CliqueRRR

March 18, 2018

This notebook is just for showcasing. The heavy lifting is in the referenced python files, which are attached

0.0.1 Loading and testing

Let's load the files.

The projections have already been tested, so let's try running RRR on a small, 5 vertex graph.

```
In [1]: from RRR import *
        from rankProject import *
        from cliqueProject import *

        n = 5

        H = np.array(
            [[0, 1, 1, 1, 1],
            [1, 0, 1, 0, 0],
            [1, 1, 0, 0, 0],
            [1, 0, 0, 0, 0],
            [1, 0, 0, 0, 0]])

        A = semiDefProject(np.random.rand(n, n))

        print A
```

```
[[ 0.74898253  0.68560315  0.53026984  0.53143793  0.60668445]
 [ 0.68560315  0.88109344  0.30792235  0.46922854  0.44142195]
 [ 0.53026984  0.30792235  0.28376392  0.47602041  0.26655964]
 [ 0.53143793  0.46922854  0.47602041  0.47692332  0.20668791]
 [ 0.60668445  0.44142195  0.26655964  0.20668791  0.44009071]]
```

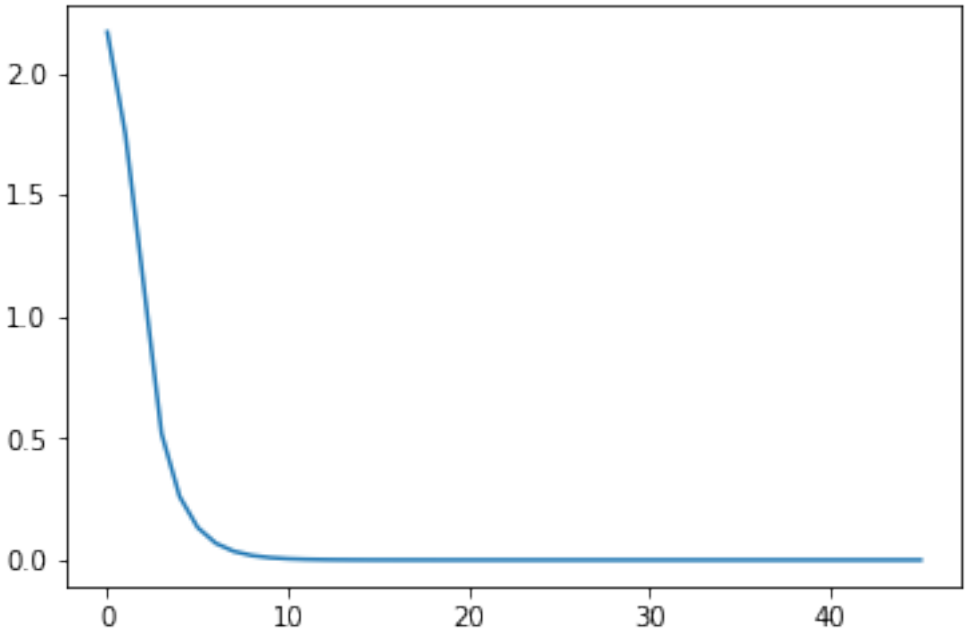
Let's try $k = 3$, which should yield a clique.

```
In [2]: k = 3

        Y, errors, sols = RRR(A,
                              lambda x: cliqueProject(x, k, H),
```

```
                        lambda x: rankProject(x, 1, False),
                        0.5, 1e-12, 10000, True)
```

In [3]:
```
plt.plot(errors)
plt.show()
```



In [4]:
```
print extractClique(sols)

#print sols
#print H
#print eig(sols)[0]


#print eig(rankProject(Y, 1, False))[0]

#print np.round(rankProject(Y, 1, False), 4)
#print np.round(cliqueProject(Y, 3, H), 4)
```

```
[0 1 2]
```

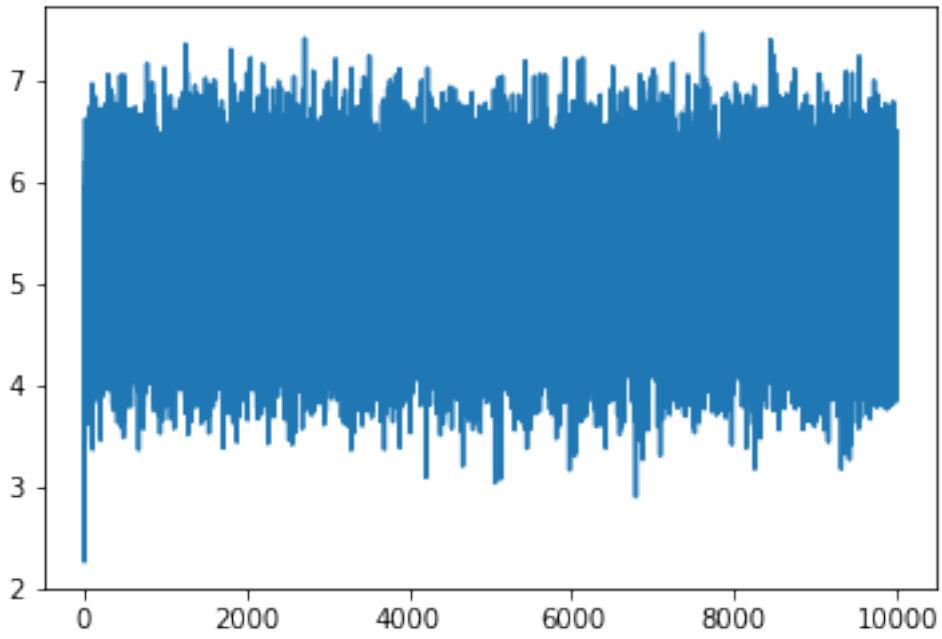
As desired, the algorithm quickly found the correct solution.
If we set k too high, of course, it will just thrash.

