SpinGlass

March 18, 2018

We first need to load the spinGlass data and modules.

```
In [2]: from RRR import *
        from linProject import *
        from spinGlassProject import *
        from rankProject import *

        n = 40

        f = open('spinglassH', 'r')
        l = f.readlines()
        f.close()
        ll = [line.split() for line in l]
        lll = [[float(s) for s in line] for line in ll]
        H = np.array(lll)

        A = semiDefProject(np.random.rand(n, n)*2 - 1)

        print spinGlassProject(A)
```

```
[[ 1. -1. -1. ..., -1. -1. -1.]
 [-1.  1.  1. ...,  1.  1.  1.]
 [-1.  1.  1. ...,  1.  1.  1.]
 ...,
 [-1.  1.  1. ...,  1.  1.  1.]
 [-1.  1.  1. ...,  1.  1.  1.]
 [-1.  1.  1. ...,  1.  1.  1.]]
```

Now, we should really test the setup. I recall from class that the true minimum energy was around -230, so let's try projecting onto -190.

```
In [24]: target = -200

         A = semiDefProject(2*np.random.rand(n, n)-1)

         Y, errors, sols = RRR(A,
                               lambda x: lowerBoundProject(x, H, 2*target), #2, because each prodc
```

```
                              spinGlassProject,
                              0.5, 1e-12, 10000, True)
```

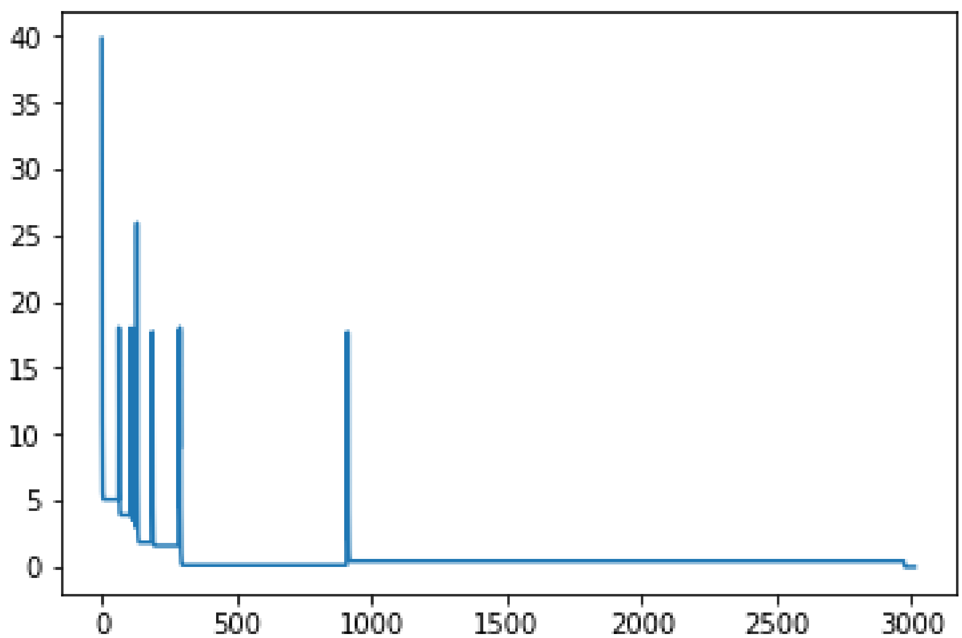
```
In [25]: print sols
```

```
[[ 1.  1.  1. ...,  1.  1.  1.]
 [ 1.  1.  1. ...,  1.  1.  1.]
 [ 1.  1.  1. ...,  1.  1.  1.]
 ...,
 [ 1.  1.  1. ...,  1.  1.  1.]
 [ 1.  1.  1. ...,  1.  1.  1.]
 [ 1.  1.  1. ...,  1.  1.  1.]]
```

```
In [26]: spins = extractSpins(sols)
         print spins

         print 0.5*np.sum(np.outer(spins, spins)*H)
```

```
[-1. -1. -1. -1. -1. -1. -1.  1.  1. -1. -1. -1. -1.  1.  1.  1. -1. -1.
 -1.  1. -1.  1.  1. -1.  1.  1. -1.  1. -1. -1. -1.  1.  1.  1.  1.  1.
 -1. -1. -1. -1.]
-212.436679919
```

```
In [27]: plt.plot(errors)
         plt.show()
```



Fantastic. We are capable of reaching a reasonably low energy.
How low can we expect to go?

