SYSTEM OF PARTICLES AND ROTATIONAL MOTION

1. **Centre of mass**
2. **Moment of a Force**
3. **Equilibrium of a rigid body**
4. **Moment of inertia**

1. Ideally, a rigid body is one for which the distances between different particles of the body do not change, even though there are forces on them.

2. A rigid body fixed at one point or along a line can have only rotational motion. A rigid body not fixed in some way can have either pure translation or a combination of translation and rotation.

3. In rotation about a fixed axis, every particle of the rigid body moves in a circle which lies in a plane perpendicular to the axis and has its centre on the axis. Every Point in the rotating rigid body has the same angular velocity at any instant of time.

4. In pure translation, every particle of the body moves with the same velocity at any instant of time.

5. Angular velocity is a vector. Its magnitude is $\omega = \frac{d\theta}{dt}$ and it is directed along the axis of rotation. For rotation about a fixed axis, this vector ω has a fixed direction.

6. The vector or cross product of two vector a and b is a vector written as a × b. The magnitude of this vector is ab sinθ and its direction is given by the right handed screw or the right hand rule.

7. The linear velocity of a particle of a rigid body rotating about a fixed axis is given by v=ω *r where r is the position vector of the particle with respect to an origin along the fixed axis. The relation applies even to more general rotation of a rigid body with one point fixed. In that case r is the position vector of the particle with respect to the fixed point taken as the origin.

8. The centre of mass of a system of particles is defined as the point whose position vector is $\mathrm{R} = \frac{\sum \mathrm{m}_1 \mathrm{r}_i}{M}$

9. Velocity of the centre of mass of a system of particles is given by V = P/M, where P is the linear momentum of the system. The centre of mass moves as if all the mass of the system is concentrated at this point and all the external forces act at it. If the total external force on the system is zero, then the total linear momentum of the system is constant.

10. The angular momentum of a system of n particles about the origin is $L = \sum_{t=1} r_1 \times p_t$

The torque or moment of force on a system of n particles about the origin is

$\sum_1 r_t \times F_1$

The force Fi acting on the i^{th} particle includes the external as well as internal forces. Assuming Newton's third law and that forces between any two particles act along the line joining the particles, we can show τ_{int}= 0 and $\frac{dL}{dt} = r_{ext}$

11. A rigid body is in mechanical equilibrium if

(1) it is in translational equilibrium, i.e., the total external force on it is zero $\sum f_1 = 0$ and

(2) it is in rotational equilibrium, i.e. the total external torque on it is zero:
$\sum t_1 = \sum r_t \times F_t$ =0

12. The centre of gravity of an extended body is that point where the total gravitationaltorque on the body is zero.

13. The moment of intertia of a rigid body about an axis is defined by the formula $\mathrm{I} = \sum m_t r_t^2$ where is the perpendicular distance of the ith point of the body from the axis. The kinetic energy of rotation is $\mathrm{K} = \frac{1}{2}\ 1\ \omega^2$.

14. The theorem of parallel axes: $I_x' = I_2 + Ma^2$ allows us to determine the moment of intertia of a rigid body about an axis as the sum of the moment of inertia of the body about a parallel axis through its centre of mass and the product of mass and square of the perpendicular distance between these two axes.

15. Rotation about a fixed axis is directly analogous to linear motion in respect of kinematics and dynamics.

16. For a rigid body rotating about a fixed axis (say, z-axis) of rotation, $\mathrm{L}_z = \mathrm{I}\omega$, where I is the moment of inertia about z-axis. In general, the angular momentum L for such a body is not along the axis of rotation. Only if the body is symmetric about the axis of rotation, L is along the axis of rotation. In that case, $|\mathrm{L}| = L_2 = \mathrm{I}\,\omega$. The angular acceleration of a rigid body rotating about a fixed axis is given by Iα = τ. If the external torque τ acting on the body is zero, the component of angular momentum about the fixed axis (say, z-axis L_z $(= \mathrm{I}\omega)$ of such a rotating body is constant.

17. For rolling motion without slipping $\mathrm{v}_{cm} = \mathrm{R}\omega$, where V_{cm} is the velocity of translation (i.e. of the centre of mass), R is the radius and m is the mass of the body. The kinetic energy of such a rolling body is the sum of kinetic energies of translation and rotation:
$\mathrm{K} = \frac{1}{2}\,\mathrm{m}\,\mathrm{v}_{cm}^2 + \frac{1}{2}\mathrm{I}\,\omega^2$.