Probability

- **Sample Space**: The set of all possible outcomes of a random experiment. It is denoted by the symbol S.
- **Sample points**: Elements of the sample space.
- **Event**: A subset of the sample space.
- **Impossible Event**: The empty set.
- **Sure Event**: The whole sample space.
- **Complementary event or "not event":** The set "S" or S - A.
- **The event A or B**: The set $A \cup B$.
- **The event A and B**: The set $A \cap B$.
- **The event A but not B**: A - B.
- **Mutually exclusive events**: A and B are mutually exclusive if $A \cap B = \phi$.
- **Exhaustive and Mutually exclusive events**: Events $E_1, E_2, \ldots\ldots, E_n$ are mutually exclusive and exhaustive if $E_1 \cup E_2 \cup \ldots\ldots \cup E_n = S$ and $E_i \cap E_j = \phi$ for all $i \neq j$.
- **Exiomatic approach to probability**: To assign probabilities to various events, some axioms or rules have been described.

Let S be the sample space of a random experiment. The probability P is a real values function whose domain is the power set of S and range is the interval [0, 1] satisfying the following axioms:

(a) For any event E, $P(E) \geq 0$

(b) P(S) = 1

(c) If E and F are mutually exclusive event, then $P(E \cup F) = P(E) + P(F)$

If $E_1, E_2, E_3 \ldots\ldots\ldots$ are n mutually exclusive events, then $P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i)$

- **Probability of an event in terms of the probabilities of the same points (outcomes):** Let S be the sample space containing n exhaustive outcomes

$W_1, W_2, W_3, \ldots\ldots W_n$ i.e., S = $(W_1, W_2, W_3, \ldots\ldots W_n)$

Now from the axiomatic definition of the probability:

(a) $0 \leq P(W_i) \leq 1$, for each $W_i \in S$.

(b) $P(W_1) + P(W_2) + \ldots\ldots + P(W_3) = P(S) = 1$

(c) For any event A, P(A) = $\sum P(W_i), W_i \in A$

- **Equally likely outcomes**: All outcomes with equal probability.
- **Classical definition of the probability of an event**: For a finite sample space with equally likely outcome, probability of an event A.

$$P(A) = \frac{n(A)}{n(S)}$$

where $n(A)$ = Number of elements in the set A. and $n(S)$ = Number of elements in set S.

- If A is any event, then P(not A) = 1 - P(A) $\Rightarrow P(\overline{A}) = 1 - P(A) \Rightarrow P(A') = 1 - P(A)$
- The conditional probability of an event E, given the occurrence of the event F is given by $P(E|F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$

  $0 \leq P(E|F) \leq 1,$
- $P(E'|F) = 1 - P(E|F)$

  $P((E \cup F)|G) = P(E|G) + P(F|G) - P((E \cap F)|G)$
- $P(E \cap F) = P(E)P(E|F), P(E) \neq 0$

  $P(E \cap F) = P(F)P(E|F), P(F) \neq 0$

  $P(E \cap F) = P(E)P(F)$

  $P(E|F) = P(E), P(F) \neq 0$

  $P(F|E) = P(F), P(E) \neq 0$

- **Theorem of total probability:**

Let $\{E_1, E_2, \ldots, E_n)$ be a partition of a sample space and suppose that each of $E_1, E_2, \ldots, E_n$ has non zero probability. Let A be any event associated with S, then

$$P(A) = P(E_1)P(A|E_1) + P(E_2) + P(A|E_2) + \ldots\ldots + P(E_n)P(A|E_n)$$

- **Bayes' theorem:** If $E_1, E_2, \ldots, E_n$ are events which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint and $E_1 4, E_2 4, \ldots, 4E_n = S$ and A be any event with non-zero probability, then,

$$P(E_i|A) = \frac{P(E_i)\, P(A|E_i)}{\sum_{j\,1}^{n} P(E_i)\, P(A|E_i)}$$

- **Random variable**: A random variable is a real valued function whose domain is the sample space of a random experiment.
- **Probability distribution**: The probability distribution of a random variable X is the system of numbers

X :	x_1	x_2	.....	x_n
$P(X)$:	p_1	p_2	......	p_n

Where, $p_i > o, \sum_{i=1}^{n} p_i = 1, \; i = 1, 2, \ldots\ldots, n$

- **Mean of a probability distribution**: Let X be a random variable whose possible values $x_1, x_2, x_3, \ldots \ldots, x_n$ occur with probabilities $p_1, p_2, p_3, \ldots \ldots, p_n$ respectively. The mean of X, denoted by μ is the number $\sum_{i=1}^{n} x_i p_i$. The mean of a random variable X is also called the expectation of X, denoted by E (X).
- **Variance**: Let X be a random variable whose possible values $x_1, x_2, x_3, \ldots \ldots, x_n$ occur with probabilities $p(x_1), p(x_2), \ldots\ldots p(x_n)$ respectively. Let $\mu = E(X)$ be the mean of X. The variance of X, denoted by Var (X) or σ_x^2 is defined as x^2

$Var\left(X\right)=\sum_{i=1}^{n}(x_i\ \mu)^2\ \ p(x_i)$ or equivalently $\sigma_x^2 = E\left(X - \mu\right)^2$. The non-negative number, $_x\sqrt{Var\left(X\right)} = \sqrt{\sum_{1=i}^{n}\ (x_i\ \mu)^2\, p(x_i)}$ is called the **standard deviation** of the random variable X.

$$Var\left(X\right) = E\left(X^2\right) - \left[E(X)\right]^2$$

- **Bernoulli Trials**: Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes: success or failure.

(iv) The probability of success remains the same in each trial.

For Binomial distribution $B(n, p)$, $P(X=x) = {}^{n}C_x q^{n-x} p^x$, $x = 0, 1, \ldots\ldots, n (q = 1 - p)$