0.0.1 How low can we go?

Well, the below calculation gives the standard deviation of a "typical" set of spins.

```
In [30]: var =  np.sum(H*H)/(40*39)
         num =  20*39. #Free variables in this thing.

         sig = (num*var)**0.5

         print sig
```

```
39.9191932285
```

What we really have is a Gumbel distribution, with 2^{40} samples.

We can approximate the minimum expected value in two common ways: the inverse error function, or with an estimate that's abundant online.

```
In [96]: exp = 0 - sig*np.sqrt(2*40*math.log(2))
         print exp

         from scipy.special import erfinv

         exp2 = sig*np.sqrt(2)*erfinv(-1+(2**(-39)))
         print exp2
```

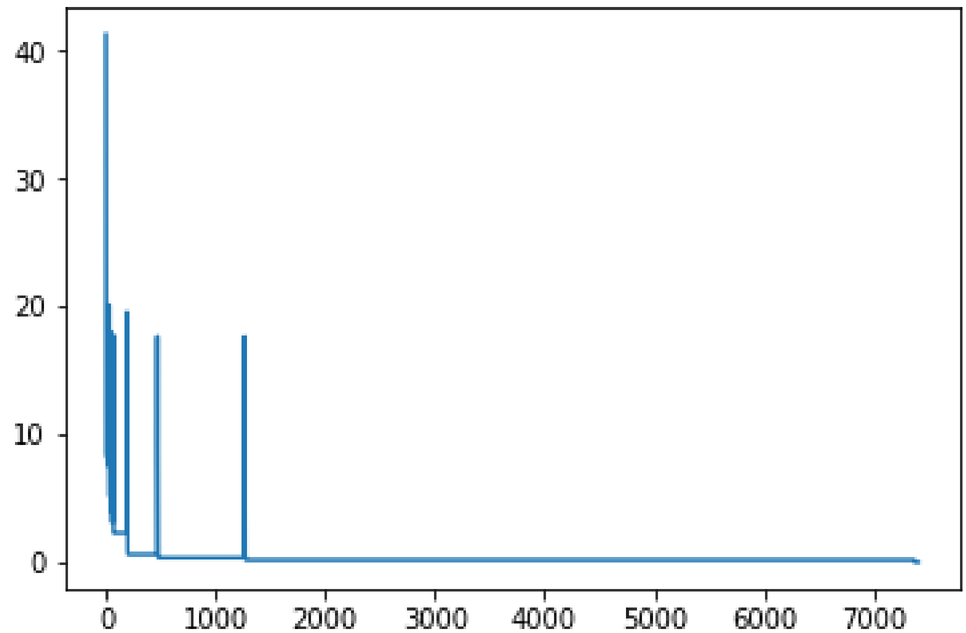
```
-297.262057598
-281.338508362
```

Let's go for that target.

```
In [55]: target = -205

         A = semiDefProject(2*np.random.rand(n, n)-1)

         Y, errors, sols = RRR(A,
                               lambda x: lowerBoundProject(x, H, 2*target),
                               spinGlassProject,
                               0.5, 1e-12, 50000, True)
```

```
In [56]: spins = extractSpins(sols)

         print spins

         print 0.5*np.sum(np.outer(spins, spins)*H)

         plt.plot(errors)
         plt.show()
```

```
[-1.  1.  1. -1.  1. -1. -1.  1. -1.  1. -1. -1. -1.  1.  1.  1. -1. -1.
  1.  1. -1.  1.  1. -1. -1.  1. -1. -1. -1. -1.  1.  1.  1. -1. -1.  1.
 -1. -1. -1. -1.]
-210.046002488
```