In [5]:
```
k = 4
```

```
        Y, errors, sols = RRR(A,
                              lambda x: cliqueProject(x, k, H),
                              lambda x: rankProject(x, 1, False),
                              0.5, 1e-12, 10000, True)
```

```
Warning: maximum iterations exceeded, no convergence
```

```
In [6]: plt.plot(errors)
        plt.show()
```



However, if we do have a 4 clique, everything works out again:

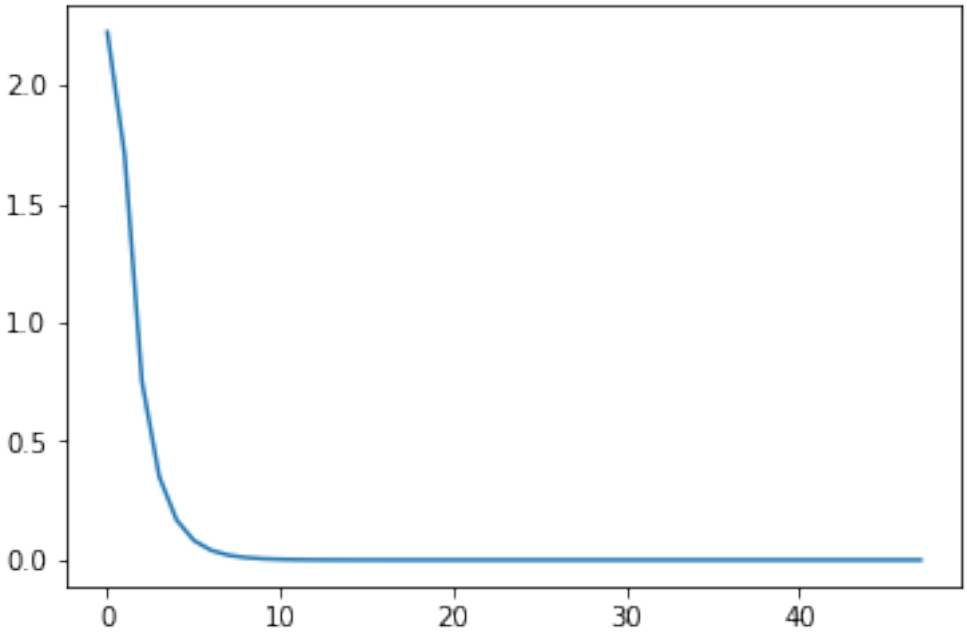
```
In [7]: H = np.array(
            [[0, 1, 1, 1, 1],
            [1, 0, 1, 1, 0],
            [1, 1, 0, 1, 0],
            [1, 1, 1, 0, 0],
            [1, 0, 0, 0, 0]])

        k = 4

        Y, errors, sols = RRR(A,
                              lambda x: cliqueProject(x, k, H),
                              lambda x: rankProject(x, 1, False),
                              0.5, 1e-12, 10000, True)
```

```
In [8]: plt.plot(errors)
        plt.show()

        print extractClique(sols)
```




```
[0 1 2 3]
```

Fantastic! On to the real problem, which takes more than 40 iterations.

0.0.2 Loading the data, and running the code again.

First, we will load the data. Then, we will start experimenting, looking for good values of k.

```
In [9]: n = 125

        from graphReader import getGraph
        H = getGraph()

        print H.shape
        print H[-1]
```

```
(125, 125)
[ 1.  1.  1.  1.  1.  0.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.
  1.  1.  1.  1.  1.  1.  1.  1.  0.  1.  1.  1.  1.  1.  1.  1.  1.  1.
  1.  1.  1.  1.  0.  1.  0.  1.  1.  1.  1.  1.  1.  1.  0.  1.  1.  1.
```

```
 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.
 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  0.  1.  1.  1.  1.  1.  1.
 1.  1.  1.  1.  0.  1.  1.  1.  1.  0.  1.  1.  1.  1.  1.  1.  1.
 1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  1.  0.]
```

Let's first be modest an set $k = 5$.

