Relation and Function

TYPES OF RELATIONS:

- **Empty Relation**: It is the relation R in X given by R = $\phi \subset X \times X$.
- **Universal Relation**: It is the relation R in X given by R = $X \times X$.
- **Reflexive Relation**: A relation R in a set A is called reflexive if (a, a) ∈ R for every a ∈ A.
- **Symmetric Relation**: A relation R in a set A is called symmetric if (a_1, a_2) ∈ R implies that (a_2, a_1) ∈ R, for all a_1, a_2 ∈
- **Transitive Relation**: A relation R in a set A is called transitive if (a_1, a_2) ∈ R, and (a_2, a_3) ∈ R together imply that all a_1, a_2, a_3 ∈ A.
- **EQUIVALENCE RELATION :** A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.
- **Equivalence Classes:** Every arbitrary equivalence relation R in a set X divides X into mutually disjoint subsets (Ai) called partitions or subdivisions of X satisfying the following conditions:

All elements of Ai are related to each other for all i.

No element of Ai is related to any element of Aj whenever i ≠ j

$$Ai \cup Ai = X \; and \; Ai \cap Ai = \Phi, \; i \neq j$$

· . These subsets ((A_i)) are called equivalence classes.

· For an equivalence relation in a set X, the equivalence class containing a ∈X, denoted by [a], is the subset of X containing all elements b related to a.

****Function: A relation f: A** $\longrightarrow$ B is said to be a function if every clement of A is correlated to a

Unique element in B.

***A is domain**
*** B is codomain**

- A **function** f : X $\rightarrow$ Y is **one-one** (or **injective**), if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2, \forall x_1, x_2 \in \mathrm{X}$.
- A **function** f : X $\rightarrow$ Y is **onto** (or **surjective**), if $y \in \mathrm{Y}, \exists x \in \mathrm{X}$ such that $f(x) = y$.
- A **function** f : X $\rightarrow$ Y is **one-one-onto** (or **bijective**), if f is both one-one and onto.
- **The composition of function** f : A $\rightarrow$ B and g : B $\rightarrow$ C is the function $gof : \mathrm{A} \rightarrow \mathrm{C}$ given by $gof(x) = g(f(x)), \forall x \in \mathrm{A}$.
- A **function** f : X $\rightarrow$ Y is **invertible**, if $\exists g : \mathrm{Y} \rightarrow \mathrm{X}$ such that $gof = \mathrm{I}_x$ and $fog = \mathrm{I}_y$.
- A **function** f : X $\rightarrow$ Y is **invertible**, if and only if f is one-one and onto.
- **Given a finite set X, a function** f : X $\rightarrow$ X is one-one (respectively onto) if and only if f is onto (respectively one-one). This is the characteristics property of a finite set. This is not true for infinite set.
- **A binary function** * on A is a function * from A x A to A.
- **An element** $e \in \mathrm{X}$ **is the identity element for binary operation *** : $\mathrm{X} \times \mathrm{X} \rightarrow \mathrm{X}$, if $a * e = a = e * a \; \forall a \in \mathrm{X}$.
- **An element** $e \in \mathrm{X}$ **is invertibel for binary operation *** : $\mathrm{X} \times \mathrm{X} \rightarrow \mathrm{X}$ if there exists $b \in \mathrm{X}$ such that $a * b * e * b * a$, where e is the binary identity for the binary operation *. The element b is called the inverse of a and is denoted by a^{-1}.
- An operation * on X is **commutative**, if $a * b = b * a, \; \forall a, b$ in X.
- An operation * on X is **associative**, if $(a * b) * c = a * (b * c), \; \forall a, b, c$ in X.