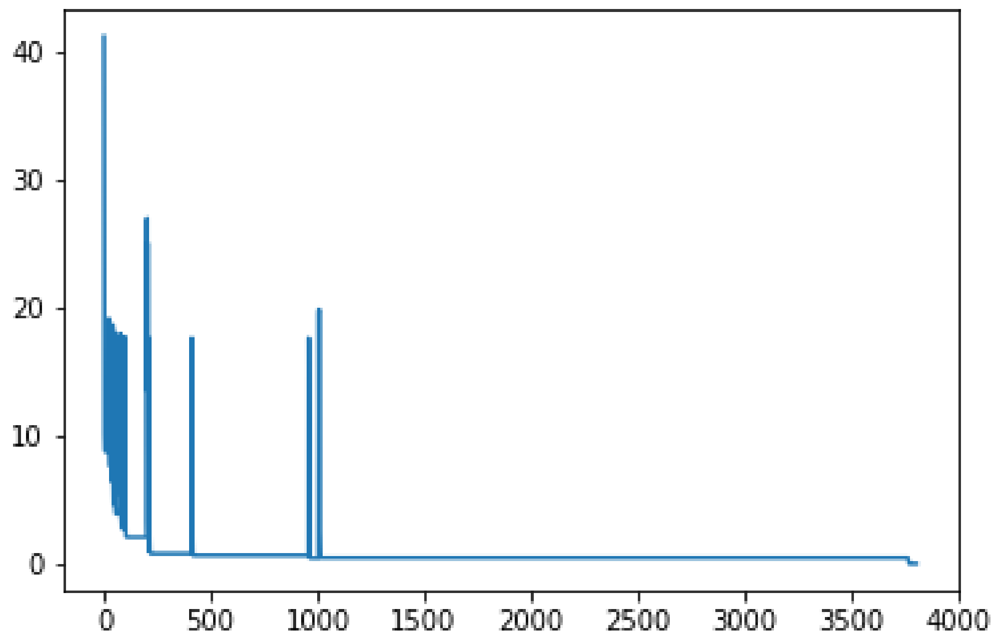
Clearly, we can go fairly low -- we have a -212 and a -210 already. Let's keep going.

```
In [80]: target = -205

         A = semiDefProject(2*np.random.rand(n, n)-1)

         Y, errors, sols = RRR(A,
                               lambda x: lowerBoundProject(x, H, 2*target),
                               spinGlassProject,
                               0.5, 1e-12, 10000, True)
```

```
In [81]: spins = extractSpins(sols)

         print spins

         print 0.5*np.sum(np.outer(spins, spins)*H)

         plt.plot(errors)
         plt.show()
```

```
[-1. -1.  1. -1.  1. -1. -1.  1. -1.  1. -1. -1. -1.  1.  1.  1. -1. -1.
  1.  1. -1.  1.  1. -1.  1.  1. -1. -1. -1. -1.  1.  1.  1. -1.  1.  1.
```

```
 -1. -1. -1. -1.]
-218.06824561
```