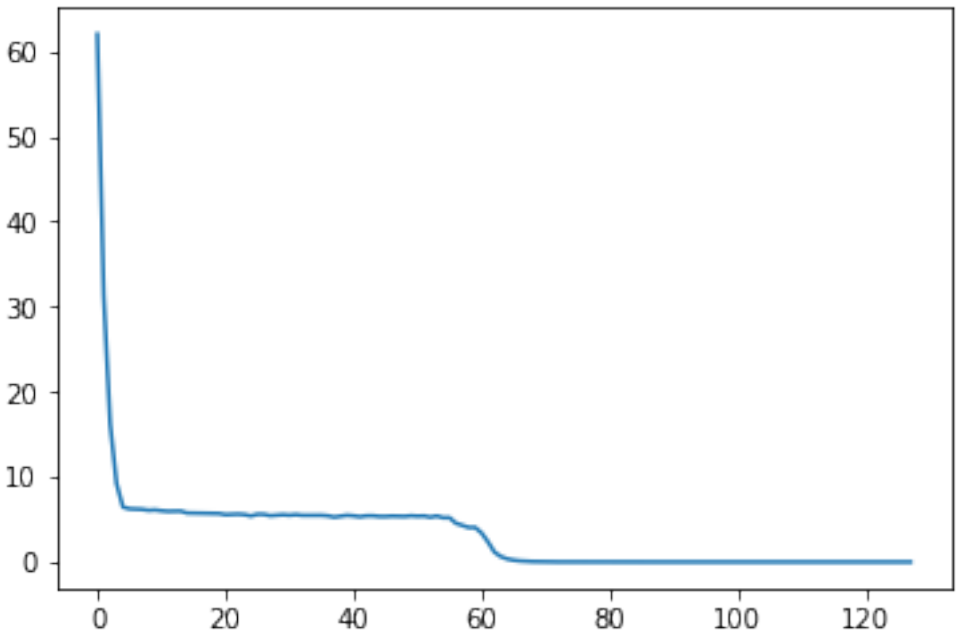
In [10]:
```
k = 5

A = semiDefProject(np.random.rand(n, n))

Y, errors, sols = RRR(A,
                      lambda x: cliqueProject(x, k, H),
                      lambda x: rankProject(x, 1, False),
                      0.5, 1e-12, 10000, True)
```

In [12]:
```
plt.plot(errors)
plt.show()
#print np.sum(np.round(cliqueProject(Y, k, H), 8))

print extractClique(sols)
```



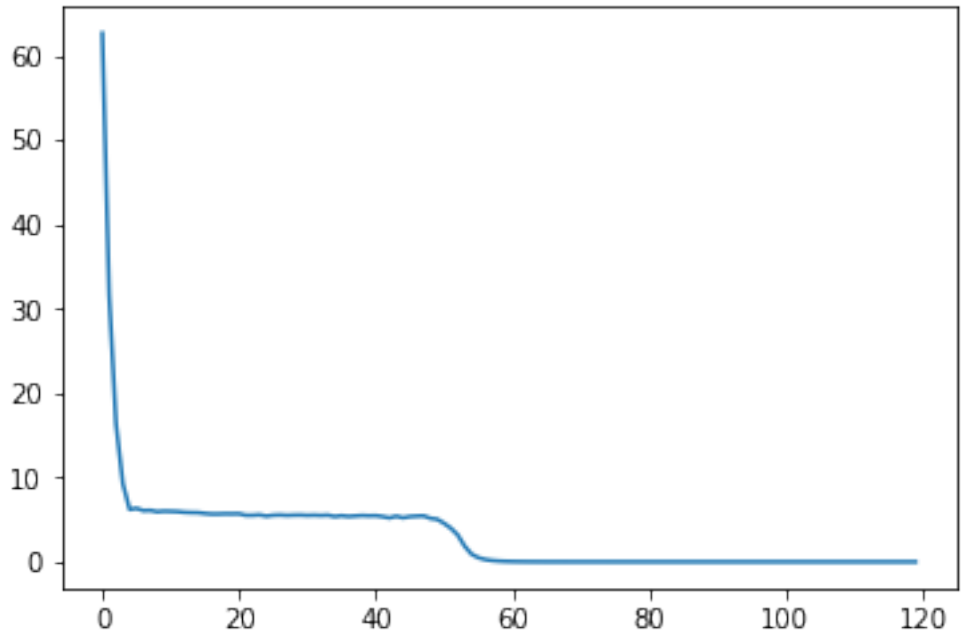

```
[  0   9  15 104 123]
```

But is this clique unique? Let's find out by running the algorithm with a different seed.

```
In [13]: A = semiDefProject(np.random.rand(n, n))

         Y, errors, sols = RRR(A,
                               lambda x: cliqueProject(x, k, H),
                               lambda x: rankProject(x, 1, False),
                               0.5, 1e-12, 10000, True)

In [14]: plt.plot(errors)
         plt.show()
         print extractClique(sols)
```




```
[ 3 38 40 45 52]
```

Clearly not.

Let's set more ambitious targets for k. The data file said that the edge probability is 0.9 That means that the probability of set of size k to be a clique is

