

RELATIONS AND FUNCTIONS

1. Cartesian Product of Sets

2. Relations

3. Functions

- **Ordered pair** A pair of elements grouped together in a particular order. Clearly, $(a, b) \neq (b, a)$.
- **Cartesian product of two sets** A and B is given by $A \times B = \{(a, b) : a \in A, b \in B\}$.

In particular $\mathbf{R} \times \mathbf{R} = \{(x, y) : x, y \in \mathbf{R}\}$ and $\mathbf{R} \times \mathbf{R} \times \mathbf{R} = \{(x, y, z) : x, y, z \in \mathbf{R}\}$

- If $(a, b) = (x, y)$, then $a = x$ and $b = y$.
- If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.
- $A \times \varnothing = \varnothing$
- In general, $A \times B \neq B \times A$.
- **Relation:** Relation A relation R from a set A to a set B is a subset of the Cartesian product $A \times B$ obtained by describing a relationship between the first element x and the second element y of the ordered pairs in $A \times B$, i.e., $R \subseteq A \times B$.
- **Number of Relations:** Let A and B be two non-empty finite sets, containing m and n elements respectively, then the total number of relations from A to B is 2^{mn}
- **Domain:** The domain of R is the set of all first elements of the ordered pairs in a relation R. $\text{Domain } R = \{a : (a, b) \in R\}$.
- The image of an element x under a relation R is given by y, where $(x, y) \in R$,
- **Range:** The range of the relation R is the set of all second elements of the ordered pairs in a relation R. $\text{Range } R = \{b : (a, b) \in R\}$.
- **Function:** Function A function f from a set A to a set B is a specific type of relation for which every element x of set A has one and only one image y in set B. We write $f: A \rightarrow B$, where $f(x) = y$.

- **Domain and Co-domain:** The set A is called the domain of function f and the set B is called the co-domain of f .
- **Range:** If f is a function from A to B , then each element of A corresponds to one and only one element of B , whereas every element in B need not be the image of some x in A . The subset of B containing the image of elements of A is called the range of the function. The range of f is denoted by $f(A)$. Mathematically, we write:

$$f(A) = \{f(x) : x \in A\}$$
- **Image:** If the element x of A corresponds to $y \in B$ under the function f , then we say that y is the image of x under f and we write, $f(x) = y$.
- **Pre-image:** If $f(x) = y$, then x is pre-image of y .