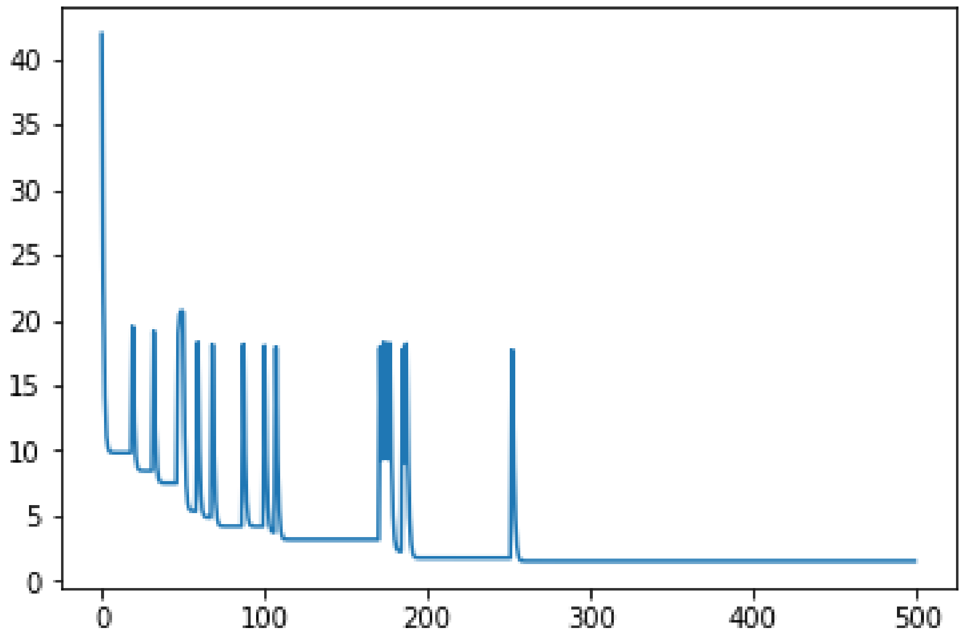
Lowest yet!
Anything further?

```
In [111]: target = -260

          A = semiDefProject(2*np.random.rand(n, n)-1)

          Y, errors, sols = RRR(A,
                                lambda x: lowerBoundProject(x, H, 2*target),
                                spinGlassProject,
                                0.5, 1e-12, 500, True)
```

```
Warning: maximum iterations exceeded, no convergence
```

```
In [112]: spins = extractSpins(sols)

          print spins

          print 0.5*np.sum(np.outer(spins, spins)*H)

          plt.plot(errors)
          plt.show()
```

```
[-1.  1.  1. -1.  1. -1. -1.  1. -1.  1. -1. -1. -1.  1.  1.  1. -1. -1.
  1.  1. -1.  1.  1.  1.  1. -1. -1. -1. -1. -1.  1. -1.  1. -1. -1.  1.
 -1. -1. -1. -1.]
-217.19843627
```



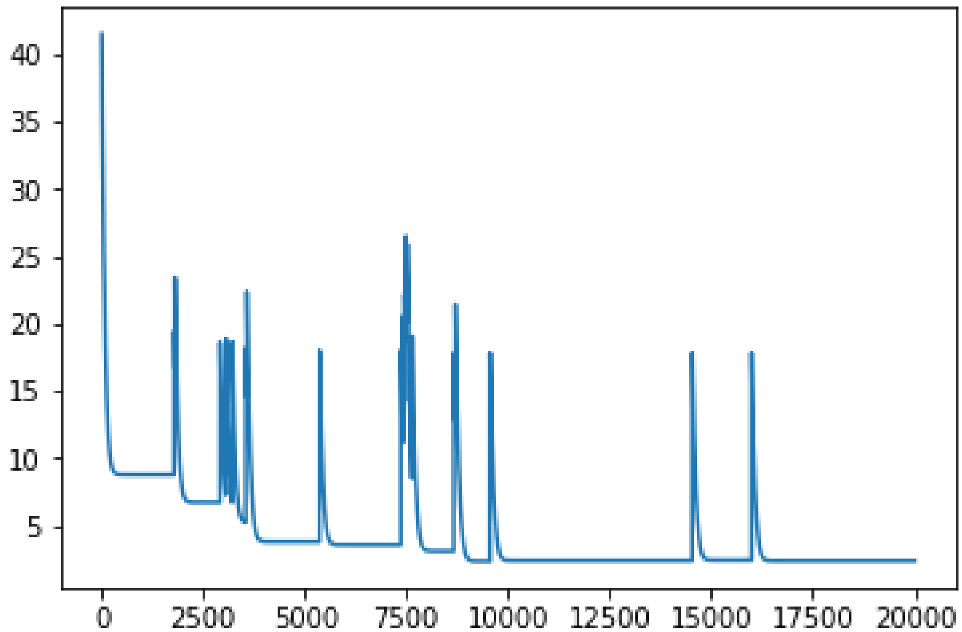
Something else close. Before moving on, let's keep this one.

```
In [164]: target = -240

          A = semiDefProject(2*np.random.rand(n, n)-1)

          Y, errors, sols = RRR(A,
                                lambda x: lowerBoundProject(x, H, 2*target),
                                spinGlassProject,
                                0.01, 1e-12, 20000, True)
```

```
Warning: maximum iterations exceeded, no convergence
```

```
In [165]: spins = extractSpins(sols)

          print spins

          print 0.5*np.sum(np.outer(spins, spins)*H)
          plt.plot(errors)
          plt.show()
```

```
[-1.  1.  1. -1. -1.  1. -1.  1. -1.  1. -1.  1.  1. -1.  1.  1. -1. -1.
  1.  1. -1.  1.  1.  1.  1. -1. -1.  1. -1. -1. -1. -1. -1. -1. -1.  1.
 -1. -1. -1. -1.]
-172.434860228
```