$$P_c = 0.9^{k(k-1)/2},$$

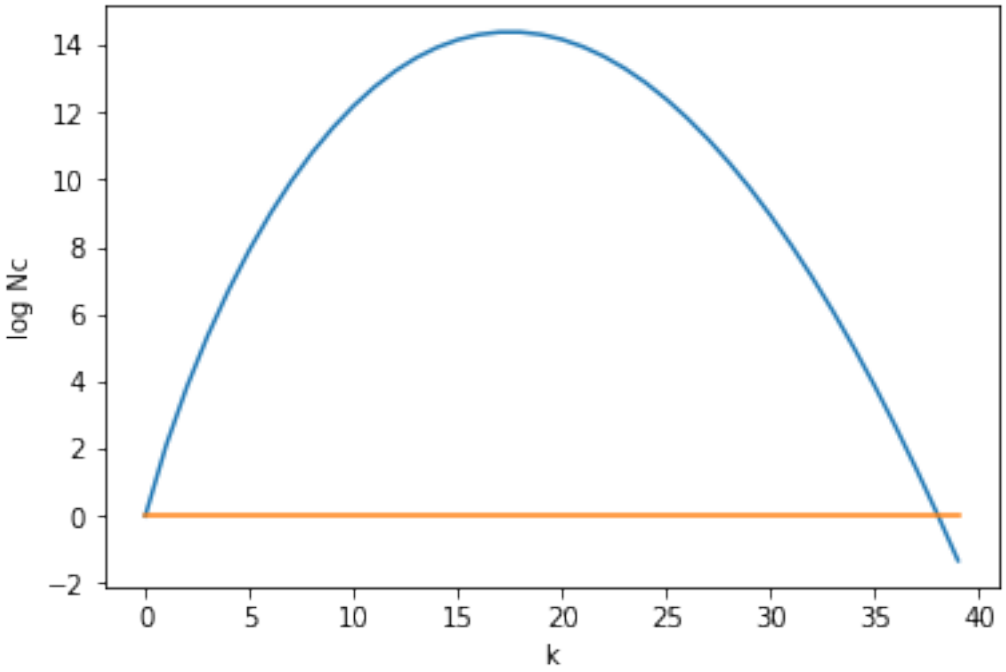
so the total expected number of such cliques is

$$N_c = \binom{125}{k} 0.9^{k(k-1)/2}$$

```
In [21]: from scipy.special import binom
         import math
```

```
def Nc(k):
    return binom(125, k)*(0.9**(k*(k-1)*0.5))

ks = range(40)
Ns = [Nc(k) for k in ks]
lNs = [math.log(N)/math.log(10) for N in Ns]
plt.plot(ks, lNs)
plt.plot(ks, [0 for k in ks])
plt.xlabel("k")
plt.ylabel("log Nc")
plt.show()
```



Here are some values of N_c for k in the high 30s:

36	460
37	25
38	1.2
39	0.05

Note that N_c comes with high variance bars. Therefore, it is reasonable to expect a max clique somewhere in the high 30s.

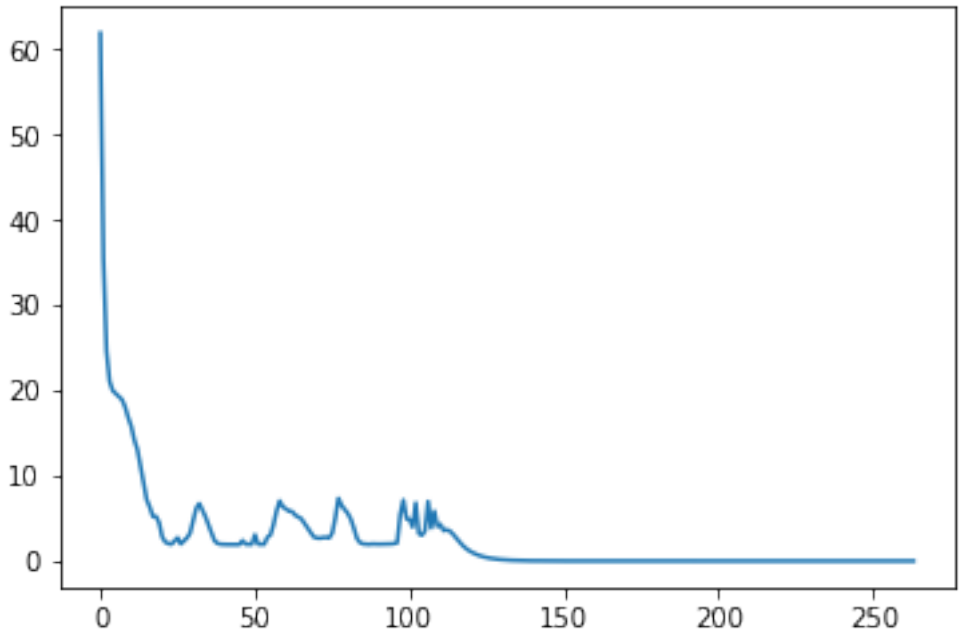
Let's start with something less ambitious: $k = 20$, which nearly maximizes N_c:

```
In [16]: k = 20
```

```
A = semiDefProject(np.random.rand(n, n))

Y, errors, sols = RRR(A,
                      lambda x: cliqueProject(x, k, H),
                      lambda x: rankProject(x, 1, False),
                      0.5, 1e-12, 10000, True)
```

In [22]:
```
plt.plot(errors)
plt.show()
print extractClique(sols)
```




```
[  3   7   9  10  18  27  35  36  38  45  48  66  69  73  88 103 113 115
 118 119]
```

