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Fast Rotation Fourier analysis of the 60h data-set

A. Chapelain[1], J. Fagin[1], D. Rubin[1], and D. Seleznev[1]
[1]Cornell University

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1 Introduction

This note presents the Cornell Fast Rotation analysis via the Fourier approach of the so-called `60h` data set collected during `RUN 1`. The Fast Rotation analysis allows to extract the frequency distribution of the stored anti-muon beam or equivalently the momentum or radial distribution. The knowledge of the radial distribution of the beam is essential in order to estimate the correction to ω_a from the electro-static quadrupoles (`ESQs`), the so-called electric field correction.

1.1 Fast Rotation signal

The so-called Fast Rotation signal corresponds to the number of positron counts as a function of time $N(t)$ corrected by the major signal features so as to leave only the bunching feature of the beam. The features corrected for are: the boosted anti-muon life-time τ, the spin precession frequency of the anti-muon ω_a so as to obtain a normalized intensity spectrum whose feature is to a really good first approximation due to the bunching feature of the beam. The correction is performed using the standard 5-parameter fit:

$$N(t) = N_0 e^{-t/\tau}[1 + Acos(\omega_a t + \phi)], \tag{1}$$

where N_0 is the number of detected positron at the start of the fit, A called the asymmetry is the amplitude of the modulation due to the spin precession and ϕ the phase of the modulation.

1.2 Fourier analysis

The Fourier analysis relies on calculating the cosine (or real part) Fourier transform of the Fast Rotation signal:

$$S(\omega, t_s, t_m) = \int_{t_s}^{t_m} S(t) cos\omega(t - t_0)dt, \tag{2}$$

where $S(t)$ is the Fast Rotation signal, t_0 the time corresponding to the center of mass of the beam passing the detector for the first time, and t_S and t_M are respectively the start and end time of the Fast Rotation signal. The parameter t_0, t_S and t_M are at the core of the Fourier analysis. In the ideal case of having access to the data from the first turn ($t_s = t_0$) when the anti-muon beam enters the ring, the equation above is exact and yield to the correct frequency distribution $\Phi(\omega)$:

$$\Phi(\omega) = S(\omega, t_s = t_0, t_m) = \int_{t_0}^{t_m} S(t) cos\omega(t - t_0)dt \tag{3}$$

Unfortunately the calorimeter data for `RUN 1` are only available after couple micro-seconds. In this case eq. (2) is only a good approximation of the frequency distribution and it needs an additional correction term, which is:

$$\Delta(\omega) = \int_{\omega^-}^{\omega^+} S(\omega') \frac{\sin(\omega - \omega')(t_s - t_0)}{\omega - \omega'} d\omega' \tag{4}$$

The correct frequency distribution can be retrived via:

$$\Phi(\omega) = \int_{t_0}^{t_m} S(t) cos\omega(t - t_0)dt + A \cdot \int_{\omega^-}^{\omega^+} S(\omega') \frac{\sin(\omega - \omega')(t_s - t_0)}{\omega - \omega'} d\omega' + B, \tag{5}$$

where A and B are optimized so that the frequency distribution vanishes in the regions outside the collimators aperture where muons cannot be stored. For more details regarding the Fourier approach, please refer to [1, 2].

2 Producing the Fast Rotation signal

3 t_0 optimization

fds

4 Frequency and radial distribution

5 E-field correction

6 Statistical uncertainty

7 Systematic uncertainty

7.1 t_0

scan around minimum jump by couple FR period

7.2 t_S

fr period effect use a maximum or minimum of fr?

7.3 t_M

results pretty steady after 100 micro-secs

7.4 Energy threshold

7.5 Calorimeters timing alignement

7.6 Field index

7.7 Frequency to radius conversion

need betatron motion assume non-uniform velocity

7.8 Binning effect

frs fourier transform wiggle fit

7.9 E-field correction approximations

References

[1] Y. Orlov et al., NIM A 482 (2002) 767-755.

[2] A. Chapelain, D. Rubin, D. Seleznev, *Extraction of the Muon Beam Frequency Distribution via the Fourier Analysis of the Fast Rotation Signal*, E989 note 130, GM2-doc-9701