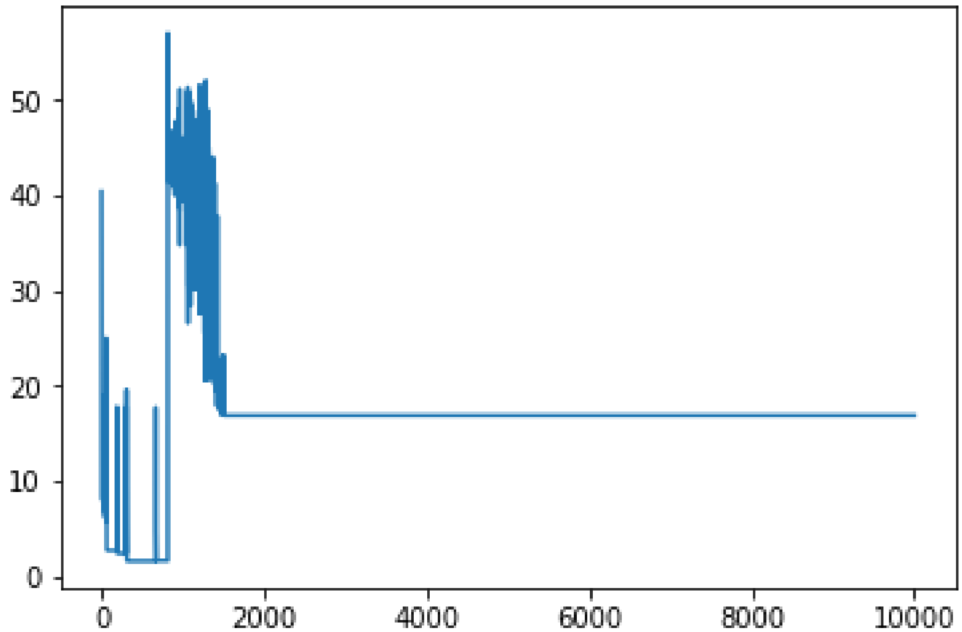
This is definitely not what we want.

It should be noted that there is an attractor for routines that do not converge. They seem to really like a certain arrangement of spins, and stay there for a while:

```
In [166]: target = -240

          A = semiDefProject(2*np.random.rand(n, n)-1)

          Y, errors, sols = RRR(A,
                                lambda x: lowerBoundProject(x, H, 2*target),
                                spinGlassProject,
                                0.5, 1e-12, 10000, True)

Warning: maximum iterations exceeded, no convergence


In [167]: spins = extractSpins(sols)

          print spins

          print 0.5*np.sum(np.outer(spins, spins)*H)
```

```
plt.plot(errors)
plt.show()
```

```
[-1.  1.  1.  1.  1.  1. -1. -1. -1.  1.  1.  1. -1.  1. -1.  1. -1. -1.
  1. -1. -1.  1.  1. -1.  1.  1.  1.  1.  1.  1.  1.  1. -1.  1.  1.  1.
  1. -1.  1. -1.]
236.561109487
```



This arrangement, and the energy 236.6, came up very frequently when the problem did not converge.

I suspect this happens when Y get's launched out of the problem space, and is projected on the nearest point of the convex hull.

0.1 Conclusion

There is clearly work to be done. There is a significant gap between the lowest expected energy and the states I am finding.

I do not think that I came close to the complete solution, at all. However, it should be noted that the ground energies I found -- $-217, -218$ -- are much better than random search, which cannot find anything below -200.