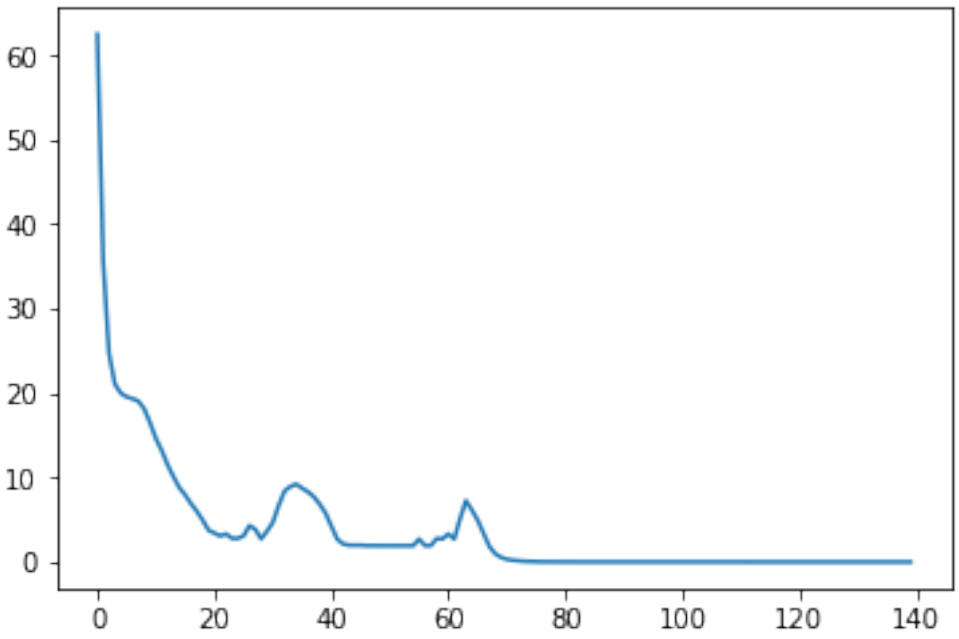
But is it unique?

In [25]:
```
k = 20

A = semiDefProject(np.random.rand(n, n))

Y, errors, sols = RRR(A,
                      lambda x: cliqueProject(x, k, H),
                      lambda x: rankProject(x, 1, False),
                      0.5, 1e-12, 10000, True)
```

```
In [26]: plt.plot(errors)
         plt.show()
         print extractClique(sols)
```




```
[  8   9  12  20  22  36  44  48  50  51  55  57  66  76  78  95  98 103
 113 116]
```

Clearly not, as is expected.

0.1 Excursion for k=33.

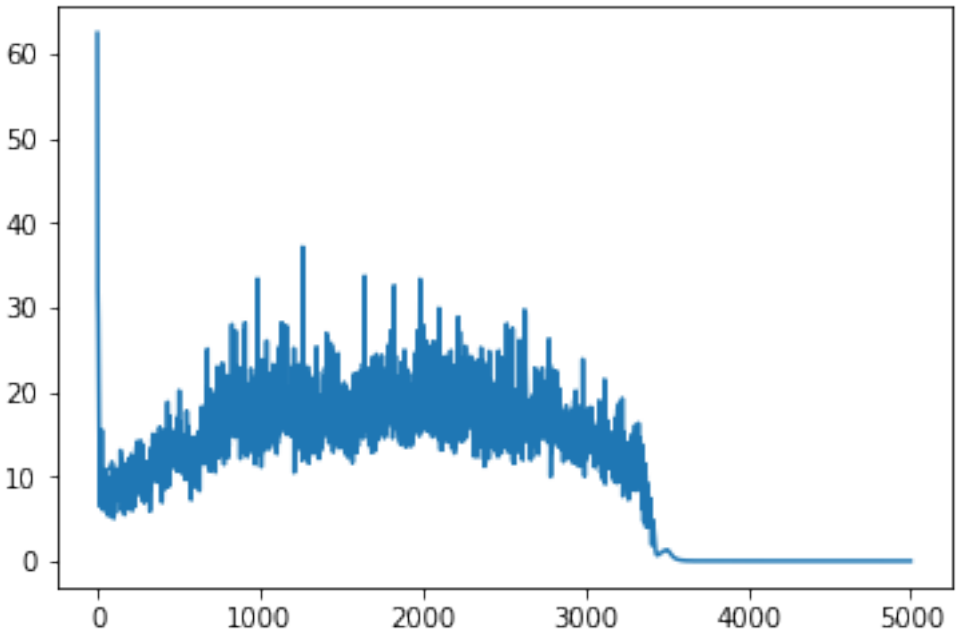
Let's push the envelope a little. There should be a million cliques of size $k = 33$; let's find one.

```
In [28]: k = 33

         A = semiDefProject(np.random.rand(n, n))

         Y, errors, sols = RRR(A,
                               lambda x: cliqueProject(x, k, H),
                               lambda x: rankProject(x, 1, False),
                               0.5, 1e-12, 10000000000, True)
```

```
In [32]: plt.plot(errors[:5000])
         plt.show()
         print extractClique(sols)
```




```
[  6   7  10  18  21  24  33  44  45  47  48  53  55  64  66  67  69  70
  73  76  78  79  81  84  95  97 101 103 113 115 116 121 124]
```

It seems we've found a solution! It didn't take too many iterations, either.
Just in case, let's verify it. We should see a "1" printed if the set is a clique.

```
In [30]: def verifyClique(c, H):
             r = 1
             for i in range(len(c)):
                 for j in range(i+ 1, len(c)):
                     r = r*H[c[i], c[j]]
             return r

         print verifyClique(extractClique(sols), H)
```

```
1.0
```

Fantastic.
I wonder, does a smaller beta have better performance?

