating medium is reduced.
2. Potential difference between the plates is thus reduced.
3. Capacitance C increases from its value when there is no medium (vacuum). where K is the dielectric constant of the insulating substance.

- **Electrostatic Shielding:** A conductor has a cavity with no charge inside the cavity, then no matter what happens outside the conductor. Even if there are intense electric fields outside the conductor, the cavity inside has, shielding whatever is inside the cavity from whatever is outside the cavity. This is called electrostatic shielding.