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## BINOMIAL THEOREM

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### 1. Binomial Theorem for Positive Integral Indices

### 2. General and Middle Terms

- **Binomial Theorem:** The expansion of a binomial for any positive integral  $n$  is given by Binomial Theorem, which is

$$(a + b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_{n-1} a b^{n-1} + {}^nC_n b^n.$$

- The coefficients of the expansions are arranged in an array. This array is called *Pascal's triangle*.
- The general term of an expansion  $(a + b)^n$  is  $T_{r+1} = {}^nC_r a^{n-r} b^r$ .
- The general term of an expansion  $(a - b)^n = (-1)^r \cdot {}^nC_r \cdot a^{n-r} \cdot b^r$
- The general term of  $(1 + x)^n = {}^nC_r \cdot x^r$
- The general term of  $(1 - x)^n = (-1)^r \cdot {}^nC_r \cdot x^r$
- In the expansion  $(a + b)^n$ , if  $n$  is even, then the middle term is the  $\left(\frac{n}{2} + 1\right)^{th}$  term.  
If  $n$  is odd, then the middle terms are  $\left(\frac{n}{2} + 1\right)^{th}$  and  $\left(\frac{n+1}{2} + 1\right)^{th}$  terms.
- $r^{th}$  term from the end in  $(a + b)^n = (n + 2 - r)^{th}$  term from the beginning.
- Method to prove Binomial Theorem:

(a) Principle of Mathematical Induction.

(b) Combinatorial Method.

- Factorial notation:

(i)  $n! = 1 \times 2 \times 3 \times 4 \dots \times n$ ;  $0! = 1$

(ii)  ${}^nC_r = \frac{n!}{r!(n-r)!}$

(iii)  ${}^nC_r = {}^nC_{n-r}$

(iv)  ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

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