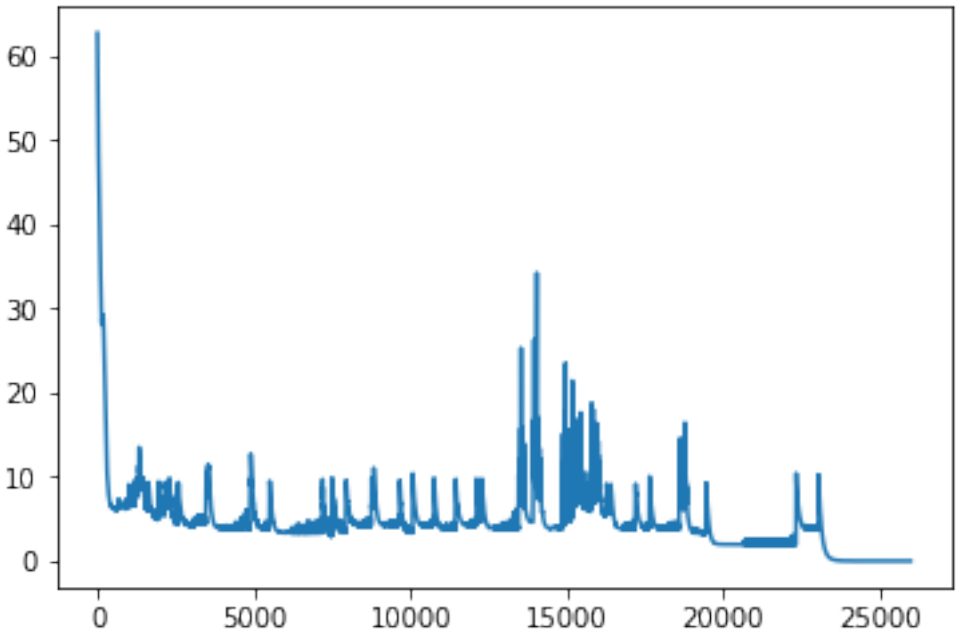
```
In [34]: k = 33

         A = semiDefProject(np.random.rand(n, n))
```

```
        Y, errors, sols = RRR(A,
                              lambda x: cliqueProject(x, k, H),
                              lambda x: rankProject(x, 1, False),
                              0.01, 1e-12, 1000, True)
```

```
Warning: maximum iterations exceeded, no convergence
```

In [44]:
```
plt.plot(errors)
plt.show()
```



In [43]:
```
while(True):
    Y, errors2, sols = RRR(Y,
                           lambda x: cliqueProject(x, k, H),
                           lambda x: rankProject(x, 1, False),
                           0.01, 1e-12, 1000, True)
    errors += errors2
    if errors[-1] < 1e-4:
        print "Close!"
    if errors[-1] <= 1e-12:
        break
```

```
Warning: maximum iterations exceeded, no convergence
Warning: maximum iterations exceeded, no convergence
Warning: maximum iterations exceeded, no convergence
```

```
Warning: maximum iterations exceeded, no convergence
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Warning: maximum iterations exceeded, no convergence
Close!
Warning: maximum iterations exceeded, no convergence
Close!
```
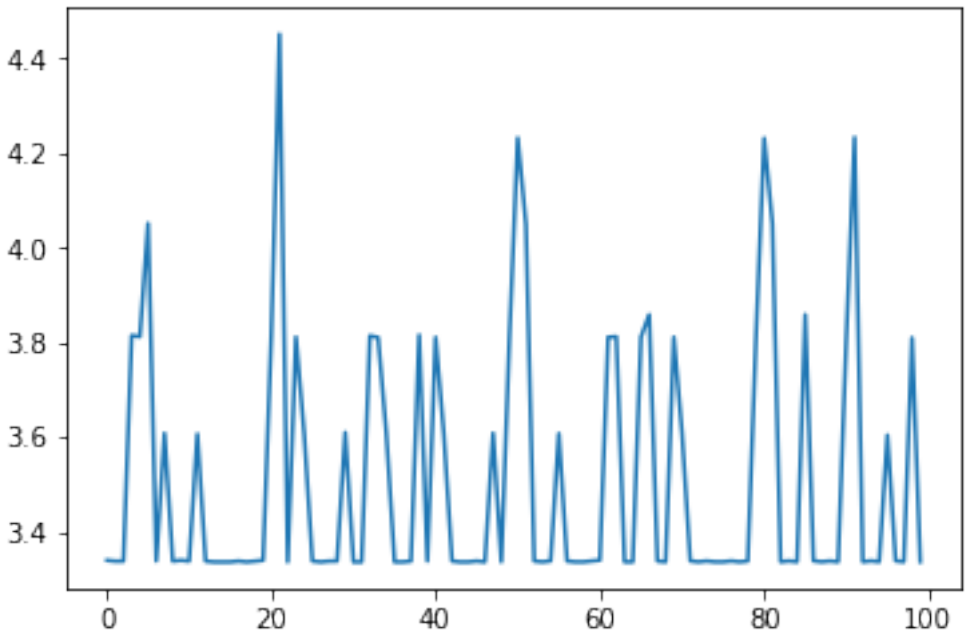

```
---------------------------------------------------------------------------

KeyboardInterrupt                         Traceback (most recent call last)

<ipython-input-43-9fd8e9ae6ed9> in <module>()
  3                     lambda x: cliqueProject(x, k, H),
  4                     lambda x: rankProject(x, 1, False),
----> 5                     0.01, 1e-12, 1000, True)
  6     errors += errors2
  7     if errors[-1] < 1e-4:


/home/atbolsh/VeitElserModule/HW4/RRR.pyc in RRR(v, proj1, proj2, beta, cutoff, maxIter,
 14     while norm(error) > cutoff and i < maxIter:
 15         i += 1
---> 16         error = RRR_error(v, proj1, proj2)
 17         v = v + beta*error
 18         if errors:


/home/atbolsh/VeitElserModule/HW4/RRR.pyc in RRR_error(v, proj1, proj2)
  4
  5 def RRR_error(v, proj1, proj2):
----> 6     b = proj2(v)
  7     a = proj1(2*b - v)
  8     return a - b


<ipython-input-43-9fd8e9ae6ed9> in <lambda>(x)
  2     Y, errors2, sols = RRR(Y,
  3                     lambda x: cliqueProject(x, k, H),
----> 4                     lambda x: rankProject(x, 1, False),
  5                     0.01, 1e-12, 1000, True)
```

