Matrices

- A **matrix** is an ordered rectangular array of numbers, real or complex or functions.
- A matrix having m rows and n columns is called a matrix of **order** m × n.
- **Column matrix**: A matrix with one column is denoted by $[a_{ij}]_{m\times 1}$.
- **Row matrix**: A matrix with one row is denoted by $[a_{ij}]_{1\times n}$.
- **Square matrix**: An m × n matrix is a square matrix if m = n.
- **Diagonal matrix**: A = $A = [a_{ij}]_{m\times n}$ is a diagonal matrix if a_{ij} =0, when $i \neq j$
- **Scalar matrix**: $A = \left[a_{ji}\right]_{n\times n}$ is a scalar matrix if $a_{ij} = 0$ when i ≠ j, $a_{ij} = k$ (k is *some constant), when I=j.*
- **Identity matrix**: $A = \left[a_{ij}\right]_{n\times n}$ is an identity matrix, if $a_{ij} = 1,\ when\ i = j,\ a_{ij} = 0,\ when\ i \neq j.$
- **Zero matrix**: A zero matrix has all its elements as zero.
- **Equality of two matrices**: $A = \left[a_{ij}\right] = \left[b_{ij}\right] = B$ if (i) A and B are of same order, (ii) for all possible values of $i\ and\ j.$
- **Scalar multiplication**: $kA = k\left[a_{ij}\right]_{m\times n} = \left[k\left(a_{ij}\right)\right]_{m\times n}$

  Also – A = (–1)A

- A – B = A + (–1) B

  A + B = B + A

  (A + B) + C = A + (B + C), where A = $[a_{ij}]$, B = $[b_{ij}]$ and C = $[c_{ij}]$ are of same order.

- k(A + B) = kA + kB, where A and B are of same order, k is constant.
- (k + l) A = kA + lA, where k and l are constant.
- $If\ A = [a_{ij}]_{m\times n}\ and\ B = [b_{jk}]_{n\times p},\ then\ AB = C = [c_{ik}]_{m\times p},$ where

$$C_{t1} = \sum_{j=i}^{n} a_{ij} b_{jk}$$

- (i) A(BC) = (AB)C,

(ii) A(B + C) = AB + AC,

(iii) (A + B)C = AC + BC

- *If* $A = [a_{ij}]_{m\times n}$, *then* A' *or* $A^{T} = [a_{ji}]_{n\times m}$
- (i) (A′)′ = A, · (ii) (kA)′ = kA′, · (iii) (A + B)′ = A′ + B′, · (iv) (AB)′ = B′A′
- **Symmetric matrix**: A is a symmetric matrix if A′ = A.
- **Skew-aymmetric matrix**: A is a skew symmetric matrix if A′ = –A.
- Any square matrix can be represented as the sum of a symmetric and a skew symmetric matrix. In fact, $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$, where $\frac{1}{2}(A + A')$ is a symmetric matrix and $\frac{1}{2}(A - A')$ is a skew-symmetric matrix.
- **Equivalent matrices**: Two matrices A and B are equivalent that is, A - B is A is obtained from the other by a sequence of elementary operations. Elementary operations of a matrix are as follows:

(i) $R_i \leftrightarrow R_j$ or $C_i \rightarrow C_j$ (interchange rows or columns)

(ii) $R_i \rightarrow kR_j$ or $C_i \rightarrow kC_j$

(iii) $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$

- If A and B are two square matrices such that AB = BA = I, then B is the inverse matrix of A and is denoted by A^{-1} and A is the inverse of B.
- Inverse of a square matrix, if it exists, is unique.