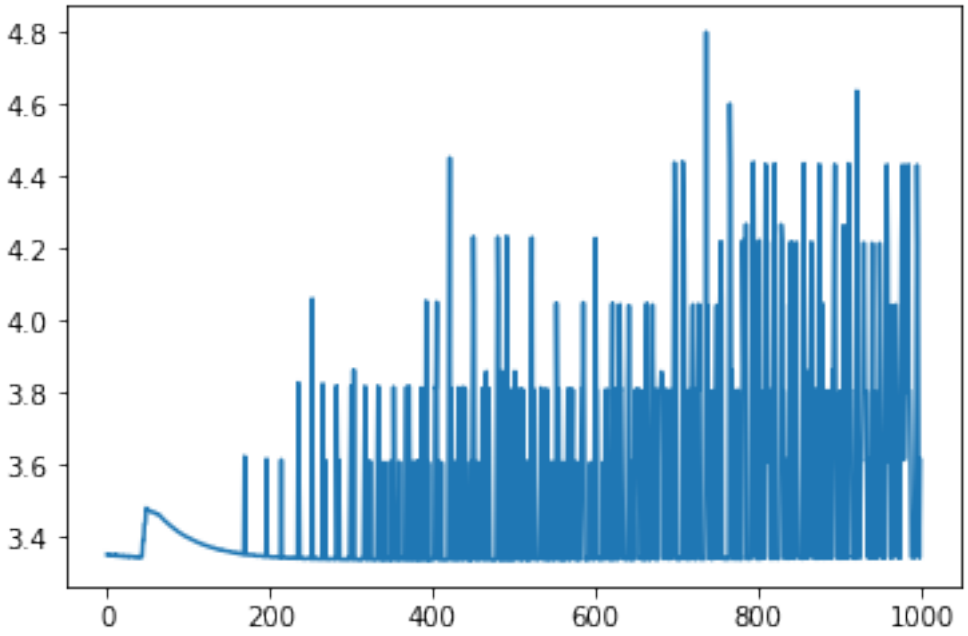
```
      6     errors += errors2


/home/atbolsh/VeitElserModule/HW4/rankProject.pyc in rankProject(X, rank, unitary)
     14     l = np.shape(X)[0]
     15     M = semiDefProject(X)
---> 16     vals, x = eig(M)
     17     #Indeces for sorting
     18     ind = np.argsort(vals)


/home/atbolsh/anaconda2/lib/python2.7/site-packages/numpy/linalg/linalg.pyc in eig(a)
   1132         _raise_linalgerror_eigenvalues_nonconvergence)
   1133     signature = 'D->DD' if isComplexType(t) else 'd->DD'
-> 1134     w, vt = _umath_linalg.eig(a, signature=signature, extobj=extobj)
   1135
   1136     if not isComplexType(t) and all(w.imag == 0.0):


KeyboardInterrupt:
```

It's worth nothing that closeups of the error profile reveal strange plateaus, as if the projections are often a set distance apart. Could these be smaller cliques? Or ranks 2, 3, etc.?

```
In [48]: plt.plot(errors[6400:6500])
         plt.show()
```



```
In [49]: plt.plot(errors[6000:7000])
         plt.show()
```




```
In [50]: print extractClique(sols)
         print verifyClique(extractClique(sols), H)
```

```
[  4   6   8  10  16  18  24  28  30  33  43  51  54  65  66  69  76  78
  79  81  84  90  92  95  97 102 103 109 113 116 120 121 124]
1.0
```

I think I will use a smaller beta. Even though it took a lot more steps, it only took a factor of 5 more, while we reduced step size by a factor of 50.

Now, let's look for truly difficult cliques.

0.2 k = 35

There should be 7000 cliques at $k = 35$. Let's find them.

```
In [51]: k = 35

         A = semiDefProject(np.random.rand(n, n))

         Y, errors, sols = RRR(A,
                               lambda x: cliqueProject(x, k, H),
                               lambda x: rankProject(x, 1, False),
                               0.01, 1e-12, 1000, True)
```

```
Warning: maximum iterations exceeded, no convergence
```

```
In [97]: while(True):
             Y, errors2, sols = RRR(Y,
                             lambda x: cliqueProject(x, k, H),
                             lambda x: rankProject(x, 1, False),
                             0.01, 1e-12, 1000, True)
             errors += errors2
             if errors[-1] < 1e-4:
                 break
```

```
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          ----------------------------------------------------------------------------
```
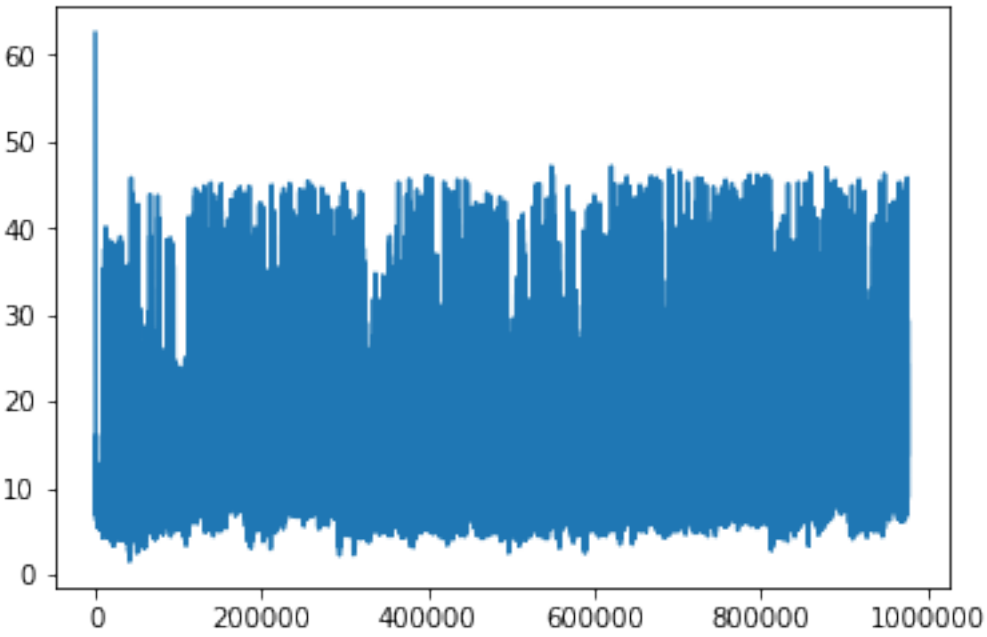

```
    KeyboardInterrupt                         Traceback (most recent call last)

    <ipython-input-97-74c51ba1a162> in <module>()
      3                     lambda x: cliqueProject(x, k, H),
      4                     lambda x: rankProject(x, 1, False),
----> 5                     0.01, 1e-12, 1000, True)
      6     errors += errors2
      7     if errors[-1] < 1e-4:


    /home/atbolsh/VeitElserModule/HW4/RRR.pyc in RRR(v, proj1, proj2, beta, cutoff, maxIter,
     14     while norm(error) > cutoff and i < maxIter:
     15         i += 1
---> 16         error = RRR_error(v, proj1, proj2)
     17         v = v + beta*error
     18         if errors:


    /home/atbolsh/VeitElserModule/HW4/RRR.pyc in RRR_error(v, proj1, proj2)
      4
      5 def RRR_error(v, proj1, proj2):
----> 6     b = proj2(v)
      7     a = proj1(2*b - v)
      8     return a - b


    <ipython-input-97-74c51ba1a162> in <lambda>(x)
      2     Y, errors2, sols = RRR(Y,
      3                     lambda x: cliqueProject(x, k, H),
----> 4                     lambda x: rankProject(x, 1, False),
      5                     0.01, 1e-12, 1000, True)
      6     errors += errors2


    /home/atbolsh/VeitElserModule/HW4/rankProject.pyc in rankProject(X, rank, unitary)
     14    l = np.shape(X)[0]
     15    M = semiDefProject(X)
---> 16    vals, x = eig(M)
     17    if sG:
     18       x = np.sign(x)


    /home/atbolsh/anaconda2/lib/python2.7/site-packages/numpy/linalg/linalg.pyc in eig(a)
   1132         _raise_linalgerror_eigenvalues_nonconvergence)
   1133     signature = 'D->DD' if isComplexType(t) else 'd->DD'
-> 1134     w, vt = _umath_linalg.eig(a, signature=signature, extobj=extobj)
   1135
   1136     if not isComplexType(t) and all(w.imag == 0.0):
```

```
        KeyboardInterrupt:
```

```
In [98]: plt.plot(errors)
         plt.show()
```




```
In [99]: print errors[-1]
```

```
13.8930809964
```

How much harder is this than $k = 34$, anyway?

```
In [96]: print Nc(34)
         print Nc(35)
         print 0.9**70
```

```
100851.385037
7292.90465632
0.000626578748218
```

Somewhere between 2000 times harder and 20 times harder. It's probably closer to 2000 because 'fake attractors' keep distracting the algorithm.

Nothing to do but to keep running.

0.2.1 Conclusion

$k = 35$ did not converge after a full nightl; clearly I need better code.
Howevere, the other notebook did compute a clique at $k = 34$.
That is the largest my code can detect at this stage.